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A new electrostatic mode in a dusty plasma due to dust charge fluctuation

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Abstract. A dusty plasma consisting of cold and hot electrons, cold ions, and charge fluctuating isolated cold dust has been considered. It has been shown by a normal mode analysis that in such a dusty plasma there exists a new type of electrostatic perturbation mode due to the charge fluctuation of the isolated dust. The basic features of this new electrostatic perturbation mode, which are different from those of the electron-acoustic waves, have also been analytically identified. The implications of these results in both the space and laboratory dusty plasma conditions are briefly discussed.

1. Introduction

Electrons of two different temperatures (viz. cold and hot electrons) are common in laboratory devices (Derfler and Simonen 1969; Henry and Treguier 1972; Hellberg et al. 2000) as well as in space environments (Dubouloz et al. 1993; Pottelette et al. 1999; Berthomier et al. 2000; Montgomery et al. 2001; Singh and Lakhina 2001). There exist different types of electrostatic modes in a two electron-temperature plasma (Derfler and Simonen 1969; Henry and Treguier 1972; Dubouloz et al. 1993; Pottelette et al. 1999; Hellberg et al. 2000; Berthomier et al. 2000; Montgomery et al. 2001; Singh and Lakhina 2001). The most important one is the electron-acoustic (EA) mode (Watanabe and Taniuti 1977; Yu and Shukla 1983; Gary and Tokar 1985). This is basically an electro-acoustic wave in which the inertia is provided by the mass of the cold electron, and the restoring force comes from the pressure of the hot electrons. The ions play the role of a neutralizing background, i.e. the ion dynamics does not influence the EA waves, since the EA wave frequency is much larger than the ion plasma frequency. The wave frequency (ω) of the linear EA waves, unlike that of the well-known Langmuir waves, extends only up to the cold electron plasma frequency $\omega_{\rm pc} = (4\pi n_{\rm c0} e^2/m_{\rm e})^{1/2}$, where $n_{\rm c0}$ is the equilibrium cold electron number density, $-e$ is the electron charge, and $m_{\rm e}$ is the electron mass. This upper wave frequency limit ($\omega \simeq \omega_{\rm pc}$) corresponds to a short-wavelength EA wave, and depends on the equilibrium cold electron number density $n_{\rm c0}$.

On the other hand, the dispersion relation of the linear EA waves in the long-wavelength limit (in comparison with the hot electron Debye radius $\lambda_{\rm Dh} = (T_{\rm h}/4\pi n_{\rm h0} e^2)^{1/2}$, where $T_{\rm h}$ is the hot electron temperature in units of the Boltzmann constant, and $n_{\rm h0}$ is the equilibrium hot electron number density) is (Watanabe and Taniuti 1977; Yu and Shukla 1983; Gary and Tokar 1985) $\omega \simeq kC_{\rm e}$, where k is the wave number, and $C_{\rm e} = (n_{\rm c0} T_{\rm h}/n_{\rm h0} m_{\rm e})^{1/2}$ is the EA speed. The propagation of the EA waves has received a great deal of renewed interest not only because the two-electron-temperature plasma is very common in both space and laboratory plasmas, but also because of the vital role of the EA mode in interpreting the electrostatic disturbances in space (Dubouloz et al. 1993; Pottelette et al. 1999; Berthomier et al. 2000; Montgomery et al. 2001; Singh and Lakhina 2001; Shukla et al. 2004) and laboratory devices (Derfler and Simonen 1969; Henry and Treguier 1972; Hellberg et al. 2000). The linear (Watanabe and Taniuti 1977; Yu and Shukla 1983; Gary and Tokar 1985; Mace and Hellberg 1990) and nonlinear (Mace et al. 1991; Mamun and Shukla 2002; El-Taibany 2005; El-Taibany and Moslem 2005) features of the EA waves have been investigated by many authors over the past few decades (Watanabe and Taniuti 1977; Yu and Shukla 1983; Gary and Tokar 1985; Mace and Hellberg 1990; Mace et al. 1991; Mamun and Shukla 2002; El-Taibany 2005; El-Taibany and Moslem 2005).

