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ADULT LONGEVITY AND GROWTH TAKEOFF

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This paper develops an overlapping-generations model in which agents make educational and fertility decisions under life-cycle considerations and retirement from work is distinguished from death. Gains in adult longevity induce agents to decrease fertility, invest in education, and achieve higher income in order to save more for retirement. Even if working life is shortened by early retirement, this mechanism works as long as adult longevity increases sufficiently. Our model can explain the positive effect of life expectancy on education without contradicting the fact that working life length has not substantially increased, because of retirement. We also provide new insights into the interaction between fertility and retirement decisions.

Keywords: Fertility, Growth, Human Capital, Life Expectancy, Retirement

1. INTRODUCTION

It is sometimes argued that increased life expectancy triggers greater investment in human capital, a demographic transition, and a takeoff from stagnation to growth.[1] This argument is based on the life-cycle model of Ben-Porath (1967): increased life expectancy prolongs the working lives over which investments in human capital are paid off, thereby positively affecting human capital investments. This intuitively appealing mechanism has attracted many growth theorists and is the foundation of several growth theories. Examples include Ehrlich and Lui (1991), Meltzer (1992), de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Blackburn and Cipriani (2002), Boucekkine et al. (2002, 2003), Kalemli-Ozcan (2002), Chakraborty (2004, 2005), Cervellati and Sunde (2005, 2013), Soares (2005), Boucekkine et al. (2007), Soares and Falcão (2008), de la Croix and Licandro (2013), and Hansen and Lostrup (2012). However, this conventional theory is challenged by the fact that even if mortality declines, working life might not be prolonged because of early retirement. Lee (2001) shows that over the last 150 years, the retirement length expected by male workers in the United States at the age of 20 has increased considerably, whereas their expected working life length has not changed significantly.[2] Hazan (2009) finds a similar trend.[3] Gendell

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and Siegel (1992) estimate that since 1950, the retirement age has fallen by four to five years for both men and women.[4] Whether increased life expectancy promotes education through the Ben-Porath channel is controversial, but there is little doubt that increased life expectancy has resulted in a considerable increase in the length of retirement.[5] This paper aims to construct a growth model focusing on the role of retirement rather than working life length in individual life-cycle behavior.

The key mechanism in our model is as follows. Gains in life expectancy induce agents to save more for retirement and to decrease fertility.[6] Decreases in fertility (child-rearing time) increase the labor supply and consequently also returns on educational investments. Thus, agents allocate resources to their own education. Even if working life is shortened by early retirement, this mechanism works as long as adult longevity increases sufficiently.[7] That is, our model reconciles the theory that gains in life expectancy trigger a growth takeoff by increasing educational investment with the observation that because of retirement, working life length has not increased substantially.[8] Our model predicts that economies (dynasties) with low longevity are likely to stagnate and those with high longevity are likely to obtain high income, and within an intermediate range of longevity, multiple long-run equilibria arise; whether an economy (a dynasty) is stuck in a poverty trap depends on its initial amount of ancestral human capital.

Hazan (2009) explores not only expected years of labor market participation but also yearly hours worked and shows that expected total hours worked over a lifetime have decreased because of a decline in hours worked per year. One may argue that our model, like most previous studies, cannot overcome the critique of Hazan (2009) because it attributes increases in education induced by increased longevity to increases in the provision of labor over a lifetime. However, taking into account the increase in female participation in paid work and the decrease in female participation in housework over the last century [see Mammen and Paxson (2000); Greenwood et al. (2005); Ramey and Francis (2009); Kimura and Yasui (2010)], as well as the fact that women have historically taken the central role in child rearing, which is the focus of this paper, our mechanism does not contradict the data on household time allocation. Ramey and Francis (2009) illustrate that females 25–54 years old worked an average 7.9 and 26.1 hours per week in 1900 and 2005, respectively, whereas the figures for females aged 65 or older were, respectively, 4.8 and 2.9 hours per week in 1900 and 2005, respectively. That is, the data suggest that over the last century, females increased their provision of labor in their earlier years and decreased their provision of labor as they got older, which is consistent with our theory.

We share with Soares (2005) the motivation to explain changes in fertility and educational attainment in terms of exogenous changes in life expectancy.[9] Soares (2005) provides cross-country data for 1960 and 1995, suggesting that for constant levels of income, life expectancy and education rose and fertility declined, and that changes in fertility and education closely followed the changes in life expectancy.[10] We are also motivated by this evidence. Our model departs from the model of Soares (2005) by explicitly considering individual life-cycle behavior. In particular, because our model distinguishes between retirement and death, we can separately explore the effects of the prolongation of working life and those of the prolongation of retirement life. This distinction is important because the prolongation of working life and that of retirement life might provide very different incentives for individuals; for example, our model suggests that a longer working life increases fertility whereas a longer retirement life decreases it.

