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CREDIT FRICTIONS AND OPTIMAL LABOR-INCOME TAXATION

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This paper studies optimal labor-income taxation in a simple model with credit constraints on firms. The labor-income tax rate and the shadow value on the credit constraint induce a wedge between the marginal product of labor and the marginal rate of substitution between labor and consumption. It is found that optimal policy prescribes a volatile path for the labor-income tax rate even in the presence of state-contingent debt and capital. In this respect, credit frictions are akin to a form of market incompleteness. Credit frictions break the equivalence between tax smoothing and wedge smoothing; therefore, as the tightness of the credit constraint varies over the business cycle, tax volatility is needed in order to counter this variation and, as a result, allow for wedge smoothing.

Keywords: Labor-Income Tax Smoothing, Credit Frictions, Static Wedge

1. INTRODUCTION

A classic result in optimal fiscal policy is that the labor-income tax rate should be virtually constant over the business cycle (labor tax smoothing). This paper studies the optimal behavior of the labor-income tax rate in a flexible-price business cycle model with capital and state-contingent debt in which firms borrow to pay factors of production in advance, and borrowing is constrained by the beginning-of-period collateral. This paper finds that the labor-income tax rate should vary over the business cycle; if firms are more constrained in hiring labor, the labor-income tax rate should be lowered to boost labor supply, thus increasing labor in equilibrium. The optimal nonconstant path of the labor-income tax rate that is uncovered in this study stands in a sharp contrast to previous studies in models with complete markets and capital. In this respect, credit frictions are akin to a form of market incompleteness. The introduction of property taxation allows for a smoother tax rate on labor, but it remains meaningfully volatile.

Besides the government, the baseline setup assumes two types of agents in the economy: A representative household and a representative firm (owned by a representative entrepreneur) that operates in a perfectly-competitive product market. The firm hires labor from the households in a neoclassical labor market and borrows in order to pay at least part of the input costs at the beginning of

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the period (the standard "working capital" requirement). Borrowing, in turn, is constrained by the firm's value of real estate. This corresponds to the usual limited enforcement problem as in Kiyotaki and Moore (1997).

The basic intuition behind the main result of the paper is as follows. Due to the binding credit constraint, labor demand is inefficiently low and it depends on the tightness of the credit constraint. When the credit constraint tightens more, labor supply should be encouraged by reducing the labor tax rate. When the credit constraint is looser, the labor tax rate is relatively higher. In either case, the labor tax rate is lower than in an otherwise model with no credit frictions. The labor-income tax rate, thus, moves in opposite of the tightness of the credit constraint in order to prevent excessive volatility in labor, hence output and consumption. More generally, the labor-income tax rate in this environment is a stabilizing tool in the face of exogenous shocks to the macroeconomy.

An alternative way of viewing the main result is by considering the implications of the credit constraint: Because of the binding credit constraint, the representative firm hires labor so that the marginal product of labor exceeds the real wage that the firm takes as given, thus generating a "markup." Optimal policy aims for offsetting this markup (at least partially) by "subsidizing" labor supply. An increase in labor supply lowers the before-tax real wage and leads to a higher quantity of labor in equilibrium.

Also, the labor tax rate and the shadow value on the credit constraint generate a static (labor) wedge, thus breaking down the full mapping between the static wedge and the labor tax rate. I show that the time-varying labor-income tax rate allows for complete smoothing of the wedge when credit frictions are present. Therefore, even though the labor-income tax rate is not fully smoothed, the notion of "static wedge smoothing" remains optimal. The finding that credit frictions enters the wedge is significant, and its significance goes beyond this particular study. Essentially, this paper suggests that if a friction enters the static wedge, then it is crucial to distinguish between labor-income tax rate smoothing and wedge smoothing. If the ultimate goal is to smooth the wedge over the business cycle, then full tax smoothing may not emerge as optimal in that environment.

I also present results of a model with incomplete markets, whereby the government has no access to state-contingent debt. Two main observations are made. First, the effects of credit frictions on the optimal path of the labor tax rate hold in this setup. In fact, credit frictions combine with market incompleteness to generate even a larger volatility in the labor tax rate. Second, comparing the model with incomplete markets but no credit frictions to the model with both complete markets and credit frictions reveals that credit frictions manifest themselves as a form of market incompleteness. This "incompleteness" reflects the nature of the credit market as opposed to the nature of the government-issued bonds.

Based on a partial equilibrium deterministic setup, Barro (1979) shows that the government wants to smooth tax distortions across periods, and that debt issuance should be adjusted to finance temporary increases in public spending, thus allowing for constant tax rates. In a stochastic environment, the model predicts random walk responses of debt and taxes to public spending. Using a model with state-contingent debt but without capital, Lucas and Stokey (1983) find that the optimal labor tax rate is not a random walk but rather largely inherits the serial correlation and the stochastic properties of government spending. On the other hand, Aiyagari et al. (2002) study the behavior of the labor tax rate in a model with incomplete markets and find that, under certain conditions on asset and debt limits, the optimal Ramsey policy entails a near random walk. However, without ad hoc constraints on the government's asset holdings, the results might significantly differ from Barro's. Chari et al. (1994) build a model with capital and state-contingent debt. It is found that the capital tax rate is roughly zero ex-ante but fluctuates ex post. As they absorb most of the shocks to government spending, the variations in the ex-post capital tax rate and the debt allow for keeping the labor tax rate virtually constant.

Some more recent studies find that tax smoothing is not optimal. Schmitt-Grohe and Uribe (2004) show that the volatility of the labor-income tax rate is very small in a model with flexible prices (with and without perfect competition in the product market), but significantly higher if prices are sticky. Andersen and Dogonowski (2004) suggest that the optimal tax rate should be procyclical to smooth leisure. Chugh (2009) shows that, following government spending shocks, the timing of the good markets/financial markets of Svensson (1985) implies zero inflation variability, and hence the Ramsey planner resorts to labor tax volatility in an attempt to smooth consumption. Arseneau and Chugh (2012) demonstrate that the result of labor tax smoothing does not hold in a model with labor market frictions: Labor tax rate volatility is optimal to induce efficient fluctuations in the labor market by keeping distortions (or wedges) constant over the business cycle. Arbex and O'Dea (2013) study optimal taxation when jobs are found through social networks, and show that optimal policy prescribes a nonconstant tax rate on labor.

The results of this paper stand mostly in contrast to Chari et al. (1994). First, in the benchmark analysis, the ex-ante (and the steady-state) capital-income tax rate is negative as opposed to zero. Second, the tax rate on labor income remains meaningfully volatile even in the presence of state-contingent debt and capital. Third, while the capital tax rate is volatile as in the aforementioned study, the volatility of the capital tax rate in the current paper follows the volatility of the tightness of the credit constraint and it *raises* (or at least does not reduce) the volatility of the labor tax rate. This occurs because capital is subsidized by, among others, labor-income taxation. Fourth, the labor-income tax rate displays a very low degree of persistence and does not inherit the cyclical properties of the shock.

In the robustness analysis, I show that labor-income tax rate volatility remains optimal in an alternative model where the ex-ante capital tax rate *is* zero. In that alternative scenario, variations in the capital-income tax rate help with smoothing the labor-income tax rate, but the existence of credit frictions does not allow for full smoothing. The labor-income tax rate remains meaningfully volatile and with low degrees of persistence. Finally, the fact that the present study finds labor tax smoothing not optimal even with flexible prices, perfect competition and no labor search and matching frictions is significant; credit frictions alone can justify meaningful variations in the labor-income tax rate.

The remainder of the paper proceeds as follows. Section 2 outlines the model and defines both the private-sector equilibrium and the optimal policy problem. Section 3 presents analytical results. Section 4 discusses the parameterization of the model and reports the main quantitative results. Section 5 presents robustness analyses and Section 6 presents the results in a model with non-state-contingent debt. Section 7 concludes.

2. THE MODEL

The economy is populated by the government, a representative household and a representative firm that is owned by a representative entrepreneur. Alternatively, we may assume [as in Iacoviello (2005) and Gerali et al. (2010), among others] that each type of agents is of measure one. Then, we can refer to each set of agents as "households" and "entrepreneurs." The representative firm pays (at least part of) its input costs before production takes place, thus giving rise to borrowing from the representative household. Borrowing is constrained by the value of real estate that the representative entrepreneur owns. This is the source of the credit friction in the model.

2.1. The Representative Household

In each period t, the representative household purchases consumption c_t, supplies labor l_t, purchases real estate h_t (e.g., in the form of housing), and provides an intraperiod loan of b_t^f to the firm at a gross real interest rate of R_t^f. In addition, the household has access to a one-period state-contingent real government bond b_t that pays a gross real interest rate of R_t. Then, the problem of the representative household is given by

$$\max_{\{c_t,l_t,h_t,b_t,b_t^f\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, l_t) \tag{1}$$

with $\mathbf{E}_0$ being the expectation operator, $\beta(< 1)$ denotes the subjective discount factor of the household and $u(c_t, h_t, l_t)$ is the period utility function from consumption, real estate, and labor. This function satisfies $\frac{\partial u}{\partial c} > 0$, $\frac{\partial^2 u}{\partial c^2} < 0$, $\frac{\partial u}{\partial h} > 0$, $\frac{\partial^2 u}{\partial h^2} < 0$, $\frac{\partial u}{\partial l} < 0$, and $\frac{\partial^2 u}{\partial l^2} \leq 0$.

Maximization is subject to the representative household's sequence of period budget constraints of the form:

$$\begin{aligned} c_t + k_{t+1} + b_{t+1} + b_t^f + q_t h_{t+1} &= (1 - \tau_t^l) w_t l_t + [1 - \delta + r_t - \tau_t^k (r_t - \delta)] k_t \\ &\quad + (1 - \tau_t^p) q_t h_t + R_t b_t + R_t^f b_t^f, \end{aligned} \tag{2}$$

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where w_t is the real wage, q_t is the market price of real estate, r_t is rental rate of capital, δ denotes the depreciation rate of capital, and τ_t^l, τ_t^k, and τ_t^p stand for the labor-income tax rate, the capital-income tax rate, and the property tax rate, respectively.

The optimal choices of consumption, labor supply, capital, real estate, lending to the firm and bond holdings yield the following optimization conditions:

$$R_t^f = 1, \tag{3}$$

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t, \tag{4}$$

$$u_{c,t} = \beta \mathbf{E}_t(R_{t+1}u_{c,t+1}), \tag{5}$$

$$u_{c,t} = \beta \mathbf{E}_t \left\{u_{c,t+1}\left[1-\delta+r_{t+1}-\tau_{t+1}^k(r_{t+1}-\delta)\right]\right\}, \tag{6}$$

$$q_t u_{c,t} = \beta \mathbf{E}_t \left[u_{h,t+1}+(1-\tau_{t+1}^p)q_{t+1}u_{c,t+1}\right], \tag{7}$$

where $u_{c,t}$ is the marginal utility of consumption, $u_{h,t}$ is the marginal utility of real estate, and $u_{l,t}$ is the marginal utility of supplying labor. Equation (3) governs the lending of the household to the firm and it implies a zero interest rate (this result reflects the intraperiod nature of the loans); as in Carlstrom and Fuerst (1998), the household is basically a passive supplier of credit to the firm. Equation (4) is the labor supply condition stating that, at the optimum, the marginal rate of substitution between labor and consumption equals the after-tax real wage. Equation (5) is the consumption Euler equation and condition (6) is the capital supply condition with capital-income taxation. Equation (7) is an asset pricing-type condition, stating that the current marginal utility from consumption equals the expected marginal gain from real estate. The latter includes a direct utility from holding real estate next period and the possibility to expand future consumption via the realized resale value of real estate (net of future property taxation).

2.2. The Representative Firm/Entrepreneur

The representative firm hires labor and rents capital from the household to produce a homogeneous good using the following production technology:

$$y_t = z_t f(k_t, l_t), \tag{8}$$

where y_t and z_t are output and total factor productivity (TFP), respectively. The firm pays part of its input costs $(w_t l_t + r_t k_t)$ before the realization of revenues, which requires borrowing at the beginning of period t. This assumption follows Carlstrom and Fuerst (1998), but with some differences in the specifics of the model.[1] Borrowing, however, is constrained by the beginning-of-period market value of the firm's real estate (which serves as collateral).[2]

The firm is owned by a representative entrepreneur who derives utility from consumption e_t and real estate x_t. For simplicity, it is assumed that she does not supply labor, but rather only manages the firm. As shown in Appendix A, the problem of the representative firm/entrepreneur can be reduced to the following maximization problem:

$$\max_{\{e_t, l_t, k_t, x_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v(e_t, x_t), \tag{9}$$

subject to the following sequence of period budget constraints:

$$e_t + q_t x_{t+1} = (1 - \tau_t^p) q_t x_t + z_t f(k_t, l_t) - w_t l_t - r_t k_t, \tag{10}$$

and the credit (collateral) constraint:

$$\phi(w_t l_t + r_t k_t) \leqslant \kappa q_t x_t, \tag{11}$$

where x_t is the firm's beginning-of-period stock of real estate, κ is the share of assets that can be used as collateral (or the loan-to-value ratio), and ϕ is the fraction of factor payments that is paid in advance. With $\phi = 0$, the model collapses to a standard business cycle model with no credit frictions. This formulation of the firm's problem is in line with Carlstrom et al. (2010) and De Paoli and Paustian (2013), but with the addition of the costs of renting capital to the left-hand side of the collateral constraint (both studies abstract from physical capital). Furthermore, γ is the subjective discount factor of the representative entrepreneur and $v(e_t, x_t)$ is her period utility function. To simplify matters, I assume that the representative entrepreneur has linear preferences in consumption: $\frac{\partial v}{\partial e} > 0$ and $\frac{\partial^2 v}{\partial e^2} = 0$. In addition, the utility function is concave in real estate: $\frac{\partial v}{\partial x} > 0$ and $\frac{\partial^2 v}{\partial x^2} < 0$.

