International Journal of Microwave and Wireless Technologies, 2017, 9(5), 1093–1102.
doi:10.1017/S1759078716001100

RESEARCH PAPER

Effects of bi-isotropic coatings and bi-isotropic background media upon gain characteristics of an axially slotted cylinder

Z. A. AWAN

An analysis about the effects of various bi-isotropic (BI) coatings and BI background media upon the gain characteristics of an axially slotted cylinder using numerical simulations is presented. It is investigated that chirality and Tellegen parameters of a coating and a background medium play a significant role in modifying the gain properties. It is further studied that an axially slotted cylinder when coated with a strong BI material and embedded in a free space background significantly enhances the gain in the forward direction. On the other hand, a strong Tellegen coating guides most of the radiated field from an axial slot toward rear side of the cylinder.

Keywords: Axial slot, Chiral material, Tellegen material, Slotted cylinder, Gain Pattern

Received 11 March 2016; Revised 24 August 2016; Accepted 25 August 2016; first published online 11 November 2016

I. INTRODUCTION

Several investigations have been carried by many authors on bi-isotropic (BI) media due to their important technical applications in the microwave and optical regimes [1–19]. The problems of wave propagation in an unbounded, half-space and multilayered BI media have been considered in [1–4]. It is studied that a BI medium affects the polarization and phase of a propagating wave inside it. The phenomenon of a negative refraction for a circularly polarized wave incident on a BI half-space has been recently studied by Monzon and Forester [5]. The scattering of electromagnetic waves from different types of BI scatterers [6–9] and BI-coated targets [10–13] have been analyzed by several authors. A general description about the radiation and scattering in a BI medium is given by Monzon [14]. He has used field decomposition method in the presence of sources to simplify derivations of dyadic Green's functions. The influences of chirality and non-reciprocity on propagation features of different types of BI filled waveguides have been investigated in [15–19]. These waveguides have interesting potential applications in the field of microwave and optical engineering. A Tellegen medium is taken to be a subclass of a BI medium. It was first suggested by Tellegen [20] how to generate macroscopic magneto-electric effect. He introduced a new passive circuit element, which he called gyrator. He also suggested a constitutive relation to explain the magneto-electric effect in these materials. Since then, Tellegen media have gained the attention of many scientists, see for example, [21–34]. It is important to note that a Tellegen medium is characterized by a spontaneous non-reciprocal magnetoelectric coupling and the most well-known naturally occurring example of such material is chromium oxide Cr_2O_3 [29, 30]. It is suggested by Mong *et al.* [31] that antiferromagnetic topological insulators also have spontaneous magnetoelectric effect similar to Cr_2O_3. Recently, some interesting applications of Tellegen media with spontaneous magnetoelectric coupling have been studied by Prudêncio *et al.* [32–34].

The radiation and gain properties of an axially slotted cylinder become more controllable if it is coated with some material and have been investigated by many authors [35–52]. Hurd [35] investigated the radiation properties of a dielectric-coated axially slotted cylinder and also made comparisons with experimental results. Wait and Mienteka analyzed the fields produced by an arbitrary slot on a circular cylinder with dielectric and permeable coatings [36]. They studied that a thin permeable coating has significant effects upon the radiation pattern as compared with a thin dielectric coating. Shafai [37] has studied the radiation properties of an axial slot coated with a homogenous material. The external admittance of a dielectric-coated axially slotted cylinder has been investigated by Knop [38]. The radiation and gain properties of a dielectric-coated cylinder with two slots have been studied by Mushref [39, 40].

The effects of different types of plasma coatings upon the radiation characteristics of an axially slotted cylinder have been investigated by some authors [41–43]. Marchin and Tyras [43] have studied that the reversal of the orientation of the magnetostatic field causes a shift of the pattern to a mirror image position. The effects of inhomogeneous coatings upon the radiation properties of an axial slot have been studied in [44, 45]. It is shown that the radiation can be made to spread out toward rear side of the cylinder by

Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan
Corresponding author:
Z. A. Awan
Email: zeeshan@qau.edu.pk

making the inhomogeneous coating more thick. An elliptic slotted cylinder with dielectric and metamaterial coatings and their radiation properties are investigated by some authors [46–48]. It is studied by Hamid in [46] that a metamaterial coating can be used to enhance the gain in the forward direction (FD) as compared with a dielectric coating. The effects of background metamaterial upon the gain pattern of a dielectric-coated axially slotted cylinder have been studied by Awan [49]. Recently, the effects of two different coating layers upon the gain properties of an axially slotted cylinder are investigated by Awan in [50].

The radiation characteristics of longitudinal and circumferential slots engraved on a circular perfectly conducting cylinder with chiral coating and free space background have been studied by Mahmoud [51]. Awan [52] has studied the effects of chiral nihility coating, various chiral coatings and chiral background media upon the gain pattern of a chiral-coated axially slotted cylinder. According to the author, no one has considered the effects of BI coatings and BI background media upon the gain of an axially slotted cylinder. Also the effects of various Tellegen coatings and Tellegen background media upon the gain of an axially slotted cylinder have not been explored previously. These effects are investigated in the current paper.

In this paper, an infinite axial slot is considered for a perfectly conducting hollow cylinder which is coated with a BI layer. This BI-coated axially slotted cylinder is embedded in another BI background medium. The effects of different BI coatings and BI background media upon the gain pattern of an axially slotted cylinder have been studied. Also the effects of Tellegen background media and coatings upon the gain pattern have been investigated. This work may be considered as an extension of the previous work of chiral media [52], in order to investigate the general BI media by incorporating the effect of non-reciprocity. It is studied that an axially slotted cylinder coated with a strong BI material and embedded in a free space background can significantly enhance the gain in the FD. Also it reduces the gain in the backward direction (BD). It is an important result and has application in designing of a new kind of antenna with maximum gain in the FD.

II. PROBLEM FORMULATION

It is known that an isotropic and homogeneous BI medium with the relative permittivity ϵ_r, relative permeability μ_r, chirality parameter κ_r, and Tellegen parameter χ_r has the following constitutive relations [1]:

$$\mathbf{D} = \epsilon_o \epsilon_r \mathbf{E} + (\chi_r - j\kappa_r)\sqrt{\epsilon_o \mu_o \epsilon_r \mu_r}\,\mathbf{H}, \tag{1}$$

$$\mathbf{B} = \mu_o \mu_r \mathbf{H} + (\chi_r + j\kappa_r)\sqrt{\epsilon_o \mu_o \epsilon_r \mu_r}\,\mathbf{E}, \tag{2}$$

where ϵ_o and μ_o represent the permittivity and permeability of the free space, respectively. The assumed time convention is $e^{j\omega t}$ and has been suppressed throughout. In the current study, all parameters ϵ_r, μ_r, κ_r, and χ_r of considered BI media are taken to be real valued, which ensures a lossless condition.

In a BI medium, the chiral parameter κ_r affects the polarization of propagating electric field, whereas the Tellegen parameter affects its phase. Due to inherent birefringence nature of the BI medium, there exists two types of waves in the medium. These waves are taken to be a right circularly polarized (RCP) wave and a left circularly polarized (LCP) wave. Also they propagate in a BI medium with different phase velocities which shows that such a BI medium has two different bulk wave numbers, i.e., k_r and k_l. These wave numbers can be expressed in mathematical form as,

$$k_r = k_o n_r, \qquad k_l = k_o n_l, \tag{3}$$

$$\begin{aligned} n_r &= \sqrt{\epsilon_r \mu_r}\left(\sqrt{1-\chi_r^2} + \kappa_r\right), \\ n_l &= \sqrt{\epsilon_r \mu_r}\left(\sqrt{1-\chi_r^2} - \kappa_r\right), \end{aligned} \tag{4}$$

where k_o is a free space wave number, and n_r, n_l are the refractive indices. The wave impedances associated with RCP and LCP waves are taken to η_r and η_l, respectively. They are defined as below:

