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Electromagnetically induced transparency in plasma

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Abstract

The coupled propagation of two electromagnetic waves in plasma is studied to establish the conditions for induced transparency. Induced transparency refers to the situation where both waves propagate unattenuated, although the frequency of one (or both) of them is below the plasma frequency so that it could not propagate in the absence of the other. The effect is due to the interaction of the waves through their beat, which modulates both the electron mass and, by exciting longitudinal plasma oscillations, their number density, and thus the plasma frequency. Starting from a relativistic fluid description, a dispersion relation for plane waves of weakly relativistic intensities is derived, which takes into account the polarization of the waves and the nonlinearities with respect to both their amplitudes. This serves as a basis for the exploration of the conditions for induced transparency and the modes of propagation.

1. INTRODUCTION

Electromagnetically induced transparency (EIT) is the condition when the propagation of an electromagnetic wave through an otherwise opaque medium is made possible by the interaction with a second wave. In plasma, the coupling mechanism between the waves is a modulation of the plasma frequency, which determines the refractive properties of the plasma. This results from a relativistic increase of the electron mass, and from a variation in electron density caused by longitudinal plasma oscillations driven by the ponderomotive potential associated with the beat between the waves.

A recent study of EIT by Harris (1996) employs a three-wave model, which incorporates two transverse electromagnetic waves—with frequencies $\omega_1 > \omega_p$, $\omega_2 < \omega_p$, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the (unperturbed) plasma frequency (n_0 is the unperturbed electron density, $-e$ the electron charge, and m their rest mass)—and a longitudinal plasma wave at the difference frequency $\omega_- = \omega_1 - \omega_2$. The model predicts transparency if ω_- is slightly lower than ω_p.

Matsko and Rostovtsev (1998) found that the conditions for EIT are affected by the excitation of the anti-stokes wave at $\omega_1 + \omega_p$ as well.

While these authors have assumed that the amplitude of the wave at the lower frequency is small, an investigation of modulational instability of two coupled waves (which involves the same interaction mechanism) by McKinstrie and Bingham (1989) allowed for finite amplitudes of both waves, while taking the relativistic electron mass into account. They use an expansion of the wave vectors in terms of the amplitudes, which, however, breaks down at the transition to transparency.

The aim of the present article is to explore the conditions of EIT for two waves of weakly relativistic amplitudes, which may be comparable. Our treatment allows us to consider co- or counterpropagating beams. Since we do not restrict the frequencies, either or both may be above or below the plasma frequency. Moreover, the present formalism can also be used to study other phenomena of nonlinear collective interaction in plasma, like Raman scattering or modulational instability.

2. COUPLED PROPAGATION IN PLASMA

2.1. Relativistic fluid equations

We start from a description of the plasma as cold electron fluid and take the relativistic corrections to the electron mass into account since these are of the same order of magnitude as the ponderomotive coupling (McKinstrie and Bingham, 1989). In one spatial dimension (z) the relevant equations read (cf. e.g., Barr *et al.*, 2000):

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} - \frac{n}{\gamma}\right)\mathbf{a} = \mathbf{0}, \tag{1}$$

$$\frac{\partial E}{\partial t} = \frac{n}{\gamma}p, \qquad \frac{\partial E}{\partial z} = 1 - n, \qquad \frac{\partial p}{\partial t} = -E - \frac{\partial \gamma}{\partial z}, \tag{2}$$

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where $\mathbf{a}(z,t)$ is the transverse vector potential, scaled by mc^2/e (c is the vacuum speed of light), $E(z,t)$ is the longitudinal electric field, scaled by $\omega_p mc^2/e$, $p(z,t)$ is the longitudinal momentum, scaled by mc, $n(z,t)$ is the electron density, scaled by the unperturbed density n_0, and $\gamma(z,t) = (1+\mathbf{a}^2+p^2)^{1/2}$ is the Lorentz factor. Time is scaled by $1/\omega_p$, and length by c/ω_p.