Denoting the Lagrange multipliers on condition (10) by π_t and on condition (11) by $\pi_t \mu_t$, profit maximization then gives the following factor demands:

$$z_t f_{l,t} = (1 + \phi \mu_t) w_t, \tag{12}$$

$$z_t f_{k,t} = (1 + \phi \mu_t) r_t. \tag{13}$$

The representative firm thus hires labor and rents capital so that the marginal product of each input is a "markup" over its respective factor price. The net markup is given by $\phi \mu_t$ and it arises solely due to the external financing needs of the firm [this result is in line with Carlstrom and Fuerst (1998), and the use of the term "markup" in this paper is borrowed from their study]. As a result, the firm generates operating profits (given by $\Pi_t = y_t - w_t l_t - r_t k_t$) in equilibrium. Using a standard Cobb–Douglas production function, $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$, these profits can be written as

$$\Pi_t = \frac{\phi \mu_t}{1 + \phi \mu_t} y_t, \tag{14}$$

which explicitly shows the size of the profits as a function of the tightness of the collateral constraint. The representative entrepreneur uses these profits to finance

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her current consumption and the purchase of the next-period real estate. The choice of next-period real estate then yields:

$$q_t v_{e,t} = \gamma \mathbf{E}_t \left[v_{x,t+1} + (1 - \tau^p_{t+1} + \kappa \mu_{t+1}) q_{t+1} v_{e,t+1} \right]. \tag{15}$$

This condition reflects the fact that real estate is valued by the representative entrepreneur because it serves as collateral and provides direct utility.

2.3. The Government

The government collects labor-income taxes, capital-income taxes, and property taxes, and issues real state-contingent debt to finance an exogenous stream of real government expenditures (g_t) in addition to the repayment of the last-period debt. The government budget constraint in period t is then given by

$$\tau^l_t w_t l_t + \tau^k_t (r_t - \delta) k_t + \tau^p_t (h_t + x_t) + b_{t+1} = g_t + R_t b_t. \tag{16}$$

2.4. Market Clearing

In addition to labor market clearing, the markets of goods and real estate clear:

$$z_t f(k_t, l_t) + (1 - \delta) k_t = c_t + e_t + k_{t+1} + g_t, \tag{17}$$

$$h_t + x_t = 1, \tag{18}$$

where condition (18) assumes that real estate is in a fixed supply, in line with Kiyotaki and Moore (1997) and Iacoviello (2005), among others. As noted by Iacoviello (2005), this is not an unrealistic assumption if real estate is given the broad interpretation of land. Alternatively, one may think that the production of real estate exactly equals the value of real-estate depreciation, thus leaving the total supply of real estate fixed.[3]

Finally, the combination of labor supply (4) and labor demand (12) gives

$$-\frac{u_{l,t}}{u_{c,t}} = \left(\frac{1 - \tau^l_t}{1 + \phi \mu_t} \right) z_t f_{l,t}, \tag{19}$$

which suggests that the labor-income tax rate and the credit friction drive a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor. We may refer to this wedge as the "static wedge" or the "labor wedge." This wedge is the difference between 1 and the term in parentheses and can be written as

$$\text{Wedge}_t = \left(\frac{\tau^l_t + \phi \mu_t}{1 + \phi \mu_t} \right). \tag{20}$$

Other things equal, a rise in the tightness of the credit constraint leads to a rise in the static wedge. As will be demonstrated later, the implications of the credit constraint for the optimal behavior of the labor tax rate are important for the degree to which the wedge can be smoothed at business cycle frequencies.

2.5. The Competitive Equilibrium

DEFINITION 1 (Competitive Equilibrium). *Given the exogenous processes* $\{z_t, g_t, \tau_t^l, \tau_t^k, \tau_t^p\}$, *the competitive equilibrium is a sequence of allocations* $\{b_t, c_t, e_t, l_t, k_t, q_t, w_t, r_t, h_t, x_t, R_t, R_t^f, \mu_t\}$ *that satisfy the equilibrium conditions (3)–(7), (10)–(13), and (15)–(18).*

2.6. Optimal Labor-Income Taxation

This subsection characterizes the optimal labor-income taxation policy. I start with a short description of the social planner's problem (i.e., the first-best problem) and then move to the optimal labor-income taxation problem of the Ramsey planner (the second-best problem). We may refer to the social planner's allocations as the "first-best allocations" or the "efficient allocations," interchangeably. These are the allocations that the planner chooses with lump-sum taxes, and they provide a good benchmark to which the optimal Ramsey taxation policy (which is the focus of this study) can be compared.

Since there are two types of agents, the objective function of the planner is given by

$$\max_{\{c_t, e_t, l_t, h_t, x_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \left[\eta\beta^t u(c_t, h_t, l_t) + (1-\eta)\gamma^t v(e_t, x_t)\right] \tag{21}$$

with η being the weight that is attached to the utility of the representative household (lender) and $1-\eta$ the weight of the utility of the representative entrepreneur (borrower). This formulation of the planner's objective function follows Monacelli (2008). With this characterization of the objective function, the optimal social planner's problem and the Ramsey planner's problem, respectively, can be stated as follows:

DEFINITION 2 (Efficient Allocations). *Given the exogenous processes* $\{z_t, g_t\}$, *the problem of the social planner is to choose* $\{c_t, e_t, l_t, h_t, x_t\}$ *to maximize (21) subject to (17)–(18).*

DEFINITION 3 (Optimal Taxation Problem). *Given the exogenous processes* $\{z_t, g_t\}$, *the Ramsey planner chooses sequences of allocations* $\{b_t, c_t, e_t, l_t, k_t, q_t, w_t, r_t, h_t, x_t, R_t, R_t^f, \mu_t, \tau_t^l, \tau_t^k, \tau_t^p\}$ *to maximize (21) subject to (3)–(7), (10)–(13), and (15)–(18).*

I proceed with some analytical analysis and then turn to the numerical results.

3. ANALYTICAL RESULTS

I first describe the solution to the social planner's problem and then turn to the solution of the Ramsey planner's problem. I close this section with some steady-state analysis.

3.1. Optimal Labor-Income Taxation: The Social Planner Problem

As Appendix B shows, the choice of labor and consumption by the social planner yields:

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}, \tag{22}$$

which states that the social planner chooses consumption and labor so that the marginal rate of substitution between labor and consumption is equalized to the marginal product of labor. This is the standard efficiency condition in this class of models.

Comparing conditions (19) and (22), the first-best labor tax rate ($\tau^l_{FB,t}$) is given by

$$\tau^l_{FB,t} = -\phi\mu_t. \tag{23}$$

Therefore, for the market allocations to be efficient, labor income should be subsidized by the size of the credit friction. More importantly, the size of this subsidy is not constant as μ_t varies over the business cycle; the market solution with a constant labor-income tax rate is not efficient. Clearly, this subsidy is eliminated when credit frictions are absent ($\tau^l_{\mathrm{FB},t} = 0$).

3.2. Optimal Labor-Income Taxation: The Ramsey Problem

This subsection outlines the solution to the second-best labor tax problem. Following Lucas and Stokey (1983) and Chari and Kehoe (1999), I use the primal approach, in which the government only chooses allocations after prices and taxes have been substituted out using the private-sector equilibrium conditions. To do so, I derive the present-value implementability constraint (PVIC) of the households by substituting their optimality conditions into their budget constraints. See Appendix C for more details.

The PVIC of the representative household reads:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[c_t u_{c,t} + l_t u_{l,t} + h_t u_{h,t} \right] = H_0, \tag{24}$$

with H_0 being a constant.

Similarly, the PVIC of the representative entrepreneur:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t \left[e_t v_{e,t} + x_t v_{x,t} \right] = F_0, \tag{25}$$

where F_0 is a constant. Condition (24) summarizes all the equilibrium conditions of the representative household and condition (25) accounts for all the equilibrium conditions of the representative entrepreneur. The resource constraint, the clearing condition of the real estate market and these two conditions constitute the

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constraints of the Ramsey planner. The problem of the Ramsey planner can now be restated as follows:

DEFINITION 4 (Optimal Taxation Problem). *Given the exogenous processes* $\{z_t, g_t\}$*, the Ramsey planner chooses sequences of allocations* $\{c_t, e_t, k_t, l_t, h_t, x_t\}$ *to maximize (21) subject to (17)–(18) and (24)–(25).*

As shown in Appendix C, the solution to Ramsey problem yields:

$$-\frac{u_{l,t} + \xi(u_{l,t} + u_{ll,t}l_t + u_{cl,t}c_t + u_{hl,t}h_t)}{u_{c,t} + \xi(u_{c,t} + u_{cc,t}c_t + u_{lc,t}l_t + u_{hc,t}h_t)} = z_t f_{l,t}, \quad (26)$$

with ξ being the Lagrange multiplier on the household's PVIC and $u_{ab,t}$ being the second derivative of u with respect to any two arguments a and b. Comparing conditions (22) and (26), if $\xi = 0$, then the Ramsey planner's solution restores the social planner's solution.

Also, the corresponding first-order condition with respect to next-period capital (k_{t+1}):

$$u_{c,t} = \beta E_t \left[u_{c,t+1}(1 - \delta + z_{t+1} f_{k,t+1})\right], \quad (27)$$

which is the standard condition in this class of models.

A comment here is in order: With either labor search and matching frictions or monetary frictions, Hagedorn (2010) shows that the first-order conditions may not be sufficient to characterize the optimum of the Ramsey problem. It is also shown that tax cycles (i.e., nontax smoothing) may emerge as optimal in that case even without extrinsic uncertainty. In Hagedorn (2010), the general lesson is that deviations between households' marginal rate of substitutions and after-tax prices are likely to generate tax cycles even without extrinsic uncertainty. Hagedorn (2010) also shows that a model with monopolistic competition in the product market does not lead to this outcome (because the implementability constraint takes the same form with and without monopolistic competition). The model with monopolistic competition differs from the models with search frictions in the labor and money markets because search frictions explicitly affect the implementability constraints.

For this reason, I first verify that the second-order conditions for optimality in the Ramsey problem are satisfied. Second, the model that I use here does not generate any deviations between the marginal rate of substitutions and the after-tax prices (of labor and capital). Third, the implementability constraints that I derive are not affected by the credit frictions. Fourth, as discussed before, credit frictions generate a "markup" and thus resemble having monopolistic competition in the product market. As in Hagedorn (2010), that case does not generate any nonconvexities in the problem of the Ramsey planner. On this basis, I safely proceed to presenting the analytical findings.

3.3. The Ramsey Solution: Labor-Income Taxation

To provide an analytical solution to the Ramsey taxation problem, I assume the following separable period utility function for the representative household:

$$u(c_t, h_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \psi \frac{h_t^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{l_t^{1+\theta}}{1+\theta}, \tag{28}$$

with ψ and χ being scaling parameters that measure the relative weights of real estate and the disutility from labor, respectively. σ is the curvature parameter of consumption, ε is the curvature parameter of real estate, and θ is the inverse of the labor supply elasticity.

Similarly, the period utility function of the representative entrepreneur is given by

$$v(e_t, x_t) = \frac{e_t^{1-\nu}}{1-\nu} + \varphi \frac{x_t^{1-\varepsilon}}{1-\varepsilon}, \tag{29}$$

where ν is the consumption curvature parameter and φ is the relative weight of real estate of the representative entrepreneur.

To maintain the assumption regarding linear preferences of the entrepreneur, I set $\nu = 0$. For simplicity, I also set $\sigma = 1$, but this assumption is not necessary for obtaining the main analytical result. Combining conditions (19) and (26) and using these functional forms then give the key expression characterizing the optimal labor tax rate in this section:

$$\tau^l_{\mathrm{SB},t} = \frac{(1+\theta)\xi - \phi\mu_t}{1 + (1+\theta)\xi}, \tag{30}$$

where $\tau^l_{\mathrm{SB},t}$ is the optimal (second-best) labor-income tax rate.[4] With no credit frictions ($\phi = 0$), the optimal labor-income tax rate is constant over the business cycle, thus re-affirming the "tax smoothing" result. However, when credit frictions are present ($\phi > 0$), the optimal labor-income tax rate varies with μ_t. In particular, the optimal labor-income tax rate will be lower when the shadow value on the credit constraint is higher and vice versa; therefore, optimal policy in this setup "leans against the wind." The optimal labor tax rate is lower than in an otherwise model with no credit imperfections, and the reduction in the labor tax rate is akin to a time-variant labor-market subsidy that is meant to raise labor supply when labor demand is distorted. This condition also makes it clear that the resulting optimal labor-income tax rate is less than 100%, regardless of whether or not credit frictions exist. Furthermore, with $\xi = 0$, the optimal labor-income tax rate equals its first-best counterpart as expected; labor income is subsidized.[5]

Important for this study, the standard deviation of the labor-income tax rate is given by

$$\mathrm{SD}(\tau^l_{\mathrm{SB},t}) = \left[\frac{\phi}{1 + (1+\theta)\xi}\right] \mathrm{SD}(\mu_t). \tag{31}$$

The volatility of the labor tax rate is rising in the credit friction parameter and the volatility of the tightness of the credit constraint. On the other hand, the volatility of the labor tax rate is decreasing in the inverse of the labor supply elasticity; the larger θ, the lower the labor supply elasticity and thus the lower the required variations in the labor tax rate to keep the fluctuations in labor small. In fact, when $\theta \to \infty$, the labor supply elasticity is zero and labor is inelastically supplied. Then, credit frictions do not affect the equilibrium level of labor. Since the equilibrium level of labor is not affected, purposeful variations in the labor-income tax rate (that otherwise would be engineered to increase labor and restore efficiency) are not required. As a result, the labor tax rate is constant.

The optimal setting of the labor tax rate allows for an important result regarding the static wedge to hold. By substituting the expression of the optimal labor tax rate (30) in the expression of the static wedge (20) we have

$$\text{Wedge}_{\text{SB},t} = \frac{(1+\theta)\xi}{1+(1+\theta)\xi}, \tag{32}$$

and thus the static wedge under optimal policy is constant. The optimal (variable) path of the labor-income tax rate enables full smoothing of the static wedge over the business cycle, which is a more general result in public finance. In Section 6, however, I show that this result hangs on the assumption of market completeness (namely, the availability of state-contingent debt).

