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VINTAGE ARTICLE

ALTERNATIVE MONETARY POLICIES IN A TURNPIKE ECONOMY

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This paper modifies a Townsend turnpike model by letting agents stay at a location long enough to trade some consumption loans, but not long enough to support a Pareto-optimal allocation. Monetary equilibria exist that are nonoptimal in the absence of a scheme to pay interest on currency at a particular rate. Paying interest on currency at the optimal rate delivers a Pareto-optimal allocation, but a different one than the allocation for an associated nonmonetary centralized economy. The price level remains determinate under an optimal policy. We study the response of the model to "helicopter drops" of currency, steady increases in the money supply, and restrictions on private intermediation.

Keywords: Credit, Fiat Money, Pareto Optimality, Friedman Rule

1. PREAMBLE

Sargent (1987) grouped together a class of incomplete markets models that feature heterogeneously endowed infinitely lived agents who solve infinite-horizon income fluctuations problems that motivate trades of fiat money and private IOUs, christening them Bewley–Townsend models. What distinguishes Bewley (1980, 1983, 1986) from Townsend (1980) models is (a) whether agents' endowments are stochastic or strictly periodic, and consequently, (b) whether they have to be solved by computer or can be solved by pencil and paper.[1] Townsend turnpike models use deterministic variations in people's locations to motivate the same kinds of market completeness found in Bewley with stochastic endowment processes.[2]

We wrote this paper in 1988 because we liked Bewley models and appreciated Townsend's spatial rationalization of market incompleteness and the transparency associated with their analytic tractability.[3] We followed Townsend in assuming strictly periodic endowments that give agents a motive to borrow and lend because

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they want consumption streams that are smoother than their endowments. We alter the spatial restrictions in order to expand the scope for private borrowing and lending relative to what Townsend's setting allowed, but our spatial restrictions still inhibit private credit enough to allow the institution of fiat money to contribute to social welfare. We continue to believe that this class of structures is useful for studying policies that impinge on how readily fiat money and private credit coexist.[4]

New Keynesian models of the type that have swept into research departments at leading central banks push into the background the forces highlighted by Townsend models, namely, the heterogeneity in agents' endowments and locations that shape demands for money and that separate markets for money from those for credit. To answer some important questions about monetary policy, these forces have to be brought back into the foreground.

2. INTRODUCTION

The answers to many questions about the proper conduct of monetary policy depend on the form and properties of the demand for money. Ever since Hicks's (1967) famous 1935 article, it has been understood that the standard general equilibrium model needs to be modified by adding frictions if an intrinsically useless asset like fiat money is to command a positive price in equilibrium. A model's predictions about the effects of alternative monetary policies and their welfare consequences depend on the friction used to generate a demand for fiat money. Furthermore, for various particular frictions that might be used, there are questions that are not usefully asked of a model. Consider, for example, the cash-in-advance restriction. If the finance constraint is interpreted as a legal restriction, then at least in models that do not display any other forms of market failures, the optimal policy is simple: just eliminate the legal restriction. When this option is ruled out, it is possible to study policy issues such as the optimal quantity of money. However, in such models, the optimal policy usually has an interpretation as a prescription to manage the money supply to undo the effect of the cash-in-advance constraint.

When the finance constraint is not explicitly motivated by legal restrictions, one would expect the answer to policy questions to depend on precisely *how* the cash-in advance restriction is motivated. In models that do not completely describe the friction that generates the demand for currency, it is difficult to study such relevant policy questions as the desirability and effects of deregulation of the banking system.

For these reasons, the last few years have seen renewed interest in model economies that are very specific about the frictions that prevent the existence of nonmonetary optimal equilibria. By studying various alternative concrete restrictions that generate a demand for money, these models provide insights into the interactions between types of frictions in the economic environment and specific rules for conducting monetary and fiscal policy.

In this paper we study an economy in which there are limited opportunities for private loan markets to effect all desired (and optimal) intertemporal transactions.

We analyze the impact of alternative monetary policies in a modified version of Townsend's (1980) turnpike model. Townsend studied the optimal quantity of money within a class of "interventionist" policies that require the government to recognize and tax different types of agents differently. Within this class, *any* stationary equal-treatment Pareto-optimal allocation can be supported as a monetary equilibrium.[5] This paper studies the optimal quantity of money under a class of less interventionist policies that restrict the government to tax all agents at a given location and date equally. Because such tax policies treat agents anonymously, the monetary systems that we analyze require less information than those studied by Townsend. Limiting the government's information in this way shrinks to a single point the set of attainable optimal allocations. This particular allocation is attained by following a particular version of Friedman's rule. However, the rule works differently than in other models for which it has been prescribed. In addition to studying the optimal rule, we study how differently situated agents fare as the rate of return on currency is set at various nonoptimal values.[6]

Studying alternative monetary policies sheds light on how models such as Townsend's, in which currency is valued because exchange is decentralized across time and space, differ from a variety of other models in which some or all exchange is centralized. Although we recover a version of Friedman's rule, our results differ in important ways from analyses of the optimal quantity of money that have been carried out in cash-in-advance models, currency-in-the-utility-function models, and overlapping-generations models. In particular, our results differ from analyses with those models with respect to the matters of (i) price level determinacy under an optimal interest on currency scheme; (ii) equivalence of an optimal interest on currency policy with permitting a "free banking" regime; and (iii) whether the allocation attained under an optimal interest on currency scheme recovers the allocation from a nonmonetary (Arrow–Debreu) equilibrium for a corresponding economy obtained by keeping endowments and preferences unaltered but relaxing the restrictions that prevent centralized trading.

We carry out our analysis in a model that is in the spirit of Townsend's, but that is altered by having agents linked with differently endowed agents for two periods before moving on. Also, to create a potential role for currency, we alter each individual's endowment sequence from the period two pattern assumed by Townsend to a period four pattern. These modifications permit one period loans to play a consumption smoothing role, and potentially permit "inside money" to coexist with "outside money." The modification to two-period trading encounters also creates a framework within which we can study how currency gets redistributed over time when the system is started up with distributions of currency across agents that do not correspond to a steady state.[7]

We also investigate the effect of changes in the growth rate of the money supply when the proceeds are used to finance a transfer program. In this case we show that, unlike other models, it is possible to determine the impact of such a policy on the real variables of the economy without much knowledge of the precise distributional effects of the transfer program. However, the consequences for the price level and the quantity of inside money are sensitive to the distributional details. We contrast this with the key role that income distribution plays in the analysis of once-and-for-all increases in the money supply. We also study the interaction between monetary policy and credit policy, defined as restrictions on private financial intermediation. We show that imposing limits on private borrowing and lending can have deflationary effects and can stabilize output while, at the same time, reducing the welfare levels of all agents.

The environment that we describe shares some properties of the transactions-demand models of money as well as models in which money is basically a store of value. The spatial and temporal separation can be viewed as an impediment to the existence of a single present-value budget constraint; this is a feature that is commonly associated with the transactions motive for holding money.[8] However, the result that sustained inflation has an impact mostly through the effect on the rate of return is sometimes associated with the store-of-value function of money.

The turnpike model has been studied, in addition to Townsend, by Hornstein and Krusell (1989), Mitsui and Watanabe (1989), and Ireland (1991). Mitsui and Watanabe present a version in which individuals move randomly and there are both a good and a bad technology. For specific functional forms they establish the existence of equilibrium as well as the impact of inflation on capital accumulation. Hornstein and Krusell show that if some insurance markets, whose existence is compatible with the physical and informational environment, are allowed to exist in the Mitsui and Watanabe environment, it is possible for the monetary equilibrium to disappear. Ireland explores the case of stochastic endowments and he studies existence of a monetary equilibrium in an example. Our model is similar in spirit to that of Scheinkman and Weiss (1986), in the sense that each time the Scheinkman–Weiss Poisson counter goes off, one could imagine that half of the agents at each location move east, the other half west. This locational interpretation shuts down the markets closed by Scheinkman and Weiss.

Several other recent models of money also assume patterns of spatial separation to support restrictions on borrowing and lending that make room for valued currency. Kiyotaki and Wright (1989) restrict preferences and matching and storage technologies in ways that force all trades to occur bilaterally outside of markets. In Kiyotaki and Wright's model, one party to each exchange values the commodity that he is acquiring only for its exchange value (i.e., its indirect utility). The Kiyotaki–Wright model is at one end of a spectrum (no centralized exchange in time and space), with the Arrow–Debreu model at the other end (all exchange centralized in time and space). Our model is somewhere in the middle.

The models of Stacey Schreft (1992a, 1992b) use spatial distance to index a verification cost involved in using credit to effect exchanges. In her models, each person conducts part of his transactions with credit at locations where verification is cheap, and part with currency, where verification is expensive. Because the credit system is socially costly to administer while the currency system is not, the optimal monetary–fiscal policy involves driving credit market out of existence via a version of Friedman's deflation or interest on currency scheme.

Finally, David Levine (1991) has created examples in the context of stochastic models such as Bewley's or Townsend's in which equilibria with inflation generated by lump-sum money infusions Pareto-dominate equilibria without inflation. In these examples, the money transfer scheme serves an insurance role that more than offsets the social costs imposed by the distortion associated with inflation.

This paper is organized as follows. Section 3 presents the basic model. In Section 4 we define a monetary equilibrium and give conditions under which one exists. Section 5 analyzes the welfare properties of different equilibria. In Section 6 we discuss the optimal quantity of money and the difference between our results and existing models. Sections 7 and 8 explore, respectively, the effects of an increase in the money supply and the consequences of an increase in the growth rate of the money supply. Section 9 studies the effects of restrictions on private intermediation. Finally, Section 10 contains some concluding comments. All proofs are relegated to the Appendices.

3. PREFERENCES, TECHNOLOGY, ENDOWMENTS, AND ITINERARIES

The economy consists of equal numbers of two types of agents, whom we label as $i = e$ and $i = o$, where e stands for even and o stands for odd. Agents of type i have preferences over streams of consumption and labor supply $\{c_t^i, \ell_t^i\}_{t=0}^{\infty} = (c^i, \ell^i)$ that are ordered by

$$U(c^i, \ell^i) = \sum_{t=0}^{\infty} \beta^t u(c_t^i, 1 - \ell_t^i), \tag{1}$$

where u is strictly concave and twice differentiable. Agent i has access to the technology for producing a single consumption good,

$$y_t^i \leq \omega_t^i \ell_t^i,$$

where $\{\omega_t^i\}_{t=0}^{\infty}$ is a sequence of labor productivities for agent i and y_t^i is agent i's output of the time t consumption good. The consumption good is nonstorable.y

Odd and even agents are identified by their productivity sequences. In particular, let $a > 0, b > 0$. Then $\{\omega_t^o\}$ and $\{\omega_t^e\}$ are the sequences

$$\{\omega_t^o\}_{t=0}^{\infty} = \{a, 0, 0, b, a, 0, 0, b, \ldots\},$$

$$\{\omega_t^e\}_{t=0}^{\infty} = \{0, b, a, 0, 0, b, a, 0, \ldots\}\ .$$

Individual productivity sequences are of period four, whereas the aggregate productivity sequence $\omega_t^o + \omega_t^e$ is of period two. Every two periods, odd and even agents experience a reversal of productivity prospects as characterized by the tails $\{\omega_s^i\}_{s=t}^{\infty}$ of their productivity sequences.