$$\begin{aligned} \eta_r &= \eta_o \sqrt{\frac{\mu_r}{\epsilon_r}}\left(\sqrt{1-\chi_r^2} - j\chi_r\right), \\ \eta_l &= \eta_o \sqrt{\frac{\mu_r}{\epsilon_r}}\left(\sqrt{1-\chi_r^2} + j\chi_r\right), \end{aligned} \tag{5}$$

where the factor η_o represents the intrinsic impedance of the free space. A BI medium becomes a Tellegen medium provided that if a chirality parameter $\kappa_r = 0$ and a Tellegen parameter is non-zero, i.e., $\chi_r \neq 0$. In this case, wave numbers associated with RCP and LCP waves become same, whereas corresponding wave impedances are different. That is why, a Tellegen medium is bi-impedant. On the other hand, for a reciprocal chiral medium where $\chi_r = 0$ and $\kappa_r \neq 0$, we have different wave numbers associated with RCP and LCP waves, whereas both waves have same wave impedances. Due to this property, a chiral medium is bi-refringent. It should be noticed that for a BI medium, the two refractive indices, i.e., n_r, n_l are positive provided that if the following condition on chirality and Tellegen parameters of the medium is satisfied [1], i.e.,

$$\chi_r^2 + \kappa_r^2 < 1. \tag{6}$$

If the condition given in equation (6) is not satisfied, then the nature of the BI medium radically changed and is discussed in Section III.

An infinite length hollow cylinder of perfectly conducting material with an axial slot of angular width ϕ_o is considered. The radius of this cylinder is taken to be a. The axis of the cylinder coincides with the z-axis of the coordinates system and this axial slot is parallel to the cylinder axis. This axially slotted cylinder is coated with a BI material. The radius of outer BI coating is taken to be b. This coating occupies a region II and lies in the range $a \leq \rho \leq b$, which is characterized by parameters ϵ_{r2}, μ_{r2}, κ_{r2}, and χ_{r2}. This BI-coated axially slotted cylinder is placed in another BI background which lies in the range $\rho > b$ and occupies a region I. This

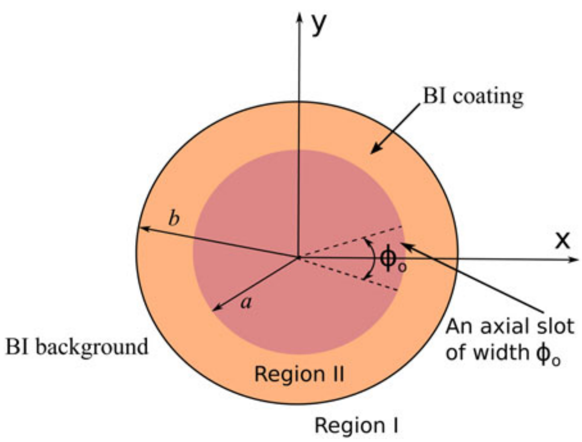

Fig. 1. The geometry of an axially slotted cylinder coated with a BI medium and embedded in another BI background.

background BI medium has parameters of ϵ_{r1}, μ_{r1}, κ_{r1}, and χ_{r1}. This geometry is shown in Fig. 1.

An axial slot is assumed to be narrow which maintains a constant electric field distribution E_o across it for $|\phi| \leq \phi_o/2$. This type of constant field distribution gives rise to only circumferential currents with no axial currents. This ensures that the ϕ-component of an electric field and z-component of a magnetic field are non-zero, i.e., $E_\phi \neq 0$ and $H_z \neq 0$. This type of excitation can be taken as a transverse electric (TE) type. Thus, the electric field distribution of an axial slot can be expressed in terms of a complex Fourier series using [42, 44] as,

$$\mathbf{E} = \hat{\phi} E_\phi = \hat{\phi}\left[\frac{E_o}{\pi}\sum_{n=-\infty}^{n=+\infty}\frac{\sin(n\phi_o/2)}{n}e^{jn\phi}\right]. \tag{7}$$

It is clear that an axial slot excites only TE wave but due to polarization rotation of a BI medium, the electric and magnetic fields in the background BI medium have both transverse electric (TE) and transverse magnetic (TM) components. Therefore, the electric and magnetic fields in the background BI medium can be written as using [13, 52],

$$E_z^I = E_o\sum_{n=-\infty}^{n=+\infty}\left[A_n^+ H_n^{(2)}(k_{1r}\rho) - B_n^+ H_n^{(2)}(k_{1l}\rho)\right]e^{jn\phi}, \tag{8}$$

$$E_\phi^I = -E_o\sum_{n=-\infty}^{n=+\infty}\left[A_n^+ H_n^{(2)'}(k_{1r}\rho) + B_n^+ H_n^{(2)'}(k_{1l}\rho)\right]e^{jn\phi}, \tag{9}$$

$$H_z^I = jE_o\sum_{n=-\infty}^{n=+\infty}\left[\frac{A_n^+}{\eta_{1r}} H_n^{(2)}(k_{1r}\rho) + \frac{B_n^+}{\eta_{1l}} H_n^{(2)}(k_{1l}\rho)\right]e^{jn\phi}, \tag{10}$$

$$H_\phi^I = -jE_o\sum_{n=-\infty}^{n=+\infty}\left[\frac{A_n^+}{\eta_{1r}} H_n^{(2)'}(k_{1r}\rho) - \frac{B_n^+}{\eta_{1l}} H_n^{(2)'}(k_{1l}\rho)\right]e^{jn\phi}, \tag{11}$$

where k_{1r} and k_{1l} represent the wave numbers associated with RCP and LCP waves in the background BI medium. Likewise, η_{1r} and η_{1l} show the respective impedances. The function $H_n^{(2)}(\cdot)$ is the nth order Hankel function of the second kind and prime $'$ shows the derivative with respect to the argument. The superscript $\pm$ indicate if the wave propagates along the $+\rho$ or $-\rho$ direction. If the wave is propagating along $+\rho$ direction then we take it as an outward propagating wave, whereas if the wave is propagating along $-\rho$ direction then it is taken to be an inward propagating wave. As the background medium is unbounded therefore there exit no reflections at all and both RCP and LCP waves are taken to be outward propagating waves. The unknown coefficient A_n^+ is associated with an outward propagating RCP wave and unknown coefficient B_n^+ is related to an outward propagating LCP wave in the background region I. As there exist no reflected waves, so no unknown coefficients with superscript $(-)$ appear in equations (8)–(11). Also these unknown coefficients A_n^+ and B_n^+ are needed to be determined.

Likewise, the electric and magnetic fields in the BI coating which is taken to be a region II can be written in the following forms as,

$$E_z^{II} = E_o\sum_{n=-\infty}^{n=+\infty}\left[C_n^+ J_n(k_{2r}\rho) - D_n^+ J_n(k_{2l}\rho) + D_n^- Y_n(k_{2r}\rho) - C_n^- Y_n(k_{2l}\rho)\right]e^{jn\phi}, \tag{12}$$

$$E_\phi^{II} = -E_o\sum_{n=-\infty}^{n=+\infty}\left[C_n^+ J_n'(k_{2r}\rho) + D_n^+ J_n'(k_{2l}\rho) + D_n^- Y_n'(k_{2r}\rho) + C_n^- Y_n'(k_{2l}\rho)\right]e^{jn\phi}, \tag{13}$$

$$H_z^{II} = jE_o\sum_{n=-\infty}^{n=+\infty}\left[\frac{C_n^+}{\eta_{2r}} J_n(k_{2r}\rho) + \frac{D_n^+}{\eta_{2l}} J_n(k_{2l}\rho) + \frac{D_n^-}{\eta_{2r}} Y_n(k_{2r}\rho) + \frac{C_n^-}{\eta_{2l}} Y_n(k_{2l}\rho)\right]e^{jn\phi}, \tag{14}$$

$$H_\phi^{II} = -jE_o\sum_{n=-\infty}^{n=+\infty}\left[\frac{C_n^+}{\eta_{2r}} J_n'(k_{2r}\rho) - \frac{D_n^+}{\eta_{2l}} J_n'(k_{2l}\rho) + \frac{D_n^-}{\eta_{2r}} Y_n'(k_{2r}\rho) - \frac{C_n^-}{\eta_{2l}} Y_n'(k_{2l}\rho)\right]e^{jn\phi}, \tag{15}$$

