

### III. BINARITY, PULSATION, ROTATION AND MIXING

## BINARY EVOLUTION AND OBSERVATIONAL CONSTRAINTS

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### ABSTRACT.

The evolution of close binaries is discussed in connection with problems concerning mass and angular momentum losses. Theoretical and observational evidence for outflow of matter, leaving the system during evolution is given: statistics on total masses and mass ratios, effects of the accretion of the mass gaining component, the presence of streams, disks, rings, circumstellar envelopes, period changes, abundance changes in the atmosphere.

The effects of outflowing matter on the evolution is outlined, and estimates of the fraction of matter expelled by the loser, and leaving the system, are given. The various time scales involved with evolution and observation are compared. Examples of non conservative evolution are discussed.

Problems related to contact phases, on mass and energy losses, in connection with entropy changes are briefly analysed. For advanced stages the disruption probabilities for supernova explosions are examined. A global picture is given for the evolution of massive close binaries, from ZAMS, through WR phases, X-ray phases, leading to runaway pulsars or to a binary pulsar and later to a millisecond pulsar.

### I. INTRODUCTION.

The aim of evolutionary computations for close binaries can be summarized as follows:

1. to discover and to describe the physical laws governing the change of:
  - a. the various parameters which determine the internal structure of both components ( $M, L, T, P, X, Y, \dots$ )
  - b. the parameters of the system: masses of the components, distance of the components, orbital period.
2. to determine for a given set of binaries with common characteristics the set of progenitors, hence to compute how sets of binaries (Algols, Wolf-Rayet binaries, X-ray binaries, cataclysmic binaries, bursters, Be-binaries and Be-X-ray binaries) can be formed, starting from sets of ZAMS-binaries.
3. to determine for a given binary system with known actual status the progenitor ZAMS system, to calculate the exact evolutionary sequence

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leading from the ZAMS system to the actual observed system.

4. to derive for a given binary system with known actual status the descendant.
5. as a combination of 3 and 4 complete sequences for close binaries from ZAMS stage to their final phases can be computed.

In the following sections we will analyse how progress in the theory of close binaries and observations has modified the picture of binary evolution. Mass loss changes the structure of the components, and in an advanced mass transfer stage also the atmospheric abundance; this is the case for the mass losing star (the loser), as well as for the mass accreting star (the gainer). The consequences of the mass exchange on the two components, loser and gainer, are investigated in section 2.

Various treatments of the Roche lobe overflow phase, conservative or not, dealing eventually with contact phases, and a discussion of the parameters governing mass and angular momentum losses are described in section 3. The different time scales involved with binary evolution are analysed in section 4, and their relevance on the evolution is discussed.

In section 5 is given a discussion of the connection between groups of binaries with given characteristics and their ancestors and descendants. Section 6 deals with different mass outflow possibilities. In section 7 is described how theory and observations have influenced the general ideas of binary evolution. Observed features such as gas streams, disks, period changes are analysed and discussed critically. Contact systems, models for the computations of such systems, and application for WUMa systems are described in section 8. In the last sections the advanced evolution of massive close binaries, leading to supernova explosions, is described; the effect of the explosion on the system, disruption or not, is discussed, and the origin of binary pulsars and runaway pulsars is explained. The millisecond pulsar can be considered as the final evolution of a binary pulsar by coalescence.

## II. CONSEQUENCES OF THE MASS EXCHANGE ON THE BINARY COMPONENTS.

From stellar evolution is known that exhaustion of a nuclear source, or the beginning of exhaustion leads to the expansion of the star. If this star is member of a close binary with a rotational period equal to the orbital period, hence showing synchronous rotation, its radius can eventually equal its Roche-radius and mass transfer can occur, at a rate  $\dot{M}$  (Kuiper, 1941).

Usually for stellar evolution it is assumed that the Roche-lobe constitutes the limiting surface for the contact component of a semi-detached system. Evolutionary computations under the assumption of conservation of mass, and angular momentum during the mass transfer stage could explain the gross features of groups of observed binaries (a.o. the Algol paradox). When the conservative assumption is no longer maintained the duration of the first stages of mass transfer can be drastically increased, and the final systems after the mass exchange phase can be different from the conservative case. During the phase of rapid mass transfer the gainer can spin up, so that synchronism it lost. It would be very important to investigate mass accretion under non-synchro-

nous conditions.

If the gainer and/or a possible accretion disk does not expand too drastically, synchronism can be maintained, and the orbit remains circular. The conservative treatment has been justified on dynamical grounds by Lubow and Shu (1975, 1976), Prendergast and Taam (1974), Flannery, 1975 a,b).

In these circumstance (Shu and Lubow, 1981) the outflowing gas with low Mach number makes a sonic transition within a small region around  $L_1$ , falls then into the Roche lobe of the gainer and is trapped within that surface.

If the flow impinges on itself or hits the surface of the gainer, orbital energy is dissipated and the gas falls even deeper in the potential well of the gainer. No mass leaves the system. Even if a fraction of the angular momentum is stored in an accretion disk, small with respect to the masses of the two stars, for a certain time, the bulk of the angular momentum remains in the orbital motion of the two stars.

The mass gaining star accretes matter on the Kelvin-Helmholtz time scale of the loser (Paczynski, 1971), which can be different of its own Kelvin-Helmholtz time scale. The effects of rapid mass transfer on the gainer have been examined a.o. by Benson (1970), Yungelson (1973), Flannery and Ulrich (1973) Kippenhahn and Meyer-Hofmeister (1977), Neo et al. (1971), who find that the gainer drastically swells in size, fills its own Roche lobe giving rise to contact systems, and the subsequent evolution could be different from earlier calculations.

Shu and Lubow (1981) argue however that they are not convinced by the results of Benson; they refer to the behaviour of protostars, where the accretion of matter does not lead to a drastic swelling of the receiving star. (Larson 1969, 1972, Appenzeller and Tscharnuter 1975; Winkler and Newman 1980 a,b, Stahler et al. 1980 a,b).

The reasoning is as follows: as boundary conditions used in binary evolution, an "ad hoc" thermal boundary condition is imposed on a hydrostatic star of variable mass. As boundary conditions is generally used

$$\left(\frac{\partial S}{\partial M}\right)_t = 0 \text{ which means that the specific entropy on the newly added shell is equal to that of the shell immediately below.}$$

or  $\left(\frac{\partial L}{\partial M}\right)_t = 0$  meaning that the newly added shell is in radiative equilibrium.

The two forms, which are physically equivalent for steady flow, are not completely correct, because the inflowing material has a very on short local thermal timescale and can behave in a non adiabatic way on falling down to the star, lowering the entropy as the gas radiates. Correct computations are very delicate and difficult (Stahler et al. 1980,b).