The locational structure of the economy is depicted in Figure 1. Each agent travels along the real line in a single direction determined at birth. Half of the agents of each type are heading east; half are heading west. All economic activity occurs at "villages" located at the integers. At each date $t \geq 0$, there are equal numbers of

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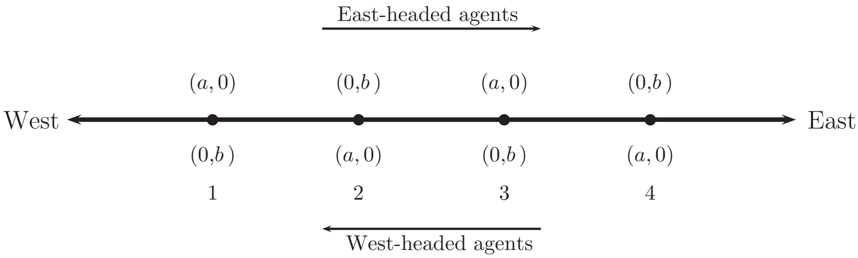

FIGURE 1. A turnpike in which east-headed agents meet west-headed agents for two periods. Agents above the line are moving west, and those below the line are moving east. At each integer, there are equal numbers of east-headed and west-headed agents. For $t = 0, 4, \ldots$, odd agents have two-period endowment $(a, 0)$, and even agents have two-period endowment $(0, b)$. For $t = 2, 6, \ldots$, even agents have two-period endowment $(a, 0)$, and odd agents have two-period endowment $(0, b)$. Agents move in their assigned directions at the start of every even period $t \geq 2$.

odd and even agents at each integer. At date $t = 0$ at odd-numbered locations, east-headed agents have a two-period productivity pattern $(a, 0)$. They are paired at that location with west-heading agents who have two-period productivity pattern $(0, b)$. At even-numbered locations, the productivity endowment pattern is the mirror image in the sense that east-headed agents have two-period productivity pattern $(0, b)$ while west-headed agents' productivity pattern is $(a, 0)$. At the beginning of each *even* $t \geq 2$, each east-headed agent moves east by one trading post, and each west-headed agent moves west by one trading post. This pattern of movement supports the situation depicted in Figure 1. Thus, at each integer for each $t = 0, 2, 4, \ldots$, half of the agents have two-period productivity $[a, 0]$, and half have $[0, b]$.

In this economy, two agents of different types (odd and even) will meet in at most one location during at most one two-period encounter. Once an odd–even pair of agents separates after having been together at a common location, neither member of the pair will ever meet another agent who has ever been or will ever be in contact with the other member of that pair. This implies that only one-period loans extended during even periods $t \geq 0$ can be repaid.

3.1. Trading Possibilities

For each $t = 0, 2, 4, \ldots$, newly arrived agents at each location j can borrow and lend with one another. Loans of duration exceeding one period will not be made because it is not feasible for them to be repaid. The only asset that can be carried across locations is a government-issued fiat currency that is initially distributed equally across all locations. We shall later consider alternative initial distributions of the currency across odd and even agents at $t = 0$.

We let p_{jt} denote the nominal price level at location j at time t. We let R_{jt} denote the gross real interest rate on consumption loans from t to $t+1$ at location j. In this paper, we restrict attention to equilibria in which prices and interest rates are identical at all locations, and we let p_t denote the common price level and R_t the common gross interest rate at t.[9]

3.2. Government

The economy starts out with a per capita quantity of currency of $H/2$ in each location. The government pays interest on currency held from t to $t+1$ at a net nominal rate of r_0 for t even and r_1 for t odd. We let $\{r_t\}$ denote this period two sequence of nominal rates on currency. For convenience, we denote the gross two-period nominal interest rate on currency as $r = (1+r_0)(1+r_1)$. Interest payments are financed by anonymous lump-sum taxes at each location: all agents at all locations bear the same tax at a given time.[10] The government budget constraint holds location by location, so that interest payments at a location are fully financed by local taxes. We study policies that hold the stock of currency constant through time. Let τ_0 be the tax on each agent in even periods and τ_1 the tax in odd periods. The government's budget constraints in nominal terms are

$$Hr_0 = 2\tau_1 p_1, \qquad Hr_1 = 2\tau_0 p_0.$$

A government policy is a four-tuple $(r_0, r_1, \tau_0, \tau_1)$. A policy of not paying interest on currency involves the setting $(0, 0, 0, 0)$. This corresponds to what has sometimes been called a "laissez faire" or "noninterventionist" policy.

3.3. Households

Let $x^i = \{x_t^i\}_{t=0}^{\infty}$ for any symbol x, where it is understood that $x = \{x_t\}_{t=0}^{\infty}$ for vectors x not depending on i. A household of type i faces the following problem: given sequences R, p, r, and subject to $c_t^i \geq 0$, $m_t^i \geq 0$, $0 \leq \ell_t^i \leq 1$ for all $t \geq 0$, choose $\{c_t^i, \ell_t^i, b_t^i, m_t^i\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i, 1 - \ell_t^i\right) \tag{2}$$

subject to

$$\frac{m_t^i}{p_t} + b_t^i + c_t^i + \tau_t \leq \omega_t^i \ell_t^i + R_{t-1} b_{t-1}^i + (1 + r_{t-1}) \frac{m_{t-1}^i}{p_t}, \tag{3}$$

$$b_t^i = 0 \text{ for } t = 1, 3, 5, \ldots, \tag{4}$$

$$m_{-1}^i = \bar{m}_{-1}^i \text{ given; } \ b_{-1}^i = 0 \text{ given.} \tag{5}$$

The statement of the household's problem leaves the household free not to hold currency, which implies that if currency is held from t to $t+1$ for t even, currency and consumption loans must bear equal real rates of return. Restriction (4) captures the feature that debt cannot be transferred across locations. Notice that it rules out explosive processes ("Ponzi schemes") for private debt.

4. EQUILIBRIUM

We employ the following:

DEFINITION 1. *An* equilibrium *is a collection of sequences* $\{\bar{p}, \bar{R}, \bar{c}^i, \bar{\ell}^i, \bar{m}^i, \bar{b}^i, i = o, e\}$ *that satisfy*

(i) *(Utility Maximization) Given* $\bar{p}$ *and* $\bar{R}$, *for* $i = o, e$, $\{\bar{c}^i, \bar{\ell}^i, \bar{m}^i, \bar{b}^i\}$ *solves the household's problem.*

(ii) *(Market Clearing)*

$$\begin{aligned} c_t^o + c_t^e &= \omega_t^o \ell_t^o + \omega_t^e \ell_t^e, \\ m_t^o + m_t^e &= m_{t-1}^o + m_{t-1}^e. \end{aligned} \tag{6}$$

(iii) *(Initial Endowments of Currency)*

$$m_{-1}^o + m_{-1}^e = H. \tag{7}$$

We call it a monetary equilibrium *if* $1/p_t > 0$ *for all* $t \geq 0$. *We call it a* nonmonetary equilibrium *otherwise.*

4.1. Periodic Monetary Equilibria

Given the physical structure of the economy, it is natural to seek equilibria in which for $t = 1, 5, \ldots$ one class of agents (even or odd) carries all of the money from t to $t+1$, and for $t = 3, 7, \ldots$, the other class of agents (odd or even) carries all of the money from t to $t+1$. Such a pattern is a counterpart to the one studied by Townsend. For most of this paper, we restrict attention to such equilibria. In particular, we consider equilibrium allocations and prices that are *symmetric* and *periodic*. By *symmetric*, we mean that identically situated agents are treated equally. By *periodic*, we mean that the allocations satisfy[11]

$$\begin{aligned} \{c_t^o\} &= \{c_0, c_1, a\ell_0 - c_0, b\ell_1 - c_1, c_0, c_1, a\ell_0 - c_0, b\ell_1 - c_1, \ldots\}, \\ \{\ell_t^o\} &= \{\ell_0, 0, 0, \ell_1, \ell_0, 0, 0, \ell_1, \ldots\}, \\ \{c_t^e\} &= \{a\ell_0 - c_0, b\ell_1 - c_1, c_0, c_1, a\ell_0 - c_0, b\ell_1 - c_1, c_0, c_1, \ldots\}, \\ \{\ell_t^e\} &= \{0, \ell_1, \ell_0, 0, 0, \ell_1\ell_0, 0, \ldots\}, \end{aligned} \tag{8}$$

and that the price level satisfies

$$p_t = \{p_0, p_1, p_0, p_1, \ldots\}. \tag{9}$$

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Note that (8) extends to the equilibrium allocation the "situation reversals" that odd and even agents experience every two periods.

Given the symmetry and periodicity in the specification of the physical aspects of the economy and the government policy, it is natural to conjecture that there are initial distributions of the currency stock for which periodic equilibria exist. Notice that a periodic equilibrium is characterized by the eight-tuple $\{c_0, c_1, \ell_0, \ell_1, p_0, p_1, r_0, r_1\}$. We now study the restriction that equilibrium imposes on this six-tuple.

4.2. Equilibrium Conditions

Evaluating the first-order conditions for the household's problems at the periodic sequences for allocations and nominal prices for $t = 0, 1$ gives

$$t=0 \qquad \text{odd:} \quad \frac{u_1(c_0, 1-\ell_0)}{p_0} = \frac{\beta(1+r_0)u_1(c_1,1)}{p_1} \qquad \text{even:} \quad \frac{u_1(a\ell_0 - c_0, 1)}{p_0} = \frac{\beta(1+r_0)u_1(b\ell_1 - c_1, 1-\ell_1)}{p_1}$$

$$u_1(c_0, 1-\ell_0)a = u_2(c_0, 1-\ell_0) \tag{10a}$$

$$t=1 \qquad \text{odd:} \quad \frac{u_1(c_1,1)}{p_1} \geq \beta(1+r_1)\frac{u_1(a\ell_0 - c_0, 1)}{p_0} \qquad \text{even:} \quad \frac{u_1(b\ell_1 - c_1, 1-\ell_1)}{p_1} \geq \beta(1+r_1)\frac{u_1(c_0, 1-\ell_0)}{p_0}$$

$$u_1(b\ell_1 - c_1, 1-\ell_1)b = u_2(b\ell_1 - c_1, 1-\ell_1). \tag{10b}$$

The inequality for $t = 1$ holds with equality if agent i chooses to set $m_1^i > 0$. Within the class of periodic equilibria, at most one of the two classes of agents will set $m_1^i > 0$, whereas the other will set $m_1^i = 0$. This follows from a minor adaptation of an argument of Townsend (1980).[12]

We consider two possible kinds of periodic equilibria, depending on which type of agent i sets $m_1^i > 0$. We use

DEFINITION 2. *In a type* I *(periodic) equilibrium,* $m_{-1}^e > 0, m_{-1}^o = 0, m_1^o > 0$.

DEFINITION 3. *In a type* II *(periodic) equilibrium,* $m_{-1}^o > 0, m_{-1}^e = 0, m_1^e > 0$.

First-order conditions (10) imply that in a type I equilibrium, the following equalities must hold:

$$u_1(c_0, 1-\ell_0) = \beta^2(1+r_0)(1+r_1)u_1(a\ell_0 - c_0, 1), \tag{11a}$$

$$u_1(c_0, 1-\ell_0)a = u_2(c_0, 1-\ell_0),$$

$$u_1(c_1, 1) = \beta^2(1+r_0)(1+r_1)u_1(b\ell_1 - c_1, 1-\ell_1), \tag{11b}$$

$$u_1(b\ell_1 - c_1, 1-\ell_1)b = u_2(b\ell_1 - c_1, 1-\ell_1).$$

Equations (11) determine the allocations in a type I equilibrium. The price level is determined by an odd agent's budget constraints for $t = 0, 1$, evaluated at the equilibrium allocation. In particular, substituting the equilibrium conditions (6) and (7) and the appropriate versions of (10a) and (10b) as selected by Definition 2 of a type I equilibrium into the budget constraints for an odd agent at $t = 0, 1$ and rearranging gives the following condition that must hold in order that $p_0 > 0$, $p_1 > 0$:

$$\beta(1 + r_0)(1 + r_1)u_1(a\ell_0 - c_0, 1)(a\ell_0 - c_0) - u_1(c_1, 1)c_1 > 0. \quad \textbf{(12)}$$

Equalities (11) and inequality (12) are the equilibrium conditions for a type I equilibrium.

