Bull. Aust. Math. Soc. **105** (2022), 161–170
doi:10.1017/S0004972721000319

ON THE CONNECTEDNESS OF THE CHABAUTY SPACE OF A LOCALLY COMPACT PRONILPOTENT GROUP

BILEL KADRI

(Received 21 March 2021; accepted 27 March 2021; first published online 17 May 2021)

Abstract

Let G be a locally compact group and let $\mathcal{SUB}(G)$ be the hyperspace of closed subgroups of G endowed with the Chabauty topology. The main purpose of this paper is to characterise the connectedness of the Chabauty space $\mathcal{SUB}(G)$. More precisely, we show that if G is a connected pronilpotent group, then $\mathcal{SUB}(G)$ is connected if and only if G contains a closed subgroup topologically isomorphic to $\mathbb{R}$.

2020 *Mathematics subject classification*: primary 54B20; secondary 20F19, 22D05, 54D05.

Keywords and phrases: Chabauty topology, locally compact group, pronilpotent group.

1. Introduction

Let G be a locally compact group whose identity element is denoted by e. The Chabauty space of G is $\mathcal{SUB}(G)$, the hyperspace of closed subgroups of G endowed with the Chabauty topology: a basis of neighbourhoods for a closed subgroup H is formed by the subsets

$$\mathcal{U}(H; C, U) = \{K \in \mathcal{SUB}(G) \mid K \cap C \subset HU \text{ and } H \cap C \subset KU\},$$

where C runs over the compact subsets of G and U runs over the neighbourhoods of e. This topology turns $\mathcal{SUB}(G)$ into a compact space and therefore the question of connectedness arises naturally. In [18], Protasov and Tsybenko proved that if the Chabauty space $\mathcal{SUB}(G)$ is connected, then G must contain a subgroup topologically isomorphic to $\mathbb{R}$, the additive group of real numbers. Moreover, they showed that the converse holds for abelian groups in light of the following statement.

PROTASOV–TSYBENKO'S STATEMENT [18, Lemma 1]. For every locally compact group G, the Chabauty space $\mathcal{SUB}(\mathbb{R} \times G)$ is connected.

However, in contrast to these facts, they conjectured a negative answer to the following question.

QUESTION 1.1. Is the Chabauty space $\mathcal{SUB}(G)$ connected for all locally compact groups G containing a subgroup topologically isomorphic to $\mathbb{R}$?

The first contribution to this question is due to Tsybenko. In fact, he proved that their conjecture is correct by showing the following result.

THEOREM 1.2 (Tsybenko's theorem; see [14, page 182]). *Let G be the group of matrices of the form*

$$[m, n; x] = \begin{pmatrix} 1 & m & x \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix},$$

where $m, n \in \mathbb{Z}$ and $x \in \mathbb{R}$. Then G contains a central subgroup topologically isomorphic to $\mathbb{R}$, but $\mathcal{SUB}(G)$ is not connected.

The second contribution is due to Cornulier.

THEOREM 1.3 (Cornulier's theorem; see [6, Proposition 8.6]). *Let G be a nonsolvable finite group. Then $\mathcal{SUB}(\mathbb{R} \times G)$ is not connected.*

Cornulier's theorem is remarkable and has many important consequences. In addition to reconfirming the conjecture, it constitutes a counterexample to the connectedness statement of Protasov and Tsybenko. Thus, the consideration of the following question was very natural.

QUESTION 1.4. Under which conditions on the group G is the Chabauty space $\mathcal{SUB}(\mathbb{R} \times G)$ connected?

In [6, Theorem 1.3], Cornulier gave an answer to this question as follows.

THEOREM 1.5. *Let G be a locally compact abelian group. Then the Chabauty space $\mathcal{SUB}(\mathbb{R} \times G)$ is connected.*

Recall that a topological group is called prosolvable (respectively, pronilpotent) if every open neighbourhood of the identity contains a closed normal subgroup whose corresponding quotient is a solvable (respectively, nilpotent) group. In [10, Theorem 7.3], the author and Hamrouni generalised the result of Cornulier as follows.

