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Rotation effects of elongated dust grains on a surface dust-acoustic wave in a semi-bounded dusty plasma

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Abstract. The surface dust-acoustic wave is investigated in a semi-bounded dusty plasma containing elongated and rotating dust grains. The dispersion relation of the low-frequency surface ion plasma wave is obtained by the plasma dielectric function with the specular reflection boundary condition. The results show that the phase and group velocities of the surface dust-acoustic wave propagating along the plasma–vacuum interface decrease with an increase of the rotation frequency. However, for long wavelengths it is found that the phase and group velocities of the surface wave are independent of the magnitude of the rotation frequency.

1. Introduction

The investigation of the surface plasma waves (Alexandrov et al. 1984; Cho et al. 1998; Aliev et al. 2000; Shokri and Jazi 2003) has attracted much attention since their frequency spectra provide useful information on plasma parameters for spatially bounded or semi-bounded plasmas. In addition the propagation of surface waves on the plasma–vacuum interface has various applications such as laser physics, plasma technology and spectroscopy. There has been considerable interest in the dynamics of gases and plasmas containing dust grains or highly charged aerosols, including collective effects and strong electrostatic interaction between the charged particles. The various physical processes in dusty plasmas have been investigated in order to obtain information on plasma parameters in dusty plasmas (Mendis and Rosenberg 1994; Shukla 2002; Shukla and Mamum 2002; Denysenko et al. 2005). Recently, it has been found that the elongated charged dust grains are ubiquitous in many cases of astrophysical and laboratory dusty plasmas (Mohideen et al. 1998). It is found that the elongated charged dust grains can be rotated due to the oscillating electric field or due to their interaction with particles and photons (Shukla and Mamum 2002). However, the surface dispersion properties of the dust-acoustic wave in a dusty plasma containing elongated and rotating dust grains have not been investigated as yet. Thus, in this paper we investigate the effects of dust grain rotation on the surface dust-acoustic wave in a semi-bounded dusty plasma. The investigation of the dispersion properties of the surface wave in dusty plasmas would be a useful tool for investigating the physical properties of dusty plasmas. Here we consider the propagation of the surface dust-acoustic wave along the plasma–vacuum interface. The specular reflection condition is known to be quite useful for investigating the dispersion relation of surface waves in semi-bounded plasmas.

In Sec. 2, we obtain the dispersion relation of a surface dust-acoustic wave in a semi-bounded dusty plasma containing elongated and rotating dust grains. We also investigate the variation of the phase velocity of the surface dust-acoustic wave propagating the plasma–vacuum interface due to the change of the angular frequency of the rotating dust grains. Our conclusions are given in Sec. 3.

2. Surface dust-acoustic wave

The specular reflection condition (Alexandrov et al. 1984; Aliev et al. 2000) for surface waves propagating in the z-direction in semi-bounded isotropic plasmas with the plasma–vacuum interface at $x = 0$ is represented as

$$\left(\frac{k_z^2 c^2}{\omega^2} - 1\right)^{1/2} + \frac{\omega}{\pi c}\int_{-\infty}^{\infty}\frac{dk_\perp}{k^2}\left[\frac{k_z^2 c^2}{\omega^2 \varepsilon_\mathrm{l}(\omega,k)} - \frac{k_\perp^2 c^2}{k^2 c^2 - \omega^2 \varepsilon_\mathrm{t}(\omega,k)}\right] = 0, \tag{1}$$

where $k^2 = k_\perp^2 + k_z^2$, $k_\perp (= k_x)$ and k_z are the perpendicular and parallel components of the wave vector $\mathbf{k}$, ω is the frequency, c is the speed of the light, $\varepsilon_\mathrm{l}(\omega,k)$ and $\varepsilon_\mathrm{t}(\omega,k)$ are the longitudinal and transverse components of the plasma dielectric function, respectively. It is known that the physical properties of electrostatic wave in plasmas can be obtained by the plasma dielectric function. Then, the plasma dielectric function $\varepsilon_\mathrm{l}(\omega,k)$ $(= 1 + \sum_{\alpha=\mathrm{e,i,d}} \chi_\alpha)$ in dusty plasmas can be obtained by the dielectric susceptibilities χ_α for electrons (e), ions (i) and dust grains (d). When the axial direction of elongated dust grains is parallel to the plasma–vacuum interface, the longitudinal component of the plasma dielectric function for $\omega \ll kc, kv_\mathrm{Te}, kv_\mathrm{Ti}$ and $|\omega - \Omega_0| \gg kv_\mathrm{Td}$, i.e. the low-frequency dust-acoustic wave, in a dusty plasma containing elongated and rotating dust grains is derived (Mahmoodi et al. 2000):