In fact the K-H-timescale is connected with the deeper, interior layers of the gainer, and is much larger than the K-H-timescale of the outer layers, where really the material is accreted. In the outer layers the material can, by radiation, lower the specific entropy of the newly accreted matter.

Hence, although the interior structure of the gainer deviates from ther-

mal equilibrium, only drastic changes of the radius occur when a very large amount of matter (comparable to the original mass) has been accreted.

In a recent study (Hellings, 1983a) the response to accretion of massive secondaries in close binaries was examined during their core H burning and shell H burning stages. In this work constant accretion rates comparable to the main loss rates of primaries during Roche lobe overflow were used. The accreted matter in these systems is assumed to fall gently on the surface of the secondary with no shock effects, and with the same chemical composition as the outer layers of the accreting star. In the binary program an instantaneous thermohaline mixing procedure (Ulrich, 1972) is used when accreted matter with a lower H abundance is added to the secondary. It is found that the accretion stars are unable to remain in equilibrium as has been found for low mass and for ZAMS stars but when accretion stops the secondaries restore their thermal equilibrium by a relaxation stage towards normal ZAMS structure. This new structure containing a longer convective core and increased core H constant is obtained by the process of semi-convection. Further evolution does not deviate from normal main sequence evolution of single stars.

The results of this work were used to derive the structure of the secondary after RLOF starting from the initial parameters  $M_{1i}$ ,  $q$  and  $\beta$  ( $M_{1i}$  is initial mass of the primary,  $q$  is the mass ratio, secondary to primary and  $\beta$  is the fraction of the mass lost by the primary and accreted by the secondary, see also section 3). This work is the basis for the mass determination of eight double lined WR + OB binaries (Hellings, 1983, b). With this method the masses of the WR components and OB components are found using the theoretical stage of the companion in the HR diagram, and the observational results  $M_{WR} \sin^3 i$  and  $q$ , the mass ratio. The resulting masses are calculated with an accuracy of about 20%, and are mostly in the range 7-10  $M_{\odot}$ , which is lower than the estimated masses by Massey (1982).

### III. GENERAL SURVEY OF CLOSE BINARY EVOLUTION.

Evolutionary computations for single stars show how the stellar radius changes, as a function of time. The presence of a companion will impose restrictions on the radius increase. Theoretically this is translated by the concept of Roche lobes, i.e. the determination of an allowed maximum volume for the primary with increasing radius imposed by a critical equipotential surface. This latter is calculated in a simplified way for two point masses for primary and secondary. It can also be done a tidal lobe concept, i.e. a maximum volume imposed by the tidal interaction of the secondary on the primary.

The easiest way to calculate binary evolution is to assume conservation of total mass and orbital angular momentum, and moreover, to assume synchronous rotation. Constraints for conservative evolution are that the mass ratio  $q = M_2/M_1$ , with  $M_1$  mass of the primary and  $M_2$  mass of the secondary, is not too small (larger than 0.3), that the separation  $A$  is not too small and that the envelope of the mass losing star is in radiative equilibrium. When these conditions are satisfied the distance  $A$  of the two components, given by

$$\frac{A}{A_0} = \left( \frac{M_1 M_2}{M_1 M_2} \right)^2$$

is not too much reduced by the mass transfer. The subscripts 0 refers to the initial (ZAMS) values.

If these conditions on mass ratio and distance are not satisfied the system will evolve into a contact system with a common envelope (Webbink 1979).

The effect of the mass loss on a convective envelope will be that this envelope expands, inducing a stronger mass loss rate leading to a catastrophic envelope growth. The envelope engulfs the companion, and also in this case a common envelope will be produced. The tendency of convective envelopes to expand further when mass loss occurs, is a consequence of the fact that for such envelopes the specific entropy of the gas decreases outwards, so that no energy supply for the restoration of the equilibrium is necessary, unlike for radiative envelopes.

The conditions for conservative evolution are not always satisfied, and computations of evolutionary sequences have been carried out with assumptions on mass and angular momentum losses (Yungelson, 1973; Plavec et al. 1973; Paczynski and Ziolkowski, 1967; Vanbeveren et al. 1979; Hellings et al. 1982). In the case of non conservative evolution the fraction  $\beta$  of the mass lost by the primary and accreted by the secondary has to be specified as well as a prescription on the fraction  $\gamma$  of the total angular momentum leaving the system.

This can be done a.o. in the following ways.

1. arbitrary, i.e. computations are carried out for a series of values of  $\beta$  ( $0 < \beta < 1$ , with  $\beta = 1$ ) the conservative case, and various values of  $\gamma$  for a set of ZAMS-binaries (Vanbeveren et al. 1979). Comparison of the post Roche-lobe overflow systems with observed evolved systems (in the sense of post-mass transfer systems) allows to impose conditions on  $\beta$  and  $\gamma$  (Vanbeveren and de Loore, 1980);
2. by assuming  $\beta = 1$  for semi-detached phases, and adopting lower values for contact phases: the fraction  $\beta$  of the mass expelled by the primary is accreted by the secondary, determined in such a way that this secondary keeps filling its Roche lobe, the rest leaving the system in a spherically symmetric way ( $\beta < 1$ ) (Plavec, 1981).

After a certain time the secondary becomes detached from the contact volume, the system becomes semi-detached and the mass transfer occurs again in a conservative way ( $\beta = 1$ ).

The mass transfer comes to an end when the primary shrinks back within its Roche lobe.

3. a different way to treat the contact phase has been investigated by Packet (1983) in the following sense: the mass transfer rate is determined such that both stars fill a common equipotential surface, outside their Roche lobes. Until now luminosity transfer has been omitted: in the future this will be investigated. Mass exchange is thus treated in a conservative way; only when the surface of the system corresponds with the equipotential surface through  $L_2$ , mass loss has to be considered. Test computations carried out by Packet for a binary system, with a primary of  $9 M_{\odot}$ , and secondaries with mass ratios of 0.9 and 0.6 show that the latter situation is not reached,

and that the total binary evolution occurs in a conservative way, at least for early B and late A-cases).

4. another possibility is that during the semidetached phase the excess material, or a fraction, leaves the system in a spherical symmetrical way, in the case of massive stars as a kind of enhanced stellar wind, driven by radiation pressure and increased by gravitational acceleration or by the effect of the changed potential by the presence of the secondary alone.

5. a detailed analysis of the parameters determining the fractional mass loss ( $\beta$ ) and the amount of the angular momentum carried along with the escaping gas  $g$  was performed by Gianuzzi (1981), assuming  $\beta$  and  $g$  functions of the mass ratio  $q$  alone.