where k_{2r} and η_{2r} represent the wave number and wave impedance associated with an RCP wave. On the other hand, k_{2l} and η_{2l} show the wave number and wave impedance associated with an LCP wave. Also $J_n(\cdot)$ and $Y_n(\cdot)$ are nth order Bessel's functions of first and second kinds, respectively. It is noticed that an outward propagating RCP wave in region II when reflected from a region II–region I interface it becomes an inward propagating LCP wave and vice versa. Therefore, an outward propagating RCP electric wave in region II, i.e., $E_o C_n^+ J_n(k_{2r}\rho)$ when strike a region II–region I interface, it becomes a reflected inward propagating LCP electric wave, i.e., $E_o C_n^- Y_n(k_{2l}\rho)$. Similar description holds true for unknowns D_n^+, D_n^-. The unknown coefficients A_n^+, B_n^+, C_n^+, D_n^+, C_n^-, and D_n^- can be determined by using the tangential

https://doi.org/10.1017/S1759078716001100 Published online by Cambridge University Press

boundary conditions at $\rho = a$ and $\rho = b$. Once these unknown coefficients are known then the electric and magnetic fields in each region can be found using equations (8)–(11) and equations (12)–(15). Using the theory outlined in [46, 47, 50, 52], the overall gain $G(\phi)$ of an axially slotted cylinder coated with a BI material and embedded in another BI background medium can be written as,

$$G(\phi) = G_R(\phi) + G_L(\phi), \quad (16)$$

$$G_R(\phi) = \frac{\sum_{n=-\infty}^{n=+\infty} |A_n^+ j^n e^{jn\phi}|^2}{\sum_{n=-\infty}^{n=+\infty} |A_n^+|^2}, \quad (17)$$

$$G_L(\phi) = \frac{\sum_{n=-\infty}^{n=+\infty} |B_n^+ j^n e^{jn\phi}|^2}{\sum_{n=-\infty}^{n=+\infty} |B_n^+|^2}. \quad (18)$$

The gain $G_R(\phi)$ is associated with an RCP wave, whereas the gain $G_L(\phi)$ is associated with an LCP wave in the background BI medium.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the effects of various coatings and background media upon the gain characteristics of an axially slotted cylinder have been shown and discussed. The considered types of coatings and background media are taken to be BI and Tellegen. The gain is computed using equations (16)–(18) provided that the unknown coefficients A_n^+ and B_n^+ are known. The radius of inner perfectly conducting cylinder is taken to $a = 0.5$ m, whereas an outer radius of the coating is assumed to be $b = 1$ m for an operating frequency of 0.3 GHz. The width of an axial slot ϕ_o is fixed at $\pi/100$ rad. This assumption ensures a thin slot. The value of E_o is taken to be unity. After several computations, it is known that the summation over n encountered in equations (17) and (18) can be truncated for $n = \pm 30$, which ensures the rapid convergence. For the present study, all BI and Tellegen media are taken to be lossless except the last Fig. 15, where BI media are taken to be lossy. Also a dielectric coating has $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$ for all the numerical results to come. It can be shown that the presented results are consistent with [36, 37, 44, 45] for a dielectric-coated axially slotted cylinder and with [51, 52] for a chiral-coated axially slotted cylinder. It should be noted that for a dielectric-coated axially slotted cylinder, an axial slot is centered at $\phi = 0^\circ$ therefore it is expected that the maximum radiation lies in this direction and is taken to be the FD. The radiated field from a slot travels around the periphery of the cylinder in both directions and is guided through coating toward rear side of the cylinder. Some of the radiated field from an axial slot penetrates into the background medium through a coating–background interface. It is noticeable that the field strength toward rear side of the cylinder is smaller than the field strength in the FD. A rear side of the cylinder is characterized by an angle $\phi = 180^\circ$. That is why, these two directions, i.e., forward and backward are emphasized for the coming analysis.

Figure 2 deals with the effects of different types of coatings, i.e., dielectric, chiral, Tellegen and BI upon the gain properties of an axially slotted cylinder embedded in a free space background. In this case, the relative permittivity and the relative permeability of all coatings are taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$, respectively. Likewise, we have $\kappa_{r2} = 0.95$ for a chiral coating, $\chi_{r2} = 0.95$ for a Tellegen coating and $\kappa_{r2} = \chi_{r2} = 0.95$ for a BI coating. It is clear from Fig. 2 that in the FD, i.e., $\phi = 0^\circ$, the gain of dielectric and Tellegen coatings is nearly same. On the other hand, a chiral coating enhances the gain in the FD as compared with a dielectric coating. The dominant enhancement in the FD gain is observed for a BI coating. In the BD, i.e., $\phi = 180^\circ$, a chiral coating reduces the gain, whereas a Tellegen coating significantly enhances the gain as compared with a dielectric coating. A BI coating has the lowest gain in the BD as compared to other coatings. From this analysis, it can be concluded that an optimum coating can be taken as a BI coating. This is because it has the highest gain in the FD and the lowest gain in the BD. It is interesting to note that for the considered chiral and Tellegen coatings, the condition given by equation (6) is satisfied and both refractive indices are positive. But in case of BI coating with $\chi_{r2} = \kappa_{r2} = 0.95$, the condition given by equation (6) is not satisfied. In this case, a refractive index associated with an RCP wave is positive, whereas a refractive index associated with an LCP wave become negative. Thus, an equivalent bulk wave number associated with an LCP wave also become negative. This shows that an LCP wave is converted into a backward wave, which is a known accepted concept. It is clear from Hamid [46] that if an axial slot is covered with a double negative or backward wave metamaterial coating then it enhances the gain in the FD as a compared with a conventional dielectric coating. Thus for a considered BI coating, the backward wave inside a coating and the surrounding free space background acts as an equivalent LC resonator for the radiated field from the slot which is centered at $\phi = 0^\circ$. In this way, a resonance phenomenon is associated with this equivalent LC resonator having a maximum value at $\phi = 0^\circ$. This allows negligible radiated field to be guided toward rear side of the cylinder through the coating region.

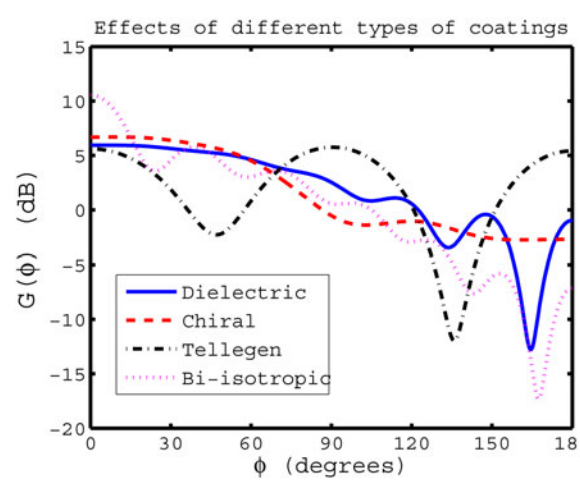

Fig. 2. Effects of different types of coatings upon the gain pattern of an axially slotted cylinder with the free space background. The relative permittivity and relative permeability of all coatings are taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$. For a chiral coating, $\kappa_{r2} = 0.95$, for a Tellegen coating, $\chi_{r2} = 0.95$ and for a BI coating, $\kappa_{r2} = 0.95$, $\chi_{r2} = 0.95$.

https://doi.org/10.1017/S1759078716001100 Published online by Cambridge University Press

Therefore, the radiated field in the BD reduces significantly. The gains associated with different coatings in the FD and the BD are tabulated in Table 1.