$$\varepsilon_\mathrm{l}(\omega, k_\perp, k_z) = 1 + \frac{1}{k^2 r_\mathrm{D}^2} - \frac{\omega_\mathrm{pd}^2}{\omega^2} - \frac{k_\perp^2}{k^2}\frac{\Omega_\mathrm{r}^2}{(\omega - \Omega_0)^2}, \tag{2}$$

where $v_{\mathrm{T}\alpha}$ is the thermal velocity of the species α, Ω_0 is the preferred angular frequency of the rotating dust grains, r_D $[= (\sum_{\alpha=\mathrm{e,i}} \omega_{\mathrm{p}\alpha}^2/v_{\mathrm{T}\alpha}^2)^{-1/2}]$ is the effective Debye radius, $\omega_{\mathrm{p}\alpha}$ is the plasma frequency of the species α, $\Omega_\mathrm{r} \equiv (4\pi d^2 n_\mathrm{d0}/2I)^{1/2}$, d is the magnitude of the dipole moment of the dust grains, n_d0 is the dust number density and I is the moment of inertia of the elongated dust grain. Thus, for the electrostatic mode $(\omega^2\varepsilon/c^2 \ll k^2)$, i.e. the quasi-static limit, the dispersion equation in a semi-bounded dusty plasma reduces to

$$\int_{-\infty}^{\infty}\frac{dk_\perp}{(k_\perp^2 + k_z^2)(1 - \omega_\mathrm{pd}^2/\omega^2) - k_\perp^2\Omega_\mathrm{r}^2/(\omega - \Omega_0)^2 + 1/r_\mathrm{D}^2} = -\frac{\pi}{k_z}. \tag{3}$$

Then the dispersion relation for the low-frequency surface dust-acoustic wave is obtained by contour integration in the $k_\perp$-plane:

$$\frac{k_z^2}{1 - \omega_\mathrm{pd}^2/\omega^2 - \Omega_\mathrm{r}^2/(\omega - \Omega_0)^2} = k_z^2\left(1 - \frac{\omega_\mathrm{pd}^2}{\omega^2}\right) + \frac{1}{r_\mathrm{D}^2}. \tag{4}$$

For low-frequency cases, $\omega \ll \Omega_0$, the dispersion relation for the surface dust-acoustic wave is found to be

$$\omega^4\left[\frac{1}{r_{\mathrm{D}}^2}-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\left(k_z^2+\frac{1}{r_{\mathrm{D}}^2}\right)\right]-\omega^2\omega_{\mathrm{pd}}^2\left[k_z^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)+\frac{1}{r_{\mathrm{D}}^2}\right]+\omega_{\mathrm{pd}}^4k_z^2=0. \tag{5}$$

After some manipulations, the low-frequency mode solution of the dispersion for the surface dust-acoustic wave in a semi-bounded dusty plasma containing elongated and rotating dust grains is sought in the form

$$\begin{aligned}
&\omega(\Omega_{\mathrm{r}}/\Omega_0, k_z r_{\mathrm{D}})/\omega_{\mathrm{pd}}\\
&\quad=\left[2-\frac{2\Omega_{\mathrm{r}}^2}{\Omega_0^2}[(k_z r_{\mathrm{D}})^2+1]\right]^{-1/2}\Biggl\{\left[(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)+1\right]\\
&\qquad-\Biggl[\left[(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)+1\right]^2-4(k_z r_{\mathrm{D}})^2\left[1-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}[(k_z r_{\mathrm{D}})^2+1]\right]\Biggr]\Biggr\}^{1/2}.
\end{aligned} \tag{6}$$