She examined two cases:  $\beta = \text{constant}$  and  $\beta = \frac{q}{1+q}$ . The change of the angular momentum depends critically on the period of mass ejection. Idealized cases, Jeans' mode and matter leaving the system through the second Lagrangian point  $L_2$  were investigated, leading to the following expression for the orbital period.

$$\frac{P}{P_0} = - \left[ \left( \frac{J}{J_0} \right)^3 / \left( \frac{M_t}{M_{t0}} \right)^6 \right] \cdot \left( \frac{q+1}{q_0+1} \right)^6 \left( \frac{q_0}{q} \right)^3.$$

#### IV. TIME SCALES.

In order to check the validity of evolutionary schemes one can compare various parameters predicted by computations with those of individually observed systems. Another possibility is to compute sets of similar systems, and to compare their averaged parameters with theoretical values. These statistical methods have the advantage that the mean values such as masses, periods and mass ratios, as well as their scatter during or after Roche-lobe overflow can be compared with those of a comparable group of non evolved systems (Pre-RLOF).

We have at our disposal a restricted number of binaries with known parameters in their specific evolutionary stage, evolving, but on timescales that are not the evolutionary timescales, for certain phases without interaction between the two components, and phases with interactions. Successive evolutionary models have time steps, completely different from the observational time scales. We consider e.g. a primary of  $\sim 20 M_\odot$ . The evolutionary timescales for the various phases, i.e. the time which enters the evolutionary code to calculate two successive models are different from the observational timescales.

The core hydrogen burning lifetime is  $\sim 7$  million years, the subsequent stages are  $\sim 10\%$  of this time. The mass transfer occurs on the Kelvin-Helmholtz time scale, which is in this case, of the order of  $10^4$  years. While the time step for successive evolutionary models during core hydrogen is of the order of  $10^4$  years, the time step for successive models during the mass transfer phase drops to the order of 10 years.

Let us now turn to the observations. Two calculated successive models, model  $N$  and model  $(N+1)$  are about 5 years apart, which means that these two models depict two phases in the life of the binary with a time interval of 5 years, corresponding to two observations, also 5 years apart. What is happening between these two phases can be observed

but not calculated and this makes the comparison of theory and observations very difficult. This becomes even clearer when we compare the time lapse between two successive models with the orbital period. In our example about 200 orbits fill the gap between two computed successive models!

Evolutionary sequences deal with nuclear timescales, given by  $\tau_n$  (the timescale related to relevant structure changes), or with thermal timescales  $\tau_k$  (Kelvin-Helmholtz timescale) given by  $3.10^7 \frac{M^2}{R}$  years. Hence evolutionary models give information on transitions from a long-living situation to another one; such transitions can be described by a hydrostatic code. Evolution on a thermal timescale gives information on changes of the surface structure, changes of the dimensions of systems.

From observations can be derived information on shortlived phenomena, e.g. changes in the flow pattern, information on the impact of the flow of matter from the primary to a disk, and the generation of a hot spot.

An observing run of two weeks for a binary with a period of 10 days reveals what is happening in one period while in an evolutionary sequence for such a system, two successive computed models cover more than 100 periods.

Hence parameters derived from observations can only be used for evolutionary purposes very carefully and with suspicion. The presence of disks can be used as evidence for mass outflow but it does not mean necessarily that the disk will not disappear afterwards, carrying its mass and angular momentum in- or outwards, so that for the overall picture of the evolution with mass exchange and the final status of the system, the storage of transferred matter into a disk is perhaps not so important.

#### V. IMPLICATION OF OBSERVATIONS OF GROUPS ON THE COMPUTATIONS OF BINARY EVOLUTION.

Observations can give checks on the validity of evolutionary sequences, can furnish information on progenitors of binary systems, or can help to discriminate between various possible scenarios: mass ratio determinations, mass determinations from spectral variations, leading to radial velocities, spectral type determination, radius-determinations from photometric observations, spectral analysis leading to atmospheric abundances, different from ZAMS abundances.

For a review on various stellar groups and their characteristics I refer to de Loore (1983) and references therein, Popper 1980 a,b; Plavec and Koch (1978); Stencel et al. (1980) Robinson (1976), Warner (1976) Plavec (1981) Paczynski (1980), Conti (1982).

For a number of groups the characteristics of the progenitors i.e. ZAMS systems are known: mass range, mass ratios, periods. As may be seen in Table 1 this is the case for Algols, the group of the W Serpenti's Stars, Wolf-Rayet binaries. Also for certain groups links with other, more advanced groups are known, i.e. the later evolutionary scheme. This is the case for Wolf-Rayet stars evolving into massive X-ray binaries. For other groups the situation is less clear: for the progenitors of WUMA stars there remain doubts, as well as for cataclysmic variables probably evolving into X-ray bursters, and for symbiotic stars.



Table 1.

Groups of close binaries with common characteristics and connected groups of ancestors and/or descendants.

Progenitors	Group	Descendants.
Intermediate mass ZAMS binaries-short periods	Algols-periods < 1d.	White dwarfs system.
Intermediate mass ZAMS binaries-long periods	W Serpentis-periods from 13 to 600 days	White dwarf systems.
Massive ZAMS systems	-Wolf-Rayet Stars -Be-binaries -Cataclysmic variables periods < 1d. One of the components is a white dwarf-disks are present.	Massive X-ray binaries Be-X-ray binaries Low mass X-ray binaries.
Fission of single stars in rapid rotation or detached or semi-detached binaries by angular momentum loss=magnetic braking.	WUMA Stars.	

## VI. MASS MOTIONS IN BINARIES.

During the lifetime of binaries various phases are present where the stellar material is moving. The following phases can be discerned:

- 1) spherical outflow, due to stellar wind in massive stars, during their hydrogen burning stage. This is not specific for binaries but occurs also for single stars. For massive close binaries this spherical outflow of material can occur during their detached, semi-detached and contact phases.
- 2) mass flow through L<sub>1</sub>: this occurs during Roche lobe overflow phases, as a flow from one of the stars towards its companion.
- 3) mass flow through L<sub>2</sub> and outflow according to Jeans' mode: these modes can be considered in the case of non conservative evolution during semi-detached phases or contact phases.
- 4) mass ejection. This type of heavy mass flow occurs at advanced evolutionary phases, when supernova explosions eject the outer layers of the evolved massive star, leaving compact objects.

Each of these types of mass flow has its repercussion on the status of the system: changes of the structure, especially on the atmospheric layers, and changes of the orbital elements. In the case of supernova explosions the influence on the system is very large: the periods of the systems can be changed drastically, and the system can even be disrupted.

## VII. INFLUENCE OF OBSERVATIONS ON THE THEORY OF CLOSE BINARY EVOLUTION.

Theoretical as well as observational arguments have modified the schemes adopted for the computation of evolutionary sequences for close binaries. As mentioned before (see section 2) the effect of rapid mass transfer could be that the accreting star swells up, leading to overcontact and ensuing mass loss. Moreover contact systems exist: e.g. Leung & Schneider (1979) have calculated absolute dimensions of the O type binaries UW Cma, AO Cas, V729 Cyg by a combination of photometric and spectroscopic data and found for each of these systems, that they have to be in contact. Determination of the evolutionary status reveals that UW Cma and AO Cas are still burning hydrogen, but evolved from the ZAMS, hence examples of case A mass exchange. A statistical study by Gianuzzi (1981) reveals a dependence of total mass and angular momentum on the mass ratio for semi-detached systems, pointing to a decrease of the total mass and the angular momentum as evolution proceeds.