Applying analogous reasoning in the context of a type II equilibrium leads to the following equilibrium conditions:

$$\begin{aligned} \beta^2(1 + r_0)(1 + r_1)u_1(c_0, 1 - \ell_0) &= u_1(a\ell_0 - c_0, 1), \\ u_1(c_0, 1 - \ell_0)a &= u_2(c_0, 1 - \ell_0), \end{aligned} \quad \textbf{(13a)}$$

$$\begin{aligned} \beta^2(1 + r_0)(1 + r_1)u_1(c_1, 1) &= u_1(b\ell_1 - c_1, 1 - \ell_1), \\ u_1(b\ell_1 - c_1, 1 - \ell_1)b &= u_2(b\ell_1 - c_1, 1 - \ell_1) \end{aligned} \quad \textbf{(13b)}$$

$$\beta^3(1 + r_0)(1 + r_1)u_1(c_1, 1)c_1 - u_1(a\ell_0 - c_0, 1)(a\ell_0 - c_0) > 0. \quad \textbf{(14)}$$

4.3. Existence of Periodic Equilibria

We break the study of existence of periodic equilibria into two parts. First, we study the system of equations (11) or (13), disregarding the inequalities (12) and (14) that guarantee a positive price level. In Propositions 1 and 2, we display conditions under which these equations always have solutions, and characterize the dependence of the solutions on $\beta^2(1 + r_0)(1 + r_1)$. In Propositions 3 and 4, we describe conditions under which the solutions of equalities (11) and (13) also satisfy inequalities (12) and (14), respectively, so that a monetary equilibrium exists.

PROPOSITION 1. *Assume that u is strictly concave and C^2, $u_{12} \geq 0$, and $\forall x \leq 1$, $\lim_{(c,\ell)\to(0,x)} u_1(c, 1 - \ell) = +\infty$. Then the system of equations (11a) and (11b) has a unique solution.*

PROPOSITION 2. *Let (c_0, ℓ_0) be the solution to (11a) and let (c_1, ℓ_1) be the solution to (11b). This means that we are temporarily confining ourselves to a type I equilibrium. Under the assumptions of Proposition 1, the following statements are true:*

- (i) *c_0 is decreasing in $\beta^2(1 + r_0)(1 + r_1)$.*
- (i′) *c_1 is increasing in $\beta^2(1 + r_0)(1 + r_1)$, and $\lim_{b\to\infty} c_1 = \infty$.*
- (ii) *ℓ_0 is increasing in $\beta^2(1 + r_0)(1 + r_1)$.*
- (ii′) *ℓ_1 is decreasing in $\beta^2(1 + r_0)(1 + r_1)$.*
- (iii) *$a\ell_0 - c_0$ is increasing in both $\beta(1 + r_0)(1 + r_1)$ and a, and $\lim_{a\to\infty}(a\ell_0 - c_0) = \infty$.*

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Counterparts to Propositions 1 and 2 exist that refer to equations (13a) and (13b), which correspond to a type II equilibrium. In particular, the counterpart to Proposition 1 is identical for system (13a)–(13b). In the counterpart to Proposition 2, the directions of dependence on $\beta^2(1 + r_0)(1 + r_1)$ are reversed in each case.

Propositions 1 and 2 and their counterparts summarize what we know about how monetary policy impinges on the allocations for type I and type II equilibria. The effects of policy on allocations are all intermediated through the two-period rate of return $\beta^2(1 + r_0)(1 + r_1)$. Below, we repeatedly use these results to trace the impact on allocations of some particular policy experiments. Note that in a type I equilibrium, increases in the interest rate $\beta^2(1 + r_0)(1 + r_1)$ increase output in even periods and decrease it in odds, and that the effects are opposite in a type II equilibrium. For a given individual, increases in the interest rate also affect the allocation of consumption and leisure across time within a meeting period.

Propositions 1 and 2 are of limited interest until we know that a monetary equilibrium exists. Existence is the topic of the next two propositions.

PROPOSITION 3. *In addition to the assumptions of Proposition* 1*, assume that* $u_1(x, 1)x$ *is monotone, that if it is monotone decreasing then* $\lim_{x\to\infty} u_1(x, 1)x = 0$*, and that if it is monotone increasing then* $\lim_{x\to\infty} u_1(x, 1)x = \infty$*. Then*

(i) $\exists\bar{a}(b)$ *such that for any given b, if* $a \geq \bar{a}(b)$ *then there exists a monetary equilibrium. If* $u_1(x, 1)x$ *is increasing, it is a type I equilibrium. If* $u_1(x, 1)x$ *is decreasing, it is a type II equilibrium.*

(ii) $\exists\bar{b}(a)$ *such that for any given a, if* $b \geq \bar{b}(a)$ *then there exists a monetary equilibrium. If* $u_1(x, 1)x$ *is increasing, it is a type II equilibrium. If* $u_1(x, 1)x$ *is decreasing, it is a type II equilibrium.*

Proposition 3 tells us that for any given monetary policy as parameterized by $r = (1+r_0)(1+r_1)$, existence of a monetary equilibrium is more likely the bigger is the discrepancy between a and b, say as measured by a/b or b/a, whichever is larger. The discrepancy between a and b measures the potential utility gains to smoothing consumption across meeting periods (see Figure 1). The discrepancy between a and b interacts with the elasticity of the marginal utility of consumption evaluated at $\ell = 1$ in determining existence of a monetary equilibrium. For example, inspection of (12) and (14) shows that no monetary equilibrium exists when $r < \beta^{-1}$ when $u(c, 1 - \ell) = \log(c) + v(1 - \ell)$ for any concave function v. This example shows that existence is not guaranteed for a "noninterventionist" ($r = 1$) monetary equilibrium. Additionally, a monetary policy characterized by higher interest on currency will make, in some cases, existence of an equilibrium more likely. In particular, if the function $u_1(x, 1)x$ is increasing in x (roughly this corresponds to an intertemporal elasticity of substitution exceeding one), the higher r is, the more likely it is for conditions (12) and (14) to be satisfied. In this case, economies in which there exist no noninterventionist monetary equilibrium ($r = 1$) may have a monetary equilibrium under an interest-on-currency scheme. Existence of a monetary equilibrium is more tenuous in the present model than in most cash-in-advance models or than in Townsend's original version of the turnpike model because of the access agents have to consumption loans as a vehicle for achieving some consumption smoothing.

5. PARETO OPTIMALITY

Consider the class of optimal allocations in which all agents in a given class (odd or even) are treated symmetrically. It is standard to show that the class of all such allocations can be characterized as the solutions to the problem of maximizing $W = \lambda U(c^o, \ell^o) + (1-\lambda)U(c^e, \ell^e)$ subject to feasibility of different choices of the welfare weight $\lambda \in [0, 1]$.

Feasible allocations that can be supported as periodic equilibria are completely summarized by (8). We can verify that a necessary condition for a periodic allocation sequence of the form (8) to solve the problem of maximizing W subject to feasibility is that the welfare criterion puts equal weights on the utilities of both types ($\lambda = \frac{1}{2}$). Thus, if any Pareto optimal allocation is supportable by a periodic monetary equilibrium, it must be the $\lambda = \frac{1}{2}$ equilibrium. At $\lambda = \frac{1}{2}$, the first-order conditions with respect to c_t^i evaluated at a periodic allocation (8) imply that $u_1(c_0, 1-\ell_0) = u_1(a\ell_0 - c_0, 1)$. From the first-order conditions (11) or (13) for a periodic monetary equilibrium, it is evident that if this ($\lambda = \frac{1}{2}$) Pareto-optimal allocation is to be supported by a periodic monetary equilibrium, it must be that $\beta^2(1+r_0)(1+r_1) = 1$.

PROPOSITION 4 (Existence of an Optimal Equilibrium). *Assume the conditions of Proposition* 1. *Also assume that* $\beta^2(1+r_0)(1+r_1) = 1$. *Then, for almost all economies, an equilibrium exists.*

There exist economies (β, a, b) for which there exist noninterventionist ($r = 1$) equilibria of both type I and type II. However, for all economies, under the optimal policy ($r = \beta^{-2}$) there exists *either* a type I *or* a type II equilibrium. Soon we shall discuss how odd and even agents fare as r changes within a type I or type II equilibrium.

When $\beta^2(1+r_0)(1+r_1) < 1$, a periodic monetary equilibrium is not optimal. The source of the nonoptimality is that in a type I (II) equilibrium, (10b) is satisfied with equality for $i = o$ (e) but with strict inequality for $i = e$ (o). For example, in a type I equilibrium the even agents are unable to borrow at the same interest rate r_1 at which the odd agents do. More precisely, because there are a large number of even agents that move together, there is a credit market for them (in this case net trades are zero). Let the interest rate on even-agent-issued securities be denoted R_t ($t = 0, 1$). Then it follows that

$$R_0 = 1 + r_0,$$
$$R_1 = (1+r_1)[\beta^2(1+r_0)(1+r_1)]^{-2}.$$

Thus whenever $\beta^2(1+r_0)(1+r_1) < 1$, private debt issued by the even agents dominates money (and debt issued by odd agents) in rate of return. To discuss the sense in which the policy $\beta^2 r = 1$ implements a version of Friedman's rule, it is useful to study a related economy.

5.1. An Associated Centralized Economy

For purposes of comparison, it is useful to consider an alternative economy in which all locational and traveling features of our model are abandoned and replaced by the assumption that all activity and trade occur at a single centralized location. In a (nonmonetary) competitive equilibrium of this economy, futures markets are open at date 0. An agent of type i maximizes (1) subject to the single budget constraint

$$\sum_{t=0}^{\infty} q_t^0 c_t^i \leq \sum_{t=0}^{\infty} q_t^0 \omega_t^i \ell_t^i,$$

where q_t^0 is the time 0 price of time t consumption goods. It is straight forward to show that a competitive equilibrium involves the period-two consumption allocation $\{c_t^o\} = \{c_0^o, c_1^o, c_0^o, c_1^o, \ldots\}\{c_t^e\} = \{c_0^e, c_1^e, c_0^e, c_1^e, \ldots\}$ and the labor allocation $\{\ell_t^o\} = \{\ell_0^o, 0, 0, \ell_1^e, \ldots\}, \{\ell_t^e\} = \{0, \ell_1^e, \ell_0^o, 0, \ldots\}$, where $c_0^e = a\ell_0^o - c_0^o, c_0^e = b\ell_1^0 - c_1^o$, where $(c_0^o, c_1^o, \ell_0^o, \ell_1^e)$ solve

$$u_1\big(c_0^o, 1-\ell_0^o\big) = \frac{\mu^o}{\mu^e} u_1\big(a\ell_0^o - c_0^o, 1\big),$$

$$u_1\big(c_1^o, 1\big) = \frac{\mu^o}{\mu^e} u_1\big(b\ell_1^e - c_1^o, 1-\ell_1^e\big),$$

$$u_2\big(c_0^o, 1-\ell_0^o\big) = u_1\big(c_0^o, 1-\ell_0^o\big)a,$$

$$u_2\big(b\ell_1^e - c_1^o, 1-\ell_1^e\big) = u_1\big(b\ell_1^e - c_1^o, 1-\ell_1^e\big)b,$$

where μ^o, μ^e are the Lagrange multipliers on odd and even agents' budget constraints, respectively.

A competitive equilibrium allocation exists, is unique, and is Pareto-optimal. It can be verified that the competitive equilibrium allocation depends on the values of (β, w, e), and that in general it does not equal the $(\lambda = \frac{1}{2})$ Pareto-optimal allocation selected by the optimal interest on currency equilibrium.