Figure 3 deals with the gain properties of a dielectric-coated axially slotted cylinder embedded in various Tellegen background media and their comparisons with a free space background. In this case, we have $\epsilon_{r1} = \epsilon_{r2} = 2$, $\mu_{r1} = \mu_{r2} = 1$, and χ_{r1} is taken to be variable. It is observed from Fig. 3 that for a relatively smaller value of $\chi_{r1} = 0.33$, the gain properties of a Tellegen background is almost same as that of a free space background for angles in the range $0^\circ < \phi \leq 61^\circ$. These gain characteristics are different for angles $\phi > 61^\circ$. As the value of χ_{r1} increases to 0.66 and 0.99, the gain pattern becomes more fluctuating with the highest value of 9.2034 dB in the FD for $\chi_{r1} = 0.99$.

The effects of various Tellegen coatings upon the gain pattern of an axially slotted cylinder embedded in the free space background are shown in Fig. 4. In this case, we have taken $\epsilon_{r2} = 2$, $\mu_{r2} = 1$ and χ_{r2} as a variable quantity. It is observed that for $\chi_{r2} = 0.33$, 0.66, the gain patterns are almost same as that of a dielectric coating for angles $0^\circ < \phi \leq 80^\circ$. These gain patterns of Tellegen coatings begin to differ from that of a dielectric coating for angles $\phi > 80^\circ$. On the other hand, when a Tellegen coating becomes a strong Tellegen medium, i.e., $\chi_{r2} = 0.99$ then the gain pattern is not fluctuating. In this case, the gain in the BD becomes large as compared to the gain in the FD. Thus, it can be concluded that a strong Tellegen coating guides most of the radiated field from an axial slot centered at $\phi = 0^\circ$ toward rear side of the cylinder.

Figure 5 shows the effects of background Tellegen parameter χ_{r1} upon the gains in the FD and BD of a dielectric-coated axially slotted cylinder. In this case, the assumed parameters are taken to be $\epsilon_{r2} = \epsilon_{r1} = 2$, $\mu_{r2} = \mu_{r1} = 1$, $\chi_{r2} = 0$, and variable χ_{r1}. It is obvious from Fig. 5 that as the value of χ_{r1} increases from 0.01 to 0.765, the gain in the FD decreases, whereas the gain in the BD increases gradually. It is also observed that for a strong Tellegen background where χ_{r1} is close to unity, the gain in the FD is large as compared with the gain in the BD.

The gains of an axially slotted cylinder in the FD and the BD as a function of Tellegen parameter of coating have been shown in Fig. 6. It is assumed that a Tellegen-coated slotted cylinder is embedded in the free space background. It is clear from Fig. 6 that gains in the FD and BD are fluctuating. The overall gain in the FD is greater than the gain in the BD for $0.01 \leq \chi_{r2} < 0.8915$. For a strong Tellegen coating, i.e., $\chi_{r2} = 0.99$, we have gain in the FD as 5.2017 dB and gain in the BD as 6.4980 dB.

Table 1. Comparative study of the gains in the FD and BD of an axially slotted cylinder coated with different coatings and embedded in the free space background.

Types of coatings	$G(\phi = 0^\circ)$ **(dB)**	$G(\phi = 180^\circ)$ **(dB)**
Dielectric coating	5.9413	−0.9612
Chiral coating	6.6806	−2.7047
Tellegen coating	5.6078	5.4195
BI coating	10.5621	−7.1280

The core radius is $a = 0.5$ m, whereas the coating layer has a radius of $b = 1$ m.

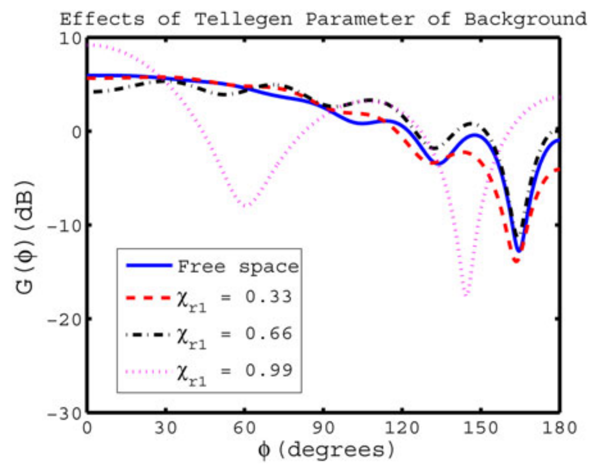

Fig. 3. Influences of Tellegen parameter of background media upon the gain pattern of a dielectric-coated axially slotted cylinder. The electromagnetic parameters of a dielectric coating is taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$. The parameters of the background Tellegen media are $\epsilon_{r1} = 2$, $\mu_{r1} = 1$ and χ_{r1} is taken to be variable.

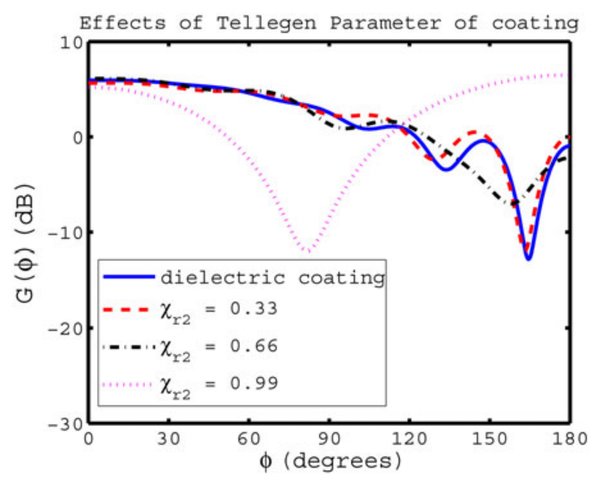

Fig. 4. Effects of Tellegen parameter of a Tellegen coating upon the gain and its comparison with a dielectric coating. The background medium is taken to be free space. For a Tellegen coating, we have assumed $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, and χ_{r2} as variable.

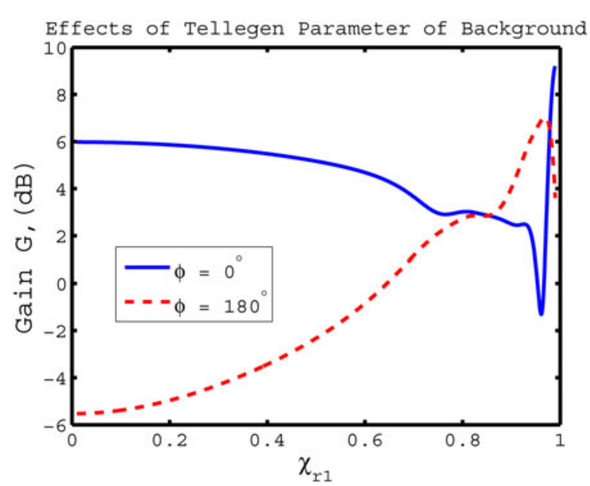

Fig. 5. Effects of Tellegen parameter of the background medium upon the gains in the FD, i.e., $\phi = 0^\circ$ and in the BD, i.e., $\phi = 180^\circ$ of a dielectric-coated axially slotted cylinder. The dielectric coating has parameters of $\epsilon_{r2} = 2$, $\mu_{r2} = 1$. The background Tellegen medium has $\epsilon_{r1} = 2$, $\mu_{r1} = 1$ and variable χ_{r1}.

https://doi.org/10.1017/S1759078716001100 Published online by Cambridge University Press

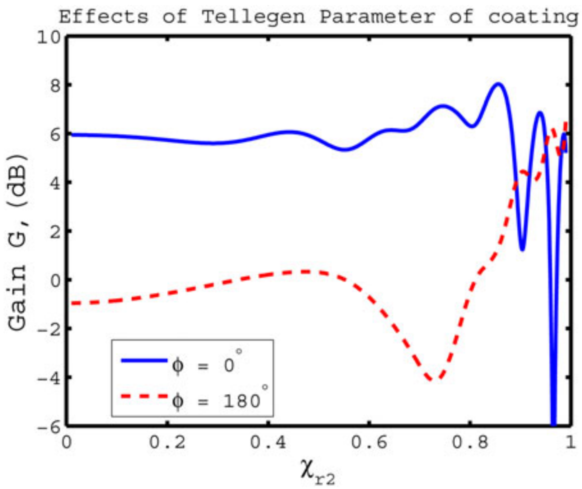

Fig. 6. Effects of Tellegen parameter of the Tellegen coating upon the gains in the FD and BD. The Tellegen coating has parameters of $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, whereas its Tellegen parameter χ_{r2} is variable. The background is taken to be a free space.