From observations (photometry and spectroscopy) we can, apart from radial velocities, light variations, periods, eclipse durations leading to mass ratios, masses, radii, also try to get information on mass motions (streams, expanding envelopes) and mass concentrations (disks, rings, clouds, circumstellar envelopes). Some examples will be given and discussed.

### 1. Gas streams in U Cep.

According to Plavec (1981) the interacting binary UCep consists of a B8V primary and a G8III secondary. Most recent mass determinations give  $4.4 M_{\odot}$  and  $2.9 M_{\odot}$  with radii of  $2.55 R_{\odot}$  and  $4.95 R_{\odot}$  respectively. The separation of the components is  $15 R_{\odot}$ . The G star is evolved and is losing mass; some returns to the G-star. The gas stream leaves the G star from the hemisphere directed towards the B star, circles around the B star; some of the gas probably falls onto this star. According to this model the rest leaves the system after orbiting  $3/4$  of the B star. Some of the matter can fall back onto the G star. The gas in the stream is hot enough to keep most of the Fe, Si and Mg atoms ionized. (Kondo et al. 1980). A model is shown in Figure 1. According to Plavec (1983) the line emitting region surrounds the hotter component and extends to several stellar radii. He proposed a hot turbulent circumstellar region.

### 2. Circumstellar cloud or disk in HR 2142.

Observations of HR 2142 (Polidan and Peters, 1980) in the UV show that most of the material that leaves the primary is not accreted immediately by the secondary. Evidence of mass ejection can be seen in CII 1335 where rather strong, sharp absorption components are present, not from interstellar origin. As most probable origin for these components a circumstellar cloud or disk is suggested. Also in HR 7085 and  $\lambda$  Tauri gas streams were detected.

### 3. Disks in W Serpentis stars.

The spectra of "Serpentides" show as common characteristics that:

- a) in the UV we see strong emission lines superposed on
- b) a hot continuum

The optical companions are cooler:

B8II for  $\beta$  Lyr, A6III for SX Cas, F511 for W Ser.

The UV is produced in a region smaller than the observed stellar surface. In these interacting binaries emission lines of C II, C IV, N V, Si II,

III,IV, Fe III, Al II,III are present, probably related to the mass flow and accretion; the ionization is most likely connected with a hot spot or a hot radiative region in the interior of the thick disk. The matter surrounding the hot component is not completely eclipsed when the star itself disappears behind the companion, and certain components of the shell lines remain visible against the background of continuous hydrogen radiation. A disk model for SX Cas is shown in Figure 2, and spectra in Figure 3.  $\beta$  Lyrae has a geometrically thick, opaque disk (Huang 1963, Wilson, 1974)  $\beta$  Lyrae is in the phase of rapid mass transfer (derived from the large rate of period change).

The interval of observations is very long  $\sim 65$  years. In the center of the disk sits a main sequence star of 10 to 15  $M_{\odot}$ .

The disk is stable, otherwise the system would have brightened during its history in the Lyrae constellation. Wilson (1979, 1981) has suggested that centrifugally limited rotation of the central star is responsible for the persistence of the disk. According to Packet (1981) a small amount of transferred matter is already sufficient to spin up an accreting star to its critical centrifugal limit.

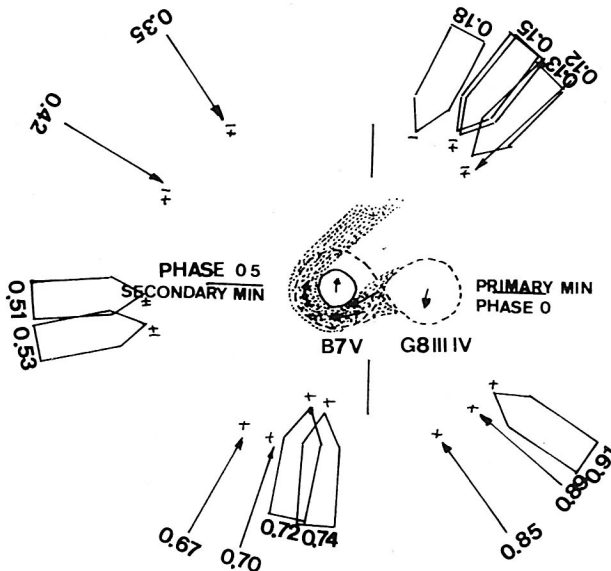


Figure 1 : Schematic representation of the gas stream as observed in the Fe II and Mg II resonance lines. The phases of observation are given. The + sign indicates that the radial velocity of the gas stream projected against the B-star is away from the observer, the - sign that the radial velocity is toward the observer; the + sign both away and toward the observer, and the  $\pm$  sign mostly toward, but with a minor component away from the observer (From Kondo et al. 1980).

$\beta$  Lyrae is, according to Wilson (1979), a contact binary.

Investigations on streams are based on particle dynamics calculations (Plavec et al. 1964; Plavec and Kriz, 1965). Prendergast 1960, Sobouti (1970), Biermann (1971), have tried to examine the hydrodynamical behaviour of circumstellar gases in binaries but they have studied velocity fields rather than trajectories. Circumstellar matter has been found around one of the components of binaries, as well as between the components, by observations of H  $\alpha$  emission in Algol (Struve and Sahade, 1957) and the line of He I  $\lambda$  4471.

From observations an estimate of the dimensions of disks can be derived. Spectra of VV Cep show emission lines of hydrogen, interpreted as generated in a structure around an invisible hotter star, a late O or B star.

We have no observational data on the lifetime of disks. In  $\beta$  Lyrae the disk was probably present for the last 65 years, but this is still a small time lapse compared to the evolutionary timescale (section 4). If the disk is resolved after a certain time it has no influence on the final outcome of the system but has just acted as a storage space for matter and angular momentum not immediately transferred. If the mass of disks is small relative to the masses of the components the mass transfer can probably still be treated as conservative. The interpretation of period changes can also lead to arguments for mass loss from the system. The period of SX Cas is variable, and decreases at a rate of  $\dot{P}/P = 7.6 \times 10^{-8} \text{ yr}^{-1}$  (Guinan and Tomczyk, 1979). This period change is comparable to those of  $\beta$  Lyrae or W Ser, but the sign is opposite. Simple mass transfer from the less massive component to the other one INCREASES the period. In SX Cas the loser is certainly the less massive one. Probably strong mass loss from the system in the form of a strong stellar wind occurs. Important information on the evolutionary status and another check on the quality of calculated evolutionary models could be found in the abundance determination, e.g. the H/He ratio in the components of binaries. A non LTE analysis of the O4 star  $\zeta$  Pup, carried out by Kudritzki et al. (1983), revealed a He-overabundance of  $\text{NHe}/(\text{NH} + \text{NHe}) = 0.14 + 0.03$ . From a similar analysis for an O-type subdwarf eclipsing binary system LB 3459 a helium abundance of 0.3 % by number was found, far below the primordial He-abundance, but caused by diffusion, not by evolution (Kudritzki et al. 1982). For WR stars abnormal abundances for the He-stars, have been found (low or zero H, abnormal C or N); for the O companions, the chemical composition is not known; and neither for the O or Of components in X-ray binaries. Such an analysis would reveal information on the accretion process, and the evolutionary status. For WR binaries it is very difficult to obtain high resolution spectra of the O components, for the X-ray binaries HD 153919 and HD 77581 however this is perfectly possible.