For all $t \geq 0$, the two-period rate of return in this associated centralized economy is β^{-2}. Thus, putting $(1+r_0)(1+r_1) = \beta^{-2}$ amounts to setting the two-period real rate of return on currency equal to the real rate of return in the equilibrium of the associated centralized economy. The following points implied by Proposition 2 about a type I equilibrium (and implied by the paragraph immediately after Proposition 2 about a type II equilibrium) are noteworthy. First, although the optimal policy can be implemented by equating r_0, r_1 to the corresponding one-period rates of return for the associated centralized economy, any other settings that satisfy $(1+r_0)(1+r_1) = \beta^{-2}$ will also work. Thus, the one-period rates r_0, r_1 are not pinned down by the requirement of optimality. Second, the optimal policy involves eliminating any discrepancy between the rates of return on currency and private securities, but only in a very special and subtle way. In this model, securities issued by odd and even agents can sell at different prices at switching times. As described earlier, the rates of return are the same at $t = 0, 2, 4, \ldots$, but $R_1 > (1+r_1)$. The optimal policy has the effect of making $R_1 = 1+r_1$ and, hence, effectively eliminating existing–rate of return dominance. Note, however, that the implementation of an optimal allocation is not guaranteed by the elimination of legal restrictions on the financial sector. In some models, it is restrictions on private intermediation that lie behind the suboptimality of the monetary equilibrium. It is not so in this model. No private institution can be created, given the environment, that is capable of eliminating the suboptimality of the equilibrium allocation. Third, the fact that the optimal policy is framed in terms of a restriction on a *two-period* rate r reveals that it is the imperfection of the monetary equilibrium as a mechanism for carrying goods from t to $t+1$ for t odd that the optimal policy is correcting. In other words, private markets are effectively myopic in this environment. Unrestricted borrowing and lending solves the short-term (one-period) borrowing needs. However, the structure of the economy is such that the demand for loans induced by the smoothing motive is more long-term (four periods). It is this limitation of private loan markets that is being solved by the optimal monetary policy.

5.2. Restricted Pareto Optimality

Although our characterization of the optimal monetary policy is very sharp, it says little about the potential problems that a government may face trying to implement it. More precisely, consider an initial situation in which the monetary policy is characterized by $r_A < \beta^{-2}$. To study the impediments that a central bank will face in implementing another policy r_B satisfying $\beta^{-2} \geq r_B > r_A$, it is necessary to evaluate how much "waste" is associated with the original policy, as well as the welfare implications of a change in the interest on currency policy.

To do this, a restricted notion of Pareto optimality is useful. This notion of optimality is induced by a Pareto problem defined over the restricted class of periodic allocations (7). Formally, we have

DEFINITION 3. *An allocation* (c^o, ℓ^o), (c^e, ℓ^e) *is restricted Pareto-optimal if it is feasible, satisfies the restrictions imposed by (8), and is such that there is no other allocation satisfying the same constraints that yields higher utility to one of the two types of agents without decreasing the utility of the other.*

Basically, restricted Pareto optimality is just simple (stationary) Pareto optimality over the set of possible monetary equilibrium allocations. The class of these allocations can be parameterized by a point $x = (c_0, \ell_0, c_1, \ell_1)$, where $x \in X \subset \mathbf{R}^4$. Let $V^o(x)$ and $V^e(x)$ be the utility functionals (1) of odd and even

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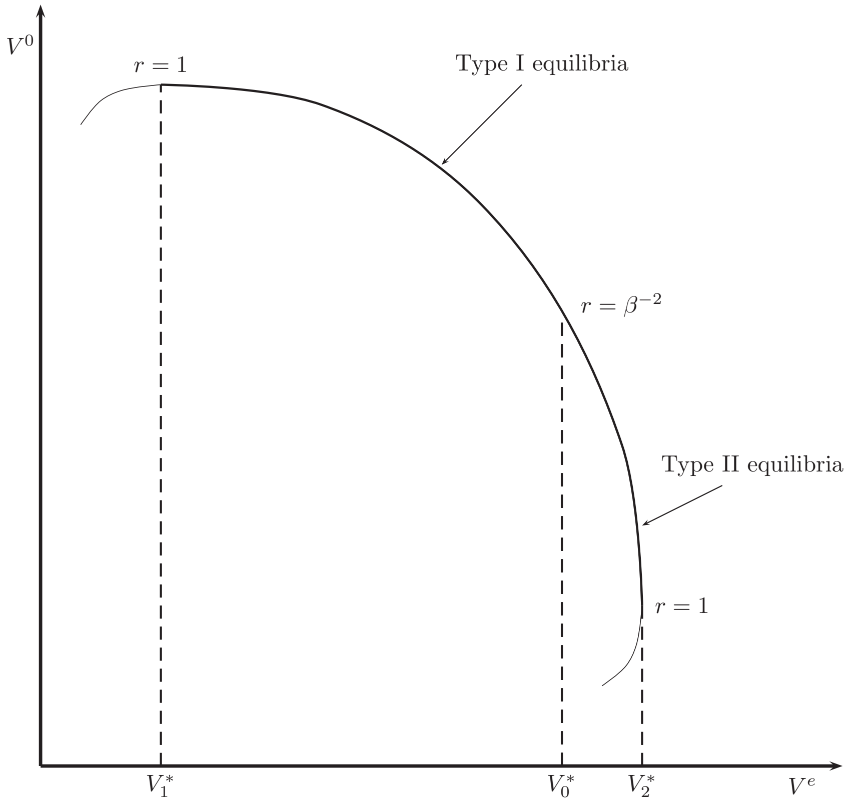

FIGURE 2. The function $F(V^*)$ depicts the set of restricted Pareto optimal utilities, where "restricted" means utilities attainable via a periodic allocation satisfying (8). The function $F(V^*)$ is inside the utility possibility frontier for the economy, except at the point $(V_0^*, F(V_0^*))$.

agents evaluated at their respective periodic allocations. Let $V^* \in [V_1^*, V_2^*]$, where V_1^* is the value of the utility functional assigned to even agents in a type I noninterventionist ($r = 1$) equilibrium allocation and V_2^* is the value of utility assigned to even agents in a type II noninterventionist ($r = 1$) equilibrium. The restricted utility possibility frontier is described by the function $F(V^*) = \max_x\{V^o(x)$ subject to $V^e(x) \geq V^*\}$.

On the interval $[V_1^*, V_2^*]$, $F(V^*)$ has the shape depicted in Figure 2. We summarize the main features of $F(V^*)$ in

PROPOSITION 5.

(i) *Let $V^e = V_0^*$ be the utility level assigned to even agents in the unrestricted optimal equilibrium with $1 = \beta^2(1 + r_0)(1 + r_1)$. Then $V^o = F(V_0^*)$ is the utility level assigned to odd agents in that unrestricted optimal equilibrium.*

(ii) $F'(V_0^*) = -1$.

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(iii) *For* $V^* \in [V_1^*, V_0^*]$, *the associated allocations* $x = (c_0, \ell_0, c_1, \ell_1)$ *solve the type* I *equalities* (11) *for some* $r > 1$.
(iv) $F'(V_1^*) = 0$.
(v) *For* $V^* \in [V_0^*, V_1^*]$, *the associated allocations* x *solve the type* II *equalities* (13) *for some* $r > 1$.
(vi) $F'(V_2^*) = +\infty$.

When $r \geq 1$, if a periodic monetary equilibrium exists, it is restricted Pareto-optimal. The $(V_1^*, F(V_1^*))$ equilibrium is a type I equilibrium with $r = 1$. Among all periodic equilibria, this one makes the odd agents best off. As we increase r from 1 toward β^{-2}, within type I equilibria the welfare of odd agents falls and the welfare of even agents rises. The $(V_2^*, F(V_2^*))$ equilibrium is a type II equilibria with $r = 1$. Among all periodic equilibria, this one makes the even agents best off. Within type II equilibria, welfare of even agents falls as that of odd agents rises as r is increased from 1 to β^{-2}.

It is then clear that in this model the proposal of following the optimal monetary rule starting from some suboptimal equilibrium does not have unanimous support. Any movement toward an equilibrium with $r = \beta^{-2}$ will result in a utility loss for either the even or odd individuals depending on the equilibrium the economy is in.[13]

In cash-in-advance and money-in-the-utility function models, proposals to increase the rate of return on currency from one toward β^{-2} are Pareto-improving, and so have unanimous support. In the present model, half of the agents are harmed by such increases, whereas half are benefitted. This feature of our model is due to the heterogeneity of agents, and is shared by other heterogeneous agent models, such as the overlapping-generations model. However, the present model differs from the overlapping generations model with respect to the impact of changing the inflation rate. We postpone the discussion of these issues until later.

6. INTERPRETATION

The optimal equilibrium under interest on currency is determinate in several relevant senses. Equilibrium allocations and therefore real interest rates are unique. The odd-period price level p_1 is uniquely determined. The even-period price level p_0 is not determined until r_0 is set. There remains enough flexibility to select r_0 so that $p_0 = p_1$, but such price stability is not needed to implement the optimal policy.

It is noteworthy that the optimal policy cannot be characterized in terms of paying interest at a "market rate" associated with some suboptimal equilibrium. In *any* equilibrium in which currency is valued, currency and *some* private loans bear identical rates of return. Of course, there are *other* loans that, in some periods, bear a higher rate of return. It is then possible to state the policy as picking a rate of return on currency holdings that eliminates the difference between the highest return riskless assets and money.

The allocation associated with the optimal policy is Pareto-optimal, but generally differs from the Pareto-optimal equilibrium allocation of the associated (centralized) Arrow–Debreu economy. In particular, note that the optimal–interest on currency policy results in the equal-weight ($\lambda = 1/2$) Pareto-optimal allocation.

These results on paying interest on currency reveal important differences between the models of the present paper and each of the cash-in-advance, money-in-the-utility function, and overlapping-generations models. In the cash-in-advance model, the price level becomes indeterminate under an optimal–interest on currency policy financed by lump sum taxes [see Sargent (1987, Chapt. 5)]. Essentially, this is because at the optimal rate of interest on currency, the cash-in-advance constraint does not bind, so that the demand for currency becomes a correspondence. In the cash-in-advance model, the equilibrium allocation that obtains under the optimal–interest on currency policy is identical with the one that would be achieved simply by repealing the law that imposes the government-issued-cash-in-advance constraint and permitting free banking. This equilibrium allocation is the same one that would be achieved if the timing-of-trade restrictions that support the cash-in-advance restriction were relaxed to permit the full time-zero Arrow–Debreu markets.[14]

In contrast, in our model, government-issued currency continues to play an essential role under the optimal–interest on currency policy. The allocation achieved under that policy differs from the one that would be associated with the corresponding economy with the locational restrictions voided and centralized time-zero trades permitted. Further, the interest-on-currency policy does not work in a way that can be replicated by permitting free banking. Also, the price level remains determinate under the optimal interest on currency policy.

In the overlapping generations model, paying interest on currency at the market rate of return, financed by lump-sum taxes, gives rise to a continuum of equilibrium real interest rates, taxes, and price levels, each equilibrium of which is Pareto-optimal [see Sargent and Wallace (1985)]. Some aspects of these results carry over to our model, but others do not. What does carry over is an element of indeterminacy. Above we constructed a continuum of equilibrium interest rate, tax rate, and price level combinations that could be indexed by the interest rate $r \in [1, \beta^{-2}]$. Subject to the existence conditions (12) and (14) holding, any two-period gross real rate of interest $r \in [1, \beta^{-2}]$ is associated with an equilibrium. In this sense, just as in Sargent and Wallace (1985), the proposal to pay interest on currency "at the market rate" is incomplete even when supplemented by a plan to finance the proposal by lump-sum taxes.[15] However, the current model differs from the overlapping-generations model in that among the continuum of equilibria under an interest on currency policy, there is a unique optimal one. This is the Friedman rule, $r = \beta^{-2}$. The uniqueness of the optimal rate for our model seems to stem from the infinite lives of the agents, which has the effect of rendering infeasible the high–interest rate equilibria that are optimal in Sargent and Wallace's (1985) model.