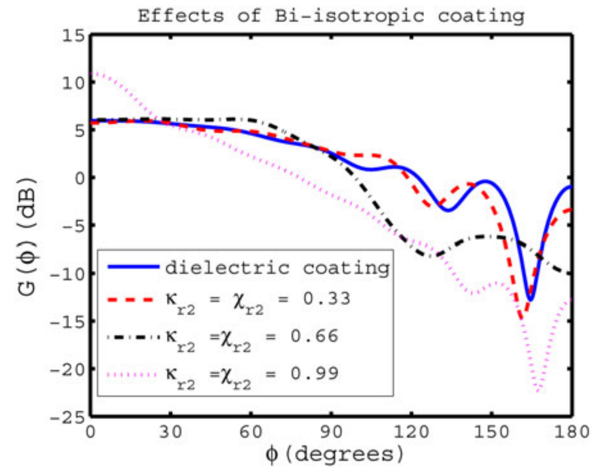

Fig. 8. Effects of different BI coatings upon the gain of an axially slotted cylinder and its comparison with a dielectric-coated axially slotted cylinder. For all these cases, the background is taken to be free space. The parameters of BI coatings are $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, whereas κ_{r2}, χ_{r2} are taken to be variable.

Figures 7 and 8 deal with the effects of different BI background media and BI coatings upon the gain pattern. The influences of various BI background media upon the gain of a dielectric-coated axially slotted cylinder have been shown in Fig. 7. The gain patterns of the free space background and a BI background with $\kappa_{r1} = \chi_{r1} = 0.33$ are almost same. As the values of κ_{r1} and χ_{r1} increase to a value of 0.66 then the gain pattern becomes different from that of the free space background and dominant difference exists for $\phi > 100^\circ$. For a strong BI background, i.e., $\kappa_{r1} = \chi_{r1} = 0.99$, the variations in the gain pattern are obvious. Also it is further observed that for a strong BI background, the gain in the BD significantly greater than the gain in the FD. Contrary to this, if an axially slotted cylinder is coated with a strong BI coating and embedded in a free space background then a significant enhancement in the FD gain and a significant reduction in the BD gain is observed as compared with a dielectric coating, which is shown in Fig. 8. This is expected and explained previously because for a strong BI coating having $\kappa_{r1} = \chi_{r1} = 0.99$, one of the eigenwave inside the BI coating becomes a backward wave. In order to highlight the gains in the FD and BD for a dielectric and the considered BI coatings, Table 2 is constructed.

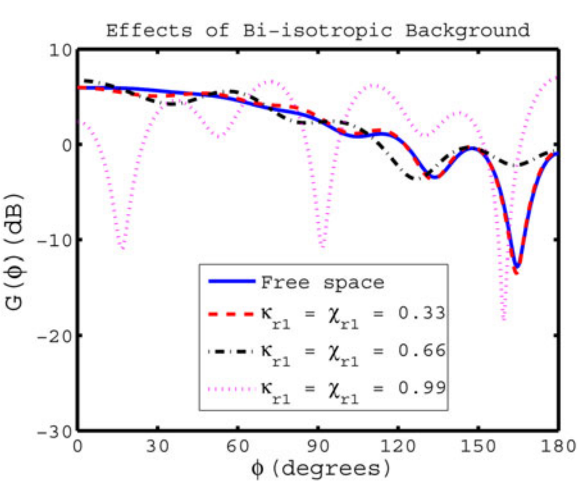

Fig. 7. Effects of different BI background media upon the gain pattern of a dielectric-coated axially slotted cylinder embedded in BI background. The background BI media have $\epsilon_{r1} = 2$, $\mu_{r1} = 1$ with variable values of κ_{r1} and χ_{r1}.

The influences of chirality and Tellegen parameters of the background BI media upon the gains in the FD and BD are shown in Figs 9 and 10. The coating is taken to be a dielectric type. It is obvious form Fig. 9 that the maximum value of the FD gain, i.e., 9.7196 dB is observed at $\chi_{r1} = 0.985$, whereas the minimum value of the BD gain of -1.506 dB occurs at $\chi_{r1} = 0.554$ for the fixed value of a chirality $\kappa_{r1} = 0.5$. In this case, there exist two noticeable ranges of Tellegen parameters χ_{r1}. The first range is $0.2476 < \chi_{r1} < 0.3393$ where the FD gain is smaller than the gain in the BD. In the second range, i.e., $0.9161 < \chi_{r1} < 0.9716$, we have fluctuating values of the gains in the FD and BD. In practical designs where it is required to have greater gain in the FD as compared with the gain in the BD, these above two ranges of Tellegen parameters can be excluded. Likewise, if we have a fixed value of $\chi_{r1} = 0.5$ and variable values of the chirality of the background BI medium, then we can exclude these ranges of κ_{r1}, i.e., $0.400 < \kappa_{r1} < 0.4328$ and $0.9768 < \kappa_{r1} < 0.99$. For these ranges of κ_{r1}, we have smaller gain in the FD as compared with the gain in the BD. This is shown in Fig. 10. In this case, the maximum value of the FD gain, i.e., 8.8223 is observed for a chirality of 0.94 and the minimum value of the BD gain, i.e., -2.2 occurs at $\kappa_{r1} = 0.01$.

Table 2. Gains in the FD and BD of a coated axially slotted cylinder embedded in the free space background having dielectric and different BI coatings.

Coating layers	$G(\phi = 0^\circ)$ **(dB)**	$G(\phi = 180^\circ)$ **(dB)**
Dielectric	5.9413	−0.9612
BI ($\kappa_{r2} = \chi_{r2} = 0.33$)	5.7133	−3.3930
BI ($\kappa_{r2} = \chi_{r2} = 0.66$)	6.0319	−9.9572
BI ($\kappa_{r2} = \chi_{r2} = 0.99$)	10.9079	−12.7980

The relative permittivity and relative permeability of all coatings are taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$, respectively.

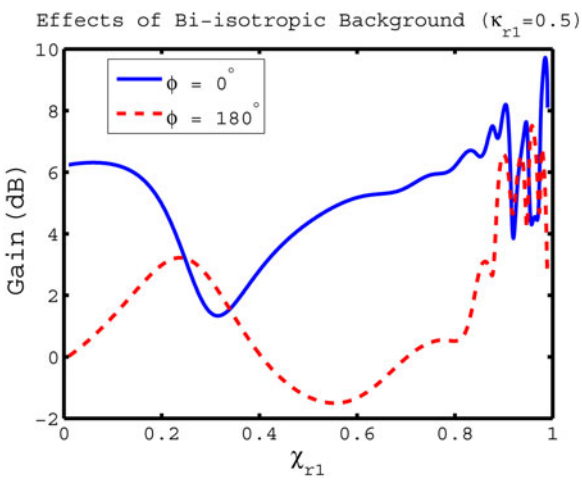

Fig. 9. Effects of background BI media upon the forward and backward gains of a dielectric-coated axially slotted cylinder. In this case, BI background media have fixed values of $\epsilon_{r1} = 2$, $\mu_{r1} = 1$, and $\kappa_{r1} = 0.5$, whereas χ_{r1} is variable.

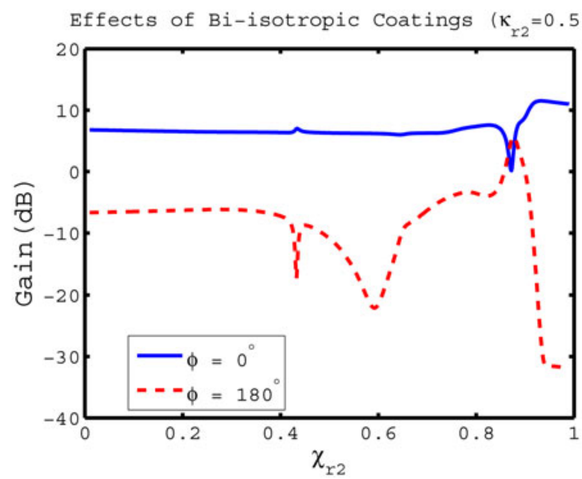

Fig. 11. Effects of different BI coatings upon the gains in the FD and BD of a BI-coated axially slotted cylinder embedded in free space. The BI coatings have fixed values of $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, $\kappa_{r2} = 0.5$, and variable values of χ_{r2}.