In addition, the group velocity of the surface dust-acoustic wave is obtained as

$$\begin{aligned}
&\frac{d(\omega/\omega_{\mathrm{pd}})}{d(k_z r_{\mathrm{D}})}\\
&\quad=\Biggl\{\Biggl\{2(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)-\Biggl[8(k_z r_{\mathrm{D}})^3\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}-8(k_z r_{\mathrm{D}})\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]\\
&\qquad-4(k_z r_{\mathrm{D}})\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\left[-1-(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\right]\Biggr]\\
&\qquad\times\Biggl\{2\Biggl[-4(k_z r_{\mathrm{D}})^2\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]\\
&\qquad+\left[-1-(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\right]^2\Biggr]^{1/2}\Biggr\}^{-1}\Biggr\}\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]^{-1}\\
&\qquad+\Biggl\{2(k_z r_{\mathrm{D}})\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\Biggl[1+(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\\
&\qquad-\Biggl[-4(k_z r_{\mathrm{D}})^2\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]+\left[-1-(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\right]^2\Biggr]^{1/2}\Biggr]\Biggr\}\\
&\qquad\times\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]^{-1}\Biggr\}\Biggl\{8^{1/2}\Biggl[\Biggl[1+(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\\
&\qquad-\Biggl[-4(k_z r_{\mathrm{D}})^2\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]+\left[-1-(k_z r_{\mathrm{D}})^2\left(2-\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right)\right]^2\Biggr]^{1/2}\Biggr]\\
&\qquad\times\left[1-[1+(k_z r_{\mathrm{D}})^2]\frac{\Omega_{\mathrm{r}}^2}{\Omega_0^2}\right]^{-1}\Biggr]^{1/2}\Biggr\}^{-1}.
\end{aligned} \tag{7}$$

The spectrum of the surface dust-acoustic wave is graphically represented as a function of the scaled wave number $(k_z r_{\mathrm{D}})$ for various values of the rotation frequency (Ω_0) in Fig. 1. As can be seen, the phase velocity of the surface dust-acoustic wave

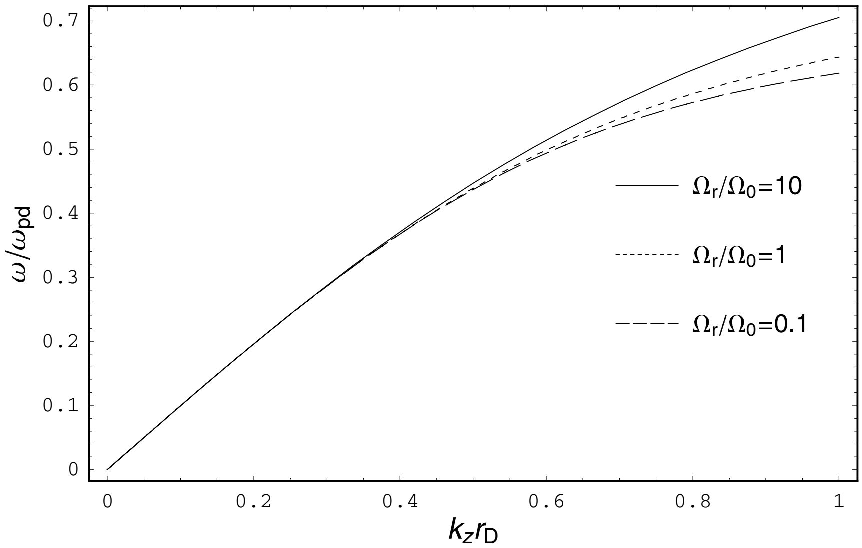

Figure 1. The dispersion relation ($\omega/\omega_{\mathrm{pd}}$) as a function of the scaled wave number ($k_z r_{\mathrm{D}}$) for various values of the ion cyclotron frequency. The solid line represents the case of $\Omega_{\mathrm{r}}/\Omega_0 = 10$. The dotted line represents the case of $\Omega_{\mathrm{r}}/\Omega_0 = 1$. The dashed line represents the case of $\Omega_{\mathrm{r}}/\Omega_0 = 0.1$.

decreases with increasing rotation frequency. i.e. with increasing rotation frequency, the rotation energy would be strongly coupled to the plasma oscillation. Hence, the rotation frequency suppresses the phase velocity of the surface dust-acoustic wave in a semi-bounded dusty plasma containing elongated and rotating dust grains, especially, for large wave numbers. Figure 2 illustrates the three-dimensional plot of the dispersion relation as a function of the frequency ratio ($\Omega_{\mathrm{r}}/\Omega_0$) and the scaled wave number. It should be noted that the phase velocity of the surface dust-acoustic wave is independent of the variation of the rotation frequency for long wavelengths since $\omega/\omega_{\mathrm{pd}} \approx k_z r_{\mathrm{D}}$ for fast rotations. It is also found that the phase velocity slightly decreases with an increase of the wave number. Figure 3 shows the scaled group velocity $(d\omega/dk_z)/(\omega_{\mathrm{pd}} r_{\mathrm{D}})$ of the surface dust-acoustic wave as a function of the scaled wave number for various values of the frequency ratio. The three-dimensional plot of the group velocity is also illustrated in Fig. 4 as a function of the frequency ratio and the scaled wave number. As we see in these figures, the group velocity of the surface dust-acoustic wave decreases with increasing the wave number and increases with increasing frequency ratio. Therefore, it can also be understood that the rotation frequency suppresses the group velocity. However, for small wave numbers the group velocity is found to be almost independent of the magnitude of the frequency ratio as in the case of the phase velocity.