#### VIII. CONTACT SYSTEMS.

Most theories for contact systems are developed for WUMa systems. The reason is that many WUMa systems have been observed, while for early type contact binaries only a few observations exist.

Methods which can be applied:

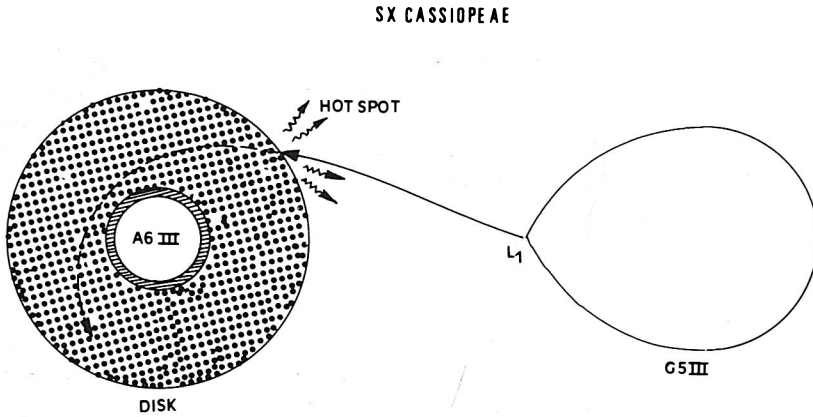


Figure 2 : Model of a close binary during the mass transfer phase, with an accretion disk and a hot spot where the flow of matter, leaving the mass losing star, impinges on the disk. Model for SX Cas by Plavec, Weiland and Koch (1981). The spectral type of the gainer is B7III, that of the loser K3III.

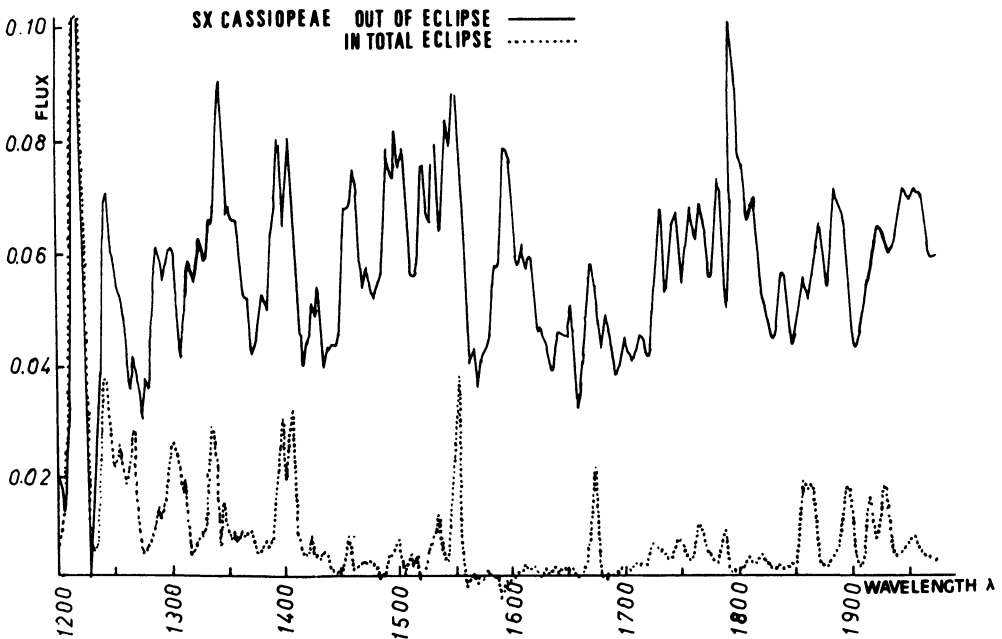


Figure 3 : IUE spectra of SX Cas in- and outside eclipse. During eclipse the continuum disappears, but the UV lines remain. (Plavec, Weiland, Koch, 1981).

- 1) one boundary condition on the equipotential surfaces  $\Omega$  for primary and secondary,  $\Omega_p = \Omega_s$ .

We can then assume  $\dot{M} = 0$  or determine  $\dot{M}$  in a theoretical way. The change in the luminosity  $\Delta L$  can then be expressed as  $\Delta L = K \cdot \Delta S \cdot d^m$  with  $\Delta S$  the entropy change and  $d$  the depth of the contact discontinuity (Hazlehurst 1970, Meyer and Meyer-Hofmeister, 1980). This is only valid for late type stars with convective outer layers.

- 2) 2 boundary conditions :

$\Omega_p = \Omega_s = \Omega_1$  (both stars fill their critical equipotential surfaces). (Biermann and Thomas, 1972).

From these two conditions constraints on the mass loss rate  $\dot{M}$  and the energy loss rate  $E$  can be derived.

Contact binaries are interesting in two respects: evolving binaries can in the course of their evolution both fill their Roche lobe (depending on their initial mass ratio and initial period), and for the explanation of WUMa stars. These stars are probably binaries on the main sequence, with photospheres of the same effective temperature, and a mass ratio of  $\sim 0.5$  (Eggen, 1961; Binnendijk, 1970). These controversial facts can be explained by assuming that the two stars have an optically thick common envelope where the different luminosities of the two stars are completely redistributed (Osaki, 1965, Lucy, 1968). Photometric and spectroscopic observations (Rucinsky, 1974, 1978; Anderson et al. 1980) sustain this model. Real difficulties arise in modelling the stellar interiors. The equations for stellar structure and the Roche model are only compatible if the two stars are identical. Solutions for this problem were presented by Lucy (1976), by a thermal relaxation model, and Shu et al (1976) by a contact discontinuity model. A discussion of these theories is given by Shu (1980).