In Figs 11 and 12, the effects of various Tellegen and chirality parameters of the BI coatings upon the FD and BD gains are depicted. The background medium is taken to be a free space. An influence of variable Tellegen parameter χ_{r2} of a BI coating having fixed value of chirality $\kappa_{r2} = 0.5$ upon the gains in the FD and BD is shown in Fig. 11. It is clear from Fig. 11 that the FD gain is greater than the BD gain for $0.01 \leq \chi_{r2} < 0.866$. Also it is observed that for this range of χ_{r2}, the FD is not very much fluctuating. The FD gain is smaller than the BD gain in the range $0.866 < \chi_{r2} < 0.8794$. It should be noted that the maximum value of the FD gain, i.e., 11.5096 dB exists at $\chi_{r2} = 0.9330$ and the minimum value of the BD gain, i.e., -31.6544 dB is observed at $\chi_{r2} = 0.946$. On the other hand, if we fix the Tellegen parameter χ_{r2} of a BI coating to a value of 0.5 and has variable values of the chirality κ_{r2}, then the maximum FD gain of 10.7551 dB occurs at $\kappa_{r2} = 0.96$ and the minimum value of the BD gain, i.e., -16 dB exists at $\kappa_{r2} = 0.538$. It is obvious from Fig. 12. In this case, the FD gain is always greater than the BD gain for all values of κ_{r2} with the exception of $0.8605 < \kappa_{r2} < 0.8660$ where the FD gain is smaller than the BD gain.

For all the numerical results considered so far, the two optimum cases are observed for a BI-coated axially slotted cylinder embedded in a free space background. These cases are taken to be optimum because they have highest gains in the FDs and are tabulated in Table 3. These two BI coatings have the optimum parameters of $\kappa_{r2} = 0.5$, $\chi_{r2} = 0.933$ and $\kappa_{r2} = \chi_{r2} = 0.99$. The relative permittivity and relative permeability in each case is taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$, respectively. In these two cases, we have significant reduction in the gains in the BDs. It should be noted that for these two BI coatings, associated LCP waves inside the BI coatings become backward waves causing a resonance at $\phi = 0^{\circ}$ which guide most of the slot radiated field in the FD. These two types of optimum BI coatings represent the most plausible effects which can be used to design a new kind of antenna with the highest gain in the FD and significantly reduced gain in the BD.

At this stage, it is interesting to investigate the effects of realistic Tellegen and BI media upon the gain pattern of an

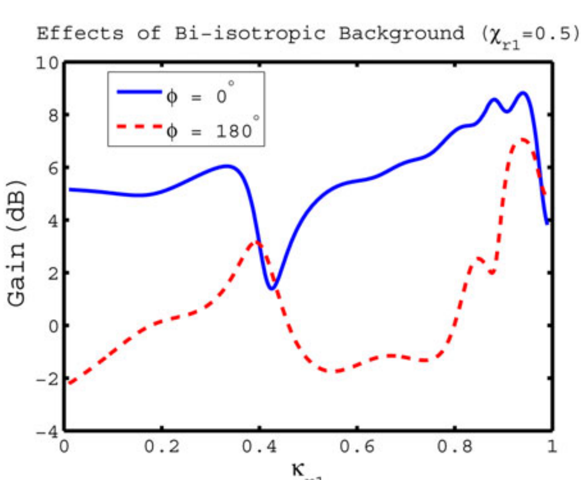

Fig. 10. Effects of background BI media upon the gains in the FD and BD of a dielectric-coated axially slotted cylinder. The BI background media have fixed values of $\epsilon_{r1} = 2$, $\mu_{r1} = 1$ and $\chi_{r1} = 0.5$, whereas κ_{r1} is variable.

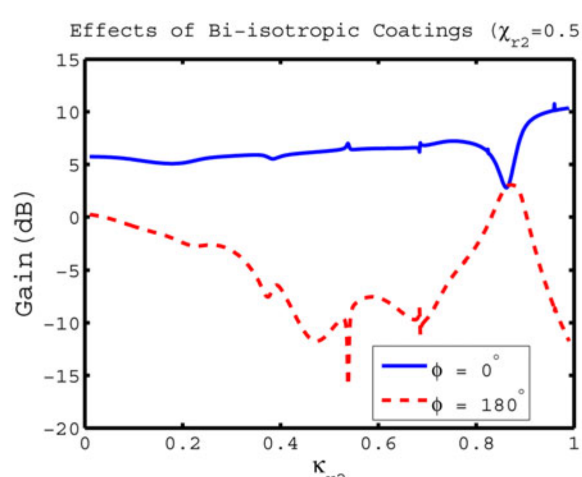

Fig. 12. Effects of different BI coatings upon the gains of a BI-coated axially slotted cylinder in the FD and BD. The background is taken to be free space. The BI coatings have fixed values of $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, and $\chi_{r2} = 0.5$. The chirality parameter κ_{r2} is taken to be variable.

Table 3. The optimum gains in the FD and BD of a BI-coated axially slotted cylinder embedded in the free space background and their corresponding Tellegen and chirality parameters.

BI coatings layers	$G(\phi = 0°)$ (dB)	$G(\phi = 180°)$ (dB)
BI ($\kappa_{r2} = 0.500$, $\chi_{r2} = 0.933$)	11.5096	−27.8191
BI ($\kappa_{r2} = 0.990$, $\chi_{r2} = 0.990$)	10.9079	−12.7980

The relative permittivity and relative permeability for both BI coatings are taken to be $\epsilon_{r2} = 2$ and $\mu_{r2} = 1$, respectively.

axially slotted cylinder. In this case, the Tellegen parameter of considered Tellegen and BI media is taken to be of the same order of magnitude as that of naturally occurring chromium oxide, i.e., 10^{-3}; see for example [32]. It should be noted that all the other structural and material parameters remain same as previously but the Tellegen parameter is taken to be 10^{-3}. In Fig. 13, the effects of realistic Tellegen and BI coatings upon the gain pattern of a coated axially slotted cylinder embedded in the free space have been shown. It is seen that the effects of these realistic coatings upon gains are very small, but closer look shows that these effects are detectable to some extent. This is because of weak nature of a magneto-electric coupling parameter, i.e., $\chi_{r2} = 10^{-3}$ of realistic Tellegen and BI coatings. The effects of realistic Tellegen and BI background media upon the gains of a dielectric-coated axially slotted cylinder are shown in Fig. 14. Once again the differences in gain patterns are very small.

In the previous analysis, it was assumed that pertinent materials were lossless, but the proposed theory can also be applied to lossy materials as well. For this one needs to assume the relative parameters, i.e., ϵ_{ri}, μ_{ri}, κ_{ri}, and χ_{ri} where $i = 1{,}2$ as complex numbers with their imaginary parts less than zero. This ensures the passivity condition for the assumed time convention. The influences of lossy BI coatings upon the gain pattern of a BI-coated axially slotted cylinder placed in the free space background medium are shown in Fig. 15. It is observed that for a smaller value of loss factor, i.e., $\delta_r = 0.05$, the gain in the FD and BD decreases as a compared with a lossless BI coating with $\delta_r = 0$. On the other hand, it is noticed that for a relatively larger values of loss factors $\delta_r = 0.1, 0.15$, the gain in the FD decreases, whereas the gain in the BD increases as a compared with a lossless BI coating. Thus, it is concluded that for a relatively larger value of loss factor associated with a lossy BI coating, the radiated field from a slot is guided toward rear side of the cylinder, which in turn reduces the gain in the FD.

