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Solution of a nonlinear system by the adaptively deaccelerated Newton method: application to shielding current analysis in HTS

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Abstract. A numerical method for a nonlinear system is presented. Formulation of the electromagnetic behavior of the shielding current density in high-T_c superconductors (HTS) gives a system of time-dependent integro-differential equations. The behavior can be determined by solving the initial-boundary-value problem of the system using the element-free Galerkin (EFG) method and the complete implicit difference method. After discretizing the problem, we obtain a nonlinear equation. In the present study, the shielding current density in HTS is calculated by applying the deaccelerated Newton method (DNM) and adaptively deaccelerated Newton method (ADNM) for the solution of the nonlinear system. The results of computation show that the DNM does not give a convergent solution in some cases. On the other hand, the ADNM gives a convergent solution in a few iterations.

1. Introduction

Recently, applications of high-T_c superconductors (HTSs) such as magnetic levitation and magnetic shielding have received a great deal of attention in various engineering fields. Evaluation of the shielding current density in a HTS is essential in analyses of its dynamic electromagnetic force and magnetic shielding. Formulation of the electromagnetic behavior of the shielding current density in HTSs gives a system of time-dependent integro-differential equations. The behavior can be easily determined by solving the initial-boundary-value problem of the system using the element-free Galerkin (EFG) method [1] and the complete implicit difference method. After discretizing the problem, we obtain a nonlinear equation. Since the equation is nonlinear, the Newton method is adopted for the solution. However, the Newton method does not give a convergent solution.

The purpose of the present study is to introduce a new method for the nonlinear system. Moreover, the new methods are applied to the solution of the shielding current density in the HTS.

2. Shielding current analysis in HTSs

Let us explain the governing equation of shielding current density in HTSs. First, we assume that the shape of the target HTS is disk-type, and the area of the circular cross-section is constant along the thickness direction. Thus, we can treat the problem as axisymmetric. Furthermore, the HTS is placed in a homogeneous AC magnetic field $\boldsymbol{B}$ whose direction is parallel to the thickness direction.

By taking the crystallographic anisotropy of the melt-powder-melt-growth processed YBCO HTS into account, we apply the multiple-thin-layer approximation [2]. Let us use the cylindrical coordinate (r, θ, z) by taking the symmetry axis of the HTS as the z-direction throughout this paper. Under these assumptions, there exists a scalar function $S_p(r,t)$ for the pth layer such that the shielding current density in this layer, $\boldsymbol{j}_p$, satisfies the following equation:

$$\boldsymbol{j}_p = \frac{1}{\varepsilon} \nabla S_p \times \boldsymbol{e}_z. \tag{2.1}$$

In addition, the flux-flow and flux-creep models for $J - E$ constitutive relation are adopted [3]. The behavior of shielding current density of axisymmetric HTS can be express as the initial-boundary-value problem. By applying the backward Euler method to the initial-boundary-value problem, the system is discretized with respect to time, and we discretize the problem with respect to space by using the EFG method [1, 4]. In this way, we can discretize the initial-boundary-problem and the following nonlinear system is derived:

$$\boldsymbol{G}(\boldsymbol{s}, \boldsymbol{\lambda}) \equiv \begin{bmatrix} W & B \\ B^{\mathrm{T}} & O \end{bmatrix} \begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{\lambda} \end{bmatrix} + \Delta t \begin{bmatrix} \boldsymbol{e}(\boldsymbol{s}) \\ \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{0}, \tag{2.2}$$

where the nodal vectors $\boldsymbol{s}$, $\boldsymbol{e}(\boldsymbol{s})$ and $\boldsymbol{\lambda}$ correspond to the scalar function S_p^m, the electric field $\boldsymbol{E}$ and the Lagrange multiplier λ, respectively. Here, superscript m indicates the number of time steps and Δt denotes the time step. The nodal vector $\boldsymbol{u}$ is evaluated from the values of the scalar function S_p^{m-1}. Moreover, the matrix W is calculated from the shape functions of the EFG and the integration kernel; the matrix B is calculated from the shape functions of the EFG as well [4]. The behavior of shielding current density in HTS is determined by solving the nonlinear system (2.2). Throughout the present paper, values of parameters are fixed as follows: $B_0 = 1.1$ T and $f = 7$ Hz, where B_0 denotes the magnitude of $\boldsymbol{B}$ and f denotes the frequency of $\boldsymbol{B}$.

3. Adaptively deaccelerated Newton method

As we discussed above, the behavior of shielding current density in HTS is determined by solving the nonlinear system (2.2). Since the equation is nonlinear, the Newton method is adopted for the solution. However, the Newton method does not give a convergent solution. Therefore, we adopt another two methods for the nonlinear system.

Let us first introduce the deaccelerated Newton method (DNM). In the DNM, the solution of the nonlinear system is determined by using following two procedures.

- *step 1.* The linear system $J(\boldsymbol{s}^{n-1})\delta\boldsymbol{s} = -\boldsymbol{G}(\boldsymbol{s}^{n-1})$ is solved to determine a $\delta\boldsymbol{s}$.
- *step 2.* The approximate solution is corrected by $\boldsymbol{s}^n = \boldsymbol{s}^{n-1} + \gamma\delta\boldsymbol{s}$.