7. REDISTRIBUTION OF INITIAL CURRENCIES

The monetary equilibria studied up to this point are ones in which initially currency has all been in the hands either exclusively of odd or exclusively of even agents. That specification was made to support the existence of periodic equilibria. We now perform experiments in which the economy starts out with a fraction $0 < \alpha < 1$ of the currency in the hands of even agents and a fraction $(1 - \alpha)$ in the hands of odd agents. We show that if a type I (II) periodic equilibrium exists, then for any $0 < \alpha < 1$, there exists a nonperiodic monetary equilibrium in which allocations and prices attain their values for the periodic type I (II) equilibrium after one meeting period. In these equilibria, the tails $\{c_s^i\}_{s=2}^{\infty}$ $\{\ell_s^i\}_{s=2}^{\infty}$ of the allocations agree with the corresponding tails of the allocations given by (7); the first two-period allocations to odd agents are denoted $(c_0^\alpha, c_1^\alpha, \ell_0^\alpha, \ell_1^\alpha)$. The nominal price sequence has the form $(p_0^\alpha, p_1^\alpha, p_0, p_1, p_0, p_1, \ldots)$. For simplicity we concentrate on noninterventionist equilibria ($r = 1$). The following propositions describe our results:

PROPOSITION 6. *Assume that a type* I *equilibrium exists. Assume that initial currency holdings are* $m_{-1}^o = (1 - \alpha)H, m_{-1}^e = \alpha H$ *for* $0 < \alpha < 1$. *Then there exists a monetary equilibrium that converges to a type* I *equilibrium after one meeting period. Further, if we denote* $(c_0^o, c_1^o, c_2^o, c_3^o)$ *by* $(c_0^\alpha, c_1^\alpha, a\ell_0 - c_0, b\ell_1 - c_1)$, *then it is true that*

(i)

$$c_0^\alpha > c_0, \qquad \ell_0^\alpha < \ell_0,$$

$$c_1^\alpha > c_1, \qquad \ell_1^\alpha > \ell_1,$$

(ii) $a = b$ *implies that* $\ell_1^\alpha > \ell_1 > \ell_0 > \ell_0^\alpha$.
(iii) $p_0^\alpha < p_0, \qquad p_1^\alpha < p_1$.

PROPOSITION 7. *Assume that a type* II *equilibrium exists. Assume that initial currency holdings are* $m_{-1}^o = (1 - \alpha)H, m_{-1}^e = \alpha H$ *for* $0 < \alpha < 1$. *Then there exists a monetary equilibrium that converges to a type* II *equilibrium after one meeting period. It is true that*

(i)

$$c_0^\alpha < c_0, \qquad \ell_0^\alpha > \ell_0,$$

$$c_1^\alpha < c_1, \qquad \ell_1^\alpha < \ell_1,$$

(ii) $a = b$ *implies that* $\ell_0^\alpha > \ell_0 > \ell_1 > \ell_1^\alpha$,
(iii) $p_0^\alpha < p_0, \qquad p_1^\alpha < p_1$.

These propositions are proved constructively by working backward from the respective type I and type II equilibria. Thus, for Proposition 6, the first-order conditions (10) for odd and even agents are evaluated at candidate values for prices and quantities of the (periodic after two periods) form described above. For

Proposition 6, we add the budget constraints of odd agents, imposing that odd agents carry all of the currency from $t = 1$ to $t = 2$. We thereby obtain a system of equations capable of uniquely determining $c_0^\alpha, c_1^\alpha, \ell_0^\alpha, \ell_1^\alpha, p_0^\alpha, p_1^\alpha$. For Proposition 7, we add the budget constraints of even agents, requiring that even agents hold all of the currency from $t = 1$ to $t = 2$, and proceed analogously.

Propositions 6 and 7 imply that a particular kind of indeterminacy emerges for our experiment in cases in which (β, a, b) are such that type I and type II equilibria both exist. In these cases, for any value of the distribution parameter $\alpha \in (0, 1)$, it is possible to jump to either a type I or a type II equilibrium after one meeting period.[16]

More interesting are the implications of this flexible-price model for the effects of a once-and-for-all monetary injection. Consider a structure in which at time zero the pretransfer money supply is given by $H/\mu, \mu > 1$, and the government finances transfers of $(\mu - 1)H/2\mu$ dollars to each individual by printing money. Consider the impact in a type I equilibrium. Before the transfer occurs each odd agent holds no money; thus, his or her post-transfer stock of currency is $(\mu - 1)H/2\mu$. In this case $\alpha = (\mu - 1)/2\mu$ and the consequences of this monetary and fiscal policy combination are summarized in Proposition 6. Note that initially output goes down. This reflects the positive income effect of the transfer, which reduces the labor supply of the odd individuals who are the only ones working at $t = 0$. Symmetrically, the income effect is negative for the even agents and this is reflected in an increase in their labor supply ($\ell_1^\alpha > \ell_1$) and the resulting increase in output at $t = 1$.[17]

An observer of this economy could erroneously conclude that prices are sticky. Formally, the full impact of the monetary shock is not felt until period 2 and in the first two periods prices are below their steady state level ($p_t^\alpha < p_t,\ t = 0, 1$). It is clear that this sluggishness is not due to frictions that prevent instantaneous price adjustments. It is the counterpart of the real effects of the monetary transfer and it is basically due to the presence of heterogeneous agents. More precisely, although we study a situation in which all individuals receive identical transfers, these transfers affect their decisions because some of them are at a corner of their budget sets. This results in real effects and nonneutrality of a standard "helicopter" monetary shock.

The results are similar to those obtained by Grossman and Weiss (1983), Rotemberg (1984), and Grossman (1987).[18] In all those models the demand for money is motivated by a cash-in-advance constraint, and specific functional forms for the utility function are studied. By assumption, only a fraction of the population is at the bank when the open-market operation occurs and hence it is this assumed heterogeneity that causes the real effects. In the case that we studied, all individuals could receive the same amount of currency and still prices would not fully respond. Moreover, an implication of the models studied in all three papers is that a policy of sustained increases in the money supply does not have effects on aggregate output, whereas in the model of this paper, it generates changes in both the mean level and the variability of output.

8. THE EFFECTS OF INFLATION

Consider the case in which the government finances a stationary transfer program by printing money. Assume, for simplicity, that the government increases the money supply at the rate μ. It is relatively straightforward to derive the analogues of equations (10) in the case of a steady-state equilibrium. By this we mean a situation in which $p_t = ((1+\pi_0)(1+\pi_1))^{t/2} p_0$ for $t =$ even, and $p_t = (1+\pi_0)(1+\pi_1)^{t-1/2} p_1$ for $t =$ odd. It follows that the set of equations (11), (12) and (13), (14) still describe the necessary and sufficient conditions for the existence of a monetary equilibrium. In the case of positive money growth, $r = (1+r_0)(1+r_1) = \mu^{-2} < 1$. It is then possible to use Propositions 1–3 to study the impact of higher rates of money growth.

There are several interesting effects associated with money growth. To simplify, we concentrate on a type I equilibrium (similar arguments are valid for a type II equilibrium) and we compare a case with $\mu > 1$ with the no-inflation ($\mu = 1$) case for the environment with no exogenous productivity fluctuations ($a = b$). From Proposition 2 it follows that

$$\ell_1(\mu) > \ell_1(1) > \ell_0(1) > \ell_0(\mu).$$

That is, a higher mean rate of inflation also causes an increase in the variance of output. Because the nominal interest rate does not respond to the higher monetary growth, the real rate of return also decreases.[19] Finally, it is interesting that, unlike many other heterogenous agent models, inflation yields Pareto-inferior outcomes. To see this, extend the analysis of the restricted Pareto-optimal allocations and define $F(V)$ by

$$F(V^*) = \max_x \{V^0(x)\},$$
$$\text{subject to } V^e(x) = V^*.$$

Note that in this case we impose the constraint with equality and, hence, the Lagrange multiplier can be negative. Following the arguments in Proposition 5, it is possible to show that inflationary equilibria (when they exist) indeed correspond to choices of V^* for which the Lagrange multiplier is negative. This corresponds to the upward sloping part of the restricted utilities possibility frontier in Figure 2. In the case of a type I equilibrium the even agents receive a utility level $V^* < V_1$ and the odd agents receive $F(V^*) < F(V_1^*)$. Unlike the case in which there is interest paid on money holdings, a decrease in the rate of money growth is unanimously preferred. This feature contrasts with the situation in models of overlapping generations of two period–lived and heterogeneous agents. Versions of those models exist in which one class of agents, borrowers, prefer inflation, whereas another class of agents, lenders, prefer price stability to inflation. We think that the outcome in our model differs from that in these overlapping-generation models because of the absence of permanent classes of borrowers and lenders in our model. In our model, each agent is sometimes a borrower and at other times a lender.

The channels through which a policy of sustained increases in the money supply affect the equilibrium are qualitatively very different from those corresponding to a once-and-for-all increase in the money supply. To see this, note that *independent* of who receives the transfer, the equilibrium allocation is completely determined by μ. It is then clear that the effects of inflation upon the real allocation are strictly substitution effects. The details of how the money supply is increased affect the level of prices as well as the amount of net credit in the economy. To make this argument precise, consider the budget constraints of an odd agent in periods 0 and 1

$$p_0 c_0 + m_0 + p_0 b_0 = p_0 a \ell_0 + T_0,$$
$$p_1 c_1 + m_1 = m_0 + p_0 b_0 + T_1,$$

where T_i are transfers at time i and in a type I equilibrium $m_1 = \mu H$, the supply of money. It is clear that the model pins down $q = m_0 + p_0 b_0$ and not the individual components. Thus, the real value of total savings by an odd individual is

$$\frac{q}{p_0} = a\ell_0 - c_0 + \frac{T_0}{p_0}.$$

Using the results in Propositions 1–3, it can be shown that increases in μ decrease $a\ell_0 - c_0$. Thus, if $T_0 = 0$ (even agents get the transfer), real savings decrease. If, on the other hand, $T_0 > 0$ and, say, $T_0 = (\mu - 1)H/2\mu$ (each individual gets one-half of the increase in the money supply), an increase in μ increases the first-period income of the odd agent, and it is possible that this will result in an increase in total savings. Thus, although it is relatively easy to predict the impact on real balances, it seems that the relationship between monetary/financial magnitudes and real variables depends on who gets the transfers.

The intuition for the different roles played by income and substitution (or rate of return) effects in the experiment studied in the previous section and in the inflationary environment analyzed here is easy to describe. Consider, for example, a situation in which the odd agents gain from the inflationary policy. This agent at $t = 2$ will be an even agent and consequently will lose in the future. What is remarkable is that, independent of preferences, the relevant present value of gains and losses does not change. This is to be contrasted with the once-and-for-all change in the money supply. In this case, there is no future date at which the experiment will be reversed and, consequently, the income effects (or details about how the money is distributed) have real consequences.

9. MONETARY AND CREDIT POLICY

We now modify our previous assumptions about the access that individuals have to private loan markets. We consider a situation in which the government has the option of restricting private borrowing and lending. We compare an extreme case of laissez-faire (the situation analyzed so far) with a complete ban on private borrowing and lending. In this second regime [which is reminiscent of the original Townsend (1980) environment], the only asset that individuals can hold is currency. As in Townsend (1980), in the case of a constant money supply, this automatically generates a well-defined demand for money and guarantees the existence of an equilibrium. The details are in Appendix B.

