Proceedings of the Edinburgh Mathematical Society (2005) **48**, 757–762 ©
Printed in the United Kingdom

BOOK REVIEWS

DOI:10.1017/S0013091505254834

SCHENCK, H. *Computational algebraic geometry* (Cambridge University Press, 2003), xiv + 193 pp., 0521 53650 2 (paperback), £18.99, 0521 82964 9 (hardback), £50.

Mathematical work thrives on examples, in both research and teaching. Now that computers are powerful and cheap enough to be widely available, the range of feasible examples has enlarged significantly. Algorithmic methods that once seemed impractical are now used routinely. In the areas of algebraic geometry and commutative algebra, one of the most widely used packages is Macaulay (due to Grayson and Stillman). This is free, available for most platforms and fairly easy to use. The foundation stone here is the notion of Gröbner bases and the associated algorithm for computing them, both due to Buchberger. Briefly speaking, suppose we are given multivariate polynomials $f_1, f_2, \dots, f_n$ with coefficients from a field and wish to know if a polynomial f is in the ideal generated by the given polynomials. Trivially, this is so if and only if $f = c_1g_1u_1 + c_2g_2u_2 + \cdots + c_mg_mu_m$, where each c_i is a constant, each g_i is one of the given generators and each u_i is a power product in the indeterminates. This suggests introducing a notion of multivariate division with the aim of showing that f is in the ideal if and only if it leaves remainder 0 when divided by the generators. Even when this notion is made precise it is easy enough to see that it is over optimistic (if the remainder is 0, then certainly we have membership of the ideal, but the converse is not guaranteed). However, a simple analysis of how things go wrong suggests the notion of a change of basis for which the optimism is fully justified: hence Gröbner bases. Buchberger's algorithm provides a way of bringing about such a change of basis. Once we have a Gröbner basis for an ideal, a great deal of information can be derived. The idea can be generalized to modules in a fairly straightforward manner and as a reward we can, for example, compute syzygies (due to Schreyer).

The book under review covers a great deal of ground, taking in topics from commutative algebra, algebraic geometry, algebraic topology and algebraic combinatorics. Aspects of theory are presented together with illustrative examples using Macaulay 2. The book is organized into 10 chapters as well as two introductory appendices on algebra and complex analysis. Gröbner bases are discussed in an early chapter. The chapters begin with *Basics of commutative algebra* and range through *Combinatorics, topology and the Stanley–Reisner ring* to *Curves, sheaves and cohomology* and finally *Projective dimension, Cohen–Macaulay modules, upper bound theorem*. This sample of chapter headings makes it clear that this short book cannot possibly cover anything like all the details, indeed to quote the author, 'Mea maxima culpa: in this chapter we have sketched material more properly studied in a semester course'. In fact this is one of the book's strengths; it gives the reader a guided and informative tour to deep material. The author suggests further reading at the end of each chapter, discussing how it fits in with the topics and approach taken in the book.

The book is not primarily concerned with the development of algorithms in algebraic geometry but much of the presentation is informed by such considerations. For example, computing the Hilbert polynomial is discussed first in terms of free resolutions. This is then revisited in the chapter on Gröbner bases and reconsidered in this context via a well known result due to Macaulay (that the Hilbert function of an ideal is the same as that of the ideal generated by the initial terms of the ideal, under an admissible order). As another example we quote again from the author (Chapter 8): 'So what? Well, this is exactly what proves that we can compute $\mathrm{Tor}_i(M, N)$ . . . '. This also indicates the informal style of the writing: friendly but not overly so. There are many exercises throughout, of both a theoretical and computational nature. They also range from the straightforward to the hard (for which the author gives helpful hints or references where the solution can be found).

To sum up, a student considering this area will find the guided-tour aspect of this book very helpful. The computing aspect is a good pointer to what can be done, but the book is not intended to act as a manual. Unless such a student is already well informed, he or she will find the suggestions for further reading essential.

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DOI:10.1017/S0013091505264830

Stopple, J. *A primer of analytic number theory* (Cambridge University Press, 2003), 0 521 01253 8 (paperback) £22.95, 0 521 81309 3 (hardback) £65.

The author makes it immediately clear in the introduction that this book is not intended to be anything like a conventional text on analytic number theory. The starting point is the observation that courses on number theory normally appear quite early in the curriculum, and consequently avoid all results that require any input from analysis. His basic thesis is that this avoidance is unnecessary. In this book, he seeks to demonstrate that one can introduce a wide range of topics within analytic number theory assuming no more than a basic exposure to 'calculus'. It is not even assumed that the reader knows what is meant by the sum of a series: this is the subject of a 30-page digression, starting with Taylor series, in which the actual definition of convergence only appears on about the twentieth page.

The general style is user-friendly and interactive. The simpler steps of proofs are often set as exercises, so that the reader is recruited as a partner in the process of discovery. Solutions to the exercises are provided at the end. There are also a large number of numerical exercises and illustrations, based on Maple or Mathematica. Readers are repeatedly assured that they can leave out proofs if they find them hard. The text is liberally interspersed with biographies, historical information and quotations. A good deal of space is given to informal discussion of famous conjectures and open problems. The claim that nothing is required beyond basic calculus is partly sustained by a rather free policy of importing unproved results, both from analysis and from number theory, when wanted.

Despite all this, the range of topics attempted is distinctly ambitious for a reader who is really starting from the position described. In the reviewer's opinion, any such reader is likely to find large parts of the book extremely challenging!

The early material covers topics like perfect numbers, convolutions, the Möbius function and the estimates for the partial sums of the divisor function τ and the sum-of-divisors function σ. Readers of the type postulated should get this far without much trouble. With strong enough motivation, they may also succeed with the next few topics: the Chebyshev estimates, the Bernoulli numbers and the Euler product. Here there is a typical example of the consequences of the author's self-imposed task. The cotangent series is needed for the evaluation of $\zeta(2n)$. Fourier series are not allowed, so a direct proof is given (to the reviewer's mind, this is one of the author's less satisfactory proofs; it is based on a rather casual use of the O-notation, and would seem to require some non-trivial further work for its completion).

After that, the level rises steeply. Mertens's theorems are presented in some detail, with the application to the maximal order of σ. Three chapters then develop the theory of the gamma