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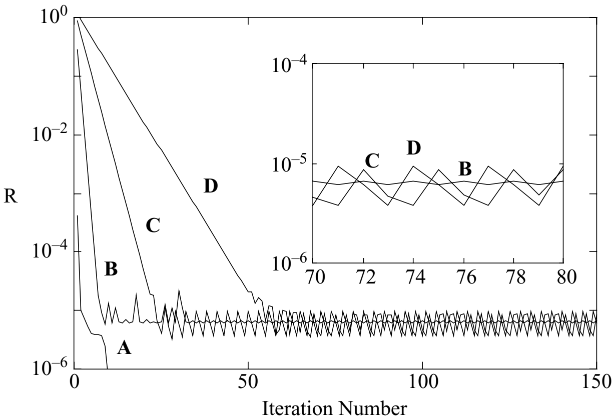

Figure 1. The residual histories of the DNM and the ADNM. Here, the nonlinear system is solved at time $t = 1/(2f)$. The inset is an enlarged plot for the same case: A, ADNM, B, $\gamma = 0.8$; C, $\gamma = 0.4$; D, $\gamma = 0.2$.

Here, the superscript n denotes an iteration number of DNM and γ is a relaxation factor whose range is $0 < \gamma \leqslant 1$. $J(\boldsymbol{s})$ denotes the Jacobian matrix of $\boldsymbol{G}(\boldsymbol{s})$. The above two steps are iterated until the series $\{\boldsymbol{s}^n\}$ converges.

Next, let us introduce the adaptively deaccelerated Newton method (ADNM). The ADNM is prepared by modifying step 2 in the DNM [5].

- *step 2a*. The minimum non-negative integer m, which satisfies $||\boldsymbol{G}(\boldsymbol{s}^{n-1}) + \beta^m \delta\boldsymbol{s}|| < ||G(\boldsymbol{s}^{n-1})||$, is determined.
- *step 2b*. The relaxation factor γ is evaluated by $\gamma = \beta^m$ and subsequently, the approximate solution is corrected by $\boldsymbol{s}^n = \boldsymbol{s}^{n-1} + \gamma\delta\boldsymbol{s}$.

Here, β is a constant which satisfies $0 < \beta \leqslant 1$.

The above two methods are adopted in the solution of the nonlinear system (2.2).

4. Convergence properties

Let us first investigate the convergence properties of the DNM and the ADNM. In Fig. 1, we show the residual norm $R = ||\boldsymbol{G}(\boldsymbol{s})||$ histories of the DNM and ADNM as functions of the iteration number. The terminate condition is fixed as 10^{-10}. We see from this figure that the residual norm of DNM decreases, monotonously to around 10^{-5} and falls into a limit cycle in both cases. As is apparent from the inset in Fig. 1, the residual norm of DNM moves only several fixed values. By the same token, the solution of the nonlinear system also moves only several fixed approximate solutions. We refer to this fixed approximate solution as equilibrium points. A similar tendency occurs when the conventional finite-element method (FEM) is adopted as the method of the simulation code. This figure also indicates that the residual of ADNM does not have an equilibrium point. The residual norm decreases rapidly and terminates in a few iterations.

Next, we investigate how the distribution of the equilibrium points will change with an increase in the relaxation factor γ. In Fig. 2, we show the equilibrium points of the DNM as a function of the relaxation factor. We see from Fig. 2 that a decrease in the relaxation factor will not always decrease the number of equilibrium points.

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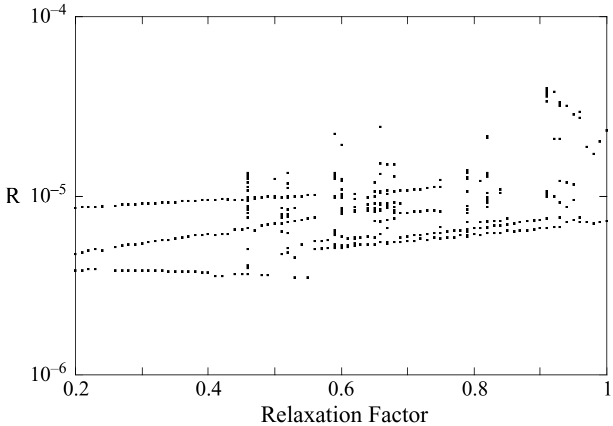

Figure 2. Equilibrium points of the DNM as a function of the relaxation factor γ. Here, the nonlinear system is solved at time $t = 1/(2f)$.

5. Conclusion

In the present study, we have introduced DNM and ADNM, and the two methods were adopted for the nonlinear system obtained from the shielding current analysis of HTS.

Our conclusions can be summarized as follows.

- The residual norm of DNM decreases monotonously to around 10^{-5} and falls into a limit cycle in both cases. That is to say, the solution of the nonlinear system moves only several equilibrium points.
- A decrease in the relaxation factor will not always decrease the number of equilibrium points.
- The residual of ADNM decreases rapidly and terminates in a few iterations.

References

[1] Belytschko, T., Lu, Y. Y. and Gu, L. 1994 Element-free Galerkin methods. *Int. J. Numer. Methods Eng.* **37**, 229–256.

[2] Kamitani, A. and Ohshima, S. 1999 Magnetic shielding analysis of axisymmetric HTS plates in mixed state. *IEICE Trans. Electron.* **E82-C**, 766–773.

[3] Yoshida, Y., Uesaka, M. and Miya, K. 1994 Evaluation of dynamic magnetic force of high T_c superconductor with flux flow and creep. *Int. J. Appl. Electromagn. Mater.* **5**, 83–89.

[4] Ikuno, S. and Kamitani, A. 2005 Shielding current density analysis of axisymmetric HTS by element-free Galerkin method. *IEEE Trans. Appl. Superconductivity* **15**, 3688–3691.

[5] Kamitani, A., Ikuno, S. and Yokono, T. 2004 High performance method for calculating shielding current density in HTS plate–application of adaptively deaccelerated Newton method. *IEICE Trans. Electron.* **E87-C**, 101–108.

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