It is useful to study the effect of a given monetary policy in two economies that have different credit policies. In particular, we will argue that allowing for a minimal amount of heterogeneity (and hence making restrictions on private borrowing and lending meaningful) dramatically alters the impact on the economy of the different monetary policies considered so far. To simplify the presentation, we consider the case of no aggregate fluctuations in productivity ($a = b$) and we concentrate on a type I equilibrium of the laissez-faire economy. A more general treatment is in Appendix B. First consider the effect of a policy of sustained increases in the money supply that are used to finance a transfer program. As described in Section 7, an increase in the rate of growth of the money supply (μ) has ambiguous effects on the average level of output but increases its volatility when there are no restrictions on private borrowing and lending. However, in the economy in which individuals do not have access to private loan markets, the results are quite different: an increase in μ *decreases* mean output and has *no effect* on volatility (which remains zero).

Next consider a once-and-for-all increase in the stock of money. In the laissez-faire case, prices are sluggish and output initially drops, followed by an increase relative to the steady state value. The adjustment process is completed after two periods. With credit restrictions, the qualitative effects are quite different. The most important features are these. First, the size of the transfer affects the real outcomes. Second, the adjustment process is completed after one period. Third, prices may over- or undershoot the steady state values, depending on the size of the increase in the money supply.

Although the details that determine the differences in the response of the economies to this monetary experiements are model-specific, the more general principle that we should expect these impacts to depend on the specific credit policy in place seems to be robust. In particular, the two aspects that account for the differential effects are the differing effects of policies on wealth distribution, and the differing opportunities that individuals have to smoothe consumption in the face of wealth shocks.

Our analysis of the optimal–interest on currency policy indicates that under the optimal policy, the government in effect chooses interest payments to "complete" the credit market by way of eliminating the differences between the intertemporal rates of substitution of different groups. As a partial confirmation of this intuitive argument, we can use the fact that the optimal policy under complete restrictions on borrowing and lending remains unchanged: the interest rate must equal the inverse of the discount factor. In other words, optimal management of the interest paid on cash balances completely makes up for the missing credit markets and yields the same optimal allocation as in the case of no restrictions.

So far we have studied the effects of a given monetary policy in two different and extreme credit environments. It is of interest to understand the consequences of a regime switch, that is, a change in credit policy holding monetary policy constant. To simplify the exposition we only consider a monetary policy that holds the supply of currency constant.

It is not uncommon for policy makers to advocate quantitative restrictions on credit in order to curb inflation.[20] It is relatively easy to study the consequences of an extreme form of credit control in this model. To be precise, consider an economy with no restrictions on private borrowing and lending in which at time zero the government institutes a complete ban on borrowing and lending. Using the analysis in Appendix B, it is easy to show that the dynamics are similar to those of an economy with no private loans and with arbitrary initial endowments of money (those corresponding to the previous equilibrium). The main results are these. First, initially output increases and prices decrease. Second, the economy converges to the new steady state in one period. This steady state is characterized by a lower mean output and no volatility.

The basic intuition for the deflationary consequences of credit controls is similar to that in Wallace (1980): the elimination of one asset (private loans) increases the demand for the other assets (in this case fiat money) and consequently increases its value. Additionally, there are real effects because the intertemporal price of consumption and leisure changes. Although it can be shown that the switch to a policy of credit controls has an adverse effect upon welfare, it delivers a lower price level.

An opposite credit policy is a form of financial liberalization. Within the context of this model, it corresponds to an unanticipated elimination of the ban on private borrowing and lending at time zero. The effects are exactly the opposite to those described in the case of credit controls: prices increase and output becomes more volatile.[21] It is interesting that in describing actual experiences with financial deregulation, their potential destabilizing effects are often emphasized. For example, in reviewing the recent experiences of financial liberalization in the Southern Cone countries, the World Bank notes that "The biggest problems began in the real sectors of the economy, but efforts to liberalize the financial sector undoubtedly contributed to the resulting instability." [22] As one of the lessons from the reform, the World Bank notes that "The clearest lesson is that reforms carried out against an unstable macroeconomic background can make that instability worse."[23] Thus, the concern shown by policy makers about the potential increase in instability associated with a liberalized credit market have a counterpart in the model as output volatility increases. However, in the model, a liberalized regime is superior from the point of view of welfare.

To summarize, the model suggests that there is a rich and complex interaction between monetary and credit policy. These effects are captured even in a model in which there is a small degree of heterogeneity across individuals that allows us to model a nontrivial credit market. This suggests that representative agent models might give misleading answers to standard questions about the effect of monetary policies insofar as they do not take into account distribution effects.

10. CONCLUDING COMMENTS

We believe that explicit modeling of the frictions that prevent a competitive equilibrium from attaining Pareto-optimal allocations provides useful insights into both the form of the demand for money and the effects of alternative monetary policies. In this paper, we have explored a particular version of Townsend's turnpike model in which inside and outside money coexist. We study a specification in which the length of an individual productivity cycle (four) exceeds that of a meeting period (two). In particular, although individuals would like to smooth their consumption across their period-four productivity cycles, private loan markets fail to provide smoothing for more than two periods. We think of the model as representing frictions that prevent credit markets from providing bonds of all maturities. It is evident in our model that if individuals stayed together for four periods, the resulting nonmonetary equilibrium allocation would be Pareto-optimal and there would be no monetary equilibrium. However, if individuals were to meet for more than four periods, then it *would be* possible for monetary equilibria to exist (except if the meeting period were a multiple of four).[24] Thus, the strong spatial separation modeled in the turnpike economy provides one way of modeling less-than-perfect credit markets and the role of money.

The heterogeneity among individuals implied by our environment influences our results in key ways. This feature is well illustrated by our results about interest on currency. On one hand, the heterogeneity implies that price-level sequences and tax rates are unique under an optimal interest-on-currency policy. In many models an optimal policy either does not exist or else leads to price-level indeterminacy. On the other hand, the heterogeneity and the pervasive wealth redistribution in our environment in general cause the optimal interest-on-currency policy to recover a different Pareto-optimal allocation than the one associated with the Arrow–Debreu economy in which all frictions have been removed.

It is possible to describe the optimal policy as one in which the government pays interest on currency in order to eliminate completely all interest rate differentials between assets of the same risk characteristics. In any monetary equilibrium, all individuals traveling together are free to issue private loans. Thus, whenever it is their turn to hold money, currency and private debt earn the same return. In general, in periods in which nobody is moving to a new village, private evidence of indebtedness and fiat money do not dominate one another. It is only during switching periods that some individuals (those not holding money) are forced to borrow at a higher interest rate. These individuals do not have access to the low–interest rate market available at their location because of the nature of their liabilities: it is impossible for any potential lenders to redeem the debt. The optimal monetary policy effectively eliminates all differences in rate of return among all types of privately issued bonds.

The welfare consequences of alternative monetary policies are somewhat surprising. Consider the class of policies that affect the rate of return (this class includes both interest on reserves and sustained inflation policies). The heterogeneity of the productivity sequences across people in general renders welfare analysis inconclusive, but for some of the policy interventions we studied we attain sharp "distribution-free" welfare conclusions. There are two welfare results that do not depend on specific properties of preferences. First, a movement from positive inflation to zero inflation is a Pareto improvement. Second, different interest-on-currency policies are all efficient in the class of stationary market-implementable policies. Thus, there is no consensus in this economy on the optimal monetary policy.

Our model shares some properties with transactions-demand models of the demand for money as well as models in which currency is a store of value. In the spirit of the former, our environment gives money-holders the ability to make payments when other forms of wealth are useless and it shares some of the implications of many of those models about the effect of a one-time increase in the money supply: there is a short-run impact on the equilibrium allocation, and prices respond sluggishly to the monetary shock. However, the study of interest on currency or sustained inflation policies reveals that the impact on the real part of the economy is mediated through their effects on the rate of return, independent of the distributive consequences of the associated fiscal policy. This exercise suggests that explicit modeling of frictions offers some advantages over the alternative of motivating the demand for money using its "store of value" or "medium of exchange" function.

The existence of inside money has important implications for the effects of monetary policies. One way of highlighting this is to consider the same model that we studied with the restriction that there is no private borrowing and lending. Even with the minimal amount of heterogeneity that we consider, the impact of different monetary policies depends on the ability that agents have to smooth out or adjust to the consequences of those policies via credit markets. Episodes of credit controls can succeed in lowering the price level and reducing fluctuations, albeit at the cost of reducing welfare. Instances of financial liberalization can be accompanied by price increases and increased output volatility that nevertheless, correspond to a better allocation from the standpoint of individuals' utility.

Much more can be done with this model. Ireland (1991) has studied a stochastic version. He concentrates on numerically finding equilibria but does not explore the welfare consequences of different monetary policies. We suspect that an optimal interest on currency policy would be much more complicated than the one found here because in Ireland's environment money not only must guarantee the "right" intertemporal marginal rate of substitution but also must help partly to make up for the missing insurance markets. It is then possible that the optimal interest paid on currency depends on the state of the economy.

Although we have taken the meeting period as given in this paper, our preliminary research indicates that the implications of some monetary policies vary with the length of the meeting period. Note that as the meeting period increases private loan markets are better suited to provide for consumption smoothing; this will change the role of money in this economy.

Finally, the turnpike structure seems an interesting one in which to study potential coordination problems. For example, consider a situation in which individuals can stay at most three periods in a location; that is, each agent can move after one, two, or three periods. It is likely that the welfare consequences of different configurations of moving strategies would give rise to different equilibria and that the optimal monetary policy might eliminate potential coordination problems.

NOTES

1. However, for a Bewley model solved with pencil and paper, see Wang (2003).

2. Ljungqvist and Sargent (2004, Chap. 17) describe a variety of fruitful applications of computer-analyzed Bewley models with ex ante identical but ex post heterogeneous agents who own stochastic stochastic income processes.

3. Chatterjee and Corbae (2006) also use a turnpike structure.

4. Papers written since 1988 have substantially deepened our understanding of environments that more or less justify assuming market completeness. Huggett and Krasa (1996) propose using mechanism design to understand the sense in which money is essential in environments like Townsend's. Kocherlakota (1998) uses a mechanism design analysis to study the role of communication and commitment technologies within a class of environments rich enough to include turnpike structures. Cole and Kocherlakota (2001) solve a mechanism design problem whose solution is an allocation equivalent to that attained by a Bewley model with natural borrowing limits imposed.

5. See Sargent (1987, pp. 220–224) for a slight modification and extension of Townsend's intervention that supports the claim made in the text.

6. For a recent survey of the literature on the optimal quantity of money see Woodford (1990). In that paper Woodford also describes settings that are versions of the Townsend Turnpike.

7. See Archibald and Lipsey (1958) for an earlier analysis of this issue.

8. Ostroy and Starr (1990) critically review models of the transactions role of money, including the turnpike model.

9. Consider a different version of our model in which agents stay at each trading post for four periods before moving on. Given the period-four individual endowment sequences, the equilibrium of this model would have allocations identical to those of the Arrow–Debreu equilibrium of an associated centralized economy (see below), and there would exist no monetary equilibria. Similarly, for Townsend's model, in which agents have period-two endowment sequences, extending to meeting session from one to two periods would eliminate monetary equilibria and make the equilibrium allocation equivalent with that of a centralized competitive equilibrium.

10. In our model, it is not feasible for the government to finance payment of interest on currency by using the proceeds from holding a portfolio of private securities. The reason is that although private evidences of indebtedness are traded every other period, they trade at a net nominal interest rate of zero, which leaves no arbitrage profits available for the government to reap. Lucas (1986) and Wallace (1989) have described and studied such a scheme as a way of financing interest on currency in various models in which initially currency is dominated in rate of return by private securities. In the models studied by Lucas and Wallace, implementing this scheme attains the same consumption allocation that would be achieved by dismantling the legal restrictions that support rate-of-return dominance (e.g., by permitting "free banking").

11. Notice that $\{c_t^e\} = S\{c_t^o\}$, $\ell_t^e = S\{\ell_t^o\}$, where S is the two-period shift operator defined by $S\{x_t\}_{t=0}^{\infty} = \{x_{t+2}\}_{t=0}^{\infty}$.