3.4. The Ramsey Solution: Capital-Income Taxation

I show the implications of credit frictions for optimal capital-income taxation in this subsection. The capital supply condition (6), capital demand condition (13), and the optimal choice of capital by the Ramsey planner (27) are part of the Ramsey equilibrium and they can be used to pin down the optimal capital-income tax rate. To fix ideas, I start by dropping the expectations operator. Then, combining these three conditions gives

$$\tau^k_{\text{SB},t} = -\frac{\phi\mu_t r_t}{r_t - \delta}, \tag{33}$$

which at the steady state reads

$$\tau^k_{\text{SB}} = -\frac{\phi\mu r}{r-\delta}. \tag{34}$$

As long as the standard assumption $r_t > \delta$ holds, the capital-income tax rate is zero in the absence of credit frictions ($\phi = 0$) but unambiguously different from zero otherwise ($\phi, \mu > 0$). Therefore, the introduction of credit frictions induces departures from the celebrated result of zero long-run capital taxation of Judd (1985) and Chamley (1986). Furthermore, since the denominator is positive, the optimal capital-income tax rate is negative. Capital income should, thus, be subsidized: If the credit constraint reduces the demand for capital, then optimal policy should encourage capital supply to position the economy closer to the first-best solution. Key to this paper is the observation that capital income may fluctuate with the fluctuations in the tightness of the collateral constraint, which resembles the observation for the labor-income tax rate. Therefore, the capital-income tax rate will ex-post be volatile as long as the tightness of the credit constraint changes following shocks.[6]

The capital-income subsidy is partly financed by labor-income taxation, and the credit friction-induced variations in the capital-income tax rate may *increase* the variations in the labor-income tax rate. In other words, ex-post variations in the capital-income tax rate will not necessarily reduce the variations in the labor-income tax rate. To see this, substitute condition (33) in condition (30) to obtain

$$\tau^{l}_{\mathrm{SB},t} = \frac{(1+\theta)\xi r_t + (r_t - \delta)\tau^{k}_{SB,t}}{r_t[1+(1+\theta)\xi]}. \tag{35}$$

In the presence of credit frictions, the capital-income tax rate may increase the labor tax rate (the ultimate outcome depends on the behavior of r_t). If confirmed, this result will constitute another departure from the results of Chari et al. (1994).

In line with the literature, I also define the ex-ante tax rate on capital ($\tau_t^{e,k}$):

$$\tau_t^{e,k} = \frac{E_t\left[u_{c,t+1}(r_{t+1}-\delta)\tau^{k}_{t+1}\right]}{E_t\left[u_{c,t+1}(r_{t+1}-\delta)\right]}, \tag{36}$$

which is the ratio of the value of capital-income tax payments across states of nature to the revenues from capital-income taxation. In other words, it is the expected value (or weighted average) of the tax rate on capital income that is received in period $t+1$. This definition follows Chari et al. (1994) and the subsequent literature. The latter study shows that the ex-ante capital-income tax rate ($\tau_t^{e,k}$) can differ from the ex-post tax rate on capital income (τ_t^k). In addition, they show that the tax rate on capital income is zero ex ante but volatile ex post, and that capital-income taxation plays an important role in smoothing the tax rate on labor. On this basis, I will discuss the ex-ante and ex-post tax rates on capital and the role of capital taxation in smoothing the tax rate on labor income.

Bringing back the expectation operator and combining conditions (6), (13), and (27) yield:

$$E_t\left[u_{c,t+1}(r_{t+1}-\delta)\tau^{k}_{t+1}\right] = -\phi E_t\left[u_{c,t+1}\mu_{t+1}r_{t+1}\right]. \tag{37}$$

Since $r_t > \delta$ and $u_{c,t}$ is positive and finite, the expected tax rate on capital income will be zero if credit frictions are absent ($\phi = 0$). However, if credit frictions exist, the expected tax rate on capital will be negative, which is a deviation from the findings of Chari et al. (1994). By extension, condition (36) then implies that the ex-ante capital tax rate is negative with credit frictions and zero otherwise.

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3.5. Tax On Private Assets

I make another definition that enables comparisons with Chari et al. (1994): the private-asset tax rate (which will be denoted by τ_t^a). To do so, the state-contingent return on bonds (R_t) can be decomposed into a state-noncontingent return on these bonds ($R_{b,t-1}$) as well as a state-contingent tax on the these returns (τ_t^b) as follows: $R_t = 1 + R_{b,t-1}(1-\tau_t^b)$. This decomposition is not essential for the analysis, but it helps in clarifying the role of the state-contingent returns on bonds in smoothing out fluctuations in the labor-income tax rate. Then, the tax rate on private assets in this model is given by

$$\tau_t^a = \frac{\tau_t^k(r_t-\delta)k_t + \tau_t^b R_{b,t-1}b_t + \tau_t^p q_t(h_t+x_t)}{(r_t-\delta)k_t + R_{b,t-1}b_t + q_t(h_t+x_t)}, \tag{38}$$

which differs from the literature due to the inclusion of the property tax rate. The government can choose capital-income tax taxes, property taxes and the returns on the state-contingent bonds (or alternatively, the tax rate on these returns) as tools for smoothing out fluctuations in the labor-income tax rate.

3.6. Steady-State Analyses

I present brief analyses about the steady state of the model. While these analyses do not provide insights about the cyclical variations in the labor tax rate, they help in fixing ideas and forming expectations about the sign and the mean of the labor tax rate. To this end, substitute $h + x = 1$ and the steady-state capital-income tax rate, $\tau^k = \frac{-\phi\mu r}{r-\delta}$ in the steady-state version of the government budget constraint (16) to get

$$\tau^l wl + \tau^p = \phi\mu rk + g + (R-1)b. \tag{39}$$

The analyses do not consider the case of ($b < 0$), which implies a budget surplus at the steady state (the benchmark assumption is $b > 0$). Condition (39) illustrates that the sign of τ^l depends on the case in hand. For example, with labor taxes only and $g = 0$, $\tau^l wl = (R-1)b$; the labor-income tax rate is either positive or zero. With all taxes, the labor-income tax rate can essentially take any sign: the right-hand side is positive, and the left-hand side can be positive even with nonpositive labor-income taxation. In this respect, the magnitudes of the property tax rate and government spending are crucial.

Furthermore, based on condition (15), the steady-state value of the Lagrange multiplier on the credit constraint is given by

$$\mu = \frac{q(1-\gamma+\gamma\tau^p) - \gamma v_x}{\gamma\kappa q}, \tag{40}$$

and, therefore, it is a function of preferences, the property tax rate, the price of real estate, the loan-to-value ratio, and the discount factor of the representative entrepreneur. A comment on the property tax rate is in order: other things equal,

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TABLE 1. Values of the parameters- benchmark analysis

α	β	σ	γ	ν	ρ_g	ρ_z	κ	θ	ε	ϕ	μ	σ_g
0.34	0.99	1.00	0.99	0.00	0.90	0.95	0.80	0.00	1.00	0.47	0.019	0.079

a higher τ^p leads to a higher μ. This occurs because higher property taxes reduce the resources that are available to the representative entrepreneur, thus leading to a tighter credit constraint. However, since the price of real estate and the marginal utility of real estate are endogenous, it is not a priori clear how changes in τ^p will change μ. If v_x is very small, then condition (40) can be approximated by $\mu \approx \frac{1-\gamma}{\gamma\kappa} + \frac{\tau^p}{\kappa}$, which clearly shows that a higher τ^p leads to a tighter credit constraint at the steady state. Dynamically, it will be interesting to see how the planner resolves the trade-off between using property taxes to smooth the labor-income tax rate and not over-taxing real estate as it could make the credit constraint tighter.

4. QUANTITATIVE RESULTS

This section starts with a brief discussion of the parameter values. I then present impulse-response functions to a government spending shock as well as first and second moments of the main variables. There are four different scenarios that are compared in this section. First, a model with no credit frictions ($\phi = 0$). Second, a model with credit frictions and labor-income tax only. Third, credit frictions with labor-income and capital-income taxes. Finally, credit frictions with labor-income, capital-income and property taxation. By following these steps, we can observe how the introduction of each additional tax instrument affects the volatility of the labor-income tax rate and the behavior of the economy.

4.1. Parameterization

The time unit is a quarter. The household's preferences are specified in condition (28) and the entrepreneur's preferences are governed by condition (29). Table 1 summarizes the parameter values that I use and Appendix E provides more details about the calibration. In what follows, I mainly elaborate on ϕ and μ, which are key to this paper, as well as γ.

To calibrate the parameter ϕ, I proceed in two steps. First, I think of $\phi\mu$ as capturing a corporate credit spread. Therefore, I match the average of a corporate credit spread over the period 1960–2015, which is roughly 3.5% annually (implying $\phi\mu = 0.0088$). Given the implied value of μ by condition (40), the parameter ϕ is then set to 0.47. The corporate credit spread is measured as the difference between Moody's Seasoned Baa Corporate Bond Yield and a 3-Month Treasury Bill Rate. Both series are obtained from the Federal Reserve Economic Data (FRED) database.[7]

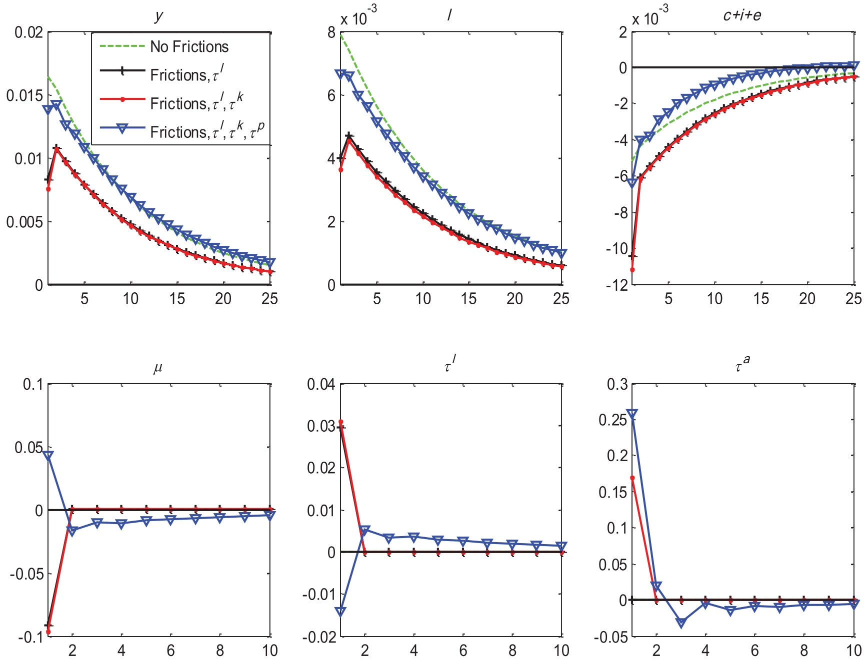

FIGURE 1. Impulse responses to a government expenditures shock. The parameter values are presented in Table 1. $c + i + e$ denotes private absorptions.

With respect to the discount factor of the representative entrepreneur (γ): in standard models with credit constraints, borrowers are assumed to be less patient than lenders. This assumption guarantees that the credit constraint binds in the steady state (essentially, the tightness of the constraint becomes a direct function of the difference between the discount factors). In this framework, however, condition (40) does not put any restriction on γ with respect to β: the credit constraint will bind in the steady state even if $\gamma = \beta$. For this reason, I assume that the discount factors of the representative household and the representative entrepreneur are equal. As shown in the analytical analysis of Appendix C, this setting also simplifies some of the derivations (but it is not necessary for the main results).

4.2. Impulse Responses

Figure 1 displays the responses of key variables to a rise in government expenditures (g_t). For this experiment, I set the standard deviation of the shock to government expenditures (σ_g) so that the standard deviation of output in the model without credit frictions is 1%. This constitutes a good benchmark against which the model with credit frictions can be judged.[8] In all cases, labor and output rise on impact while private absorptions (consumption of both types of agents and investment) fall, which is the standard "crowding out" result in a model with separable preferences. The rise in output, however, is not uniform: Output rises the most when credit frictions are absent. In this case, the tax rate on labor is constant and the tax rate on capital is zero. In addition, the decline in private absorptions is relatively muted, which allows for a stronger response of output. These observations also apply to labor, but the differences between the models with and without credit frictions are bigger: The decline in investment affects capital and limits the rise in output, which leads to a smaller rise in output than labor.

When credit frictions are introduced, the behavior of the labor-income tax rate depends on the set of tax instruments that the government uses. When the government uses labor-income taxation only, the labor-income tax rate rises on impact and its path follows the path of the shadow value on the credit constraint. As a consequence, labor and output rise by less than in the credit-frictionless model while private absorptions fall significantly more. With labor and capital taxes, the results are very similar, but the taxation of capital income make matters slightly worse. The reason behind this result is that capital is subsidized, which entails higher taxation of labor income (recall the discussion in Section 3.4). In this respect, the taxation of capital income does not help in smoothing the labor tax rate, which is a clear deviation from Chari et al. (1994).

Consider now the case with taxes on labor, capital, and property. There are two effects that property taxes have on the labor tax rate, and they create a trade-off for the planner. On the one hand, property taxes allow the government to partly rely on this form of taxation to finance the rise in government spending and, thus, reduce the reliance on labor taxes (the "direct effect"). On the other hand, property taxes reduce the resources of the representative entrepreneur and may trigger a rise in the tightness of the credit constraint, and in turn pushes for a higher labor tax rate (the "indirect effect"). In sum, the direct effect dominates: the labor tax rate falls, and labor and output rise by more than under the scenario without property taxes. As suggested by condition (30), the fall in the labor tax rate is consistent with the rise in the tightness of the credit constraint (which in turn reflects either the taxation of real estate or a bigger rise in the demand for external funds than the rise in the value of collateral, or both). Overall, the behavior of the economy with all forms of taxes and credit frictions becomes closer to the case without credit frictions.