2.2. Expansion in powers of the vector potential

In this section, we determine the coupling of two transverse waves, driven by applied fields of frequencies ω_1 and ω_2, through longitudinal oscillations to the lowest (second) order in an expansion in powers of the vector potential. For the latter, we try an ansatz of the form

$$\mathbf{a} = \Re[\mathbf{a}_1 \exp(i\theta_1) + \mathbf{a}_2 \exp(i\theta_2)], \tag{3}$$

with complex amplitude vectors $\mathbf{a}_{1,2}$, and phases $\theta_{1,2} = k_{1,2}z - \omega_{1,2}t$. (Corresponding to the scalings of time and length, the frequencies $\omega_{1,2}$ are scaled by ω_p, and the wave numbers $k_{1,2}$ by ω_p/c.)

From Eqs. (2) we find that the ponderomotive force $-\partial\gamma/\partial z$ drives longitudinal plasma waves, characterized by E, p, and $n-1$. To the lowest order these quantities are proportional to the (scaled) intensity of the transverse waves

$$I = \mathbf{a}^2 = I_0^{(1)} + I_0^{(2)} + \Re(I_+ + I_- + I_{11} + I_{22}), \tag{4}$$

where the individual terms are given by

$$I_0^{(1,2)} = |\mathbf{a}_{1,2}|^2/2, \qquad I_{11,22} = \mathbf{a}_{1,2}^2 \exp(2i\theta_{1,2})/2,$$
$$I_+ = \mathbf{a}_1\cdot\mathbf{a}_2 \exp(i\theta_+), \qquad I_- = \mathbf{a}_1\cdot\mathbf{a}_2^* \exp(i\theta_-), \tag{5}$$

with sum and difference phases $\theta_\pm = \theta_1 \pm \theta_2$.

The Lorentz factor, to this order, is $\gamma \approx (1+I)^{1/2} \approx 1 + I/2$, so that the density modulations are determined by

$$\left(\frac{\partial^2}{\partial t^2} + 1\right)(n-1) \approx \frac{1}{2}\frac{\partial^2 I}{\partial z^2}. \tag{6}$$

Accordingly, they may be decomposed as

$$n - 1 = \Re(n_+ + n_- + n_{11} + n_{22}), \tag{7}$$

with

$$n_\pm = k_\pm^2 I_\pm/D_\pm, \qquad n_{11,22} = 4k_{1,2}^2 I_{11,22}/D_{11,22}, \tag{8}$$

and $k_\pm = k_1 \pm k_2$, $D_\pm = \omega_\pm^2 - 1$, $D_{11,22} = 4\omega_{1,2}^2 - 1$, $\omega_\pm = \omega_1 \pm \omega_2$.

2.3. Dispersion relation

We substitute n and γ into Eq. (1) for the vector potential and retain only terms oscillating with the frequencies ω_1, ω_2 of the applied fields. (This assumes that the frequencies $3\omega_1, 3\omega_2, |2\omega_1+\omega_2|$, and $|\omega_1+2\omega_2|$ of the discarded terms differ from ω_1 and ω_2.) We thus arrive at coupled dispersion relations for the wave numbers k_1 and k_2:

$$\left[\left(D_1 - k_1^2 + \frac{I_0^{(1)}+I_0^{(2)}}{2}\right)\vec{\mathbf{1}} + \left(1 - \frac{k_+^2}{D_+}\right)\frac{\mathbf{a}_2^*\mathbf{a}_2}{4} + \left(1 - \frac{k_-^2}{D_-}\right)\frac{\mathbf{a}_2\mathbf{a}_2^*}{4} + \left(1 - \frac{4k_1^2}{D_{11}}\right)\frac{\mathbf{a}_1^*\mathbf{a}_1}{8}\right]\cdot\mathbf{a}_1 = 0, \tag{9}$$