12. The argument is by contradiction. Assume that (9b) holds with equality for each type of agent. For the case in which $r < \beta^{-2}$, this implies a price-level path such that real balances are growing without bound over time, which violates the hypothesis that households are solving their choice problems. For the case in which $r = \beta^{-2}$, the argument is subtler but similar.

13. For another class of models that display rate-of-return dominance and in which paying interest on currency does not Pareto-dominate a noninterventionist equilibrium, see Freeman (1985).

14. With respect to the issues being discussed here, results for the currency-in-the-utility-function model are quite close to those for the cash-in-advance model. Some technical differences may emerge, depending on whether there is utility satiation in currency. See Brock (1974, 1975).

15. This is one of the points made by Tobin (1965).

16. The indeterminacy is undoubtedly even greater. In particular, we suspect that there are equilibria in which for any $\alpha \in (0, 1)$, convergence to a type I or type II equilibrium occurs after N meeting sessions for various values of $N \geq 2$.

17. These results are not robust, as they are reversed in the case of a type II equilibrium as shown in Proposition 7.

18. The mechanism at play is also closely related to that studied by Levine (1991).

19. There is some ambiguity in this statement, because in the switching periods there is no well-defined interest rate, and hence we use the one-period rate given by $u_1(c_0, 1 - \ell_0)/\beta u_1(c_1, 1)$.

20. For a description of the history of credit controls in the U.S. with special emphasis on the Credit Control Act of 1969 and its use by the Carter Administration, see Schreft (1990). For an analysis of the European experience see Hodgman (1973) and the Joint Economic Committee (1981).

21. Formally, the analysis follows from that in section 6 where the initial endowment (α in the notation of that section) is given by the steady state of the no borrowing and lending case.

22. World Bank (1989), p. 124. For an analysis that concentrates on developed countries, see Blundell-Wignall and Browne (1991).

23. Ibid, p. 127.

24. This was established in an earlier version of this paper.

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APPENDIX A

PROOF OF PROPOSITION 1

Let us rewrite (10) in a slightly more general notation. It is immediate to check that the following equations reduce to (10) for appropriate choices of z and v:

$$u_1(c, 1-\ell) = z\, u_1(v\ell - c, 1),$$
$$u_1(c, 1-\ell)v = u_2(c, 1-\ell).$$

Let the values of (c, ℓ) that satisfy this system (if they exist) be given by the functions $\hat{c}(z, v)$ and $\hat{\ell}(z, v)$, Then it is clear that

$$c_0 = \hat{c}\left(\beta^2(1+r_0)(1+r_1), a\right) \quad \ell_0 = \hat{\ell}\left(\beta^2(1+r_0)(1+r_1), a\right)$$

and

$$c_1 = b\hat{\ell}\left(\beta^{-2}(1+r_0)^{-1}(1+r_1)^{-1}, b\right) - \hat{c}\left(\beta^{-2}(1+r_0)^{-1}(1+r_1)^{-1}, b\right),\ \ell_1 = \hat{\ell}\left(\beta^{-2}(1+r_0)^{-1}(1+r_1)^{-1}, b\right).$$

We now go back to the general case. For a given v, let $\bar{c}(v)$ be given by

$$u_1[\bar{c}(v), 1]v \equiv u_2[\bar{c}(v), 1].$$

Given $u_{12} > 0$ and strict concavity, it follows that $u_1(c, 1)v/u_2(c, 1) > 1$ for all $c \in (0, \bar{c}(v))$. This implies that for any $c \in (0, \bar{c}(v))$ the equation

$$\frac{u_1(c, 1-\ell)v}{u_2(c, 1-\ell)} = 1$$

has a solution $\ell > 0$. The assumptions that $u_{12} > 0$ and that u is strictly concave guarantee that the solution ℓ is unique and decreasing in c. Denote such a solution by

$$f(c, v).$$

Finally, let $\bar{\ell}(v)$ be given by

$$\bar{\ell}(v) \equiv \lim_{c \to 0} f(c, v).$$

Note that $\exists \bar{\bar{c}}(v) < \bar{c}(v)$ such that $f(\bar{\bar{c}}, v)v - \bar{\bar{c}} = 0$. Next, define

$$F(c; z, v) \equiv u_1[c, 1 - f(c, v)] - z u_1[v f(c, v) - c, 1].$$

It follows that $\lim_{c \to 0} F(c; z, v) = +\infty$ and $\lim_{c \to \bar{\bar{c}}(v)} F(c; z, v) = -\infty$. Since F is monotone decreasing in c, there is a unique c, that satisfies $F(c; z, v) = 1$. Denote such a c by $\hat{c}(z, v)$. Then, defining $\hat{\ell}(z, v) = f(\hat{c}(z, v), v)$ it follows that the pair $(\hat{c}, \hat{\ell})$ solves the version of (10) described in this proof. ■

PROOF OF PROPOSITION 2

From the proof of Proposition 1, it suffices to describe how the optimal choices $\hat{c}(z, v)$ and $\hat{\ell}(z, v)$ vary with (z, v). By the implicit function theorem it follows that $(\hat{c}, \hat{\ell})$ are C^1. Total differentiation with respect to z gives

$$\frac{\partial \hat{c}}{\partial z} = \frac{-u_1(v u_{12} - u_{22})}{\Delta} < 0,$$

$$\frac{\partial \hat{\ell}}{\partial z} = \frac{-u_1(v u_{11} - u_{12})}{\Delta} > 0,$$

where

$$\Delta = u_{11}u_{22} - u_{12}^2 + z u_{11} u_{22} - 2 z v u_{12} u_{11} + z v^2 u_{11}^2 > 0.$$

Define $m(z, v)$ by

$$m(z, v) \equiv v \hat{\ell}(z, v) - \hat{c}(z, v).$$

Tedious but straightforward calculations show that

$$\frac{\partial m}{\partial v} = \hat{\ell} + v \, \frac{\partial \hat{\ell}}{\partial v} - \frac{\partial \hat{c}}{\partial v} = \frac{\hat{\ell}(u_{11}u_{22} - u_{12}^2) - 2 z v \hat{\ell} u_{12} u_{11} - v u_1 u_{11} + u_1 u_{12}}{\Delta} > 0.$$

Finally, to establish that $c_1 \to \infty$ as $b \to \infty$ and that $a\ell_0 - c_0 \to \infty$, as $a \to \infty$, it is sufficient to show that $\lim_{v \to \infty} m(z, v) = \infty$. We prove this by contradiction.

Consider a sequence $\{v_t\}$ such that $v_t \to \infty$ and suppose to the contrary that $m(z, v_t) \leq m^* < \infty$. Then, by definition of $m(z, v)$, we must have $\lim_{v \to \infty} \hat{\ell}(z, v) = 0$. Recall that from the proof of Proposition 1, $\hat{c}, \hat{\ell}$ satisfy

$$u_1[\hat{c}(z, v), \ 1 - \hat{\ell}(z, v)]v = u_2[\hat{c}(z, v), \ 1 - \hat{\ell}(z, v)].$$

Thus, the left-hand side goes to infinity as v goes to infinity, where the right-hand side converges to $u_2(\hat{c}(z, \infty), 1) < \infty$. This completes the proof. ■

PROOF OF PROPOSITION 3

In the proofs of Propositions 1 and 2, the existence and properties of the functions $\hat{c}(z, v)$, $\hat{\ell}(z, v)$, and $m(z, v)$ were established. It is convenient to rewrite the inequality conditions (12) and (14) corresponding to the two types of equilibria using these functions as

$$\beta^{-1} z\, u_1[m(z, a), 1]\, m(z, a) > u_1[m(z^{-1}, b), 1] m(z^{-1}, b), \tag{12'}$$

$$\beta z u_1[m(z, b), 1] m(z, b) > u_1[m(z^{-1}, a), 1] m(z^{-1}, a), \tag{14'}$$

where $z = \beta^2(1 + r_0)(1 + r_1)$. Consider first the case in which $u_1(x, 1)x$ is increasing in x. In (12′), fix b, and from Proposition 2 $\lim_{a\to\infty} m(z, a) = \infty$. Thus, given this assumption, the left-hand side of (12′) increases without bound. Hence, for any b, let $\bar{a}_1(b)$ be the value of a such that (12′) holds as an equality. Then for $a > \bar{a}_1(b)$ (12′) is a strict inequality. This proves the first part of (i).

Consider next the case of $u_1(x, 1)x$ decreasing and, as before, let b be fixed. Consider the right-hand side of (13′) as a is increased. By assumption it converges to zero. Thus, there exists $\bar{a}_2(b)$ such that $a > \bar{a}_2(b)$ implies that (14′) holds. Therefore, for $a > \max(\bar{a}_1(b), \bar{a}_2(b)) \equiv \bar{a}(b)$, either (12′) or (14′) are satisfied.

To prove (ii), simply fix a and choose b just as in the previous argument. ■

PROOF OF PROPOSITION 4

The existence conditions (12) and (14) can be more easily studied when transformed as in the proof of Proposition 3. In particular, (12′) and (14′), when evaluated at $z = 1$ (this corresponds to $\beta^2(1 + r_0)(1 + r_1) = 1$), are

$$\beta^{-1} u_1[m(1, a), 1] m(1, a) > u_1[m(1, b), 1] m(1, b),$$
$$\beta u_1[m(1, b), 1] m(1, b) > u_1[m(1, a), 1] m(1, a).$$

Thus, unless we are in the exceptional case in which both conditions are satisfied with equality, one of them must hold. ■

PROOF OF PROPOSITION 5

Consider the problem defining $F(V)$. It is clear that F is concave and continuous. Moreover, given the convexity of the program as a function of $x = (c_0, \ell_0, c_1, \ell_1)$, the first-order conditions are necessary and sufficient. It is immediate to observe the following conditions:

$$u_1(c_0, 1 - \ell_0) = \frac{\lambda + \beta^2}{1 + \lambda\beta^2}\, u_1(a\ell_0 - c_0, 1 - \ell_0),$$
$$u_1(c_0, 1 - \ell_0)a = u_2(c_0, 1 - \ell_0),$$
$$u_1(c_1, 1) = \frac{\lambda + \beta^2}{1 + \lambda\beta^2}\, u_1(b\ell_1 - c_1, 1 - \ell),$$
$$u_1(b\ell_1 - c_1, 1 - \ell_1)b = u_2(b\ell_1 - c_1,\ 1 - \ell_1).$$

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First note that if $\lambda = 1$, the allocation corresponds to the monetary equilibrium when $r = \beta^{-2}$. Thus, (i) follows. Similarly, the Envelope Theorem implies that $F'(V_0^*) = -\lambda$ and, in this case, $\lambda = 1$. This establishes (ii).