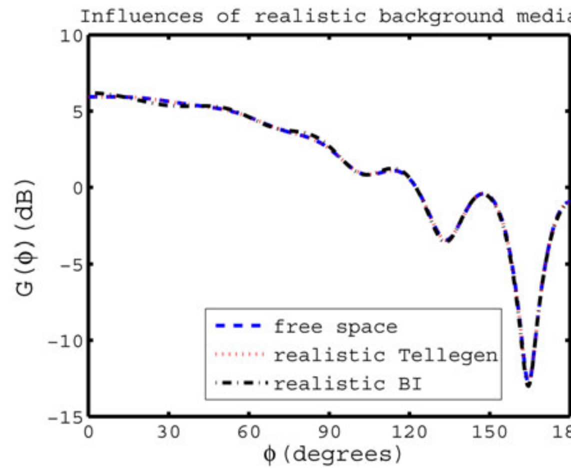

Fig. 14. Effects of realistic Tellegen and BI background media upon the gain pattern of a dielectric-coated axially slotted cylinder and their comparison with the free space background. In this case, a realistic Tellegen background medium has of $\epsilon_{r1} = 1$, $\mu_{r1} = 1$, and $\chi_{r1} = 10^{-3}$, whereas for a realistic BI background, we have $\epsilon_{r1} = 1$, $\mu_{r1} = 1$, $\chi_{r1} = 10^{-3}$, and $\kappa_{r1} = 0.1$.

IV. CONCLUDING REMARKS

The effects of various BI coatings and BI background media upon the gain pattern of an axially slotted cylinder have been investigated. It is known that a Tellegen medium is a sub-class of a general BI medium; therefore the effects of several Tellegen coatings and background media upon the gain properties of an axially slotted cylinder have also been studied. It is shown that by varying the electromagnetic parameters of a

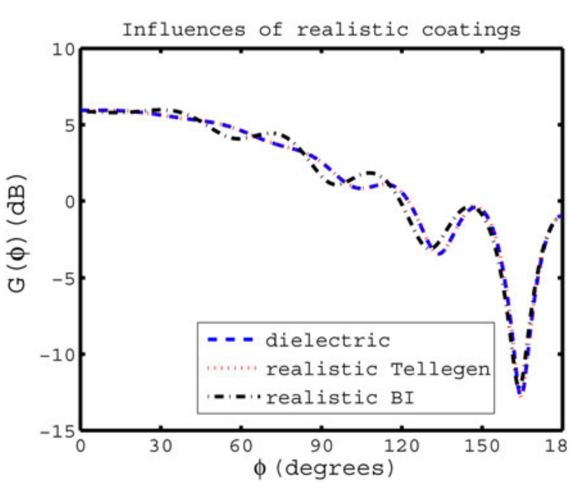

Fig. 13. Influences of realistic Tellegen and BI coatings and their comparison with a dielectric coating, whereas the background medium is taken to be the free space. For a realistic Tellegen coating, we have $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, and $\chi_{r2} = 10^{-3}$, whereas for a realistic BI coating, the electromagnetic parameters are taken to be $\epsilon_{r2} = 2$, $\mu_{r2} = 1$, $\chi_{r2} = 10^{-3}$, and $\kappa_{r2} = 0.1$.

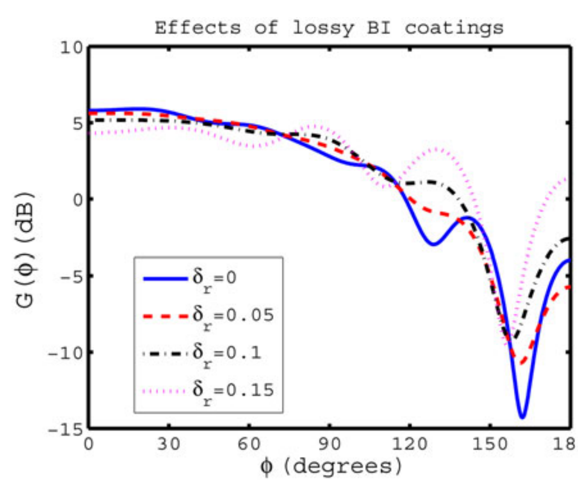

Fig. 15. Effects of different lossy BI coatings upon the gain patterns of an axially slotted cylinder embedded in the free space background. In this case, we have $\epsilon_{r2} = 2 - j\delta_r$, $\mu_{r2} = 1 - j\delta_r$, $\kappa_{r2} = 0.35 - j\delta_r$ and $\chi_{r2} = 0.35 - j\delta_r$ for BI lossy coatings where the factor δ_r is a loss factor.

https://doi.org/10.1017/S1759078716001100 Published online by Cambridge University Press

coating and a background medium, one can control the gain of an axially slotted cylinder. For a dielectric-coated axially slotted cylinder embedded in a Tellegen background, it is noticed that for a Tellegen parameter close to unity, the gains in the FD and BD become greater than their corresponding gains with the free space background. Likewise, it is shown that a strong Tellegen coating guides most of the radiated field from an axial slot toward rear side of the cylinder. Also it is further observed that for a strong BI background, the gain in the BD significantly greater than the gain in the FD. On the other hand, if an axially slotted cylinder is coated with a strong BI coating and embedded in the free space background then a significant enhancement in the FD gain and a significant reduction in the BD gain is observed as compared to a dielectric coating. This is an important finding which can be used to enhance the gain in the FD and has applications in wireless communications. It is also noticed that realistic Tellegen and BI media have weaker effects upon the gain pattern. In these realistic media, the Tellegen parameter is taken to have a magnitude comparable with a magnetoelectric coupling parameter of a naturally occurring chromium oxide.

ACKNOWLEDGEMENT

The author is thankful to the anonymous reviewers for improving the quality of the manuscript.

REFERENCES

[1] Lindell, I.V.; Sihvola, A.H.; Tretyakov, S.A.; Viitanen, A.J.: Electromagnetic Waves in Chiral and Bi-Isotropic Media, Artech House, Norwood, MA, 1994.

[2] Ougier, S.; Chenerie, I.; Sihvola, A.; Priou, A.: Propagation in bi-isotropic media: effect of different formalisms on the propagation analysis. Progr. Electromagn. Res., **9** (1994), 19–30.

[3] Lindell, I.V.; Sihvola, A.; Viitanen, A.J.: Plane wave reflection from a bi-isotropic (nonreciprocal chiral) interface. Microw. Opt. Technol. Lett., **5** (2) (1992), 79–81.

[4] He, S.; Hu, Y.: Electromagnetic scattering from a stratified bi-isotropic (nonreciprocal chiral) Slab: numerical computations. IEEE Trans. Antennas Propag., **41** (8) (1993), 1057–1062.

[5] Monzon, C.; Forester, D.W.: Negative refraction and focusing of circularly polarized waves in optically active media. Phys. Rev. Lett., **95** (2005), 123904-1–123904-4.

[6] Bohren, C.F.: Light scattering by an optically active sphere. Chem. Phys. Lett., **29** (1974), 458–462.

[7] Belichenko, V.P.; Fisanov, V.V.: Scattering of electromagnetic waves by a bi-isotropic sphere. Russ. Phys. J., **37** (10) (1994), 162–166.

[8] Monzon, J.C.: Scattering by a biisotropic body. IEEE Trans. Antennas Propag., **43** (11) (1995), 1288–1296.

[9] He, S.; Cao, J.: Scattering from a bi-isotropic object of arbitrary shape. J. Electromagn. Waves Appl., **12** (1998), 1547–1574.

[10] Bohren, C.F.: Scattering of electromagnetic waves by an optically active spherical shell. J. Chem. Phys., **62** (4) (1975), 1566–1571.

[11] Cheng, D.; Lin, W.; Zhao, Y.: Anisotropic impedance boundary condition for a cylindrical conductor coated with a bi-isotropic medium. J. Phys. D: Appl. Phys., **26** (1993), 517–521.

[12] Wang, D.X.; Lau, P.Y.; Yung, E.K.; Chen, R.S.: Scattering by conducting bodies coated with bi-isotropic matrials. IEEE Trans. Antennas Propag., **55** (8) (2007), 2313–2318.