Observations of binaries with periods below 0.5 days reveal that the contact systems are in large majority in disagreement with the fact that the time spent by these binaries during detached and contact phases should be the same according to the theory of Lucy. Lucy assumed that the specific entropies in the separate convection zones below the inner critical surface (through  $L_1$ ) are equal to the entropy in the common envelope, condition that only can be satisfied if hydrogen burning in one of the stars occurs via the PP-chain, and in the other via the CNO-cycle.

In the contact discontinuity model, a nearly discontinuous behaviour across the inner critical surface is allowed. In the outer layers above the critical surface the closed equipotential surfaces are common to both stars, and horizontal interchange of matter and heat can occur. Below the critical surface, the two stars have to be decoupled. Pressure has to be continuous across the inner critical surface; density and temperature and their gradients are allowed to be discontinuous. From mechanical considerations can then be derived that a contact discontinuity allows only a discontinuity for one of the stars: for the other star the behaviour across its Roche lobe occurs continuously. This model is valid for radiative as well as for convective conditions.

Lucy's models and Shu's model remain controversial.

### IX. SUPERNOVA EXPLOSION-PROBABILITIES FOR DISRUPTION.

Each of the two components of a massive close binary undergoes a supernova explosion. The effects of these events on the status of the system are completely different for the first or the second SN explosion. In the case of the first explosion the less massive star explodes, and consequently the probability that the system is disrupted is extremely low.

Hence most systems remain bound so that many massive X-ray binaries exist. For the second supernova explosion at a later stage, the disruption probability is very large, so that two runaway neutron stars are produced, an old one, remnant of the first explosion and a young active one, just formed.

Computations of the disruption probabilities were carried out by De Cuyper (1982), de Loore et al. (1975). Assuming an extra kick  $100 \text{ kms}^{-1}$ , a shell expansion velocity of  $10000 \text{ kms}^{-1}$  and a post-supernova remnant of  $1.5 M_{\odot}$  these probabilities are of the order of 70-80% for ZAMS primaries between 40 and  $100 M_{\odot}$  for all mass ratios. For lower mass ZAMS primaries ( $20-40 M_{\odot}$ ) the systems always remain bound.

As well in the case of Roche lobe overflow with reverse mass transfer and mass loss as in the case of spiralling in the outer layers with the original composition leave the primary star. The mass losing O star is "on its way" to become a He-star, hence shows more and more WR characteristics. The object SS433 is possibly an example of a binary in this stage (van den Heuvel et al. 1980), as well as Cir X-1. According to Firmani and Bisiacchi (1980) and Shlovski (1981) the mass loss rate is  $\sim 10^{-4} M_{\odot} \text{ yr}^{-1}$ , with an expansion velocity of  $1000 \text{ kms}^{-1}$ , and the spectral type ranges between Of and WR (from the relative strengths of Balmer- and Pickering lines). This agrees with the conclusions of van den Heuvel et al. (1980) about the mass loss rate. Probably the X-ray system is surrounded by an envelope of matter expelled by the non compact companion, and not by remnants of the supernova shell as is discussed by Shlovski (1981). An argument is given by the lifetime of the object according to evolutionary computations ( $11 \cdot 10^6$  years) and the large distance ( $3.7 - 4.7 \text{ kps}$ ). Hence the picture of SS433 by van den Heuvel et al. (1980) seems very attractive: the system is in a second X-ray stage, mass is expelled by the primary, and a large part of this matter is stored in a disk.

Table 2 Binary pulsars.

Name	$P_{\text{orb}}$	$P_{\text{pulse}} \text{ (s)}$	Eccentricity
PSR 0656+64	$24^{\text{h}}41^{\text{m}}$	0.196	0.06
PSR 0820+02	1100d	0.865	0
PSR 1913+16	$7^{\text{h}}75^{\text{m}}$	0.059	0.617
PSR 1937+215		0.0015	

As more and more matter is expelled by the companion, more He-enriched layers will appear at the surface. Hence the next evolutionary stage should be a second Wolf-Rayet stage, i.e. a helium star with a neutron

star companion (de Loore et al., 1975)(Table 3). Since such systems are descendants from binaries through a supernova event, they are assumed to have a large runaway velocity, and are supposed to be found at large distance of their place of birth (Moffat and Seggewiss, 1979).

Further evolution with further spiralling in could then lead to ultra-short period binaries like Cyg X-3 (van den Heuvel and de Loore, 1973). Finally these systems undergo a new supernova explosion, and are in nearly all cases disrupted, leaving two neutron stars, an old one and a young pulsar.

In exceptional cases binary pulsars are formed.

A list of binary pulsars is given in Table 2.

#### X. GLOBAL PICTURE OF THE EVOLUTION OF MASSIVE CLOSE BINARIES.

Considering typical objects such as SS433, Cyg X-3, PSR 1913 + 16 and PSR 1937 + 214, and by comparing their characteristics with those of the standard X-ray sources, a general picture for the evolution of massive stars emerges, starting from a pair of OB ZAMS components through successive stages of quiet and explosive mass loss, either leading to a binary pulsar, later on coalescing into a very short pulsar, or into two runaway neutron stars (Henrichs and van den Heuvel, 1983). Two massive stars with a period of  $\sim 10$  days start their mass transfer phase during shell hydrogen burning of the primary. The subsequent history is sketched in table 3.

After the mass exchange stage a Wolf-Rayet binary is formed, later on evolving, after a SN explosion into an OB runaway, i.e. a massive star with a neutron star companion.

As the massive star is nearly filling its Roche lobe; the enhanced stellar wind will be sufficient to produce X-rays, hence a first X-ray stage occurs.

The secondary evolves through the shell hydrogen burning phase, expands, fills its Roche lobe. Matter has to leave the system since the neutron star is not able to accrete the material expelled by its companion. The optical star is on its way to become a Wolf-Rayet star. An accretion disk could be formed.

This could represent a second X-ray; SS433 and Cir X-1 could be in this phase, a phase of slow mass transfer.

At the end of this second mass loss stage, the He-remnant and the orbiting neutron star represent runaway Wolf-Rayet stars.

Due to spiral in of the neutron star in the atmosphere of the WR star, a third X-ray stage could be produced, a WR-star and a neutron star with a very short period, such as Cyg X-3.

The He-star continues its subsequent nuclear burning phases, and finally explodes. If the system remains bound a binary pulsar is produced, such as PSR 1913 + 16. The two neutron stars coalesce, hence a very short period pulsar is formed, with a period of  $\sim 1.5$  ms. If the system is disrupted two runaway stars are formed.