THEOREM 1.6. *For every locally compact prosolvable group G, the Chabauty space $\mathcal{SUB}(\mathbb{R} \times G)$ is connected.*

In particular, we have the following characterisation.

PROPOSITION 1.7. *Let G be a finite group. Then the Chabauty space $\mathcal{SUB}(\mathbb{R} \times G)$ is connected if and only if G is solvable.*

In the same direction, we recall that Bridson *et al.* provided an interesting result about the connectedness of Chabauty spaces in the class of nilpotent Lie groups. More precisely, in [5, Theorem 1.3], they proved the following result.

THEOREM 1.8. *Let $\mathbb{H}_3$ be the three-dimensional Heisenberg group. Then $\mathcal{SUB}(\mathbb{H}_3)$ is arc connected but not locally connected.*

In light of these facts and by taking Tsybenko's theorem into account, the consideration of the following statement is very reasonable and one might well ask whether it is true.

QUESTION 1.9. Is the Chabauty space $\mathcal{SUB}(G)$ connected when G is a noncompact connected nilpotent group?

In this paper, our main theorem responds positively to this question. Namely, the following stronger result is proved.

THEOREM 1.10 (Main theorem). *Let G be a connected locally compact pronilpotent group. Then the Chabauty space $\mathcal{SUB}(G)$ is connected if and only if G contains a subgroup topologically isomorphic to $\mathbb{R}$.*

2. Background on the Chabauty topology

The main objective of this section is to compile several facts about the Chabauty topology and to present them in the manner in which they are used in the paper.

2.1. Chabauty topology. Let G be a topological group and let $\mathcal{SUB}(G)$ be the hyperspace of all closed subgroups of G. The Chabauty topology on $\mathcal{SUB}(G)$ has the sets

$$\mathcal{O}_1(C) = \{H \in \mathcal{SUB}(G) \mid H \cap C = \varnothing\},$$

$$\mathcal{O}_2(U) = \{H \in \mathcal{SUB}(G) \mid H \cap U \neq \varnothing\},$$

as an open subbase, where U and C run respectively over all open and compact subsets of G. The Chabauty space of a locally compact group is very remarkable. In fact, we have the following well-known result.

PROPOSITION 2.1 (Compactness of Chabauty spaces). *If G is a locally compact group, then the Chabauty space $\mathcal{SUB}(G)$ is compact.*

PROOF. See [4, Théoréme 1, page 181] or [3, Lemma E.1.1]. □

In light of this compactness, it will be helpful to introduce the following important result about the convergence of nets of closed subgroups in the Chabauty topology.

PROPOSITION 2.2. *If a net $(H_\alpha)_{\alpha\in J}$ of closed subgroups of a locally compact group G converges to a closed subgroup H, then the following assertions are equivalent.*

(1) *The points $x_1, x_2, \ldots, x_n$ belong to H.*
(2) *There exists a subnet $(H_{\alpha_\beta})_{\beta\in I}$ of the given net $(H_\alpha)_{\alpha\in J}$ and there exist nets $\{x_1^\beta\}_{\beta\in I}, \ldots, \{x_n^\beta\}_{\beta\in I}$ of G converging, respectively, to $x_1, \ldots, x_n$ in the topology of G, with $x_1^\beta, \ldots, x_n^\beta \in H_{\alpha_\beta}$ for all $\beta \in I$.*

PROOF. This follows from Remark 1.1 and Lemma 1.2 in [17]. □

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press

REMARK 2.3. Besides its compactness, if G is second countable, then $\mathcal{SUB}(G)$ is also second countable and therefore metrisable by Urysohn's metrisation theorem (see [3, Lemma E.1.1]).