$$\left[\left(D_2 - k_2^2 + \frac{I_0^{(1)}+I_0^{(2)}}{2}\right)\vec{\mathbf{1}} + \left(1 - \frac{k_+^2}{D_+}\right)\frac{\mathbf{a}_1^*\mathbf{a}_1}{4} + \left(1 - \frac{k_-^2}{D_-}\right)\frac{\mathbf{a}_1\mathbf{a}_1^*}{4} + \left(1 - \frac{4k_2^2}{D_{22}}\right)\frac{\mathbf{a}_2^*\mathbf{a}_2}{8}\right]\cdot\mathbf{a}_2 = 0, \tag{10}$$

where $\vec{\mathbf{1}}$ denotes a unit tensor, and $D_{1,2} = \omega_{1,2}^2 - 1$.

The first term in each equation corresponds to a reduction of the plasma frequency by a factor $1 - (I_0^{(1)} + I_0^{(2)})/2$ due to the time-averaged relativistic increase of the electron mass. It is independent of the polarizations of the waves, in contrast to the subsequent terms with the dyads of amplitude vectors.

2.4. Polarizations

We split the amplitude vectors into scalar amplitudes and unit vectors, $\mathbf{a}_{1,2} = a_{1,2}\mathbf{e}_{1,2}$, and find that the ansatz (3) works for four different combinations of polarizations:

- LP: Linear polarizations, parallel: $\mathbf{e}_1 = \mathbf{e}_2 = \mathbf{e}_x$;
- LO: Linear polarizations, orthogonal: $\mathbf{e}_1 = \mathbf{e}_x$, $\mathbf{e}_2 = \mathbf{e}_y$;
- CS: Circular polarizations, same senses of rotation: $\mathbf{e}_1 = \mathbf{e}_2 = \mathbf{e}_+ = (\mathbf{e}_x + i\mathbf{e}_y)/\sqrt{2}$;
- CO: Circular polarizations, opposite senses of rotation: $\mathbf{e}_1 = \mathbf{e}_+$, $\mathbf{e}_2 = \mathbf{e}_- = \mathbf{e}_+^*$.

In each case, the dispersion relation takes the form of two coupled bilinear equations in the wave numbers k_1, k_2:

$$A_{11}k_1^2 + 2A_{12}k_1k_2 + A_{22}k_2^2 = A,$$
$$B_{11}k_1^2 + 2B_{12}k_1k_2 + B_{22}k_2^2 = B, \tag{11}$$

with coefficients, depending on the polarizations:

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LP:

$$A_{11} = (D_{11} + I_0^{(1)})D_+ D_- + A_{22},$$

$$A_{12} = I_0^{(2)} D_{11}(D_- - D_+)/2,$$

$$A_{22} = I_0^{(2)} D_{11}(D_+ + D_-)/2,$$

$$A = D_{11} D_+ D_- (D_1 + 3I_0^{(1)}/4 + 3I_0^{(2)}/2),$$

$$B_{11} = I_0^{(1)} D_{22}(D_+ + D_-)/2,$$

$$B_{12} = I_0^{(1)} D_{22}(D_- - D_+)/2,$$

$$B_{22} = (D_{22} + I_0^{(2)})D_+ D_- + B_{11},$$

$$B = D_{22} D_+ D_- (D_2 + 3I_0^{(1)}/2 + 3I_0^{(2)}/4);$$

LO:

$$A_{11} = D_{11} + I_0^{(1)}, \qquad A_{12} = A_{22} = 0,$$

$$A = D_{11}(D_1 + 3I_0^{(1)}/4 + I_0^{(2)}/2),$$

$$B_{11} = B_{12} = 0, \qquad B_{22} = D_{22} + I_0^{(2)},$$

$$B = D_{22}(D_2 + I_0^{(1)}/2 + 3I_0^{(2)}/4);$$

CS/CO:

$$A_{11} = D_\mp + I_0^{(2)}/2, \qquad A_{12} = \mp A_{22} = \mp I_0^{(2)}/2,$$

$$A = D_\mp (D_1 + I_0^{(1)}/2 + I_0^{(2)}),$$

$$B_{11} = \mp B_{12} = I_0^{(1)}/2, \qquad B_{22} = D_\mp + I_0^{(1)}/2,$$

$$B = D_\mp (D_2 + I_0^{(1)} + I_0^{(2)}/2).$$

In the last set of coefficients, the upper sign is for CS, the lower for CO.