The tax rate on private assets (τ^a) responds strongly to a government spending shock, which in turn allows for a reduction in the labor tax rate. It does not, however, enable full smoothing of the tax rate on labor income. This finding reflects the existence of credit frictions: since the hiring of labor and the rental of capital are constrained, the model in essence has *two* credit frictions (one on each input). Therefore, introducing property taxation does not enable full smoothing of the labor tax rate. This is another aspect where the results of this paper differ from Chari et al. (1994); the latter shows that state-contingent debt and capital

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Table 2. Optimal fiscal policy: First and second moments

	Mean	Standard deviation	Auto-correlation
No credit frictions: Labor taxes only			
τ^l	0.3181	0.0000	1.0142
y	0.6754	0.0100	0.7443
Wedge	0.3181	0.0000	1.0142
Credit frictions: Labor taxes only			
τ^l	0.3113	0.0488	−0.0389
y	0.6678	0.0070	0.7757
Wedge	0.3180	0.0000	1.0138
Credit frictions: Labor and capital taxes			
τ^l	0.3122	0.0497	−0.0389
$\tau^{e,k}$	−0.0364	0.0008	0.7050
τ^k	−0.0365	0.2719	−0.0389
y	0.6655	0.0072	0.7812
Wedge	0.3101	0.0000	1.0002
Credit frictions: Labor, capital, and property taxes			
τ^l	0.3036	0.0081	−0.1203
$\tau^{e,k}$	−0.0358	0.0450	0.0131
τ^k	−0.0359	0.1492	0.2051
τ^a	0.0344	0.1693	0.0737
y	0.6693	0.0087	0.5490
Wedge	0.3103	0.0000	1.0128

Note: A shock to government expenditures. The parameter values are presented in Table 1.

taxation are sufficient to smooth the tax rate. Relative to the aforementioned study, however, introducing one tax instrument (in addition to the state-contingent debt and capital taxes) does not overcome the two distortions that I introduce in this paper.

4.3. Optimal Labor-Income Taxation Policy: Model's Moments

Table 2 presents the mean and second moments of the labor-income tax rate and other variables following a shock (of one standard deviation) to government expenditures. The standard deviation of the shock to government expenditures is set as in the previous subsection. Without credit frictions, the labor-income tax rate is constant, which restores former findings about tax smoothing and re-affirms the analytical findings (condition (30)). With credit frictions and labor taxes only, optimal policy calls for a time-varying path of the labor tax rate. The standard deviation of the labor tax rate is very high, higher than in standard models with

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neoclassical labor markets, and the labor tax rate is significantly more volatile than output. The labor tax rate displays virtually no persistence over the business cycle and does not resemble a random walk, but rather an independently and identically distributed (I.I.D.) process: as the shadow value of the binding credit constraint fluctuates, the labor-income tax rate fluctuates as well, leaving little room for persistence. Indeed, the labor tax rate inherits the cyclical properties of the credit distortion as opposed to the cyclical properties of the shock to government expenditures. The lack of persistence in the labor-income tax rate occurs even though both output and government expenditures display relatively high degrees of persistence (recall that $\rho_g = 0.90$).

The introduction of capital taxation makes the volatility of the labor tax rate slightly higher, but overall the differences are minuscule. This result is consistent with the impulse responses. On the other hand, the introduction of property taxation renders the standard deviation of the labor tax rate significantly smaller and the labor tax rate becomes (roughly) as volatile as output. This occurs with a very volatile tax rate on capital income and on private assets. Still, the labor-income tax rate is not fully smoothed as the tax instruments are not sufficient to overcome the credit frictions that are embedded in this study.

The volatility of the labor-income tax rate in this paper allows for the more general result of "static wedge smoothing," which is a very central result in optimal taxation, to hold. In the absence of credit frictions, labor taxation is the only source of the wedge [see condition (20)]; therefore, smoothing the wedge is equivalent to smoothing the labor tax rate. In this model, however, the credit friction is another source of the wedge, and full smoothing of the labor tax rate is not translated into full smoothing of the wedge. Under the optimal tax policy, the wedge is completely smoothed following exogenous shocks. This numerical finding is in line with condition (32).

The table also presents the optimal capital-income tax rate. Since the credit constraint affects the demand for capital, the ex-ante and ex-post optimal capital-income tax rates are negative, in line with the discussion above. These numerical results confirm the departure from the result of zero capital taxation of Judd (1985) and Chamley (1986). Capital taxation have implications for the labor-income tax rate from another perspective: The set of instruments that are available to the government are not sufficient to replicate the first-best solution. Recall that, from condition (23), the solution to the problem of the social planner implies a negative tax rate on labor income ($\tau^l_{\mathrm{FB},t} = -\phi\mu_t$). Since capital income is subsidized and labor income taxation is used to finance this subsidy, the combination of all tax instruments reduces the volatility of the labor-income tax rate but does not lead to the first-best labor-income tax rate. In Section 5.2, however, I discuss an alternative setup for which the first-best solution is attainable.

The findings also point to an important difference between this paper and Aiyagari (1995), which leads to different policy recommendations. In this paper, the firm's ability to rent capital is limited, thus generating a lower demand for capital. This leads to a policy that is designed to boost capital accumulation.

In Aiyagari (1995), however, the demand for capital is not distorted. Instead, households (i.e., capital suppliers) tend to over-accumulate capital in order to insure against future periods in which they are credit-constrained (a behavior that is re-enforced by the lack of complete insurance markets). To prevent households from so doing, it is optimal to levy a positive tax rate on capital. Therefore, one conclusion that can be drawn from this comparison is that the sector (demand side vs. supply side) on which credit constraints are imposed can matter for the optimal capital taxation policy.

Because the capital-income tax is negative and heavy taxation of real estate may make the constraint tighter, the planner will not accumulate enough assets to make the credit constraint irrelevant. Furthermore, for a meaningful discussion about the implications of credit constraint for optimal taxation, we ought to consider the case when this constraint is effective. However, as Section 5.3 shows, the results also hold when the credit constraint only occasionally binds. Namely, even if there are periods (e.g., economic booms) during which the planner can accumulate surpluses and transfer them to the constrained agents so that the constraint does not bind, that may not be feasible in bad times (e.g., in periods of war) during which net taxes are positive. Then, the credit constraint will bind and the labor-income tax rate will fluctuate with the tightness of this constraint.

4.4. Persistence of the Optimal Labor-Income Tax Rate

In this subsection, I elaborate on the persistence of the labor-income tax rate when the persistence of the government expenditures process ρ_g is varied. To this end, I compare the persistence of the labor-income tax rate to the persistence of output when all taxes (labor, capital, and property) are available to the government.

Without credit frictions, the labor tax rate is constant and technically it has a unit root for all values of ρ_g (not shown). With credit frictions, the degree of persistence is dramatically reduced (just above 10% in absolute value) and it is essentially the same for all levels of ρ_g (Figure 2). On the other hand, output displays a stronger persistence and the persistence of output rises with the persistence of the government spending process. For this reason, the persistence of the labor-income tax rate relative to the persistence of output is decreasing in ρ_g and stands at roughly 0.20–0.25. The fact that the labor-income tax rate is considerably less persistent than output stands in contrast to the fact that output and the labor-income tax rate have similar standard deviations (the right panel of Figure 2); credit frictions generate volatility in the labor-income tax rate but reduce its persistence.

5. ROBUSTNESS ANALYSIS

This section first shows results when the steady-state level of government spending is zero. It then presents the results with an alternative specification of the credit constraint and with an occasionally-binding credit constraint.

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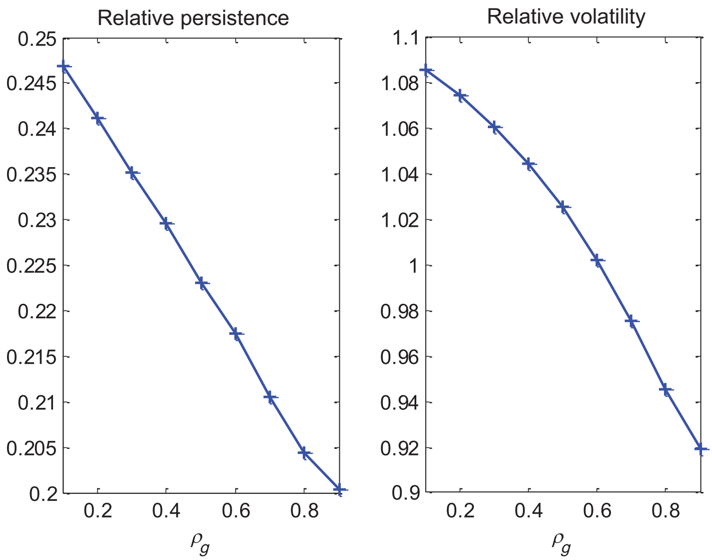

FIGURE 2. The persistence and volatility of the labor-income tax rate relative to the persistence and volatility of output for various values of the persistence parameter of the government expenditures process (ρ_g). The model with credit frictions and all tax instruments.

5.1. Zero Government Spending at the Steady State ($g = 0$)

As noted in Section 3.6, the steady-state level of government spending (g) can matter for the sign of the labor-income tax rate. This subsection presents the results with $g = 0$ in two different scenarios. First, the government keeps spending at zero and the economy is subject to a TFP shock. Second, beginning from a zero spending at the steady state, the government raises spending (with a constant TFP). While $g = 0$ is not an empirically plausible scenario, one can view it as illustrating the behavior of tax rates when the government spending-gross domestic product (GDP) ratio is sufficiently small.

The impulse responses are summarized in Figure E.1 where, to economize in presentation, I only present the labor tax rate, the tax rate on private assets and the tightness of the credit constraint. In addition, since the labor tax rate is fully constant in the absence of credit frictions, I only present the results with credit frictions in place. Overall, the results are similar for both types of shocks and they are consistent with the results from the benchmark calibration. With labor taxation only, the labor tax rate increases on impact. When capital-income taxation is introduced, the credit friction-induced subsidy of capital leads to a bigger rise in the labor tax rate. On the other hand, introducing property taxation leads to a significantly smaller response by the labor-income tax rate. As before, the slight

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increase in μ in the model with property taxes reflects the taxation of real estate that the representative entrepreneur uses as collateral.

Table E.1 and Table E.2 report the model's moments with a TFP shock and a government expenditures shock, respectively. Under a TFP shock and no property taxation, the volatility of the labor-income tax rate roughly equals the volatility of output. The addition of property taxes makes the labor-income tax rate considerably less volatile. Following a government expenditures shock, the labor-income tax rate is significantly more volatile than output (with labor taxes only as well as with labor and capital taxes) but becomes more stable when property taxation is introduced. The standard deviation of the labor tax rate, however, remains meaningful (nearly half the standard deviation of output).

Comparing both tables also reveals that a shock to government expenditures generates, in general, considerably higher volatility in the labor tax rate than a shock to TFP. This may reflect the direct impact of government expenditures on the government budget constraint and, consequently, the labor tax rate. The assumption regarding the size of government expenditures-GDP ratio matters: The volatility of the labor tax rate in the benchmark model ($g > 0$) is significantly higher than in the alternative model ($g = 0$). Also, all scenarios indicate that the first-best is not achieved (the labor-income tax rate is not negative). Although the model with zero steady-state government expenditures brings us closer to that scenario, the property tax is not sufficient to fully counter the credit friction.

5.2. Alternative Specification of the Credit Constraint

Following one scenario that Jermann and Quadrini (2012) consider in their analysis, I assume that the firm borrows working capital to finance only part of the wage payment. The credit constraint is then given by

$$\phi w_t l_t \leqslant \kappa q_t x_t. \tag{41}$$

The demand conditions for labor and capital, respectively, read

$$z_t f_{l,t} = (1 + \phi\mu_t) w_t, \tag{42}$$

$$z_t f_{k,t} = r_t. \tag{43}$$

Therefore, only the capital demand condition differs from its benchmark counterpart. And, since this modification does not alter the conditions involving the choice of labor, the assumption about what input is subject to the working capital requirement does not change the key qualitative results regarding tax smoothing; for this reason, condition (30) continues to hold. On the other hand, based on the analysis in Section 3.4, the tax rate on capital income is expected to be zero (on average).

Table E.3 shows that the numerical results have high similarities to and some differences from the benchmark analyses. With labor taxation only or with labor and capital taxation, the labor tax rate is significantly more volatile than output and displays little persistence. One difference from the benchmark analysis is that the addition of capital taxation actually reduces the volatility of the labor tax rate. This occurs because capital rental is not constrained and, consequently, not subsidized. Instead, the tax rate on capital income is ex-ante zero but varies markedly ex post, which contributes to smoothing the labor-income tax rate [these findings are in line with Chari et al. (1994)]. In addition, property taxation is very significant in this setup too as it induces a large decline in the standard deviation of the labor-income tax rate, but the volatility of the labor tax rate remains meaningful (nearly 42% of the volatility of output). Property taxation does not deliver a fully smoothed path for the tax rate on labor income as the planner balances between the desire to smooth the labor tax rate and not heavily taxing real estate.

With this specification of the collateral constraint, the set of tax instruments enables the replication of the first-best solution: the average labor-income tax rate is negative and equals -0.84%, which is consistent with condition (23). With the labor-income subsidy and since the capital-income tax rate is zero, labor is at the efficient level, which is higher than the level of labor at the first-best with positive labor-income tax rate. Consequently, output emerges as higher than under other cases with credit frictions. In addition, since the labor-income tax rate is at its first-best level, the static wedge is zero, in line with condition (20).

Figure E.2 shows the responses of the tightness of the credit constraint, labor-income tax rate, and the tax rate on private assets to a government expenditures shock when credit frictions are present. The strongest response of the labor tax rate occurs when the government has access to labor taxation only. When capital taxation is introduced, the response of the labor-income tax rate is considerably smaller. And, when property taxes are available, the response of the labor-income tax rate to the government expenditures shock is even smaller. Notice also that the adjustment of the tax rate back to the steady state is faster than in the benchmark model with constraints on hiring labor and renting capital.