This study is not the first to focus on incentives to save as a channel through which increased longevity decreases fertility. This channel was observed by Zhang and Zhang (2005) and Chen (2010). What differentiates our work from theirs is our detailed investigation of life-cycle behavior. We employ a continuous-time overlapping-generations model in which the working life and retirement periods have positive lengths. In contrast, Zhang and Zhang (2005) and Chen (2010) use a discrete-time overlapping-generations model in which an individual's life consists of childhood, adulthood, and old age, and increased longevity is modeled as an increased probability of surviving to old age. Because increased longevity is tantamount to prolonged retirement in their model, they could not separately explore the effects of longer working and retirement. We can conduct such an exploration and consider the endogeneity of working life length (Section 3.1) and the effects of increased longevity in earlier stages of development where there is little retired life (Section 3.3).

This paper follows many studies that address the interaction between education and fertility in economic development and growth [e.g., Becker et al. (1990); Tamura (1996); Dahan and Tsiddon (1998); Galor and Weil (2000); Blackburn and Cipriani (2002); Galor and Moav (2002); Greenwood and Seshadri (2002); Hazan and Berdugo (2002); Lucas (2002); de la Croix and Doepke (2003, 2004); Kalemli-Ozcan (2003); Lagerlöf (2003); Doepke (2004, 2005); Doepke and Zilibotti (2005); Soares (2005); Moav (2005); Kimura and Yasui (2007); Soares and Falcão (2008); de la Croix and Licandro (2013)]. With respect to the transmission of poverty across generations and its relationship to fertility, this paper is similar to Hazan and Berdugo (2002) and Moav (2005). Both their and our models predict that dynasties within a country can converge to one of two long-run equilibria: poor dynasties with human capital below a threshold converge to a low-education and high-fertility equilibrium, whereas wealthy dynasties with human capital above the threshold converge to a high-education and low-fertility equilibrium.[11] This prediction implies that poverty can persist in rich countries, whereas wealthy households can endure in poor countries. Furthermore, our model predicts that gains in adult longevity can trigger escape from the poverty trap.

Our paper is related to Boucekkine et al. (2002), Cervellati and Sunde (2013), and de la Croix and Licandro (2013) in that the role of adult longevity is considered in a continuous-time overlapping-generations framework. Our paper differs from these studies, as our model jointly considers the endogeneity of fertility and the retirement period.[12] The joint consideration of fertility and retirement enables us to derive some interesting results: for example, assuming that retirement is exogenous, the prolongation of working life and retirement might provide very different incentives for individuals; assuming that retirement is endogenous, a reduction in fertility induced by gains in adult longevity lowers the retirement age.

This paper also provides new insights concerning the individual choice of when to retire. Explaining the decline in the labor force participation of the elderly has been a major concern for economists. For example, the development of the public pension system, income effects of rising wages, and technological progress that makes the skills of the elderly obsolete are well-known possible causes of the decline in retirement age. Our work is related to Boucekkine et al. (2002), Ferreira and Pessôa (2007), and Kalemli-Ozcan and Weil (2010), which explore the effects of increased longevity on the retirement decision.[13] We show that declining fertility reduces the retirement age, an effect not identified in the literature as a possible explanation for early retirement. Decreases in fertility mean increases in working time and earnings when agents are young adults, thereby enabling them to save more and retire early via the income effect. We refer to this as the "fertility effect" on individual retirement decisions. This effect allows our model to simultaneously generate increases in education and decreases in fertility and the retirement age.

The remainder of this paper is organized as follows. Section 2 presents the basic model and the main results of our paper. In Section 3, we consider the individual retirement decision and explore the joint determination of retirement and fertility, introduce a quantity–quality trade-off as an extension, and discuss the possibility that our model can be used to explain the long-run changes in fertility. Section 4 concludes.

2. BASIC MODEL

2.1. Economic Environment

Consider a small open economy in which time is continuous and the population consists of a discrete number of overlapping generations.

An individual's life consists of three periods, childhood, adulthood, and old age, each of which has a time interval with positive measure. In childhood, agents do not make decisions and consume a fixed quantity of time from their parents. In adulthood, they invest in their own education, raise children, supply labor to the market, and consume goods. In old age, they only consume goods. We denote the lengths of childhood, adulthood, and old age by D, W, and R, respectively. That is, the length of the period over which agents consume goods is $T \equiv W + R$. We treat T as a proxy for adult longevity: "increases in life expectancy" and "lifetime prolongation" are defined as rises in T in the model.[14] To keep our primary mechanism as simple as possible, we assume that there is no uncertainty about the length of each period. All decisions are made at the beginning of adulthood (adult age 0): agents decide the number of children, the amount of investment in their own education, and the consumption plan over their lifetime.