Mathematically, Eqs. (11) represent conic sections in the k_1-k_2-plane, centered at the origin, and can be solved analytically. It should be noted that for circular polarizations, the longitudinal waves providing the coupling are excited only at the difference frequency ω_- for CS (as assumed by Harris, 1996, although for linear polarizations), and at the sum frequency ω_+ for CO. For LO, the equations are decoupled.

3. RESULTS

3.1. Conditions for transparency

The plasma is transparent for both waves if—for real ω_1, ω_2—the wave numbers k_1 and k_2 are both real. We have checked this condition for the solution of the dispersion relations (11) for different amplitudes a_1, a_2, and plotted the areas where it is satisfied for a frequency range $\omega_1 = 0 \ldots 2.5$, $\omega_2 = 0 \ldots 1.1$ in Figure 1 for parallel linear polarizations, and in Figure 2 for circular polarizations.

In Figure 1a, a_2 is set to zero; this corresponds to the treatment by Harris (1996), and Matsko and Rostovtsev (1998), except that we take the second harmonic $2\omega_1$ into account. We notice a region of transparency, and, for $\omega_{1,2} \geq 1$, a complimentary one of induced opacity, each bounded by the resonance of the difference frequency with the plasma frequency, $\omega_- = 1$, as found by these authors.

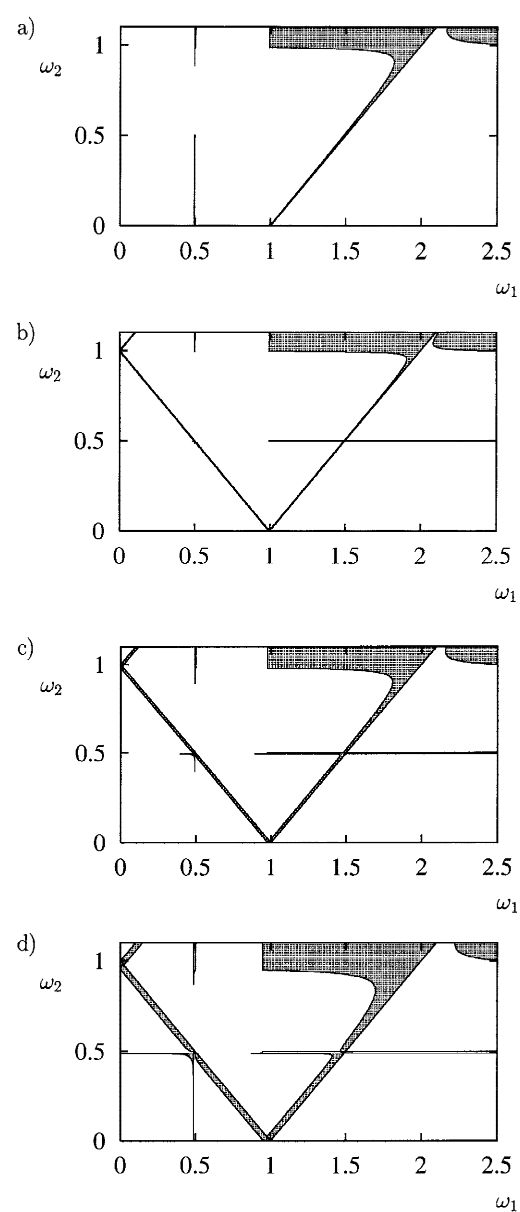

Fig. 1. Regions of transparency (shaded) in the ω_1-ω_2-plane, for parallel linear polarizations. (a) $a_1 = 0.2, a_2 = 0$; (b) $a_1 = a_2 = 0.1$; (c) $a_1 = a_2 = 0.2$; (d) $a_1 = a_2 = 0.3$.