2.2. Examples of Chabauty spaces. In this section, we present some examples of Chabauty spaces of familiar groups.

PROPOSITION 2.4 [8, Proposition 1.7]. *The mapping* $\phi_{\mathbb{R}} : [0, \infty] \longrightarrow \mathcal{SUB}(\mathbb{R})$ *defined by*

$$\phi_{\mathbb{R}}(r) = \begin{cases} \{0\} & \text{if } r = 0, \\ \dfrac{1}{r}\cdot\mathbb{Z} & \text{if } 0 < r < \infty, \\ \mathbb{R} & \text{if } r = \infty, \end{cases}$$

is a homeomorphism.

We leave the proof of the following two propositions to the reader.

PROPOSITION 2.5. *The mapping* $\phi_{\mathbb{T}} : (\mathbb{N}\backslash\{0\}) \cup \{\infty\} \longrightarrow \mathcal{SUB}(\mathbb{T})$ *defined by*

$$\phi_{\mathbb{T}}(n) = \begin{cases} \{0\} & \text{if } n = 1, \\ \dfrac{1}{n}\cdot\mathbb{Z}/\mathbb{Z} & \text{if } n > 1, \\ \mathbb{T} & \text{if } n = \infty, \end{cases}$$

is a homeomorphism.

PROPOSITION 2.6. *The mapping* $\phi_{\mathbb{Q}_p} : \mathbb{Z} \cup \{\pm\infty\} \longrightarrow \mathcal{SUB}(\mathbb{Q}_p)$ *defined by*

$$\phi_{\mathbb{Q}_p}(k) = \begin{cases} \{0\} & \text{if } k = -\infty, \\ \dfrac{1}{p^k}\cdot\mathbb{Z}_p & \text{if } k \in \mathbb{Z}, \\ \mathbb{Q}_p & \text{if } k = +\infty, \end{cases}$$

is a homeomorphism.

In contrast with the previous examples, the situation is subtle even for apparently simple examples of groups like the additive group $\mathbb{R}^n$. For instance, the first nontrivial description of Chabauty spaces is due to Pourezza and Hubbard in [16], where they proved the following result.

PROPOSITION 2.7. *The Chabauty space* $\mathcal{SUB}(\mathbb{R}^2)$ *is homeomorphic to* $\mathbb{S}^4$*, the four-dimensional sphere.*

REMARK 2.8. For $n \geq 3$, the Chabauty space $\mathcal{SUB}(\mathbb{R}^n)$ is connected and simply connected. However, it is not a topological manifold and its structure is far from understood (see [15]).

2.3. Chabauty segments. Let G be a topological group. For two closed subgroups H and K of G such that $H \subseteq K$, we define the Chabauty segment of H and K by

$$[\![H, K]\!] := \{L \in \mathcal{SUB}(G) \mid H \subseteq L \subseteq K\}.$$

DEFINITION 2.9 (Proper mapping). A continuous mapping $f : X \longrightarrow Y$ is said to be proper if X is a Hausdorff space, f is a closed mapping and all fibres $f^{-1}(y)$ are compact subsets of X.

PROPOSITION 2.10 [9, Proposition 2.1]. *If $\phi : G_1 \longrightarrow G_2$ is a proper morphism between locally compact topological groups, then the mapping*

$$\mathcal{SUB}(\phi) : \mathcal{SUB}(G_1) \longrightarrow \mathcal{SUB}(G_2), \quad H \longmapsto \phi(H)$$

is continuous. Moreover, if ϕ is surjective, then $\mathcal{SUB}(\phi)$ is surjective.

COROLLARY 2.11. *Let Γ be a closed subgroup of a locally compact group G and let $i : \Gamma \longrightarrow G$ be the canonical injection. Then the mapping*

$$\mathcal{SUB}(\Gamma) \longrightarrow [\![\{e\}, \Gamma]\!], \quad H \longmapsto i(H)$$

is a homeomorphism.