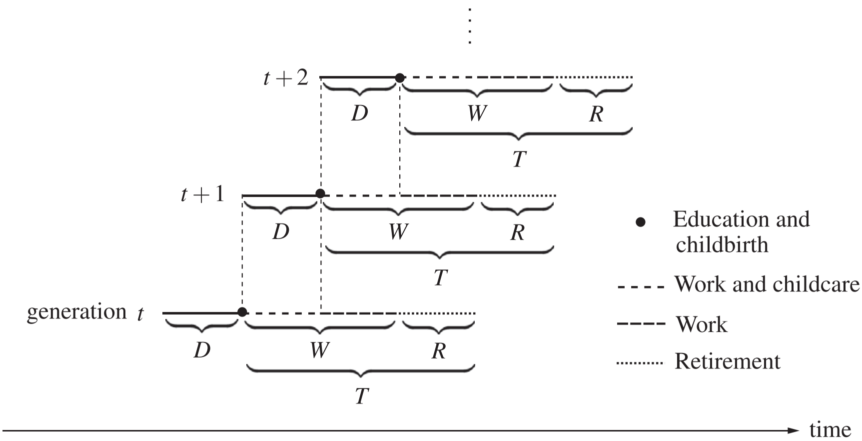

FIGURE 1. Timing of events.

For simplicity, the time required for education and childbirth is assumed to be zero: both events are instantaneously completed at adult age 0. Educational investment entails only pecuniary costs and education expenditures must be repaid during the remaining lifetime.[15] The world interest rate equals $r > 0$ and is constant over time. Individuals can lend and borrow any amount at this rate. Having children entails only time costs for child rearing. Each adult is endowed with a unit of time that can be devoted to market work or child rearing at each time. Rearing a child requires a fraction $z \in (0, 1)$ of the time endowment at every time from adult age 0 to D, when children are dependent family members. Agents retire at adult age W and die at adult age $T \equiv W + R$. We assume that $D < W$, which seems realistic. Figure 1 illustrates the timing of the model.

Agents receive utility from the number of children that they have and the consumption stream over their lifetime. For simplicity, we assume that the subjective discount rate equals r. The utility function of agents in generation t is

$$\gamma \ln n_t + (1 - \gamma) \int_0^T e^{-r\tau} \ln c_t(\tau)\, d\tau, \tag{1}$$

where n_t and $c_t(\tau)$, respectively, represent the number of children and consumption at adult age τ, and $\gamma \in (0, 1)$ denotes the relative weight given to children. The first term of (1) denotes the utility that parents derive from children and the second term denotes the utility that they derive from consumption. The main result of this paper comes directly from this specification of preferences: lifetime prolongation implies that more weight is given to own consumption.[16] Alternatively, if parents simply regard their children as consumer durables from which they receive utility at each time, the first term must also be integrated from 0 to T and discounted at rate r. Given that there are motives for demanding children other than the consumption motive, however (e.g., the procreation motive), it seems reasonable to assume the

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utility function given by (1) to concentrate on our main mechanism.[17] For example, Soares (2005), Zhang and Zhang (2005), Doepke et al. (2007), and Chen (2010) also assume a utility function in which utility from children is not discounted in the same way as utility from consumption. Note that our main result is robust as long as lifetime prolongation raises the relative weight given to own consumption (i.e., the log utility is employed only for convenience).

Denote the wage rate by $w > 0$. Agents with human capital h_t and n_t children earn $(1 - zn_t)wh_t$ at each time from adult age 0 to D and earn wh_t at each time from D to W. The production function for human capital is given by

$$h_t = \max\left\{\eta x_t^{\theta} h_{t-1}^{\zeta}, \underline{h}\right\}, \tag{2}$$

where x_t and h_{t-1} are educational investment and parental human capital, respectively, and $\eta > 0, \theta \in (0, 1), \zeta \in (0, 1-\theta]$, and $\underline{h} > 0$ are the parameters.[18,19] This formulation of human capital production technology implies that the minimum level of human capital, $\underline{h}$, is guaranteed irrespective of educational investment and parents' human capital. Alternatively, we can interpret that there exist two sectors, a modern market sector and a traditional agriculture sector (or home production), and agents can freely decide which sector to choose. Human capital matters, and earnings depend on the amount of human capital in the former sector but not in the latter.