5.3. Occasionally-Binding Credit Constraint

This subsection presents the results when the credit constraint may only occasionally bind. To solve the model with the occasionally-binding credit constraint, I use the piecewise linear solution method of Guerrieri and Iacoviello (2015), which can be applied with *Dynare* using the *OccBin* library of numerical routines. According to this solution methodology, a model with an occasionally binding constraint is equivalent to a model with two regimes: under one regime the constraint is slack, and under the other regime it is binding, with the switch between both regimes being endogenous. Under each regime, the model is linearized around its non-stochastic steady state. Guerrieri and Iacoviello (2015) compare the piecewise linear solution to a high-quality numerical solution, which can be viewed as the "exact solution" of the model, and show that the former can essentially replicate the latter.

I summarize the main insights in Figure E.3, where simulations of the labor tax rate and the tightness of the credit constraint are displayed (I use relatively short simulations for scale reasons). The credit constraint may not bind in certain occasions, but it mostly does. More importantly, when the credit constraint is not binding, the labor-income tax rate is constant. When the constraint binds, the labor-income tax rate fluctuates with the shadow value on the credit constraint, in line with condition (30). Therefore, even if there exist periods where the constraint is not binding, the key result of the paper holds.

An implication of this result is the following: in "good times" (when the credit constraint is slack), the government can set a higher labor-income tax rate. By so doing, the government can afford reducing the tax rate during "bad times" (when the credit constraint is binding) to stimulate economic activity. Then, the difference in the tax rates across the two scenarios can be seen as "front loading" of taxes. Higher tax rates in good times open more room for an accommodating fiscal policy during downturns as it allows for more reduction in the labor tax rate without compromising on spending and/or allowing for an excessive deterioration in the budget balance.

6. A MODEL WITH NON-STATE-CONTINGENT DEBT

This section outlines a model with non-state-contingent debt, whereby the returns on bonds are predetermined at time t. It is well known that the assumption regarding the contingency of government debt and market completeness is crucial for tax smoothing; see, for example, a discussion in Angeletos (2002). As noted by the latter, with short-term non-state-contingent debt, variations in government spending require the government to vary the tax rates (including on labor income) accordingly. The addition of credit frictions makes smoothing the labor tax rate even more challenging; there is another distortion that has to be corrected for. On the other hand, this model includes another tax instrument (property taxation) that, to the best of my knowledge, has not been accounted for in previous studies on labor tax smoothing. Therefore, it is interesting to study how an otherwise incomplete-market model will be affected when the government can levy property taxes. I study the extent to which this setup with property taxation manages in completing the markets (or at least in making them less incomplete) when debt is not state contingent. I close this section with a discussion about the equivalence between credit frictions and market incompleteness.

6.1. Analytical Results

I show the implications of credit frictions using a special case that has received attention in the literature with incomplete markets [e.g., Aiyagari et al. (2002)]: quasilinear preferences. This form of preferences also simplifies the analysis. Specifically, the utility function of the representative household is linear in consumption:

$$u(c_t, h_t, l_t) = c_t + \psi \frac{h_t^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{l_t^{1+\theta}}{1+\theta}. \tag{44}$$

As shown in Appendix D, the optimal labor-income tax rate is given by

$$\tau^l_{\mathrm{SB},t} = \frac{\theta\Psi_t - (1+\Psi_t)\phi\mu_t}{1+(1+\theta)\Psi_t}, \tag{45}$$

with Ψ_t being the Lagrange multiplier on the time-t measurability constraint. This condition shows that the negative effect of credit frictions on the labor-income tax rate continues to exist in this incomplete-markets setup. Therefore, the main finding of the paper is robust to the assumption regarding market incompleteness.

Condition (45) illustrates an interaction between credit frictions and the Lagrange multiplier on the measurability constraint (put differently, an interaction between credit frictions and market incompleteness) in generating a volatile path of the labor tax rate. One way to view this interaction is the following. When the credit constraint tightens more, labor and capital demands are reduced. The planner would then want to reduce the labor and capital tax rates to boost their supply. In the absence of state-contingent bonds, the planner cannot engineer variations in the rate of return on the debt, which leads to greater variations in the labor tax rate than in the complete-markets model. This effect is stronger when credit frictions are more severe.

Market incompleteness has effects on the static wedge under optimal policy:

$$\mathrm{Wedge}_{\mathrm{SB},t} = \frac{\theta\Psi_t}{1+(1+\theta)\Psi_t}. \tag{46}$$

One implication of this setup is that, due to the unavailability of state-contingent debt, the static wedge will optimally vary over the business cycle. Therefore, unlike the complete-markets setting [recall condition (32)], the optimal setting of the labor-income tax rate does not lead to full smoothing of the static wedge, and this result holds regardless of the assumption about credit frictions.

Market incompleteness has also implications for the optimal tax rate on capital income. Combining capital supply, capital demand and the optimal choice of capital by the Ramsey planner gives the following expected capital tax rate:

$$\begin{aligned} &E_t\left[(r_{t+1}-\delta)\tau^k_{t+1}\right] \\ &\quad = -E_t\left\{\phi\mu_{t+1}r_{t+1} + \left[\frac{\beta-1+\beta(r_{t+1}-\delta)}{\beta}\right]\Psi_{t+1} + \phi\mu_{t+1}r_{t+1}\Psi_{t+1}\right\}. \end{aligned} \tag{47}$$

Condition (47) is a generalization of the condition that has been obtained in the complete-markets framework (37) when the marginal utility of consumption is $u_{c,t} = 1$. The first term on the right-hand side of condition (47) captures the effect of credit frictions, the second term captures the effect of market incompleteness

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and the third term shows the interaction between credit frictions and market incompleteness; both factors reinforce each other. If β is sufficiently close to 1 and since $r_{t+1} > \delta$, the expected capital-income tax rate will be negative and, other things equal, likely to be more negative than in the complete-markets model. In what follows, I numerically verify this conjecture.

In addition, with quasilinear preferences and nonbinding constraints on the debt limits, the multiplier Ψ_t satisfies

$$\Psi_t = E_t(\Psi_{t+1}), \tag{48}$$

and thus Ψ_t is a martingale. Without credit frictions ($\phi = 0$), the labor tax rate is a ratio of two martingales and thus likely to be persistent. The addition of credit frictions, however, breaks that relationship and may affect the persistence of the labor-income tax rate.

6.2. Numerical Analysis

This subsection briefly describes the numerical results in the model with non-state-contingent debt using the benchmark paramaterization of the model (Table E.4). Without credit frictions and when only labor-income taxes (and bonds) are available, the labor-income tax rate is volatile. This result constitutes a clear deviation from the complete-markets model. The addition of all taxes markedly reduces the size of the labor-income tax rate and its volatility (in this regard, property taxation plays a major role). Essentially, the tax rate becomes very stable. In addition, since the static wedge equals the labor tax rate, the latter is not fully smoothed, which is another departure from the complete-markets setup.

With credit frictions, the volatility of the labor-income tax rate is very high (and higher than the corresponding volatility in the model with state-contingent debt), reflecting the combination of two factors that induce variations in the labor-income tax rate (credit frictions and market incompleteness). When capital and property taxes are added to the model, the volatility of the labor tax rate is cut significantly, but it remains quite volatile (the volatility of the labor tax is more than 70% of the volatility of output). Property taxation clearly helps in obtaining a more stable path of the labor-income tax rate, but it is not sufficient to bring about a fully smoothed path. In addition, the very negative values of the actual and the ex-ante capital tax rates reflect the combination of market incompleteness and credit frictions, in line with condition (47). The observation that non-state-contingent debt can lead to negative capital tax rates may also be found in Chari et al. (1994).

To assess the effects of property taxes on welfare, the last column presents the consumption equivalence, which is the percentage by which consumption of the representative household should increase so that welfare under labor and capital taxes equals welfare when all tax instruments are available. Without credit frictions, a rise of 0.8% in consumption is required, while with credit frictions a significantly larger rise in consumption (2.05%) is needed. Therefore, the availability of an additional tax instrument (i.e., property taxation) not only renders the path of the labor-income tax rate more stable but also leads to meaningful welfare gains.

6.3. Discussion

The comparisons between the incomplete-markets and the complete-markets setups reveal the following: In the absence of credit frictions, the additional constraints with incomplete markets lead to differences (relative to the complete-markets model) regarding the optimality of labor-income tax smoothing; in particular, the labor-income tax rate will optimally vary over the business cycle if state-contingent debt is not available. In many ways, credit frictions play the role of an additional constraint in that even if markets are complete, full smoothing of the labor tax rate is unattainable. In the incomplete-markets model of Aiyagari et al. (2002), the additional constraint reflects the nature of the government bonds, while in this model the additional "constraint" reflects the nature of the credit markets.

In this respect, the comparisons between the complete-markets and incomplete-markets models generate an important result. In particular, the optimal labor tax rate in the model with incomplete markets only (i.e., with no credit frictions) is given by

$$\tau^{l}_{\mathrm{SB},t} = \frac{\theta \Psi_t}{1 + (1+\theta)\Psi_t} \tag{49}$$

and the optimal labor tax rate with both complete markets and credit frictions reads

$$\tau^{l}_{\mathrm{SB},t} = \frac{\theta\xi - (1+\xi)\phi\mu_t}{1 + (1+\theta)\xi}. \tag{50}$$

If one defines

$$\Psi_t = \frac{\theta\xi - (1+\xi)\phi\mu_t}{\theta + (1+\theta)(1+\xi)\phi\mu_t}, \tag{51}$$

then the two models will deliver the same labor-income tax rate even though they differ with respect to both market completeness and the existence of credit frictions. Put differently, in the complete-markets model, credit frictions manifest themselves as a form of market incompleteness. The only source of this "incompleteness" is credit frictions, and in the absence of these frictions the variable Ψ_t collapses into ξ, thus restoring the standard outcome with complete markets. This observation suggests that credit frictions not only have implications for the quantitative variations in the labor-income tax rate but that they also affect the entire environment, leading to deviations from some of the classic results in the literature of optimal taxation, including the tax-smoothing literature.

7. CONCLUSION

The optimal cyclical behavior of the labor-income tax rate in a model with credit frictions is studied in this paper. Firms' borrowing to finance the hiring of labor at the beginning of the period is constrained by the value of their collateral, which induces an inefficiently low demand for labor. In this environment, full smoothing of the labor-income tax rate over the business cycle is not optimal even in the presence of capital-income taxation, state-contingent government debt and flexible prices. Instead, the labor-income tax rate moves in the opposite direction of the tightness of the credit constraint. Furthermore, credit frictions break the full mapping between tax smoothing and static wedge smoothing; in fact, the optimal volatility of the labor-income tax rate allows for smoothing the wedge.

The volatility of the labor-income tax rate in the face of shocks allows for stabilizing labor (hence output) over the business cycle, which in turn induces a smoothed path of consumption. Moreover, the credit constraint induces inefficiently low demand for labor as the firm hires labor at a point where the marginal product of labor is a "markup" over the real wage rate. The tax reduction in tighter credit markets helps in offsetting this markup, thus positioning the economy closer to the efficient state. Quantitatively, the volatility of the labor-income tax rate is meaningfully high and, depending on the exact set of tax instruments, may be significantly higher than the volatility of output.

The implications of credit frictions for the optimal labor-income tax rate are robust to the assumption about market incompleteness. In the robustness analysis, I assume that the government issues non-state-contingent debt only, which in itself induces variations in the labor tax rate. In this setup, credit frictions constitute another source of variations in the labor tax rate, leading to significant deviations from labor tax smoothing. Another important observation is that, in a model with complete markets, credit frictions are manifested as a form of market incompleteness that renders tax volatility optimal.

NOTES

1. Carlstrom and Fuerst (1998) use a costly state-verification setup with asymmetric information between lenders and borrowers that induces monitoring by lenders. In their study, monitoring (agency) costs are the source of financial frictions in the model. These costs induce differences between the marginal products of capital and labor and their respective factor prices; firms hire labor and rent capital so that their marginal products are markups over their respective prices. This paper, on the other hand, employs a limited enforcement framework where borrowers are constrained and default is not possible.

2. Assuming that firms use real estate as collateral is common: For example, Kiyotaki and Moore (1997) assume that borrowing is tied to the value of land, and Iacoviello (2005) assumes that entrepreneurs use real estate as collateral. Chaney et al. (2012) show that, for US firms over 1993–2007, appreciation in firms' real estate values led to increases in investment that is mainly financed through additional debt issuance (this effect is stronger for credit-constrained firms). Furthermore, if we assume another type of asset that can be used as collateral, with the collateral constraint continuing to read $\phi(w_t l_t + r_t k_t) \leqslant \kappa n w_t$ and nw_t is net worth, then the results will hold qualitatively. Compared to financial assets, real estate is easier to incorporate and it enables keeping the real structure of the

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model, which both simplifies matters and makes the model closer in nature to the basic framework in which the "tax smoothing" result was originally obtained.

3. In this regard, based on the Fixed Assets Tables of the Bureau of Economic Analysis, the US depreciation rate of housing is relatively low, averaging roughly 2.1% per annum over the period 1960–2015. This estimate is in line with Harding et al. (2007).

4. In general, the optimal labor-income tax rate reads $\tau^l_{\mathrm{SB,t}} = \frac{(\sigma+\theta)\xi-(1+\xi-\sigma\xi)\phi\mu_t}{1+(1+\theta)\xi}$, which, to extent that $1+\xi-\sigma\xi > 0$, show that an increase in the tightness of the collateral constraint will reduce the labor-income tax rate. Setting $\sigma = 1$ restores the value of $\tau^l_{\mathrm{SB,t}}$ from condition (30).

5. Assuming that the credit constraint always binds is not crucial for the main insights of this subsection. If the constraint is assumed to only occasionally bind, μ_t will be either zero or positive, hence not constant. In turn, the labor-income tax rate will not be constant. In order to simplify matters and to make the computational solution more tractable, the benchmark analyses assume an always-binding constraint. Furthermore, Section 5.3 shows that the key results hold when the credit constraint may only occasionally bind.