3. Conclusions

We investigate the effects of dust grain rotation on the surface dust-acoustic wave in a semi-bounded dusty plasma containing elongated and rotating dust grains. The specular reflection boundary condition is applied to obtain the dispersion relation of the low-frequency surface ion plasma wave from the plasma dielectric

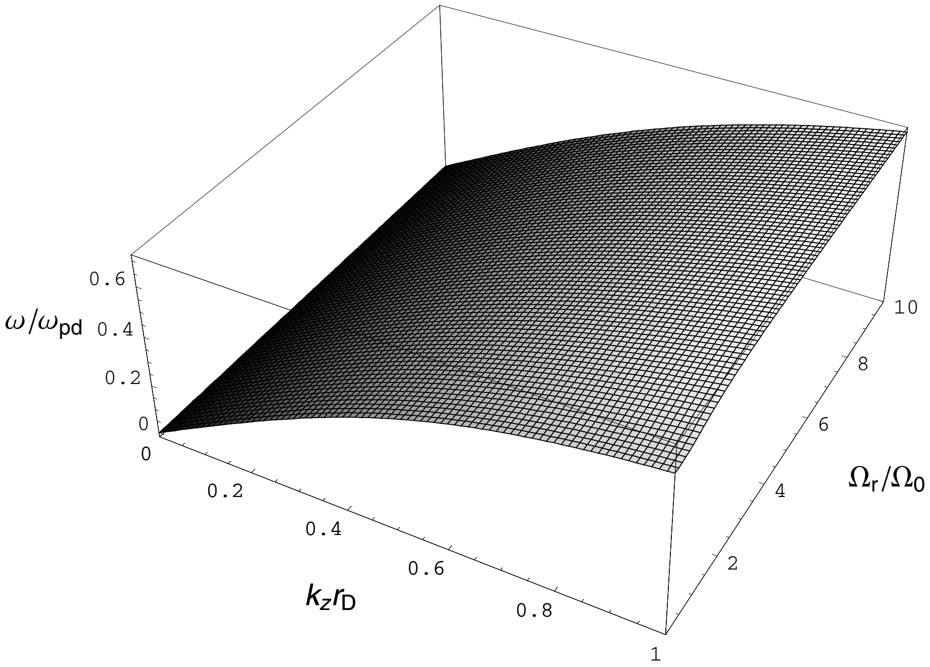

Figure 2. The three-dimensional plot of the dispersion relation as a function of the frequency ratio (Ω_r/Ω_0) and the scaled wave number $(k_z r_D)$.

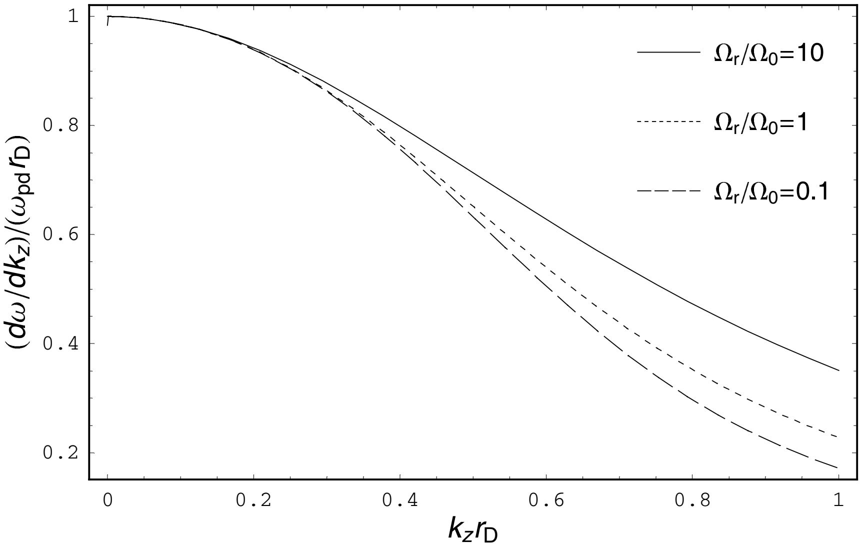

Figure 3. The scaled group velocity $(d\omega/dk_z)/(\omega_{pd} r_D)$ as a function of the scaled wave number $(k_z r_D)$. The conditions are the same as in Fig. 1.