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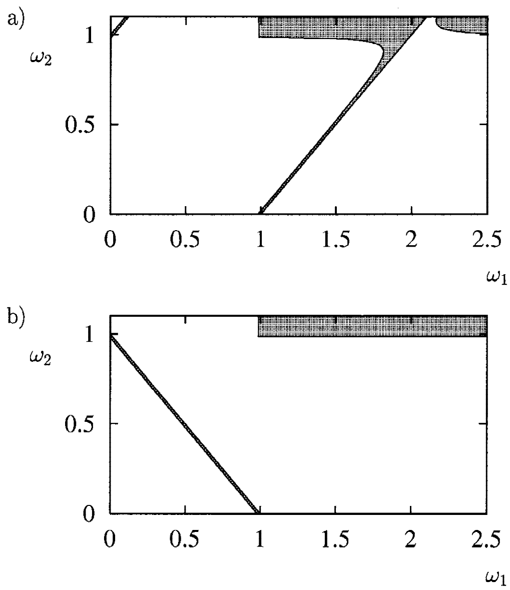

Fig. 2. Regions of transparency (shaded) in the ω_1-ω_2-plane, for circular polarizations, $a_1 = a_2 = 0.2$. (a) corotating; (b) counterrotating.

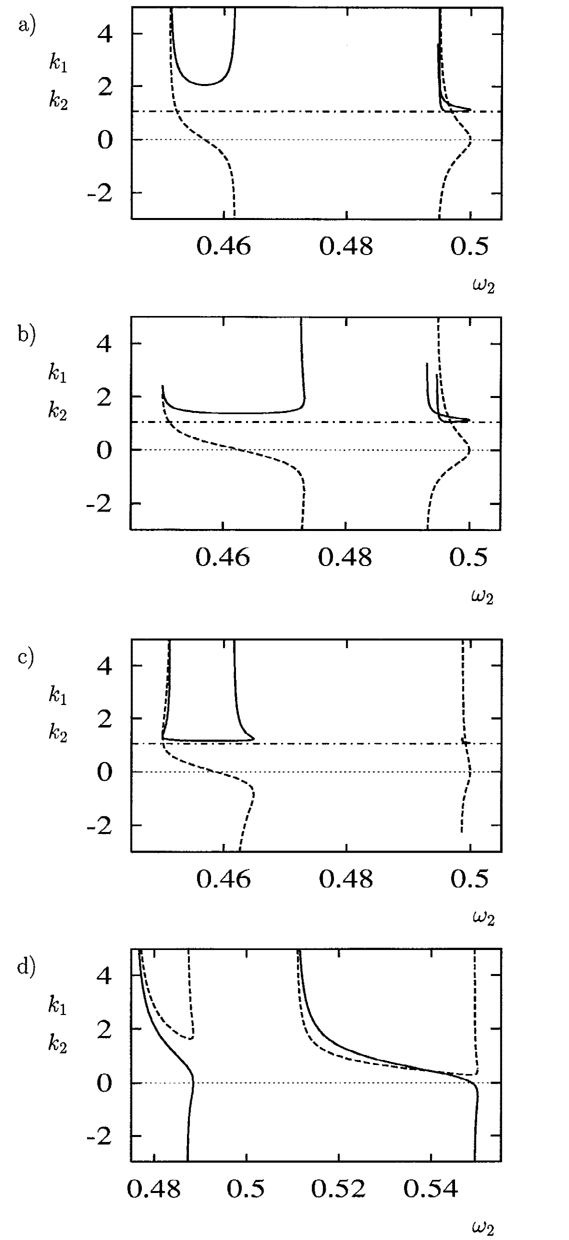

Fig. 3. Dispersion curves $k_1(\omega_2)$ (solid), $k_2(\omega_2)$ (dashed) for fixed ω_1, parallel linear polarizations. (a–c) $\omega_1 = 1.45$, dot-dashed line: $k_1^{(0)}$; (a) $a_1 = 0.1$, $a_2 = 0.2$; (b) $a_1 = a_2 = 0.2$; (c) $a_1 = 0.2$, $a_2 = 0.1$; (d) $\omega_1 = 0.45$, $a_1 = a_2 = 0.3$.