PROPOSITION 2.12 [9, Proposition 2.5]. *Let $\phi : G_1 \longrightarrow G_2$ be an open continuous morphism between locally compact topological groups. Then the mapping*

$$\mathcal{SUB}(G_2) \longrightarrow \mathcal{SUB}(G_1), \quad H \longmapsto \phi^{-1}(H)$$

is an embedding.

COROLLARY 2.13. *Let N be a closed normal subgroup of a locally compact group G and let $\pi : G \longrightarrow G/N$ be the canonical morphism. Then the mapping*

$$\mathcal{SUB}(G/N) \longrightarrow [\![N, G]\!], \quad H \longmapsto \pi^{-1}(H)$$

is a homeomorphism.

REMARK 2.14. Chabauty segments of a locally compact group are compact.

3. Connectedness of the Chabauty space of a pronilpotent group

We begin our investigation of the connectedness by providing two lemmas about the additive group of real numbers. The first one provides a criterion for the existence of a closed copy of $\mathbb{R}$ in locally compact groups.

LEMMA 3.1 [19, page 156]. *Let G be a locally compact group. Then the following statements are equivalent.*

(1) *G contains a subgroup topologically isomorphic to $\mathbb{R}$.*
(2) *The connected component of the identity is not compact.*

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press

The second lemma deals with the structure of an extension of a compact normal subgroup by a group topologically isomorphic to $\mathbb{R}$.

LEMMA 3.2 [12, Remark 1.9]. *Let K be a compact normal subgroup of a locally compact group G such that $G/K \cong \mathbb{R}$. Then $G \cong \mathbb{R} \times K$.*

For a topological group G, we shall denote by $\mathcal{Z}(G)$ the centre of G and by $\mathcal{E}(G)$ the connected component of the singleton subgroup in the Chabauty space $\mathcal{SUB}(G)$.

PROPOSITION 3.3. *Let G be a noncompact connected nilpotent Lie group. Then $\mathcal{Z}(G)$ belongs to $\mathcal{E}(G)$.*

PROOF. If $\mathcal{Z}(G)$ is not compact, the result follows immediately from combining Lemma 3.1 and Theorem 1.5. Now let us suppose that $\mathcal{Z}(G)$ is compact. In view of Lemma 3.1, G contains a closed subgroup N that is topologically isomorphic to $\mathbb{R}$. Then, by combining Lemma 3.2 and Theorem 1.5, $\mathcal{SUB}(N\mathcal{Z}(G))$ is connected and therefore $\mathcal{Z}(G) \in \mathcal{E}(G)$. □

REMARK 3.4. If G is a compact connected nilpotent group, then it is a pro-torus. It follows then that the Chabauty space $\mathcal{SUB}(G)$ is totally disconnected. For more details about the total disconnectedness of Chabauty spaces, we refer to [11].

PROPOSITION 3.5. *Let G be a locally compact group and let $\mathcal{N}(G)$ be the compact space of closed normal subgroups of G. If K is a compact subgroup of G, then the mapping*

$$\mathcal{SUB}(G) \times (\mathcal{SUB}(K) \cap \mathcal{N}(G)) \longrightarrow \mathcal{SUB}(G), \quad (H, L) \longmapsto HL$$

is continuous.