It is now well established that dust (Fortov et al. 1996; Fortov et al. 1998; Shukla and Mamun 2002; Ishihara 2007) is an omnipresent ingredient of most space and laboratory plasmas, i.e. in most cases plasma and dust always coexist (Shukla and Mamun 2002; Ishihara 2007). The dust in a plasma is not neutral, but is charged either negatively or positively depending on the plasma environments (Shukla and Mamun 2002; Ishihara 2007). The dust charge is also not constant, but fluctuates with space and time (Shukla and Mamun 2002; Ishihara 2007), and the effect of the dust charge fluctuation is important for the investigation of any kind of the electrostatic perturbation mode whose frequency is comparable to the dust charging frequency. Therefore, in the present work, we consider a dusty plasma containing cold and hot electrons, cold ions, and charge fluctuating isolated cold dust, and investigate the propagation of an electrostatic perturbation mode whose frequency is comparable to the dust charging frequency. This paper mainly consists of two parts, namely 'governing equations', which are presented in Sec. 2, and 'mode analysis and discussions', which are provided in Sec. 3.

2. Governing equations

We consider the propagation of a one-dimensional (1D) electrostatic perturbation mode (ω, k) in a dusty plasma consisting of cold and hot electrons, cold ions, and charge fluctuating isolated dust. We assume that (i) $\omega/k \ll V_{\mathrm{Th}}$ (where V_{Th} is the thermal speed of the hot electrons) i.e. the hot electrons follow the Boltzmann distribution, (ii) $\omega \gg \omega_{\mathrm{pi}} \gg \omega_{\mathrm{pd}}$ (where ω_{pi} (ω_{pd}) is the ion (dust) plasma frequency), i.e. the dynamics of the ions and dust do not influence the perturbation mode, i.e. the cold ions and the charged dust maintain only the background charge neutrality condition, $n_{\mathrm{c0}} + n_{\mathrm{h0}} = n_{\mathrm{i0}} + q_{\mathrm{d0}} n_{\mathrm{d0}}/e$, where n_{i0} (n_{d0}) is the constant number density of the stationary ions (dust), and q_{d0} is the dust charge at equilibrium, and (iii) the fluctuation of the current flowing on the dust grain surface is mainly due to the hot electrons, i.e. the cold electrons and ions (fluctuating currents due to cold electrons and ions are assumed to be negligible in comparison with the fluctuating current due to the hot electrons) maintain only the equilibrium current condition, $\sum_j I_{j0} = 0$ (where I_{j0} is the equilibrium current of the plasma species j in which j = c for the cold electrons, j = h for the hot electrons, and j = i for the cold ions). The macroscopic state of such a dusty plasma system is, therefore, described by

$$\frac{\partial n_{\mathrm{c}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{c}} u_{\mathrm{c}}) = 0, \tag{2.1}$$

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$$\frac{\partial u_{\rm c}}{\partial t}+u_{\rm c}\frac{\partial u_{\rm c}}{\partial x}=\frac{e}{m_{\rm e}}\frac{\partial\phi}{\partial x}, \tag{2.2}$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi(n_{\rm c}e+n_{\rm h}e-n_{\rm i0}e-n_{\rm d0}q_{\rm d}), \tag{2.3}$$

where $n_{\rm c}$ is the cold electron number density, $u_{\rm c}$ is the cold electron fluid speed, ϕ is the electrostatic wave potential, $n_{\rm h}$ is the number density of the hot electrons which are assumed to follow the Boltzmann distribution,

$$n_{\rm h}=n_{\rm h0}\exp\left(\frac{e\phi}{T_{\rm h}}\right), \tag{2.4}$$

where $q_{\rm d}$ is the dust charge, which is not constant, but varies with time according to

$$\frac{\partial q_{\rm d}}{\partial t}=I_{\rm h}+I_{\rm c0}+I_{\rm i0}. \tag{2.5}$$

$I_{\rm h}$ is the total hot electron current flowing on the dust grain surface, and is given by (Shukla and Mamun 2002)

$$I_{\rm h}=-r_{\rm d}^2 n_{\rm h}e\left(\frac{8\pi T_{\rm h}}{m_{\rm e}}\right)^{1/2}\exp\left(\frac{q_{\rm d}e}{r_{\rm d}T_{\rm h}}\right), \tag{2.6}$$

for $q_{\rm d}<0$, where $r_{\rm d}$ is the radius of the dust which is assumed to be spherical.