#### XI. CONCLUSIONS.

Close binary evolution does not necessarily occur in a conservative way. From statistical research on the total mass of close binaries and its dependence on the mass ratio of the components can be deri-



Table 3

Primary S <sub>1</sub> type	Secondary S <sub>2</sub> type	M <sub>1</sub>	M <sub>2</sub>	Age (age 10 <sup>6</sup> yrs)	
OB	OB	20	8	0	P=4.56d
OB	OB	20	8	6.17	P=4.56d
	fills Roche lobe				
	Direct mass exchange S1→S2				
He-star	OB	5.4	22.6	6.2	P=10.86d
	End of the first phase of mass transfer				
	Wolf-Rayet binary				
	Evolution to final stage of S1				
Neutron Star	OB	2	22.6	6.78	
	OB Runaway				
	1st.supernova explosion				
	OB Runaway - Remnant of the Further				
	Evolved Helium star is a young neutron				
	star (Pulsar)				
Neutron Star	OB	2	22.6	11.186	P=11.7d
	X-ray Binary				
	OB star nearly fills its roche lobe				
	enhanced stellar wind,				
	accretion on the neutron star generates				
	X-rays				
	1st X-ray phase.				
Neutron Star	OB	2	22.6	11.209	P=11.70
	Reverse Mass-transfer				
	Roche lobe overflow-mass leaves the system				
Neutron Star	Ob→He				
	S2 on its way to WR star, Slow phase of mass				
	accretion disk				
	2nd X-ray phase				
					transfer;optical star
					it 'on its way' to be-
					come a Wolf-Rayet star.
					(Ex.SS433, Cir X-1) <sup>6</sup>
Neutron star	He	2	6.3	t=11.239 10 <sup>6</sup> yr	
	Wolf-Rayet runaway, end of the				
	second mass loss stage.				
	He-star + neutron in common (expanding) envelope.				
Neutron star	He	2	6.3		
	Supercritical disk accretion; 3d X-ray phase(Ex.CYG X-3)				
	2n supernova explosion				
	system remains bound				
Neutron star	Neutron star				P=7.75hrs
	Ex. psr 1913 + 16 binary pulsar				
Neutron star	Neutron star				P=1.5 ms.
	Coalescence of two neutron stars. ex.PSR 1937+214.				
	Millisecond Pulsar				
	OR: system disrupted, two runaway neutron stars.				

ved that mass leaves the system during the evolution. For massive close binaries this happens already during their detached phases, as stellar wind mass losses, and for the most massive ones, only by stellar wind, since they probably have no mass transfer phase.

Observations give us information on the flow of matter: indications for outstreaming material, disks, circumstellar material are present, as well as period changes, pointing to outstreaming material. However the timescales of the observations and the evolutionary timescales are different, and the parameters derived from these observations cannot be used as general parameters for the modelling of successive evolutionary models.

Relevant parameters, allowing discrimination among various possible options (conservative or not, fractions of outflowing matter and angular momentum, validity of the treatment of mass transfer during contact phases) are the mass ratios, masses, radii, determined by photometry in eclipsing systems, periods, period changes, chemical composition, determination of the evolutionary status, lifetimes. Only for very well determined systems it can be tried to trace back the evolutionary history, and in theory to determine the progenitor ZAMS system. But such systems are very rare, e.g. S Cnc, an Algol system with period of 9.5 days, (HD 74307) studied carefully by Awadalla and Budding (1981).

Hence we are still left with the embarrassing situation that we know more or less well how certain groups of binaries evolve into their groups, but that we remain unable to produce individual observed systems.

For massive stars the situation is less uncomfortable than for intermediate and low mass stars. Within certain limits we can produce Wolf-Rayet binaries and we know that they will evolve finally into X-ray binaries. Exact evolutionary computations reproducing well defined individual system are until now impossible. Also detailed calculations for final stages are still lacking.

The combination of evolutionary tracks and observational data has led in some cases to the determination of absolute dimensions of binaries. Doom and de Loore (1984) used evolutionary tracks for massive stars, observations, and the fact that both components of a binary system have the same age and the same  $\sin i$  to determine for a number of O stars their masses and ages.

Research on the disruption probabilities of massive binaries during supernova explosions has reasonably well advanced during the last ten years and the existence of binary pulsars and runaway pulsars can be explained. A plausible explanation for the millisecond pulsar by coalescence of the two neutron stars of a binary pulsar has also been published, so that we have an acceptable (although not for all evolutionary phases completely calculated) picture of the evolution of two zero age main sequence components of a close binary to a millisecond pulsar, or to runaway neutron stars.

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#### REFERENCES.

- Anderson, L., Raff, M., Shu, F.H.: 1980 in "Close Binary Stars: Observations and Interpretation", IAU Symp. 88, eds. M.J. Plavec, D. M. Popper, R.K. Ulrich.
- Appenzeller, I., Tscharnuter, W.: 1975, *Astron. Astrophys.* 40, 397.
- Awadella, Budding, 1981.
- Benson, R.S.: 1970, Ph. D. Thesis Univ. California, Berkeley.
- Biermann, P.: 1971, *Astron. Astrophys.* 10, 205.
- Biermann, P., Thomas, H.-C.: 1972, *Astron. Astrophys.* 16, 60.
- Binnendijk, L.: 1970, *Vistas in Astron.* 12, 217.
- Conti, P.S.: 1982 in "Wolf-Rayet Stars: Observations, Physics, Evolution" eds. C. de Loore and A. Willis.
- De Cuyper, J.P.: 1982, in "Binary and Multiple Stars as Tracers of Stellar Evolution" IAU Coll. 69, eds. Z. Kopal and J. Rahe.
- Doom, C., de Loore, C.: 1984, *Astrophys. J.* in press.
- Eggen, O.J., Roy, Ols. Bull. 31, 101.
- Eggen, O.J.: 1967, *Mem. Roy. Astron. Soc.* 70, 111.
- Firmani, C.: Bisiacchi, F.: 1980, Proc. 5th. IAU Regional Meeting, Liège Belgium.
- Flannery, B.P.: 1975a, *Mon. Nat. Roy. Astron. Soc.* 170, 325.
- Flannery, B.P.: 1975b, *Astrophys. J.* 201, 661.
- Flannery, B.P., Ulrich, R.K., 1977, *Astrophys. J.* 212, 533.
- Gianuzzi, M.A.: 1981, *Astron. Astrophys.* 103, 111.
- Guinan, E.F., Tomczyk, S.: 1979, *Inform. Bull. Var. Stars*, 1623.
- Hazlehurst, J.: 1970, *Mon. Not. Roy. Astron. Soc.* 149, 129.
- Hellings, P.: 1983a, *Astrophys. Space Sci.* in press.
- Hellings, P.: 1983b, *Astrophys. Space Sci.* submitted.
- Hellings, P., Vansina, F., Packet, W., Doom, C., De Grève, J.P., de Loore C.: 1982 in IAU Symp. 99 "Wolf-Rayet Stars: Observations Physics, Evolution" eds. C. de Loore and A. Willis, p. 397.
- Henrichs, H.F., van den Heuvel, F.P.J.: 1983, *Nature*, vol. 303.
- van den Heuvel, E.P.J., Ostriker, J.P., Petterson, J.A.: 1980, *Astron. Astrophys.* 81, L7.
- van den Heuvel, E.P.J., de Loore, C.: 1973, *Astron. Astrophys.* 25, 387.
- Huang, S.: 1963: *Astrophys. J.* 138, 342.
- Kippenhahn, R., Meyer-Hofmeister, E.: 1977, *Astron. Astrophys.* 54, 539.
- Kondo, Y., McCluskey, G.E., Stencel, R.E.: 1980 in "Close Binary Stars: Observations and Interpretation, IAU Symp. 88 eds. M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 237.
- Kuiper, G.P.: 1941, *Astrophys. J.* 93, 133.
- Kurditzki, R.P., Simon, K.P., Hamann, W.R.: 1983, *Astron. Astrophys.* 118, 245.
- Kurditzki, R.P., Simon, P.K., Lynas-Gray, A.E., Kilkenny, D., Hill, P.W.: 1982, *Astron. Astrophys.* 106, 245.
- Larson, R.B.: 1969, *Mon. Not. Roy. Astron. Soc.* 145, 271.
- Larson, R.B.: 1972, *Mon. Not. Roy. Astron. Soc.* 157, 121.
- Leung, K.C., Schneider, D.P.: 1979, in "Mass Loss and Evolution of O-type