PROOF. Let $(H_i, L_i)_{i\in I}$ be a net of $\mathcal{SUB}(G) \times (\mathcal{SUB}(K) \cap \mathcal{N}(G))$ converging to (H, L). In order to prove the continuity, we shall prove that the net $(H_iL_i)_{i\in I}$ converges to HL in $\mathcal{SUB}(G)$. So, let U be an open set of G such that $HL \in \mathcal{O}_2(U)$ and let $(h, k) \in HL \cap U$. Taking an open neighbourhood $U_h \times U_k$ of (h, k) such that $U_hU_k \subset U$ shows that the net $(H_i, L_i)_i$ is eventually in $\mathcal{O}_2(U_h) \times \mathcal{O}_2(U_k)$. Clearly, this implies that the net $(H_iL_i)_i$ is eventually in $\mathcal{O}_2(U)$. Now let C be a compact subset of G such that $HL \in \mathcal{O}_1(C)$. Aiming for a contradiction, assume that there is an infinite subset J of I such that $H_jL_j \cap C \neq \varnothing$ for every $j \in J$. So, let $(g_j)_j = (h_jk_j)_j$ be a net of elements of G such that for every $j \in J$, we have $(h_j, k_j) \in H_j \times L_j$ and $g_j \in C$. Moreover, in light of the compactness of C and K, we may assume that the net $(g_j, k_j)_j$ is convergent to some $(g, k) \in C \times K$. Thus, $k \in L$ in view of Proposition 2.2. On the other hand, by applying Proposition 2.2 again, we deduce that the net $(h_j)_j$ converges to gk^{-1} in H and therefore g belongs to HL. This contradiction proves the claim and finishes the proof. □

REMARK 3.6. In the previous result, the normality of compact subgroups was needed only to show that the products are also subgroups. Thus, with the same hypothesis as

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press

above, the mapping

$$\mathcal{N}(G) \times \mathcal{SUB}(K) \longrightarrow \mathcal{SUB}(G), \quad (H, L) \longmapsto HL$$

is also continuous.

DEFINITION 3.7 (Compact-free group). A topological group is called compact-free if it has no compact subgroup except the trivial one.

PROPOSITION 3.8. *Let G be a noncompact connected nilpotent Lie group. If $\mathcal{Z}(G)$ is not compact-free, then the Chabauty space $\mathcal{SUB}(G)$ is connected.*

PROOF. We proceed by induction on the nilpotency class c of G. If G is abelian, then the connectedness follows from Theorem 1.5. Now suppose that G is of class $c > 1$ and the connectedness is guaranteed for lower nilpotency classes. Since the centre of G is not compact-free, one can find $(K_n)_{n\in\mathbb{N}}$, a sequence of nontrivial compact central subgroups converging to the identity subgroup. Moreover, it is clear that the subgroups K_n are not cocompact. By the induction hypothesis, all the Chabauty segments $[\![K_n, G]\!]$ are connected. Thus, in view of Proposition 3.3, $(K_n)_n \subset \mathcal{E}(G)$. On the other hand, if H is a closed subgroup of G, then $HK_n \in [\![K_n, G]\!] \subset \mathcal{E}(G)$ for every $n \in \mathbb{N}$. Thus, Proposition 3.5 makes $H \in \mathcal{E}(G)$. Hence, the Chabauty space $\mathcal{SUB}(G)$ is connected, as desired. □

Before treating the compact-free case, we need to deal with the following concept.

DEFINITION 3.9 ($\mathcal{Z}$-Group). We say that a locally compact group G is a $\mathcal{Z}$-group if the factor group of G modulo its centre is compact.

PROPOSITION 3.10 (Structure theorem for $\mathcal{Z}$-groups; [7, Theorem 4.4]). *Every $\mathcal{Z}$-group is the direct product of a vector group and a locally compact group having a normal compact open subgroup.*

As an immediate consequence of this result and Theorem 1.6, we obtain the following generalisation of Theorem 1.5.

COROLLARY 3.11. *Let G be a prosolvable $\mathcal{Z}$-group. Then the following properties are equivalent.*

(1) *The Chabauty space $\mathcal{SUB}(G)$ is connected.*
(2) *G contains a subgroup topologically isomorphic to $\mathbb{R}$.*

We shall also need the following lemma.

LEMMA 3.12 [2, Lemma 1.1]. *Let H and N be two closed subgroups of a connected and simply connected nilpotent Lie group G. If N is connected and normal, then the product subgroup HN is closed in G.*

NOTATION 3.13 (Derived subgroup). If G is a group, we denote its derived subgroup by $\mathcal{D}(G)$.