3. Mode analysis and discussions

We considered negatively charged dust. Therefore, using $q_{\rm d}=-z_{\rm d}e$ (where $z_{\rm d}$ is the number of electrons residing on the dust grain surface), $n_{\rm c}=n_{\rm c0}+\tilde{n}_{\rm c}$, $u_{\rm c}\simeq\tilde{u}_{\rm c}$, $\phi\simeq\tilde{\phi}$, and $z_{\rm d}=z_{\rm d0}+\tilde{z}_{\rm d}$ (where $\tilde{n}_{\rm c}$ is the perturbed part of $n_{\rm c}$, and $\tilde{z}_{\rm d}$ ($z_{\rm d0}$) is the perturbed (equilibrium) part of $z_{\rm d}$), one can linearize (2.1)–(2.6) to a first-order approximation, and can express them as

$$\frac{\partial\tilde{n}_{\rm c}}{\partial t}+n_{\rm c0}\frac{\partial\tilde{u}_{\rm c}}{\partial x}=0, \tag{3.1}$$

$$\frac{\partial\tilde{u}_{\rm c}}{\partial t}=\frac{e}{m_{\rm e}}\frac{\partial\tilde{\phi}}{\partial x}, \tag{3.2}$$

$$\frac{\partial^2\tilde{\phi}}{\partial x^2}=4\pi e\left(\tilde{n}_{\rm c}+n_{\rm h0}\frac{e\tilde{\phi}}{T_{\rm h}}+n_{\rm d0}\tilde{z}_{\rm d}\right), \tag{3.3}$$

$$\frac{\partial\tilde{z}_{\rm d}}{\partial t}=r_{\rm d}^2 n_{\rm h0}\left(\frac{8\pi e^2}{m_{\rm e}T_{\rm h}}\right)^{1/2}\left(\tilde{\phi}-\frac{e\tilde{z}_{\rm d}}{r_{\rm d}}\right). \tag{3.4}$$

Then, assuming that all perturbed quantities (viz. $\tilde{n}_{\rm c}$, $\tilde{u}_{\rm c}$, $\tilde{\phi}$, and $\tilde{z}_{\rm d}$) vary as $\exp(-i\omega t+kx)$, i.e. substituting $\partial/\partial t=-i\omega$ and $\partial/\partial x=ik$ into (3.1)–(3.4), one can derive the linear dispersion relation for the electrostatic perturbation mode (ω, k) as

$$1+k^2\lambda_{\rm Dh}^2-\frac{k^2C_{\rm e}^2}{\omega^2}+\frac{\alpha}{1-i\beta(\omega/\omega_{\rm pc})}=0, \tag{3.5}$$

where $\alpha=n_{\rm d0}r_{\rm d}T_{\rm h}/n_{\rm h0}e^2$ and $\beta=\sqrt{2}r_{\rm d}n_{\rm h0}e/\sqrt{T_{\rm h}n_{\rm c0}}$. Equation (3.5) implies that for no dust ($\alpha=0$) $\omega\simeq kC_{\rm e}$ for a long wavelength limit ($k^2\lambda_{\rm Dh}^2\ll 1$), and $\omega\simeq\omega_{\rm pc}$

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for a short wavelength limit ($k^2\lambda_{\rm Dh}^2 \gg 1$). This completely agrees with the works of Watanabe and Taniuti (1977), Yu and Shukla (1983), and Gary and Tokar (1985). However, in a plasma with isolated negatively charged dust, $\alpha \simeq 60$ and $\beta \simeq 10^{-10}$ for the laboratory plasma (Merlino et al. 1998; Nakamura et al. 1999; Hellberg et al. 2000) conditions ($n_{\rm h0} \simeq 10^{13}$ m^{-3}, $n_{\rm c0} \simeq 3\times 10^{13}$ m^{-3}, $T_{\rm h} \simeq 10$ eV, $r_{\rm d} \simeq 1$ μm and $n_{\rm d0} \simeq 10$ m^{-3}), whereas $\alpha \simeq 125$ and $\beta \simeq 10^{-14}$ for the space plasma (Dubouloz et al. 1993; Shukla and Mamun 2002) conditions ($n_{\rm h0} \simeq 2.5\times 10^6$ m^{-3}, $n_{\rm c0} \simeq 5\times 10^5$ m^{-3}, $T_{\rm h} \simeq 250$ eV, $r_{\rm d} \simeq 0.2$ μm and $n_{\rm d0} \simeq 10^{-6}$ m^{-3}). Since $\beta\omega/\omega_{\rm pc} \ll 1$, which is valid for both the space and laboratory dusty plasma conditions, (3.5) can be rewritten as

$$i\frac{\alpha\beta}{\omega_{\rm pc}}\omega^3 + (1 + k^2\lambda_{\rm Dh}^2 + \alpha)\omega^2 - k^2C_{\rm e}^2 = 0. \tag{3.6}$$

