BULGE AND DISK: A SIMPLE SELF-GRAVITATING MODEL

S.A. KUTUZOV and L.P. OSSIPKOV
St. Petersburg University, Russia

A method of finding the distribution function of some steady-state axially symmetrical mass models was suggested by Ossipkov, Kutuzov (1987). The models consist of a disk embedded in a bulge (halo). The total potential $\phi(R,z) = \phi_0\varphi(\xi)$, with $\phi_0 = \phi(0,0)$, $\varphi(\xi)$ arbitrary,

$$\xi^2 = \rho^2 + 2\mu|\zeta| + \zeta^2, \qquad \mu = const \geq 0 \tag{1}$$

and $(R, z) resp. (\rho, \zeta)$ dimensional resp. dimensionless cylindrical coordinates respectively. The total dimensionless configuration density has the following form:

$$\nu(\rho,\zeta) = \nu_b(\xi) + \delta(\zeta)\sigma_d(\rho) \tag{2}$$

where ν_b is the bulge density, σ_d is the disk surface density and $\delta(\zeta)$ is the Dirac delta function. The bulge lenslike equidensity surfaces coincide with equipotential ones. The parameter μ determines their flattening: they are spherical if $\mu = 0$ and flat if $\mu = \infty$. The following expressions are found for bulge and disk densities:

$$\nu_b(\xi) = 3\omega^2(\xi) + 2(\mu^2 + \xi^2)\frac{d\omega^2}{d(\xi^2)}, \qquad \sigma_d(\rho) = 2\mu\omega^2(\rho) \tag{3}$$

where $\omega^2(\xi) = -2d\varphi(\xi)/d(\xi^2)$, $\omega(\rho)$ is a dimensionless circular frequency. Both densities are connected with each other by means of the equations (3).

The distribution function has the following form:

$$\Psi(E,e,h) = \Psi_b(E) + \delta(\zeta)\delta(\mathcal{U}_\zeta)\Psi_d(e,h) \tag{4}$$

where $E = \varphi(\xi) - (\mathcal{U}_\rho^2 + \mathcal{U}_\theta^2 + \mathcal{U}_\zeta^2)/2$ and $e = \varphi(\rho) - (\mathcal{U}_\rho^2 + \mathcal{U}_\theta^2)/2$ are the energy integrals of spatial and flat motion respectively, $h = \rho\mathcal{U}_\theta$ is an integral of angular momentum, $\mathcal{U}_\rho, \mathcal{U}_\theta, \mathcal{U}_\zeta$ are dimensionless velocity components in cylindrical coordinates. The bulge does not rotate and its velocity distribution is isotropic. The bulge distribution function can be found as a solution of the integral equation:

$$\sqrt{8}\pi^2\Psi_b(E) = \frac{d^2}{dE^2}\left(\int_0^E (E-\varphi)^{\frac{1}{2}}G(\varphi)d\varphi\right) \tag{5}$$

Here $G(\varphi) = \nu_b(\xi(\varphi))$ is an augmented density (Dejonghe 1987) but the inverted potential law $\xi(\varphi)$ is supposed to be a single-valued function.
The disk phase density is decomposed into even and odd components with respect to the azimuthal velocity (and h):

$$\Psi_d(e,h) = \Psi_+(e,h) + \Psi_-(e,h) \tag{6}$$

The even component can be found on the basis of the surface density by known methods (Kalnajs 1976, Ossipkov 1978). In particular if it depends on the e only then (Dekker 1976)

H. Dejonghe and H. J. Habing (eds.), Galactic Bulges, 367–368.

$$2\pi\Psi_+(e) = \frac{dg(e)}{de} \tag{7}$$

where $g(\varphi) = \sigma_d(\rho(\varphi))$ is an augmented surface density. The odd component is determined by a rotation law $\langle \mathcal{U}_\theta \rangle = \rho\,\Omega(\rho)$. It is assumed to be separable in e and h, that gives rise to

$$\Psi_-(e,h) = u(e)h, \qquad 2\pi u(e) = \frac{d^2 f(e)}{de^2} \tag{8}$$

where $f(\varphi) = g(\varphi)\Omega(\rho(\varphi))$.

As an example we consider the particular cases of the Kuzmin-Malasidze (1969) potential law

$$\varphi(\xi) = \alpha\left(\alpha - 1 + (1 + \kappa\xi^2)^{\frac{1}{2}}\right)^{-1}, \qquad \alpha,\ \kappa = const > 0 \tag{9}$$

and suggest the rotation law

$$\Omega(\rho(\varphi)) = a[1 - b\varphi^p(1-\varphi)^q]\varphi^\tau, \qquad a, b, p, q, \tau > 0 \tag{10}$$

which allows a rotation curve with one minimum and flat outer part. Some restrictions on the functions and the parameters are established from the condition of non-negativity of the $\Psi_d(e,h)$. The rotation velocity $\langle \mathcal{U}_\theta \rangle$ has to be considerably smaller everywhere than the circular one $\mathcal{U} = \rho\,\omega(\rho)$. In the case of $\alpha = \kappa\mu^2 = 1$ there is a pure Kuzmin-Toomre disk (Kuzmin 1953, Toomre 1963) without bulge. Then $\tau > 3/2$ and $\alpha \leq 0.30$ for $\tau = 1.6$, $b = 0$.

References

Dejonghe, H., 1987, *Inst.Adv.Studies Preprint*, IASSNS-AST 87/10
Dekker, E., 1976, *Phys.Rep.*, **24c**, 315
Kalnajs, A., 1976, *Ap.J.*, **205**, 751
Kuzmin, G.G., 1953, *Isv.Akad.Nauk Est.SSR*, **5**, 369 (Tartu Teated, No 1)
Kuzmin, G.G., Malasidze, G.A. 1969 (1970), *Tartu Publ.*, **38**, 181
Ossipkov, L.P. 1978, *Pis'ma Astr.Zh.*, **4**, 70
Ossipkov, L.P., Kutuzov, S.A. 1987, *Astrofizika*, **27**, 523
Toomre, A. 1963. *Ap.J.*, **138**, 385

https://doi.org/10.1017/S0074180900123630 Published online by Cambridge University Press