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press

PROPOSITION 3.14. *Let G be a noncompact connected nilpotent Lie group. If $\mathcal{Z}(G)$ is compact-free, then the Chabauty space $\mathcal{SUB}(G)$ is connected.*

PROOF. We argue by induction on the nilpotency class c of G. If G is abelian, then the connectedness follows from Theorem 1.5. Now suppose that G is of class $c > 1$ and the connectedness is ensured for lower nilpotency classes. Let N be a closed central subgroup of G that is topologically isomorphic to the group of real numbers $\mathbb{R}$ and let H be an arbitrary closed subgroup of G. In order to show that H belongs to the connected component $\mathcal{E}(G)$, we shall distinguish two cases.

Case 1: $N \cap \mathcal{D}(H) \neq \{e\}$. If the subgroup $N \cap \mathcal{D}(H)$ is cocompact, then the connectedness follows immediately from Corollary 3.11. If this is not the case, then in light of the induction hypothesis, H belongs to the connected Chabauty segment $[\![N \cap \mathcal{D}(H), G]\!]$. Hence, the claim follows immediately from Proposition 3.3.

Case 2: $N \cap \mathcal{D}(H) = \{e\}$. According to Lemma 3.14 in [13], we know that G is simply connected. Then, by Lemma 3.12, the subgroups NH and $N\mathcal{D}(H)$ are closed in G. Let π be the canonical morphism from NH onto $NH/\mathcal{D}(NH)$. It is clear that $\mathcal{D}(NH) = \mathcal{D}(H)$. Then $\pi(N)$ is isomorphic to N and therefore the Chabauty segment $[\![\mathcal{D}(H), NH]\!]$ is connected. On the other hand, in light of the induction hypothesis and Corollary 3.11, the Chabauty segment $[\![N, G]\!]$ is also connected. Hence, $H \in \mathcal{E}(G)$ since HN belongs to $[\![\mathcal{D}(H), NH]\!] \cap [\![N, G]\!]$. □

By combining Propositions 3.8 and 3.14, we obtain the following result.

THEOREM 3.15. *Let G be a connected nilpotent Lie group. Then the Chabauty space $\mathcal{SUB}(G)$ is connected if and only if G contains a subgroup topologically isomorphic to $\mathbb{R}$.*

We say that a subgroup N of a topological group G is a co-Lie subgroup if it is normal and the quotient G/N is a Lie group. Furthermore, we shall denote by $\mathcal{N}_c(G)$ the set of all compact co-Lie subgroups of G.

PROPOSITION 3.16 [9, Theorem 4.7]. *Let G be a locally compact pro-Lie group. Then the following statements are equivalent.*

(1) *The space $\mathcal{SUB}(G)$ is connected.*
(2) *For every $N \in \mathcal{N}_c(G)$, the space $\mathcal{SUB}(G/N)$ is connected.*

Finally, by combining Theorem 3.15 and Proposition 3.16, we deduce the main theorem of the paper.

THEOREM 3.17 (Main theorem = Theorem 1.10). *Let G be a connected locally compact pronilpotent group. Then the Chabauty space $\mathcal{SUB}(G)$ is connected if and only if G contains a subgroup topologically isomorphic to $\mathbb{R}$.*

We conclude the paper by taking a look at simple groups. We start by dealing with the following concept.

DEFINITION 3.18 (Topologically perfect group). We say that a topological group G is topologically perfect if the derived subgroup $\mathcal{D}(G)$ is dense in G.

It is shown, in Proposition 2.2 of [1], that the Chabauty space of a nontrivial topologically perfect group is never connected. More precisely, the following characterisation is given.

PROPOSITION 3.19. *Let G be a connected Lie group. Then G is an isolated point in $\mathcal{SUB}(G)$ if and only if G is topologically perfect.*

On the other hand, we recall that a topological group is said to be topologically simple if it has no proper closed normal subgroups. Then, by combining Proposition 3.19 and Lemma 5 in [18], we obtain the following result.