Now, substituting $\omega = \omega_{\rm r} + i\omega_{\rm i}$ into (3.6) and assuming $|\omega_{\rm i}| \ll \omega_{\rm r}$ (where $\omega_{\rm r}$ ($\omega_{\rm i}$) is the real (imaginary) part of the wave frequency ω), one can find

$$\omega_{\rm r} \simeq \frac{kC_{\rm e}}{\sqrt{1 + k^2\lambda_{\rm Dh}^2 + \alpha}}, \tag{3.7}$$

$$\omega_{\rm i} \simeq -\frac{\alpha\beta k^2 C_{\rm e}^2}{2\omega_{\rm pc}(1 + k^2\lambda_{\rm Dh}^2 + \alpha)^2}. \tag{3.8}$$

Equations (3.7) and (3.8) clearly indicate that for large, but not extremely large values of α (in comparison with 1 or $k^2\lambda_{\rm Dh}^2$), the effect of the dust charge fluctuation significantly modifies the dispersion properties of the EA waves, and that for extremely large values of α (in comparison with 1 or $k^2\lambda_{\rm Dh}^2$), the effect of the dust charge fluctuation introduces a new low (in comparison with the EA wave frequency) electrostatic perturbation mode with very negligible damping since β varies from 10^{-14} (space plasma conditions) to 10^{-10} (laboratory plasma conditions), and α varies from 60 (space plasma conditions) to 125 (laboratory plasma conditions). The assumption $|\omega_{\rm i}| \ll \omega_{\rm r}$, which is proved to be valid by (3.7) and (3.8), reduces to

$$\frac{\omega_{\rm i}}{\omega_{\rm r}} \simeq -\frac{\alpha\beta k\lambda_{\rm Dh}}{2(1 + k^2\lambda_{\rm Dh}^2 + \alpha)^{3/2}}. \tag{3.9}$$

Finally, for $\alpha \gg 1$ and $\alpha \gg k^2\lambda_{\rm Dh}^2$, which are valid for both the space (Dubouloz et al. 1993; Shukla and Mamun 2002) and laboratory (Merlino et al. 1998; Nakamura et al. 1999; Hellberg et al. 2000) dusty plasma conditions, the spectrum of this new low-frequency perturbation mode is defined as

$$\frac{\omega}{k} \simeq \left(\frac{n_{\rm c0}e^2}{n_{\rm d0}r_{\rm d}m_{\rm e}}\right)^{1/2}. \tag{3.10}$$

This represents a simple form of the dispersion relation for the new low-frequency electrostatic perturbation mode, where the restoring force comes from the electrostatic pressure on the dust grain surface, and the inertia is provided by the cold electron mass. The phase speed of this mode is directly proportional to the square root of $n_{\rm c0}/n_{\rm d0}$. This mode is different from the EA mode in which the restoring force comes from the hot electron thermal pressure, and the phase speed is directly proportional to the square root of $n_{\rm c0}/n_{\rm h0}$. However, in both the modes the inertia is provided by the cold electron mass.

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To summarize, a low-frequency (in comparison with the EA wave frequency) electrostatic mode is found to exist in a plasma containing cold and hot electrons, stationary ions, and charge fluctuating isolated static dust. The basic features of this new mode, which is due to the dust charge fluctuation, are different from those of the EA waves. A new laboratory experiment with a double plasma (DP) device (Hellberg et al. 2000) modified by the dust dispersing set-up (Nakamura et al. 1999), or with a Q-machine surrounded at its end position by a rotating dust dispenser (Merlino et al. 1998), can be designed to detect this new electrostatic perturbation mode, and to identify their basic features predicted in this theoretical investigation.

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