Additionally, we find transparency in a narrow region bounded by the second harmonic resonance, $2\omega_1 \leq 1$. Plots for equal amplitudes $a_1 = a_2 = 0.1 \ldots 0.3$, Figures 1b–d, show additional regions of transparency near the resonances of the sum frequency, $\omega_+ = 1$, and also of the second harmonics, $2\omega_1 = 1$ and $2\omega_2 = 1$. All these regions grow in width as the amplitudes increase.

In Figure 2 we see that for the cases of co- and counterrotating circular polarizations, transparency is induced only near the resonances of the difference and sum frequencies, respectively.

3.2. Dispersion curves

Figure 3 shows examples of dispersion curves for the wave numbers k_1 and k_2 in dependence of ω_2 for fixed ω_1 in the case of parallel linear polarizations. Parts a–c are cuts through the transparency regions near $\omega_- = 1$ and $2\omega_2 = 1$ for $\omega_1 = 1.45$ for different ratios of amplitudes. Also indicated in these plots is the wave number corresponding to the higher frequency, $k_1^{(0)} = (\omega_1^2 - 1)^{1/2}$. Figure 3d is a cut through the regions near $\omega_+ = 1$ and $2\omega_2 = 1$ for $\omega_1 = 0.45$, and equal amplitudes $a_1 = a_2 = 0.3$. We note the existence of two modes with different wave numbers for certain frequency ranges, and the possibility of transparency for counterpropagating waves, indicated by opposite signs of k_1 and k_2.

4. CONCLUSIONS

We have studied the coupled propagation of two transverse electromagnetic plane waves through weakly relativistic cold plasma, taking into account longitudinal plasma waves up to second order in the amplitudes of the transverse fields. These waves, which are excited at the sum and difference of the frequencies of the applied fields, and at their second harmonics, lead to transparency if either of these frequencies lies in a narrow band below the plasma frequency. These bands broaden as the amplitudes increase. For the difference frequency this has been described earlier by Harris (1996), and Matsko and Rostovtsev (1998). By contrast, transparency induced through plasma oscillations at the sum fre-

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quency is of particular interest, since in this case neither transverse wave can propagate in the plasma on its own.

An important issue that remains to be addressed is the evolution of the field amplitudes for pulses of finite duration, in particular when they are switched on—how do the waves penetrate from the surface into the plasma? Will the interaction of the evanescent waves near the surface gradually establish the longitudinal oscillations necessary for transparency? From this point of view, transparency mutually induced by counterpropagating waves both below the plasma frequency, although theoretically possible, is very unlikely to be realized.

Since there is no restriction on the frequencies in the present formalism, it can equally be used to study other phenomena of nonlinear collective interaction in plasma, like Raman scattering or modulational instability.

We hope to be able to demonstrate the phenomena and confirm the theory in experiments at the Strathclyde Electron and Terahertz to Optical Pulse Source (TOPS) in the near future.

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REFERENCES

Barr, H.C., Mason, P. & Parr, D.M. (2000). *Phys. Plasmas* **7**, 2604.

Harris, S.E. (1996). *Phys. Rev. Lett.* **77**, 5357.

Matsko, A.B. & Rostovtsev, Y.V. (1998). *Phys. Rev.* E **58**, 7846.

McKinstrie, C.J. & Bingham, R. (1989). *Phys. Fluids* B **1**, 230.

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