BOOK REVIEWS

NOVIKOV, P. S., *Elements of Mathematical Logic* (Oliver and Boyd, Edinburgh, 1964), 308 pp., 50s.

This textbook, originally published in Russian in 1959, concerns three main topics: propositional logic, first-order predicate logic, and number-theory. Propositional logic is presented initially as propositional algebra, and then as a propositional calculus along the lines of Hilbert and Bernays' *Grundlagen der Mathematik*, I. Similarly, predicate logic is considered first from a model-theoretic (or set-theoretic) point of view, and then as an axiomatic system (again following Hilbert and Bernays). In the penultimate chapter two systems of number-theory are developed: Axiomatic Arithmetic (comparable to standard systems of arithmetic including unrestricted mathematical induction), and Restricted Arithmetic (obtained from Axiomatic Arithmetic by deleting the axiom-schema of mathematical induction). In the final chapter Novikov illustrates the techniques of proof-theory by constructing metamathematical proofs of (1) the consistency of Restricted Arithmetic, and (2) the independence of mathematical induction in Axiomatic Arithmetic.

The four chapters on propositional and predicate logic are straightforward, containing few innovations and covering most of the important topics. Novikov confines his attention to classical logic, saying nothing about intuitionist. One suspects that this omission is quite intentional, for in his introductory remarks, and elsewhere in the book, Novikov makes it clear that he accepts the formalist finitism of Hilbert as opposed to the intuitionist finitism of Brouwer. Within the ambit of classical logic, however, the book provides a clear, rigorous and generally reliable exposition, with many examples and fully elaborated proofs. Indeed it might be complained that examples are too numerous and proofs excessively complete—in short, that Novikov should have left more for the reader to do. This would have allowed him to devote more space to subjects which he sketchily considers (e.g. the decision-problem), and some space to subjects which he ignores altogether (e.g. Gentzen-type systems).

Turning to the chapter on number-theory, Novikov's system of Axiomatic Arithmetic is a cross-breed of axiomatic number-theory and formalised recursive number-theory. It consists of six axioms adjoined to first-order predicate logic—two equality axioms, three order axioms, and the axiom-schema of mathematical induction—together with all equations defining primitive recursive functions. As well as illustrating the adequacy of this system for defining number-theoretic concepts and for proving theorems of number-theory, Novikov also discusses very briefly the ideas of calculable (effectively computable) and general recursive functions.

By far the most difficult part of the book is the chapter on proof-theory. Novikov's consistency-proof for Restricted Arithmetic (RA) follows the pattern of other such proofs: to prove the existence of a non-deducible formula, and hence the consistency of RA, it is shown that every formula deducible in RA must have a property, called " weak regularity ", which the formula $0 \neq 0$ lacks. Similarly, in order to prove the independence in Axiomatic Arithmetic of mathematical induction, it is shown that the schema $A(0)$ & $(x)(A(x) \rightarrow A(x')) \rightarrow A(y)$ is not weakly regular, and so cannot be deduced in RA (i.e. from the other axioms of Axiomatic Arithmetic).

The book opens with a useful, though occasionally obscure, introductory chapter in which Novikov expounds the aims and methods of Hilbert's finitism. Gödel's incompleteness theorem is mentioned in this chapter, and once again later in the book; but nowhere is Gödel's proof described. This omission, like others, would not be serious if the book contained an adequate bibliography. Unfortunately it does not. Hilbert and Ackermann's *Grundzüge der theoretischen Logik* and Kleene's *Introduction to Metamathematics* are cited in the author's foreword, and papers by Kolmogorov and Gödel on intuitionist logic are cited in the final chapter, but otherwise the book gives the reader no guidance as to sources or further reading. Mr L. F. Boron, who did the translation, or Professor R. L. Goodstein, who contributed a preface and several notes, should have undertaken to provide such guidance. The quality of translation and printing is generally high, but there are several lapses, most of them obvious, which probably should be attributed to Mr Boron or the printers, rather than the author.

R. H. STOOTHOFF

MENDELSON, ELLIOTT, *Introduction to Mathematical Logic* (van Nostrand, 1964), x+300 pp., 56s. 6d.

This book is a compact introduction to some of the principal topics of mathematical logic. Although the text is rather condensed, it is nevertheless quite readable, and the book would be a valuable addition to the range of text books on mathematical logic if it were not spoiled by the editorial faults mentioned later.

The book begins with the propositional calculus and then, under the heading " Quantification Theory ", deals with the first-order calculus. There is a chapter on formal number theory that begins with axioms based on the Peano postulates and then develops recursive functions and recursive undecidability. The last two chapters, on axiomatic set theory and effective computability take up a hundred pages of the book. The set theory is developed very quickly from the von Neumann-Bernays-Gödel axioms: without a good knowledge of informal set theory this would probably be difficult to understand. The chapter on effective computability contains descriptions of Markov productions, Turing machines, Herbrand-Gödel computability and the connexions between them. There is an appendix in which the consistency of the axioms for number theory is proved.

There are no essential prerequisites for reading this book but, of course, much of the point of this subject is lost unless the reader has a fair knowledge of mathematics. The book is clearly written and contains a good deal of useful material; in short, it achieves its object. Unfortunately the book is not at all suitable for " dipping into ". The main reason for this is that the list of notation is useless: what is needed is an index of definitions and axioms, or a summary of them. Furthermore, the reason why this is needed is that the numbering or lettering of statements is inadequate and has no logical pattern. As the author says, in a footnote on page 75, " The numbering here is a continuation of the numbering of the Logical Axioms on page 57."

The layout is poor. The exercises may easily be mistaken for text, and there is no attempt either to make important statements stand out or to use a space between paragraphs. For instance, the Introduction falls naturally into two parts. They should have been separated. Mention must also be made of smaller details. Symbols in roman type are used in certain places. This has the dangerous advantage of a natural mapping into italic type that is used but not made explicit. The result is not a success—especially in the case of integer subscripts! Finally, if we excuse the troublesome problem of punctuation, there is one other interesting point. The abbreviation wf is used instead of well-formed formula: it behaves like an ordinary word except that it does not go into italics (except in error in the appendix).

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