[13] Hsu, C.G.; Chiu, C.N.: Oblique plane wave scattering from a general bi-isotropic cylindrical shell with an interior advanced composite-material backing. IEEE Trans. Electromagn. Compat., **48** (4) (2006), 614–620.

[14] Monzon, J.C.: Radiation and scattering in homogeneous general biisotropic regions. IEEE Trans. Antennas Propag., **38** (2) (1990), 227–235.

[15] Koivisto, P.K.; Tretyakov, S.A.; Oksanen, M.I.: Waveguide filled with general biisotropic media. Radio Sci., **8** (5) (1993), 675–686.

[16] Paiva, C.R.; Topa, A.L.; Barbosa, A.M.: Novel propagtion features of dielectric planar chirowaveguides due to nonreciprocity. Microw. Opt. Technol. Lett., **6** (3) (1993), 182–185.

[17] Meshcheryakov, V.A.; Mudrov, A.E.: Characteristic electromagnetic waves in a circular waveguide filled with a biisotropic medium. Russ. Phys. J., **40** (2) (1997), 162–166.

[18] Ioannidis, A.D.; Kristensson, G.; Sjöberg, D.: On the dispersion equation for a homogeneous, bi-isotropic waveguide of arbitrary cross-section. Microw. Opt. Technol. Lett., **51** (11) (2009), 2701–2705.

[19] Ioannidis, A.D.; Kristensson, G.; Sjöberg, D.: The propagation problem in a bi-isotropic waveguide. Progr. Electromagn. Res. B, **19** (2010), 21–40.

[20] Tellegen, B.D.F.: The gyrator, a new electric network element. Phillips Res. Rep., **3** (2) (1948), 81–101.

[21] Lakhtakia, A.: The Tellegen medium is "a Boojum, you see". Int. J. Infrared Millim. Waves, **15** (10) (1994), 1625–1630.

[22] Lakhtakia, A.: On the genesis of post constrain in modern electromgnetism. Int. J. Light Electron. Opt., **115** (4) (2004), 151–158.

[23] Sihvola, A.H.; Tretyakov, S.A.; Serdyukov, A.N.; Semchenko, I.V.: Duality once more applied to Tellegen media. Electromagnetics, **17** (2) (1997), 205–211.

[24] Lakhtakia, A.; Weiglhofer, W.S.: On the application of duality to Tellegen media. Electromagnetics, **17** (2) (1997), 199–204.

[25] Tretyakov, S.A.; Maslovski, S.I.; Nefedov, I.S.; Viitanen, A.J.; Belov, P.A.; Sanmartin, A.: Artificial Tellegen particle. Electromagnetics, **23** (8) (2003), 665–680.

[26] Altan, B.S.: A uniqueness theorem for initial boundary value problems in Tellegen medium. Progr. Electromagn. Res. C, **1** (2008), 73–85.

[27] Kamenetskii, E.O.; Sigalov, M.; Shavit, R.: Tellegen particles and magnetoelectric metamaterials. J. Appl. Phys., **105** (2009), 013537-1–013537-15.

[28] Prudêncio, F.R.; Matos, S.A.; Paiva, C.R.: The most general classes of Tellegen media reducible to simple reciprocal media: a geometrical approach, in *General Assembly and Scientific Symp.* (*URSI, GASS*), XXXIth URSI, Beijing, 16–23 August 2014, 1–4.

[29] Astrov, D.N.: Magnetoelectric effect in chromium oxide. Sov. Phys. – JETP, **13** (4) (1961), 729–733.

[30] Coh, S.; Vanderbilt, D.: Canonical magnetic insulators with isotropic magnetoelectric coupling. Phys. Rev. B, **88** (2013), 121106-1–121106-5.

[31] Mong, R.S.K.; Essin, A.M.; Moore, J.E.: Aniferromagnetic topological insulators. Phys. Rev. B, **81** (2010), 245209-1–245209-10.

[32] Prudêncio, F.R.; Matos, S.A.; Paiva, C.R.: Asymmetric band diagrams in photonic crystals with a spontaneous nonreciprocal response. Phys. Rev. A, **91** (2015), 063821-1–063821-13.

[33] Prudêncio, F.R.; Matos, S.A.; Paiva, C.R.: Exact image method for radiation problems in stratified isorefractive Tellegen media. IEEE Transa. Antennas and Propag., **62** (9) (2014), 4637–4646.

[34] Prudêncio, F.R.; Silveirinha, M.G.: Optical isolation of circularly polarized light with a spontaneous magnetoelectric effect. Phys. Rev. A, **93** (2016), 043846-1–043846-11.

[35] Hurd, R.A.: Radiation patterns of a dielectric coated axially slotted cylinder. Can. J. Phys., **34** (1956), 638–642.

[36] Wait, J.R.; Mienteka, W.: Slotted-cylinder antenna with a dielectric coating. J. Res. Natl. Bur. Stand., **58** (6) (1957), 287–296.

[37] Shafai, L.: Radiation from an axial slot antenna coated with a homogenous material. Can. J. Phys., **50** (23) (1972), 3072–3077.

[38] Knop, C.M.: External admittance of an axial slot on a dielectric coated metal cylinder. Radio Sci., **3** (1968), 803–818.

[39] Mushref, M.A.: Radiation from a dielectric–coated cylinder with two slots. Appl. Math. Lett., **17** (2004), 721–726.

[40] Mushref, M.A.: Electromagnetic radiation from a coated cylinder with two arbitrary axial slots. High Freq. Electron., **9** (2) (2010), 47–54.

[41] Rusch, W.V.T.: Radiation from a plasma-clad axially-slotted cylinder. J. Res. Natl. Bureau Stand., **67D** (2) (1963), 203–214.

[42] Chen, H.C.; Cheng, D.K.: Radiation from an axially slotted anisotropic plasma clad cylinder. IEEE Trans. Antennas Propag., **13** (3) (1965), 395–401.

[43] Marchin, P.D.; Tyras, G.: Radiation from an infinite axial slot on a circular cylinder clad with magnetoplasma. Radio Sci., J. Res., **69D** (4) (1965), 529–538.

[44] Yeh, C.; Kaprielian, Z.A.: Radiation from an axially slotted cylinder coated with an inhomogeneous dielectric sheath. Br. J. Appl. Phys., **14** (1963), 677–681.

[45] Tyras, G.: Field of an axially slotted circular cylinder clad with an inhomogeneous dielectric. IEEE Trans. Antennas Propag., **AP–15** (2) (1967), 222–226.

[46] Hamid, A.K.: Axially slotted antenna on a circular or elliptic cylinder coated with metamaterials. Progr. Electromagn. Res., **51** (2005), 329–341.

[47] Richmond, J.: Axial slot antenna on a dielectric-coated elliptic cylinder. IEEE Trans. Antennas Propag., **37** (10) (1989), 1235–1241.

[48] Hamid, A.K.: Elliptic cylinder with slotted antenna coated with magnetic metamaterials. Int. J. Antennas Propag., **2011** (2011), 842863-1–842863-5.

[49] Awan, Z.A.: Directive gain from an axially slotted dielectric clad cylinder embedded in a metamaterial. J. Mod. Opt., **62** (7) (2015), 560–568.

[50] Awan, Z.A.: Gain properties of an axially slotted cylinder with two coating layers. Int. J. Microw. Wireless Technol., (2015) DOI:10.1017/S1759078715001038, 8–pp.

[51] Mahmoud, S.F.: Characteristics of a chiral coated slotted cylindrical antenna. IEEE Trans. Antennas Propag., **44** (1996), 814–821.

[52] Awan, Z.A.: Gain of an axially slotted cylinder covered with a chiral coating and embedded in a chiral medium. Appl. Opt., **54** (19) (2015), 5889–5896.

Zeeshan Akbar Awan received a Ph.D. degree in Electronics from the Quaid-i-Azam University, Islamabad in 2013. He is presently an assistant professor at the same university. He has published more than 10 research articles. His research interests include metamaterials, metasurfaces and antenna theory.

https://doi.org/10.1017/S1759078716001100 Published online by Cambridge University Press