6. See Chahrour and Svec (2014) for a study that finds that capital income should be optimally subsidized when consumers face model uncertainty. Using a neoclassical growth model with imperfectly-competitive product markets, Guo and Lansing (1999) show that the capital-income tax may be negative, zero or positive. The outcome depends on parameter values, the degree of monopoly power in the product market and the level of government spending. On the other hand, Reis (2011) finds the optimal capital-income tax rate to be positive in a model with entrepreneurial labor. All of these studies abstract from credit frictions.

7. This value of ϕ is consistent with evidence on the difficulty of US firms in obtaining external funds. Carlstrom et al. (2010), among others, think of "small" firms as being subject to credit constraints. Census data for 1991–2008 show that roughly 50% of the workers were employed in small firms (of up to 500 employees), and thus Carlstrom et al. (2010) set a value of 0.5 for their credit friction parameter. Campello et al. (2010) show that 57% of the US firms have been either "somehow affected" or "very affected" by credit constraints. Using the firm-level World Business Environment Survey (WBES) Dataset, Beck et al. (2005) show that the US "financial obstacle" index is 2.39 on a scale of 1 (no obstacle) to 4 (major obstacle), which is roughly 0.46 on a scale of zero to one.

8. Alternatively, one may match the quarterly standard deviation of US GDP (which is 0.015 for 1960:Q1-2015:Q4). That approach, however, implies that the latter is the relevant standard deviation for the Ramsey planner, which may not be the case. In any case, both approaches yield similar qualitative results.

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APPENDIX A: THE ENTREPRENEUR'S PROBLEM

At the beginning of the period, the representative entrepreneur obtains a loan b_t^f from the representative household, which is repaid at the end of the period at a gross interest rate of R_t^f. Borrowing is constrained by the beginning-of-period firm's collateral. Formally, the representative entrepreneur chooses consumption, labor, capital, real estate, and loans to

$$\max_{\{b_t^f, e_t, l_t, k_t, x_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v(e_t, x_t), \tag{A.1}$$

subject to

$$e_t + q_t x_{t+1} + R_t^f b_t^f + w_t l_t + r_t k_t = (1 - \tau_t^p) q_t x_t + z_t f(k_t, l_t) + b_t^f, \tag{A.2}$$

$$b_t^f - \phi(w_t l_t + r_t k_t) \geq 0, \tag{A.3}$$

$$\kappa q_t x_t - b_t^f \geq 0. \tag{A.4}$$

Combining conditions (A.3) and (A.4) and substituting $R_t^f = 1$ in condition (A.2), as it is the equilibrium result from the solution to the households' problem, give the following optimization problem:

$$\max_{\{e_t, l_t, k_t, x_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v(e_t, x_t) \tag{A.5}$$

subject to

$$e_t + q_t x_{t+1} + w_t l_t + r_t k_t = (1 - \tau_t^p) q_t x_t + z_t f(k_t, l_t), \tag{A.6}$$

and

$$\phi(w_t l_t + r_t k_t) \leqslant \kappa q_t x_t, \tag{A.7}$$

which is the problem that is described in the text (Section 2.2).

APPENDIX B: EFFICIENT ALLOCATIONS

The problem of the social planner is to maximize:

$$\max_{\{c_t, e_t, k_t, l_t, h_t, x_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \left[\eta \beta^t u(c_t, h_t, l_t) + (1 - \eta) \gamma^t v(e_t, x_t)\right], \tag{B.1}$$

subject to the sequence of resource constraints.

$$z_t f(k_t, l_t) + (1 - \delta) k_t = c_t + e_t + k_{t+1} + g_t, \tag{B.2}$$

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and the clearing condition of the real estate market:

$$h_t + x_t = 1. \tag{B.3}$$

Letting ϱ_t be the Lagrange multiplier associated with condition (B.2), the first-order conditions with respect to c_t and l_t, respectively, read

$$\beta^t \eta u_{c,t} = \varrho_t \omega^t, \tag{B.4}$$

$$\beta^t \eta u_{l,t} = -\varrho_t \omega^t z_t f_{l,t}, \tag{B.5}$$

where, following Monacelli (2008), $\omega = \beta^\eta \gamma^{1-\eta}$ is the discount factor from the perspective of the planner. This geometric mean of the discount factors is used to discount the clearing conditions (that are relevant to both types of agents). Combining conditions (B.4) and (B.5) gives condition (22) in the text.

APPENDIX C: THE RAMSEY PROBLEM

I show here the derivation of the PVICs and the key conditions of the Ramsey problem.

C.1. THE PRESENT-VALUE IMPLEMENTABILITY CONSTRAINT: HOUSEHOLDS

Since $R_t^f = 1$, the representative household's budget constraint becomes

$$\begin{aligned}(1-\tau_t^l)w_t l_t + [1-\delta + r_t - \tau_t^k(r_t - \delta)]k_t + R_t b_t + (1-\tau_t^p)q_t h_t \\ = c_t + k_{t+1} + b_{t+1} + q_t h_{t+1}.\end{aligned} \tag{C.1}$$

By introducing $\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t}$ to (C.1) and re-arranging, we have

$$\begin{aligned}\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t}(1-\tau_t^l)w_t l_t + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t}[1-\delta + r_t - \tau_t^k(r_t-\delta)]k_t + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} R_t b_t \\ + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t}(1-\tau_t^p)q_t h_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} c_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} k_{t+1} \\ - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} b_{t+1} - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} q_t h_{t+1} = 0.\end{aligned} \tag{C.2}$$

Recall that, from the solution to the problem of the representative household, we have

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t, \tag{C.3}$$

$$u_{c,t} = \beta \mathbf{E}_t (R_{t+1} u_{c,t+1}), \tag{C.4}$$

$$u_{c,t} = \beta \mathbf{E}_t \left[u_{c,t+1}\left(1-\delta + r_{t+1} - \tau_{t+1}^k(r_{t+1}-\delta)\right)\right], \tag{C.5}$$

$$q_t u_{c,t} = \beta \mathbf{E}_t \left[u_{h,t+1} + (1-\tau_{t+1}^p) q_{t+1} u_{c,t+1}\right]. \tag{C.6}$$

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To eliminate all prices (with the exception of the price of real estate) and taxes, substitute (C.3) in the first term of (C.2), (C.4) in the third term of (C.2), (C.5) in the second term of (C.2), (C.6) in the fourth term of (C.2) and use the law of iterated expectations to get

$$\begin{aligned}
&\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} \left(-\frac{u_{l,t}}{u_{c,t}}\right) l_t + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} u_{c,t-1} k_t + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} u_{c,t-1} b_t \\
&\quad + \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} \left[u_{c,t-1} q_{t-1} h_t - \beta u_{h,t} h_t\right] - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} c_t \\
&\quad - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} k_{t+1} - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} b_{t+1} - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} q_t h_{t+1} = 0.
\end{aligned} \tag{C.7}$$

Combining the second and sixth terms of (C.7) yields

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} u_{c,t-1} k_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} k_{t+1} = \beta^{-1} k_0 u_{c,-1}, \tag{C.8}$$

where common terms have been canceled using summation rules.

Combining the third and seventh terms of (C.7) gives

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} u_{c,t-1} b_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} b_{t+1} = \beta^{-1} b_0 u_{c,-1}. \tag{C.9}$$

Combining the fourth and last terms of (C.7) yields

$$\begin{aligned}
&\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} \left[u_{c,t-1} q_{t-1} h_t - \beta u_{h,t} h_t\right] - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} q_t h_{t+1} \\
&\quad = \beta^{-1} q_{-1} h_0 u_{c,-1} - \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{h,t} h_t.
\end{aligned} \tag{C.10}$$

Also, the first term of (C.7) can be written as

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{c_t} \left(-\frac{u_{l,t}}{u_{c,t}}\right) l_t = -\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_{l_t} l_t. \tag{C.11}$$

Finally, substituting (C.8)–(C.11) into (C.7) and re-arranging give the PVIC:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t} c_t + u_{l,t} l_t + u_{h,t} h_t\right] = \beta^{-1} u_{c,-1} \left[b_0 + k_0 + q_{-1} h_0\right], \tag{C.12}$$

which is condition (24) in the text with $H_0 = \beta^{-1} u_{c,-1} \left[b_0 + k_0 + q_{-1} h_0\right]$.

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C.2. THE PRESENT-VALUE IMPLEMENTABILITY CONSTRAINT: ENTREPRENEURS

I proceed with similar steps to derive the PVIC of the representative entrepreneur. Assuming a Cobb–Douglas production function, her budget constraint reads

$$z_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - r_t k_t + (1-\tau_t^p) q_t x_t = q_t x_{t+1} + e_t, \tag{C.13}$$

the optimality conditions

$$(1-\alpha) z_t k_t^{\alpha} l_t^{-\alpha} = (1+\phi\mu_t) w_t, \tag{C.14}$$

$$\alpha z_t k_t^{\alpha-1} l_t^{1-\alpha} = (1+\phi\mu_t) r_t, \tag{C.15}$$

$$q_t v_{e,t} = \gamma \mathbf{E}_t \left(v_{x,t+1} + q_{t+1} v_{e,t+1} (1 - \tau_{t+1}^p + \kappa\mu_{t+1}) \right), \tag{C.16}$$

and the credit constraint

$$\phi(w_t l_t + r_t k_t) = \kappa q_t x_t. \tag{C.17}$$

The goal is to derive an implementability constraint that accounts for all of the representative entrepreneur's constraints, namely (C.13)–(C.17). To do so, I first multiply condition (C.14) by l_t and condition (C.15) by k_t to get

$$(1-\alpha) z_t k_t^{\alpha} l_t^{1-\alpha} = (1+\phi\mu_t) w_t l_t, \tag{C.18}$$

$$\alpha z_t k_t^{\alpha} l_t^{1-\alpha} = (1+\phi\mu_t) r_t k_t. \tag{C.19}$$

Combining conditions (C.18) and (C.19) then gives

$$z_t k_t^{\alpha} l_t^{1-\alpha} = (1+\phi\mu_t)(w_t l_t + r_t k_t). \tag{C.20}$$

Substituting this result into the budget constraint (C.13), we have

$$\phi\mu_t(w_t l_t + r_t k_t) + (1-\tau_t^p) q_t x_t = q_t x_{t+1} + e_t, \tag{C.21}$$

but since $\phi(w_t l_t + r_t k_t) = \kappa q_t x_t$, condition (C.21) can be re-written as

$$(1 - \tau_t^p + \kappa\mu_t) q_t x_t = q_t x_{t+1} + e_t. \tag{C.22}$$

This condition combines three of the four conditions that characterize the representative entrepreneur's problem: (C.14)–(C.15) and (C.17). Therefore, in deriving the PVIC of the entrepreneur, I make use of conditions (C.16) and (C.22).

Introducing $\mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{e_t}$ to (C.22) and re-arranging yields

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{e_t} (1 - \tau_t^p + \kappa\mu_t) q_t x_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{e_t} q_t x_{t+1} - \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{e_t} e_t = 0. \tag{C.23}$$

Substituting condition (C.16) into the first term of condition (C.23) and canceling common terms give

$$\gamma^{-1} v_{e,-1} q_{-1} x_0 - \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{x,t} x_t - \mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t v_{e_t} e_t = 0. \tag{C.24}$$

Re-arranging then gives the PVIC:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \gamma^t \left[v_{e_t} e_t + v_{x,t} x_t \right] = \gamma^{-1} v_{e,-1} q_{-1} x_0, \tag{C.25}$$

which is condition (25) in the text with $F_0 = \gamma^{-1} v_{e,-1} q_{-1} x_0$.

Condition (C.12) accounts for all the conditions of the representative household and condition (C.25) summarizes all the conditions of the representative entrepreneur. The economy-wide resource constraint, the clearing condition in the real estate market and these two PVIC conditions constitute the set of constraints of the Ramsey planner in the model with state-contingent bonds.

C.3. SOLUTION TO THE RAMSEY PROBLEM

Denoting λ_t as the Lagrange multiplier on the resource constraint and ξ as the Lagrange multiplier on the representative household's PVIC, the first-order conditions of the Ramsey planner with respect to c_t and l_t, respectively, yield

$$\beta^t \eta u_{c,t} + \beta^t \eta \xi \left(u_{c,t} + u_{cc,t} c_t + u_{lc,t} l_t + u_{hc,t} h_t \right) = \lambda_t \omega^t, \tag{C.26}$$

$$\beta^t \eta u_{l,t} + \beta^t \eta \xi \left(u_{l,t} + u_{ll,t} l_t + u_{cl,t} c_t + u_{hl,t} h_t \right) = -\lambda_t \omega^t z_t f_{l,t}. \tag{C.27}$$

Dividing both sides of each equation by $\beta^t \eta$ gives

$$u_{c,t} + \xi \left(u_{c,t} + u_{cc,t} c_t + u_{lc,t} l_t + u_{hc,t} h_t \right) = \frac{\lambda_t \omega^t}{\beta^t \eta}, \tag{C.28}$$

$$u_{l,t} + \xi \left(u_{l,t} + u_{ll,t} l_t + u_{cl,t} c_t + u_{hl,t} h_t \right) = -\frac{\lambda_t \omega^t z_t f_{l,t}}{\beta^t \eta}. \tag{C.29}$$

Then dividing condition (C.29) by condition (C.28) gives condition (26) in the text. Notice that this result holds regardless of the assumption about γ vs. β.

In addition, the first-order condition of the Ramsey planner with respect to next-period capital reads

$$\lambda_t = \omega E_t \left[\lambda_{t+1} (1 - \delta + z_{t+1} f_{k,t+1}) \right]. \tag{C.30}$$

Given the logarithmic preferences in consumption (which imply $u_{c,t} = c_t^{-1}$) and the separability of preferences of the households, condition (C.26) becomes

$$\beta^t \eta u_{c,t} = \lambda_t \omega^t. \tag{C.31}$$

Then, by combining conditions (C.30) and (C.31), condition (C.30) can be re-written as

$$u_{c,t} = \beta E_t \left[u_{c,t+1} (1 - \delta + z_{t+1} f_{k,t+1}) \right], \tag{C.32}$$

which is the standard condition in this class of models. A similar condition can be obtained if the (separable) utility function is not logarithmic in consumption.