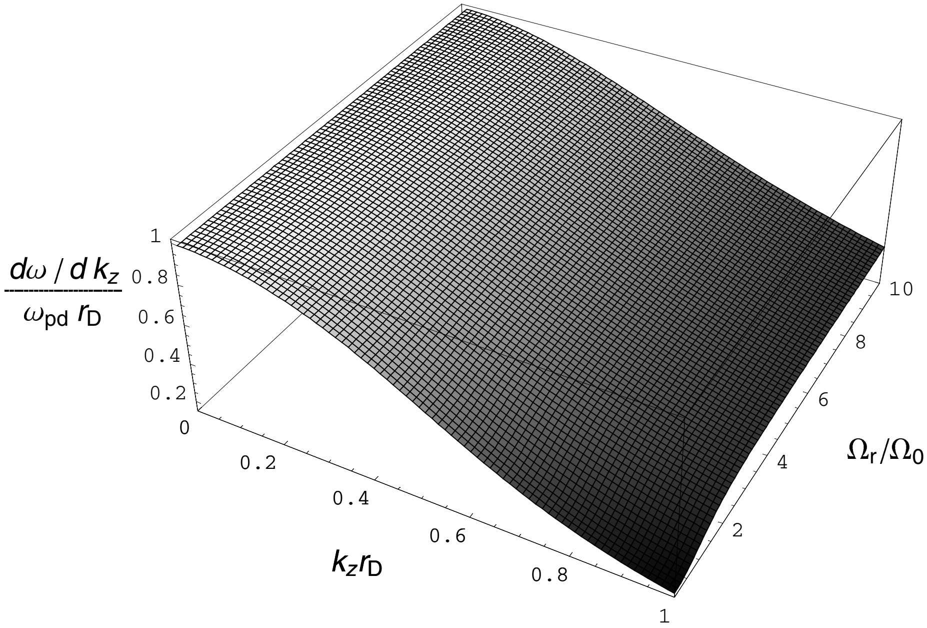

Figure 4. The three-dimensional plot of the scaled group velocity $(d\omega/dk_z)/(\omega_{\rm pd} r_{\rm D})$ as a function of the frequency ratio $(\Omega_{\rm r}/\Omega_0)$ and the scaled wave number $(k_z r_{\rm D})$.

function. It is found that the phase velocity of the surface dust-acoustic wave propagating along the plasma–vacuum interface decreases with increasing rotation frequency. However, for long wavelengths the phase velocity of the wave is found to be independent of the magnitude of the rotation frequency. The phase velocity is also found to be slightly decreased with increasing wave number. In addition, the group velocity of the surface dust-acoustic wave is decreased with an increase of the wave number and is also decreased with increasing rotation frequency. These results provide useful information on the rotation effects of elongated dust grains on the surface waves in semi-bounded dusty plasmas containing elongated and rotating dust grains.

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References

Alexandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A. 1984 *Principles of Plasma Electrodynamics*. Berlin: Springer.

Aliev, Yu M., Schlüter, H. and Shivarova, A. 2000 *Guided-Wave-Produced Plasmas*. Berlin: Springer.

Cho, S.-H., Lee, H. J. and Kim, Y.-S. 2000 *Phys. Rev.* E **61**, 4357.

Denysenko, I, Yu, M. Y. and Xu, S. 2005 *J. Phys. D: Appl. Phys.* **38**, 4003.

Mahmoodi, J., Shukla, P. K., Tsintsadze, N. L. and Tskhakaya, D. D. 2000 *Phys. Rev. Lett.* **84**, 2626.

Mendis, D. A. and Rosenberg, M. 1994 *Ann. Rev. Astron. Astrophys.* **32**, 419.

Mohideen, U., Rahman, H. U., Smith, M. A., Rosenberg, M. and Mendis, D. A. 1998 *Phys. Rev. Lett.* **81**, 349.

Shokri, B. and Jazi, B. 2003 *Phys. Plasmas* **10**, 4622.

Shukla, P. K. 2002 *Dust Plasma Interaction in Space*. New York: Nova.

Shukla, P. K. and Mamum, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol: IOP Publishing.