Consider next the solution to the first-order conditions for $\lambda \in [0, 1)$. If $\lambda = 0$, it corresponds precisely to the equations (11) for $(1 + r_0)(1 + r_1) = 1$; for $\lambda \in (0, 1)$, it also solves (11) for $(1 + r_0)(1 + r_1) = r$ given by $(1 + \lambda/\beta^2)/(1/\beta^2 + \lambda) < \beta^{-2}$. This proves (iii) and (iv). Similarly, for $\lambda > 1$ the associated allocation solves the system (13) and $\lim_{\lambda\to\infty}(\lambda + \beta^2)/(1 + \lambda\beta^2) = \beta^{-2}$ implies that the solution corresponds to a noninterventionist ($r = 1$) type II equilibrium. This completes the proof. ■

PROOF OF PROPOSITION 6

The first-order conditions corresponding to the conjectured equilibrium satisfy the relevant version of (10). Specifically,

$t = 0$ odd

$$\frac{u_1(c_0^\alpha, 1 - \ell_0^\alpha)}{p_0^\alpha} = \frac{\beta u_1(c_1^\alpha, 1)}{p_1^\alpha}$$
$$u_1(c_0^\alpha, 1 - \ell_0^\alpha)\, a = u_2(c_0^\alpha, 1 - \ell_0^\alpha)$$

even

$$\frac{u_1(a\ell_0^\alpha - c_0^\alpha, 1)}{p_0^\alpha} = \frac{\beta u_1(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha)}{p_1^\alpha}$$

$t = 1$ odd

$$\frac{u_1(c_1^\alpha, 1)}{p_1^\alpha} = \frac{\beta u_1(a\ell_0 - c_0, 1)}{p_0}$$

even

$$\frac{u_1(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha)}{p_1^\alpha} \geq \frac{\beta u(c_0, 1 - \ell_0)}{p_0}$$
$$u_1(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha)b$$
$$= u_2(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha),$$

where (c_0, ℓ_0, p_0) correspond to the steady state value with a money supply equal to H. Additionally, the budget constraints of the odd agents for periods 0 and 1 are

$$p_0^\alpha c_0^\alpha + b_0^\alpha + m_0^\alpha = p_0^\alpha a\ell_0^\alpha + (1 - \alpha)H,$$
$$p_1^\alpha c_1^\alpha = b_0^\alpha.$$

The full implications of these first-order conditions (after we impose the conjectured equilibrium condition $m_0^\alpha = H$) are summarized by

$$\frac{u_1\left(c_0^\alpha, 1 - \ell_0^\alpha\right)}{u_1\left(c_1^\alpha, 1\right)} = \frac{u_1(a\ell_0^\alpha - c_0^\alpha, 1)}{u_1(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha)}, \quad \textbf{(A.1.a)}$$

$$u_1(c_0^\alpha, 1 - \ell_0^\alpha)a = u_2(c_0^\alpha, 1 - \ell_0^\alpha), \quad \textbf{(A.1.b)}$$

$$u_1(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha)b = u_2(b\ell_1^\alpha - c_1^\alpha, 1 - \ell_1^\alpha), \quad \textbf{(A.1.c)}$$

$$\frac{u_1(c_0^\alpha, 1 - \ell_0^\alpha)}{p_0^\alpha} = \beta^2 \frac{u_1(a\ell_0 - c_0, 1)}{p_0}, \quad \textbf{(A.1.d)}$$

$$\frac{\beta^2 u_1(a\ell_0 - c_0, 1)}{p_0} = \frac{\beta^2 u_1(a\ell_0 - c_0, 1)\,(a\ell_0 - c_0) - \beta u_1(c_1, 1)c_1}{H}, \quad \textbf{(A.2.a)}$$

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$$\frac{u_1(c_0^\alpha, 1-\ell_0^\alpha)}{p_0^\alpha} = \frac{u_1(c_0^\alpha, 1-\ell_0^\alpha)\,(a\ell_0^\alpha - c_0^\alpha) - \beta u_1(c_1^\alpha, 1)c_1^\alpha}{\alpha H}. \qquad \textbf{(A.2.b)}$$

Let $\hat{c}(z, v)$, $\hat{\ell}(z, v)$, and $m(z, v)$ be as in the proofs of Propositions 1–3. We next conjecture that $\exists\, z_\alpha$ such that the equilibrium allocation is given by

$$\begin{aligned} c_0^\alpha &= \hat{c}(z_\alpha, a), \\ \ell_0^\alpha &= \hat{\ell}(z_\alpha, a), \\ c_1^\alpha &= b\hat{\ell}(z_\alpha^{-1}, b) - \hat{c}(z_\alpha^{-1}, b), \\ \ell_1^\alpha &= \hat{\ell}(z_\alpha^{-1}, b). \end{aligned}$$

Recall that the steady state allocation is obtained by choosing $z_\alpha = \beta^2$. By construction our candidate equilibrium satisfies (A.1.a)–(A.1.c) (for details see the proof of Proposition 1). We need to show that (A.1.d) holds.

Define $T(z)$ by

$$T(z) \equiv\; z u_1(m(z, a), 1)m(z, a) - \beta u_1(m(z^{-1}, b), 1)m(z^{-1}, b).$$

Then since by construction $u_1(c_0^\alpha, 1 - \ell_0^\alpha) = z_\alpha u_1(m(z_\alpha, a), 1)$ we have that (A.2.a) is simply

$$\frac{\beta^2 u_1(a\ell_0 - c_0, 1)}{p_0} = \frac{T(\beta^2)}{H} > 0$$

(given that a type I equilibrium exists). Similarly, (A.2.b) is

$$\frac{u_1(c_0^\alpha, 1-\ell_0^\alpha)}{p_0^\alpha} = \frac{T(z_\alpha)}{\alpha H}.$$

To complete the existence argument, we need to find $z_\alpha > 0$ such that

$$\frac{T(z_\alpha)}{\alpha H} = \frac{T(\beta^2)}{H}.$$

Thus, this is equivalent to finding z_α such that

$$T(z_\alpha) = \alpha\, T(\beta^2).$$

From the properties of the function $m(z, v)$ and of the utility function assumed in Propositions 1–3, it follows that

$$T(\beta^2) > \alpha T(\beta^2),$$

$$\lim_{z_\alpha \to 0} T(z_\alpha) = -u_1(\bar{m}, 1)\bar{m} < 0,$$

where $\bar{m} = \lim_{z\to\infty} m(z, b)$ satisfies $0 < \bar{m} \le b$. Because $T(z_\alpha)$ is a continuous function, there exists $z_\alpha^* \in (0, \beta^2)$ such that

$$T(z_\alpha^*) = \alpha T(\beta^2).$$

Because in the proofs of Propositions 1–3 it was established that $\hat{c}_z < 0$, $\hat{\ell}_z > 0$, and $m_z > 0$, parts (i) and (ii) follow from $z_\alpha^* < \beta^2$.

Next we prove (iii). From the first-order conditions it follows that

$$\frac{u_1(c_1^\alpha, 1)}{p_1^\alpha} = \frac{u_1(c_1, 1)}{p_1}.$$

Thus, since $c_1^\alpha > c_1$ and u is strictly concave, $p_1^\alpha < p_1$. Using (A.1.d) and $u_1(c_0^\alpha, 1 - \ell_0^\alpha |= z_\alpha^* u_1(m(z_\alpha^*, a), 1)$, it follows that

$$\frac{p_0^\alpha}{p_0} = \frac{z_\alpha^* u_1(m(z_\alpha^*, a), 1)}{\beta^2 u_1(m(\beta^2, a), 1)}.$$

Thus, to show that $p_0^\alpha < p_0$, it suffices to show that $z u_1(m(z, a), 1)$ is increasing as $z_\alpha^* < \beta^2$. This follows, as the proof of Proposition 3 shows that

$$u_1 + u_{11} m_z z = \frac{u_1(u_{11}u_{22} - u_{12}^2)}{\Delta} > 0,$$

where Δ is as in the proof of Proposition 2. ∎

PROOF OF PROPOSITION 7

This is omitted because it parallels that of Proposition 6.

APPENDIX B

In this appendix we describe the equilibrium in the absence of borrowing and lending. If the utility function satisfies the Inada conditions, we know that an individual who is not productive at time t must hold currency between periods $t - 1$ and t. Given our productivity sequence, this implies the, following pattern of money holdings for the two individuals at the beginning of each period.

Time	Odd	Even
0	$(1-\theta)H$	θH
1	H	0
2	θH	$(1-\theta)H$
3	0	H
4	$(1-\theta)H$	θH

Note that as in the unrestricted–borrowing and lending economy, there is a situation reversal at $t = 2$ in which even individuals start out with the same level of wealth and productive opportunities as odd individual's faced at $t = 0$. From the odd individual's first two budget constraint's it follows that

$$\tau_0 + p_0 c_0 + H = p_0 a \ell_0 + (1-\theta) H (1 + r_1),$$
$$\tau_1 + p_1 c_1 + \theta H = H(1 + r_0),$$

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and this implies that prices are given (using $\tau_i = r_i H/2$) by

$$p_0 = \frac{(\theta(1+r_1) - r_1/2)}{a\ell_0 - c_0} H,$$

$$p_1 = \frac{p(1-\theta+r_0/2)}{c_1} H.$$

The analog of equations (9) in the text yields allocations given by

$$\begin{aligned} c_0 &= \hat{c}(z,a), \\ \ell_0 &= \hat{\ell}(z,a), \\ c_1 &= \hat{m}(z,b), \\ \ell_1 &= \hat{\ell}(z,b), \end{aligned}$$

where the functions $\hat{c}, \hat{m}$, and $\hat{\ell}$ are as in the proof of Proposition 1 and $z = \beta^2 r = \beta^2(1+r_0)(1+r_1)$. It follows that

$$p_1/p_0 = \frac{(1-\theta+r_0/2)}{(\theta(1+r_1) - r_1/2)} \; \frac{m(z,a)}{m(z,b)}.$$

Using the expressions for (p_0, p_1) from the budget constraints it follows that θ satisfies

$$\frac{1-\theta+r_0/2}{\theta r - (1+r_0)r_1/2} = A(z) \equiv \frac{\beta u_1[m(z,b),1]m(z,b)}{z u_1[m(z,a),1]m(z,a)},$$

or

$$\theta = \frac{1}{2} \; \frac{2 + r_0 + (1+r_0)r_1 A(z)}{1 + rA(z)}.$$

For this to guarantee positive prices, it is necessary and sufficient that

$$(1+r_0)(1+r_1A(z)) > -1, \tag{A.3}$$

$$(1+r_0)(1-A(z)) < 1 + rA(z). \tag{A.4}$$

Note that if $r_0 = r_1 = 0$ these conditions are automatically satisfied and hence a monetary laissez-faire equilibrium always exists. It is also possible to show that inflationary equilibria always exist.

Let a bar denote equilibrium quantities in this credit-constrained economy. It follows that output is given by

$$\begin{cases} \bar{y}_0 = a\hat{\ell}(z,a) \\ \bar{y}_1 = b\hat{\ell}(z,b). \end{cases} \tag{A.5}$$

As in the economy with no restrictions, a constant inflation path corresponds to $z = (\beta^2\mu^2)^{-1}$. Consider next what happens if, at $t = 0$, the odd individual starts with fraction $(1-\alpha)$ of the stock of currency. For simplicity we assume that the money supply is kept constant afterward. The result depends on whether $\alpha > \theta$ or $\alpha \leq \theta$. It is possible to show, following the arguments in Propositions 6 and 7, that the economy converges to the steady state after one period (if the shock occurs at $t = 1$ it takes two periods), and that the new allocation (identified with a prime) is

$$\bar{c}_0' = \hat{c}(z', a),$$
$$\bar{\ell}_0' = \hat{\ell}(z', a),$$
$$\bar{c}_1' = m(\beta^2, b) = \bar{c}_1,$$
$$\bar{\ell}_1' = \hat{\ell}(\beta^2, b) = \bar{\ell}_1,$$

where $z' < \beta^2$ if $\alpha < \theta$ and $z' > \beta^2$ if $\alpha > \theta$. Of course, depending on whether $\alpha < \theta$ (>), the odd individuals consume more (less) than their steady state consumption. Similarly, their labor supply and the economy's output decrease (increase). In both cases, what is behind these results in a standard income effect, because $\alpha < \theta$ (>) corresponds to a case in which odd individuals have a higher (lower) fraction of wealth relative to the steady state. Using the budget constraints, it is possible to show that if $\alpha < \theta$ (>), $\bar{p}_0' < \bar{p}_0$ ($\bar{p}_0' > \bar{p}_0$) and, thus, that prices can over- or undershoot their steady state values. Finally, note that if the economy is in a steady state of constant–money supply equilibrium and legal restrictions prohibiting private borrowing and lending are imposed at $t = 0$, this is equivalent to studying the equilibrium described in this appendix with $\alpha = 1$.