Also, given the functional forms that I use in this paper, $u(c_t, h_t, l_t) = \ln c_t + \psi \frac{h_t^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{l_t^{1+\theta}}{1+\theta}$, $v(e_t, x_t) = e_t + \varphi \frac{x_t^{1-\varepsilon}}{1-\varepsilon}$, and $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$, the Lagrangian of the Ramsey problem

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reads

$$\mathcal{L}_t = \mathbf{E}_0 \sum_{t=0}^{\infty} \Big\{ \Big[\eta\beta^t \Big(\ln c_t + \psi \frac{h_t^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{l_t^{1+\theta}}{1+\theta} \Big) + (1-\eta)\gamma^t \Big(e_t + \varphi \frac{(1-h_t)^{1-\varepsilon}}{1-\varepsilon} \Big) \Big]$$
$$+ \xi\eta\beta^t \Big(1 + \psi h_t^{1-\varepsilon} - \chi l_t^{1+\theta} - H_0 \Big) + \varpi(1-\eta)\gamma^t \Big(e_t + \varphi(1-h_t)^{1-\varepsilon} - F_0 \Big)$$
$$+ \omega^t \lambda_t \big[z_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta)k_t - c_t - e_t - k_{t+1} - g_t \big] \Big\}.$$

The first-order conditions with respect to c_t, l_t, k_{t+1}, respectively, are

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = \beta^t \frac{\eta}{c_t} - \omega^t \lambda_t = 0, \tag{C.33}$$

$$\frac{\partial \mathcal{L}_t}{\partial l_t} = -\eta\beta^t \chi [1 + (1+\theta)\xi] l_t^{\theta} + \omega^t (1-\alpha)\lambda_t z_t k_t^{\alpha} l_t^{-\alpha} = 0, \tag{C.34}$$

$$\frac{\partial \mathcal{L}_t}{\partial k_{t+1}} = -\lambda_t \omega^t + \omega^{t+1} E_t \left[\lambda_{t+1} (1 - \delta + \alpha z_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha}) \right] = 0. \tag{C.35}$$

And the second-order conditions with respect to c_t, l_t, k_{t+1}, respectively, are

$$\frac{\partial^2 \mathcal{L}_t}{\partial c_t^2} = -\beta^t \frac{\eta}{c_t^2} < 0, \tag{C.36}$$

$$\frac{\partial^2 \mathcal{L}_t}{\partial l_t^2} = -\eta\beta^t \chi\theta [1 + (1+\theta)\xi] l_t^{\theta-1} - \omega^t \alpha (1-\alpha) \lambda_t z_t k_t^{\alpha} l_t^{-\alpha-1} < 0, \tag{C.37}$$

$$\frac{\partial^2 \mathcal{L}_t}{\partial k_{t+1}^2} = -\omega^{t+1} E_t \left[\lambda_{t+1} \alpha (1-\alpha) z_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} \right] < 0, \tag{C.38}$$

and, therefore, the second-order conditions for optimum are satisfied.

APPENDIX D: A MODEL WITH NON-STATE-CONTINGENT DEBT

This appendix outlines the derivations of condition (45) for the model with incomplete markets. As shown in Aiyagari et al. (2002), the unavailability of state-contingent debt affects the problem of the Ramsey planner by introducing additional constraints.

With non-state-contingent bonds, the budget constraint of the households is given by

$$c_t + k_{t+1} + b_{t+1} + q_t h_{t+1} = (1-\tau_t^l) w_t l_t + [1 - \delta + r_t - \tau_t^k (r_t - \delta)] k_t + (1-\tau_t^p) q_t h_t + R_{t-1} b_t, \tag{D.1}$$

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where the main difference is the timing of the interest rate on bonds (it becomes independent of the state at time t).

The corresponding optimality conditions read

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t, \tag{D.2}$$

$$u_{c,t} = \beta R_t \mathbf{E}_t(u_{c,t+1}), \tag{D.3}$$

$$u_{c,t} = \beta \mathbf{E}_t\left[u_{c,t+1}\big(1-\delta+r_{t+1}-\tau_{t+1}^k(r_{t+1}-\delta)\big)\right], \tag{D.4}$$

$$q_t u_{c,t} = \beta \mathbf{E}_t\left[u_{h,t+1}+(1-\tau_{t+1}^p)q_{t+1}u_{c,t+1}\right]. \tag{D.5}$$

With non-state-contingent debt, Aiyagari et al. (2002) show that the problem of the Ramsey planner includes not only the single period-0 implementability constraint [as in (C.12)] but also implementability constraints at each node. In particular, incomplete-markets add the following sequence of measurability constraints:

$$\mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \Big[u_{c,t+j}c_{t+j}+u_{l,t+j}l_{t+j}+u_{h,t+j}h_{t+j}\Big] = \beta^{-1}u_{c,t-1}\left[b_t+k_t+q_{t-1}h_t\right], \tag{D.6}$$

$$\mathbf{E}_t \sum_{j=0}^{\infty} \gamma^j \Big[v_{e,t+j}e_{t+j}+v_{x,t+j}x_{t+j}\Big] = \gamma^{-1}v_{e,t-1}q_{t-1}x_t, \tag{D.7}$$

where (C.12) and (C.25) are essentially the time-0 versions of (D.6) and (D.7), respectively.

Next, let Γ_t be the Lagrange multiplier on the time-t version of (D.6) and then introduce summation $\mathbf{E}_0 \sum_{j=0}^{\infty} \beta^t \Gamma_t$ to condition (D.6) to get

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t \left\{\mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \Big[u_{c,t+j}c_{t+j}+u_{l,t+j}l_{t+j}+u_{h,t+j}h_{t+j}\Big]\right\} - \beta^{-1}\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t u_{c,t-1}(k_t+b_t+q_{t-1}h_t) = 0, \tag{D.8}$$

where Γ_0, the Lagrange multiplier on the time-0 constraint is equivalent to ξ in the complete-markets setup.

Define

$$\Gamma_t = \Psi_t - \Psi_{t-1}. \tag{D.9}$$

Substituting condition (D.9) in the first term of condition (D.8) gives

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t (\Psi_t-\Psi_{t-1}) \left\{\mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \Big[u_{c,t+j}c_{t+j}+u_{l,t+j}l_{t+j}+u_{h,t+j}h_{t+j}\Big]\right\} - \beta^{-1}\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t u_{c,t-1}(k_t+b_t+q_{t-1}h_t) = 0. \tag{D.10}$$

Then, by eliminating common terms (see Section D.2), condition (D.10) becomes

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\Psi_t\Big[u_{c,t}c_t+u_{l,t}l_t+u_{h,t}h_t\Big]-\beta^{-1}\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\Gamma_t u_{c,t-1}(k_t+b_t+q_{t-1}h_t)=0, \quad \textbf{(D.11)}$$

which is the sequence of constraints that incomplete markets impose on the Ramsey planner. Similar constraints are derived for the entrepreneurs. Intuitively, since the beginning-of-period debt (b_t) is the same for any two histories, then for any two realizations of the state at time t that had the same histories until period $t-1$, we shall impose equality on the right-hand side of their budget constraint. In addition, it is assumed that there are limits on government debt so that $\underline{M}\leq b_t\leq\overline{M}$.

D.1. SOLUTION TO THE RAMSEY PROBLEM WITH QUASILINEAR PREFERENCES

With quasilinear preferences, the problem of the planner is to maximize:

$$\begin{aligned}\mathcal{L}_t=\mathbf{E}_0\sum_{t=0}^{\infty}\Bigg\{&\Bigg[\eta\beta^t\left(c_t+\psi\frac{h_t^{1-\varepsilon}}{1-\varepsilon}-\chi\frac{l_t^{1+\theta}}{1+\theta}\right)+(1-\eta)\gamma^t\left(e_t+\varphi\frac{(1-h_t)^{1-\varepsilon}}{1-\varepsilon}\right)\Bigg]\\&+\Psi_t\eta\beta^t\left(c_t+\psi h_t^{1-\varepsilon}-\chi l_t^{1+\theta}\right)+\Lambda_t(1-\eta)\gamma^t\left(e_t+\varphi(1-h_t)^{1-\varepsilon}\right)\\&-\eta\beta^t\frac{1}{\beta}\Gamma_t\big[b_t+k_t+q_{t-1}h_t\big]+\eta\omega^tU_{1,t}(\overline{M}-b_{t+1})+\eta\omega^tU_{2,t}(b_{t+1}-\underline{M})\\&+\omega^t\lambda_t\big[z_tf(k_t,l_t)+(1-\delta)k_t-c_t-e_t-k_{t+1}-g_t\big]\Bigg\}.\end{aligned}$$

The first-order conditions with respect to c_t, l_t, b_{t+1}, k_{t+1} are given, respectively, by

$$\eta\beta^t[1+\Psi_t]=\omega^t\lambda_t, \quad \textbf{(D.12)}$$

$$\eta\beta^t[1+(1+\theta)\Psi_t]\chi l_t^{\theta}=\omega^t\lambda_t z_t f_{l,t}, \quad \textbf{(D.13)}$$

$$\beta^tE_t(\Gamma_{t+1})=\omega^t(U_{2,t}-U_{1,t}), \quad \textbf{(D.14)}$$

$$\omega^t\lambda_t+\eta\beta^tE_t(\Gamma_{t+1})=\omega^{t+1}E_t\left[\lambda_{t+1}(1-\delta+z_{t+1}f_{k,t+1})\right], \quad \textbf{(D.15)}$$

where Λ_t is the equivalent to Ψ_t for the entrepreneurs.

Combining conditions (D.12) and (D.13) gives

$$\frac{[1+(1+\theta)\Psi_t]\chi l_t^{\theta}}{1+\Psi_t}=z_tf_{l,t}. \quad \textbf{(D.16)}$$

Using the specified functional forms, the combination of labor supply (4) and labor demand (12) gives

$$\chi l_t^{\theta}=\left(\frac{1-\tau_t^l}{1+\phi\mu_t}\right)z_tf_{l,t}. \quad \textbf{(D.17)}$$

Then, combining conditions (D.16) and (D.17) gives the optimal labor-income tax rate:

$$\tau_{\mathrm{SB},t}^l=\frac{\theta\Psi_t-(1+\Psi_t)\phi\mu_t}{1+(1+\theta)\Psi_t}, \quad \textbf{(D.18)}$$

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which is condition (45) in the text. In addition, based on condition (D.9), we have

$$E_t(\Gamma_{t+1}) = E_t(\Psi_{t+1}) - \Psi_t. \tag{D.19}$$

Then, combining (D.12), (D.15) and (D.19) gives

$$1 + E_t(\Psi_{t+1}) = \beta E_t\left[(1+\Psi_{t+1})(1-\delta+z_{t+1}f_{k,t+1})\right]. \tag{D.20}$$

The combination of capital supply (6), capital demand (13), and the choice of capital by the Ramsey planner (D.20) then gives condition (47) in the text.

Finally, if neither debt limit constraint binds ($U_{1,t} = U_{2,t} = 0$) then $E_t(\Gamma_{t+1}) = 0$. Condition (D.19) then yields

$$\Psi_t = E_t(\Psi_{t+1}), \tag{D.21}$$

which is condition (48) in the text.

Finally, with the general form of preferences, $u(c_t, h_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \psi\frac{h_t^{1-\varepsilon}}{1-\varepsilon} - \chi\frac{l_t^{1+\theta}}{1+\theta}$, we obtain

$$\tau^l_{\mathrm{SB},t} = \frac{(\sigma+\theta)\Psi_t - [1+(1-\sigma)\Psi_t]\phi\mu_t}{1+(1+\theta)\Psi_t}, \tag{D.22}$$

which collapses to (D.18) with $\sigma = 0$.