COROLLARY 3.20. *Let G be a locally compact group. If G is topologically simple, then $\mathcal{SUB}(G)$ is not connected.*

Finally, an important example arises out of this discussion.

EXAMPLE 3.21. For every $n \geq 2$, the Chabauty space of the special linear group $\mathrm{SL}_n(\mathbb{R})$ is not connected.

References

[1] M. Abert, N. Bergeron, I. Biringer, T. Gelander, N. Nikolov, J. Raimbault and I. Samet, 'On the growth of L^2-invariants for sequences of lattices in Lie groups', *Ann. Math.* **185** (2017), 711–790.

[2] M. Bekka and E. Kaniuth, 'Topological Frobenius properties for nilpotent groups', *Math. Scand.* **63** (1988), 282–296.

[3] R. Benedetti and C. Petronio, *Lectures on Hyperbolic Geometry* (Springer, Berlin, 1992).

[4] N. Bourbaki, *Intégration, Éléments de Mathématique* (Springer, Heidelberg, 2007), Chs. 7–8.

[5] M. Bridson, P. de la Harpe and V. Kleptsyn, 'The Chabauty space of closed subgroups of the three-dimensional Heisenberg group', *Pacific J. Math.* **240** (2009), 1–48.

[6] Y. Cornulier, 'On the Chabauty space of locally compact abelian groups', *Algebr. Geom. Topol.* **11** (2011), 2007–2035.

[7] S. Grosser and M. Moskowitz, 'On central topological groups', *Trans. Amer. Math. Soc.* **127** (1967), 317–340.

[8] T. Haettel, 'L'espace des sous-groupes fermés de $\mathbb{R} \times \mathbb{Z}$', *Algebr. Geom. Topol.* **10** (2010), 1395–1415.

[9] H. Hamrouni and B. Kadri, 'On the compact space of closed subgroups of locally compact groups', *J. Lie Theory* **23** (2014), 715–723.

[10] H. Hamrouni and B. Kadri, 'On the connectedness of the Chabauty space of a locally compact prosolvable group', *Adv. Pure Appl. Math.* **6** (2015), 97–111.

[11] H. Hamrouni and B. Kadri, 'Locally compact groups with totally disconnected space of subgroups', *J. Group Theory* **22** (2019), 119–132.

[12] W. Hazod, K. H. Hofmann, H. P. Scheffler, M. Wüstner and H. Zeuner, 'Normalizers of compact subgroups, the existence of commuting automorphisms, and application to operator semistable measures', *J. Lie Theory* **8** (1998), 189–209.

[13] K. Iwasawa, 'On some types of topological groups', *Ann. of Math. (2)* **50** (1949), 507–558.

[14] E. I. Khukhro and V. D. Mazurov, *Unsolved Problems in Group Theory, The Kourovka Notebook 18* (Russian Academy of Sciences Siberian Division, Institute of Mathematics, Novosibirsk, 2014).

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press

[15] B. Kloeckner, 'The space of closed subgroups of $\mathbb{R}^n$ is stratified and simply connected', *J. Topol.* **2** (2009), 570–588.
[16] I. Pourezza and J. Hubbard, 'The space of closed subgroups of $\mathbb{R}^2$', *Topology* **18** (1979), 143–146.
[17] I. V. Protasov, 'Local theorems for topological groups', *Math. USSR Izv.* **15** (1980), 625–633.
[18] I. V. Protasov and Y. V. Tsybenko, 'Connectedness in the space of subgroups', *Ukraïn. Mat. Zh.* **35** (1983), 382–385 (in Russian); English translation: *Ukrainian Math. J.* **35** (1983), 332 –334.
[19] M. Stroppel, *Locally Compact Groups* (European Mathematical Society, Berlin, 2006).

BILEL KADRI, Sfax Preparatory Engineering Institute,
Department of Mathematics, Sfax University, 3018 Sfax, Tunisia
e-mail: bilel.kadri@yahoo.fr

https://doi.org/10.1017/S0004972721000319 Published online by Cambridge University Press