- pe Stars", IAU Symp. 83, eds. P.S. Conti and C. de Loore, p. 265
- de Loore, C.: 1983, in "Double Stars, Physical Properties and Genetic Relations", IAU Coll. 80, ed. Z. Kopal.
- de Loore, C., De Grève, J.P., De Cuyper, J.P.: 1975, Mem. Soc. Astron. Italiana 45, 893.
- Lubow, S.H., Shu, F.H.: 1975, *Astrophys. J.*: 198, 383.
- Lubow, S.H., Shu, F.H.: 1976, *Astrophys. J. Letters* 207, L53.
- Lucy, L.B.: 1976, *Astrophys. J.* 205, 208.
- Lucy, L.B.: 1968, *Astrophys. J.* 151, 1123.
- Massey, P.: 1982, in "Wolf-Rayet Stars: Observations, Physics, Evolution IAU Symp. 99, eds. C. de Loore and A. Willis, p. 251.
- Meyer, F., Meyer-Hofmeister, E.: 1980 in "Close Binary Stars: Observation and Interpretation" IAU Symp. 88, eds. M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 145.
- Neo, S., Miyaji, S., Nomoto, K., Sugimoto, D.: *Publ. Astron. Soc., Japan* 29, 249.
- Osaki, Y.: 1965, *Publ. Astron. Soc. Japan*, 17, 97.
- Packet, W.: 1983, preprint.
- Paczynski, B.: 1971, *Ann. Rev. Astron. Astrophys.* 9, 183.
- Paczynski, B.: 1980, *Astron. Astrophys.* 82, 349.
- Paczynski, B., Ziolkowski, J.: 1967, *Acta Astron.* 17, 7.
- Plavec, M.J., Koch, R.H.: 1978, *Inform. Bull. Var. Stars*, No 1482.
- Plavec, M.J.: 1981, in "Effects of Mass Loss on Stellar Evolution", eds. C. Chiosi and R. Stalio.
- Plavec, M.J., Ulrich, R.K., Polidan, R.S.: 1973, *Publ. Astron. Soc. Pacific*, 85, 508.
- Plavec, P.J.: 1983, *Astrophys. J.* Submitted.
- Plavec, M.J., Sehnal, L., Mikulas, J.: 1964, *Bull. Astron. Inst. Czech* 15, 171.
- Plavec, M.J., Weiland, J.L., Koch, R.H.: 1981, *Astrophys. J.*
- Plavec, M.J., Kriz, S.: 1965, *Bull. Astron. Inst. Czech.* 16, 297.
- Polidan, R.S., Peters, G.J.: 1980 in "Close Binaries: Observations and Interpretations" eds. M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 203.
- Popper, D.M.: 1980a in "Close Binaries: Observations and Interpretations" eds. M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 203.
- Popper, D.M.: 1980b, *Ann. Rev. Astron. Astrophys.* 18, 115.
- Prendergast, K.H.: 1960, *Astrophys. J.* 132, 162.
- Prendergast, K.H., Taam, R.E.: 1974, *Astrophys. J.* 189, 125.
- Robinson, E.L.: 1976, *Ann. Rev. Astron. Astrophys.* 14, 119.
- Rucinski, S.M.: 1974, *Acta Astron.* 24, 119.
- Rucinski, S.M.: 1978 in "Non stationary Evolution of Close Binaries" ed. A. Zytkov, p. 117.
- Schlovski, I.S.: 1981, *Proc. 5th. IAU Reg. Meeting, Liège, Belgium.*
- Shu, F.H.: 1980 in "Close Binary Stars: Observations and Interpretations" IAU Symp. 88, eds. M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 477.
- Shu, F.H., Lubow, S.H.: 1981 in *Ann. Rev. Astron. Astrophys.* p. 277.
- Shu, F.H., Lubow, S.H., Anderson, L.: 1976, *Astrophys. J.* 209, 536.
- Sobouti, Y.: 1970, *Astron. Astrophys.* 5, 149.
- Stahler, S.W., Shu, F.H., Taam, R.E.: 1980a, *Astrophys. J.* 241, 637.
- Stahler, S.W., Shu, F.H., Taam, R.E.: 1980b, *Astrophys. J.* 242, 226.

- Stencel, R.E., Kondo, Y., Bernat, A.P., Mc. Clusky, G.: 1980 in "Close Binary Stars: Observations and Interpretation", IAU Symp. No. 88 eds, M.J. Plavec, D.M. Popper, R.K. Ulrich, p. 555.
- Struve, O., Sahade, J.: 1957, Publ. Astron. Soc. Pac. 69, 41.
- Vanbeveren, D., De Grève, J.P., van Dessel, E.L., de Loore, C.: 1979, Astron. Astrophys. 73, 19.
- Vanbeveren, D., de Loore, C.: 1980, Astron. Astrophys. 86, 21.
- Warner, B.: 1976, Structure and Evolution of Close Binary Systems, IAU Symp. 73, eds. Mitton, S. Whelan, J., p. 85.
- Webbinck, R.F.: 1979, IAU Coll. 49 "Changing trends in variable star Research", eds. F.M. Bateson, J. Smak, I.H. Urch, p. 102.
- Wilson, R.E.: 1974, Astrophys. J. 189, 319.
- Wilson, R.E.: 1979, Astrophys. J. 234, 1054.
- Wilson, R.E.: 1981, Astrophys. J. 251.
- Winkler, K.H., Newman, M.J.: 1980a, Astrophys. J. 236, 201.
- Winkler, K.H., Newman, M.J.: 1980b, Astrophys. J. 238, 311.
- Yungelson, L.R.: 1973, Nauch. Inf. 27, 93.