D.2. PROOF OF CONDITION (D.11)

The goal is to show the derivation of condition (D.11) from condition (D.10). Since the second term is the same in both conditions, I focus on the first term only. Therefore, we need to show that

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t(\Psi_t - \Psi_{t-1})\left\{\mathbf{E}_t\sum_{j=0}^{\infty}\beta^j\left[u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j} + u_{h,t+j}h_{t+j}\right]\right\}$$
$$= \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\Psi_t\left[u_{c,t}c_t + u_{l,t}l_t + u_{h,t}h_t\right]. \tag{D.23}$$

To simplify the notation, let $M_{t+j} = u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j} + u_{h,t+j}h_{t+j}$. Then, the left-hand side of (D.23) becomes

$$\mathbf{E}_0\sum_{t=0}^{\infty}\beta^t(\Psi_t - \Psi_{t-1})\left\{\mathbf{E}_t\sum_{j=0}^{\infty}\beta^j M_{t+j}\right\}$$
$$= \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\Psi_t\left\{\mathbf{E}_t\sum_{j=0}^{\infty}\beta^j M_{t+j}\right\} - \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\Psi_{t-1}\left\{\mathbf{E}_t\sum_{j=0}^{\infty}\beta^j M_{t+j}\right\}. \tag{D.24}$$

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The proof makes use of Abel's formula for summation by parts. Then, using the law of iterated expectations and opening condition (D.24), we have

$$\begin{aligned}
&\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t (\Psi_t - \Psi_{t-1}) \Big\{ \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j M_{t+j} \Big\} = \\
&\mathbf{E}_0(\beta^0 \Psi_0) \sum_{j=0}^{\infty} \beta^j M_j - E_0(\beta^0 \Psi_{-1}) \sum_{j=0}^{\infty} \beta^j M_j + \\
&\mathbf{E}_0(\beta^1 \Psi_1) \sum_{j=0}^{\infty} \beta^j M_{j+1} - E_0(\beta^1 \Psi_0) \sum_{j=0}^{\infty} \beta^j M_{j+1} + \\
&\mathbf{E}_0(\beta^2 \Psi_2) \sum_{j=0}^{\infty} \beta^j M_{j+2} - E_0(\beta^2 \Psi_1) \sum_{j=0}^{\infty} \beta^j M_{j+2} + \ldots \\
&\mathbf{E}_0(\beta^s \Psi_s) \sum_{j=0}^{\infty} \beta^j M_{j+s} - E_0(\beta^s \Psi_{s-1}) \sum_{j=0}^{\infty} \beta^j M_{j+s} + \ldots,
\end{aligned} \tag{D.25}$$

where the second line applies to period $t = 0$, the third line applies to period $t = 1$, etc. Letting $\Psi_{-1} = 0$ [as in Aiyagari et al. (2002)] and cancelling common terms, (D.25) simplifies to

$$\begin{aligned}
&\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t (\Psi_t - \Psi_{t-1}) \Big\{ \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j M_{t+j} \Big\} = \beta^0 \mathbf{E}_0(\Psi_0 M_0) + \beta^1 \mathbf{E}_0(\Psi_1 M_1) \\
&+ \beta^2 \mathbf{E}_0(\Psi_2 M_2) + \ldots + \beta^s \mathbf{E}_0(\Psi_s M_s) + \ldots = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t M_t.
\end{aligned} \tag{D.26}$$

Then, since $M_{t+j} = u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j} + u_{h,t+j} h_{t+j}$, condition (D.23) follows. Also, since $\Psi_{-1} = 0$, we have $\Gamma_0 = \Psi_0$. Therefore, Ψ_0 becomes the equivalent to ξ of the complete-markets model. *Q.E.D.*

APPENDIX E: CALIBRATION, FIGURES, AND TABLES

E.1. CALIBRATION

The time unit is a quarter and hence the discount factor β is set to 0.99, implying an annual interest rate of roughly 4%. The parameter θ is set to zero, implying a linear disutility function of labor. The implied labor supply elasticity helps in capturing the volatility of total hours in a model with no extensive margin, as is the case in this paper. The consumption curvature parameters are set to $\sigma = 1$ and $\nu = 0$ to numerically evaluate the analytical results. The parameter χ is set so that the steady-state value of l is 0.21 (which corresponds to a workweek of 35 h, the average number of weekly hours worked over the period 1964–

2015, for which data are available at the FRED database of the Federal Bank of St. Louis). The parameter ψ is set so that the steady-sate share of household's real estate in total real estate h is 45%. This value corresponds to the share of gross private residential fixed assets in total gross private fixed assets from the Fixed Assets Tables (FAT) of the Bureau of Economic Analysis (BEA) over the period 1960–2015.

The firm produces using the following production technology:

$$f(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha}, \tag{E.1}$$

with $\alpha = 0.34$ being the elasticity of output with respect to capital, in line with the literature. Following Cooley and Prescott (1995), the deprecation rate of capital is set in order to match the investment-capital ratio in the National Income and Product Accounts (NIPA). For the period 1960–2015, the implied annual capital depreciation rate 11%; therefore, $\delta = 0.026$.

Government expenditures evolves according to the following AR(1) process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + u_{g,t}, \tag{E.2}$$

with $u_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$. The coefficient ρ_g is set to 0.90 and the standard deviation σ_g is set to 0.079. The deterministic steady-state value of government spending g is set so that the deterministic steady-state value of government spending is 20% of deterministic steady-state GDP, which corresponds to the average government spending-GDP ratio over the period 1960–2015. The steady-state value of b is obtained so that b/y is 39.2%, which is the average of the gross federal debt held by the public as percentage of GDP for 1960–2015. These figures are based on Table 18 of the 2016 Economic Report of the President.

Similarly, TFP is governed by the following AR(1) process:

$$\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + u_{z,t}, \tag{E.3}$$

with $u_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$. The coefficient ρ_z is set to 0.95 and the standard deviation σ_z is set to 0.006. The deterministic steady state value of z is normalized to 1.

Finally, the loan-to-value ratio κ is set to 0.8, which is widely acceptable in the literature, but the main results of the paper hold with lower and higher values of this parameter.

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E.2. FIGURES

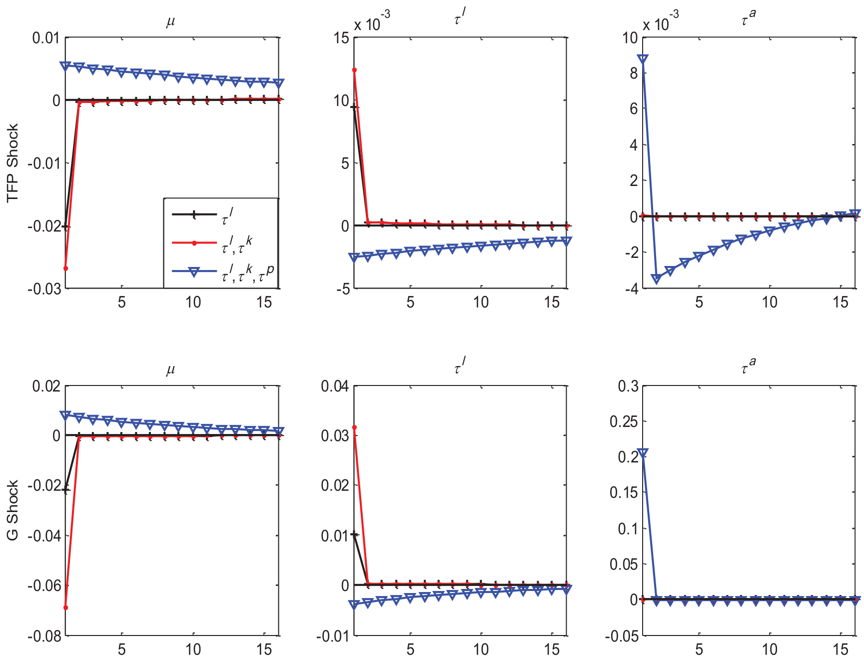

FIGURE E.1. Impulse responses to a TFP shock (top panel) and a government expenditures shock (bottom panel). $g = 0$, $\sigma_g = 0.013$, $\sigma_z = 0.006$.

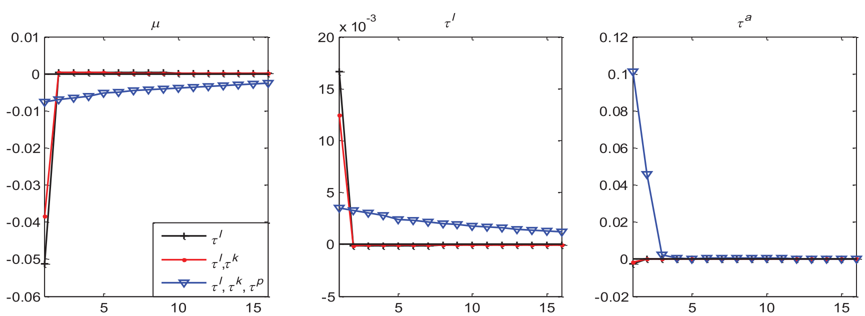

FIGURE E.2. Impulse responses to a government expenditures shock in the model with credit frictions. Only the wage payments are subject to the credit constraint.

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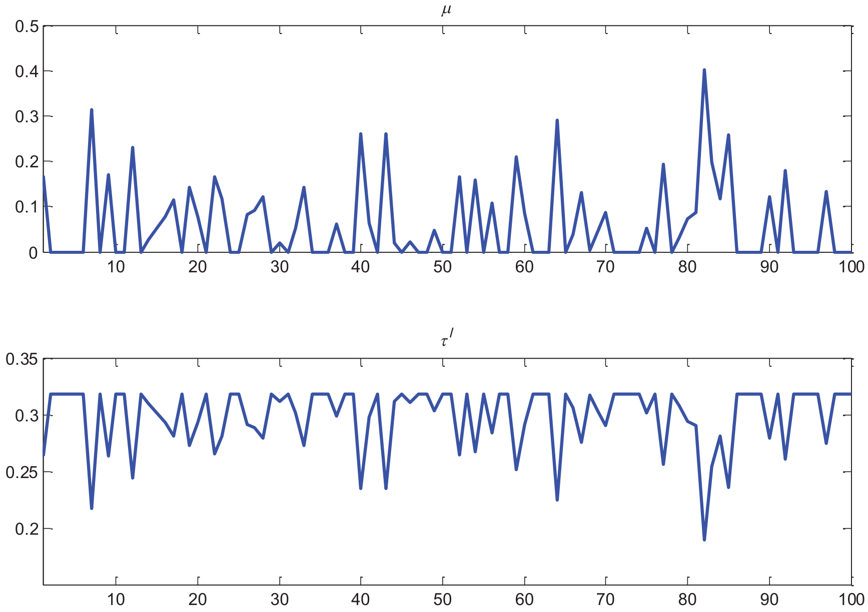

FIGURE E.3. The labor-income tax rate and the tightness of the credit constraint. A shock to government expenditures and an occasionally-binding credit constraint.

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E.3. TABLES

TABLE E.1. Optimal fiscal policy: First and second moments

	Mean	Standard deviation	Auto-correlation
No credit frictions: Labor taxes only			
τ^l	0.0062	0.0000	1.0072
y	0.6774	0.0100	0.7405
Wedge	0.0062	0.0000	1.0072
Credit frictions: Labor taxes only			
τ^l	0.0062	0.0088	−0.0589
y	0.6746	0.0102	0.7125
Wedge	0.0113	0.0000	1.0119
Credit frictions: Labor and capital taxes			
τ^l	0.0173	0.0089	−0.0587
$\tau^{e,k}$	−0.0216	0.0019	0.7162
τ^k	−0.0214	0.0102	−0.0641
y	0.6759	0.0099	0.7128
Wedge	0.0231	0.0000	1.0095
Credit frictions: Labor, capital, and property taxes			
τ^l	0.0019	0.0022	0.0333
$\tau^{e,k}$	−0.0329	0.0012	0.7823
τ^k	−0.0329	0.0076	−0.0458
τ^a	0.0211	0.0060	0.0041
y	0.6780	0.0102	0.7250
Wedge	0.0107	0.0000	1.0195

Note: TFP shock. $g = 0$, $\sigma_z = 0.006$. The values of other parameters are presented in Table 1.

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TABLE E.2. Optimal fiscal policy: First and second moments

	Mean	Standard deviation	Auto-correlation
No credit frictions: Labor taxes only			
τ^l	0.0062	0.0000	1.0112
y	0.6786	0.0100	0.7236
Wedge	0.0062	0.0000	1.0112
Credit frictions: Labor taxes only			
τ^l	0.0060	0.0173	−0.0520
y	0.6759	0.0107	0.6776
Wedge	0.0113	0.0000	1.0159
Credit frictions: Labor and capital taxes			
τ^l	0.0172	0.0179	−0.0524
$\tau^{e,k}$	−0.0215	0.0021	0.7221
τ^k	−0.0219	0.0680	−0.0538
y	0.6772	0.0106	0.6775
Wedge	0.0231	0.0000	0.9657
Credit frictions: Labor, capital, and property taxes			
τ^l	0.0018	0.0051	0.0242
$\tau^{e,k}$	−0.0328	0.0025	0.7019
τ^k	−0.0329	0.0187	0.0002
τ^a	0.0221	0.2013	−0.0512
y	0.6776	0.0104	0.6999
Wedge	0.0107	0.0000	1.0132

Note: A shock to government expenditures. $g = 0$, $\sigma_g = 0.013$. The values of other parameters are presented in Table 1.

TABLE E.3. Optimal fiscal policy: First and second moments

	Mean	Standard deviation	Auto-correlation
No credit frictions: Labor taxes only			
τ^l	0.3181	0.0000	1.0142
y	0.6754	0.0100	0.7443
Wedge	0.3181	0.0000	1.0142
Credit frictions: Labor taxes only			
τ^l	0.3071	0.0181	−0.0711
y	0.6692	0.0097	0.6644
Wedge	0.3101	0.0000	0.9975
Credit frictions: Labor and capital taxes			
τ^l	0.3071	0.0161	−0.0711
$\tau^{e,k}$	0.0000	0.0000	0.7100
τ^k	−0.0006	1.3317	−0.0697
y	0.6692	0.0086	0.6644
Wedge	0.3101	0.0000	0.7100
Credit frictions: Labor, capital, and property taxes			
τ^l	−0.0084	0.0038	0.3468
$\tau^{e,k}$	0.0000	0.0000	0.7100
τ^k	−0.0008	1.6789	−0.0697
τ^a	0.1809	0.1476	−0.0670
y	0.7456	0.0089	0.6280
Wedge	0.0000	0.0000	0.7100

Note: A shock to government expenditures. Only wage payments are subject to the collateral constraint. $g > 0$, $\sigma_g = 0.079$. The values of other parameters are presented in Table 1.

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TABLE E.4. Optimal fiscal policy with non-state-contingent bonds: First and second moments

	Mean	Standard deviation	Mean	Standard deviation	CE
			No credit frictions		
	Labor taxes only		Labor, capital, and property taxes		
τ^l	0.2934	0.0328	0.2715	0.0020	
$\tau^{e,k}$	–	–	0.0001	0.2417	
τ^k	–	–	−0.0004	1.6789	
τ^a	–	–	0.0137	0.0399	
y	0.6659	0.0236	0.6742	0.0081	
Wedge	0.2934	0.0328	0.2715	0.0020	
					0.0080
			With credit frictions		
	Labor taxes only		Labor, capital, and property taxes		
τ^l	0.2962	0.0608	0.2619	0.0109	
$\tau^{e,k}$	–	–	−0.4193	0.4122	
τ^k	–	–	−0.4174	3.9119	
τ^a	–	–	0.0988	0.0542	
y	0.6635	0.0746	0.7461	0.0141	
Wedge	0.3004	0.0608	0.2689	0.0000	
					0.0205

Note: A shock to government expenditures. The values of the parameters are presented in Table 1. CE is consumption equivalence: the rise in the consumption of the households (c_t) that is needed to make welfare with labor and capital taxes equals welfare with all taxes.

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