The accumulation of assets is described by

$$\dot{a}_t(\tau) = ra_t(\tau) + (1 - zn_t)wh_t - c_t(\tau) \quad \text{for} \quad \tau \in [0, D], \tag{3}$$

$$\dot{a}_t(\tau) = ra_t(\tau) + wh_t - c_t(\tau) \quad \text{for} \quad \tau \in [D, W], \tag{4}$$

and

$$\dot{a}_t(\tau) = ra_t(\tau) - c_t(\tau) \text{ for } \tau \in [W, T], \tag{5}$$

where $a_t(\tau)$ is the quantity of assets at adult age τ and $\dot{a}_t(\tau)$ denotes its time derivative. The differences between (3), (4), and (5) indicate that agents can supply labor in adulthood but not in old age, and that they must spend time zn_t on child rearing until adult age D.

The initial and terminal conditions for assets are, respectively, given by

$$a_t(0) = 0 \quad \text{and} \quad a_t(T) = 0. \tag{6}$$

Agents start their working lives without any debts and bequests and cannot leave debts or bequests to their offspring.

The intertemporal budget constraint for agents of generation t is

$$\int_0^T e^{-r\tau} c_t(\tau)\, d\tau + x_t = \int_0^D e^{-r\tau}(1 - zn_t)\, wh_t d\tau + \int_D^W e^{-r\tau} wh_t d\tau. \tag{7}$$

The present discounted value of expenditures on consumption and education must equal that of lifetime earnings.

2.2. Agent's Problem

Consider the agent's problem. It follows from the log utility and the assumption that the subjective discount rate equals the interest rate that the optimal consumption path is constant over the life cycle,

$$c_t(\tau) = C_t \quad \text{for} \quad \tau \in [0, T].$$

Therefore, the agent's problem can be written as

$$\max_{n_t, C_t, x_t} \gamma \ln n_t + (1-\gamma) \int_0^T e^{-r\tau} \ln C_t d\tau, \tag{8}$$

$$\text{s.t.} \int_0^T e^{-r\tau} C_t d\tau + x_t = \int_0^D e^{-r\tau} (1 - z n_t) w h_t d\tau + \int_D^W e^{-r\tau} w h_t d\tau, \tag{9}$$

$$\text{and } h_t = \max\left\{\eta x_t^\theta h_{t-1}^\zeta, \underline{h}\right\}.$$

We focus on the interior solutions for the fertility choice throughout this paper. Depending on the parameters, the time constraint at each time might be binding for $\tau \leq D$, that is, $z n_t = 1$. We concentrate on the parameter configurations such that the time constraint does not bind.[20]

The solution to this problem can either be interior or be at a corner for educational choice; that is, $x_t > 0$ or $x_t = 0$.[21] First, consider an interior solution. The first-order conditions imply that

$$n_t^{\mathrm{E}} = \frac{\gamma\left(1 - e^{-rW}\right)}{z\left(1 - e^{-rD}\right)\left(\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right)}, \tag{10}$$

$$C_t^{\mathrm{E}} = \frac{r\theta^{\frac{\theta}{1-\theta}}(1-\theta)}{1-e^{-rT}}\left(\frac{1-e^{-rW}}{r}\frac{\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}w\eta}{\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}}\right)^{\frac{1}{1-\theta}} h_{t-1}^{\frac{\zeta}{1-\theta}}, \tag{11}$$

and

$$x_t^{\mathrm{E}} = \left(\frac{1-e^{-rW}}{r}\frac{\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}w\eta\theta}{\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}}\right)^{\frac{1}{1-\theta}} h_{t-1}^{\frac{\zeta}{1-\theta}}, \tag{12}$$

where the superscript E represents "educated." Consumption and education increase with parental human capital, h_{t-1}, whereas fertility does not depend on h_{t-1} because the positive income and negative substitution effects on fertility cancel each other out.

The important comparative-static results are that fertility, n_t^{E}, is decreasing in lifetime length, T, increasing in working life length, W, and decreasing in child-rearing period length, D; in addition, educational investment, x_t^{E}, is increasing in T and W. Let us elaborate on the mechanism behind these results. Lifetime prolongation stimulates the demand for consumption relative to that for children because agents must consume goods over a longer period.[22] In response to this

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change, agents shift their time from child rearing to work.[23] The resulting increases in working time raise the return on education and the amount of educational investment increases. Conversely, working life prolongation increases fertility. Because agents have already been liberated from child care upon retirement (adult age W), working life prolongation only has positive income effects on fertility. Increases in W have two opposing effects on education: working life prolongation directly increases working time, whereas the increase in fertility induced by working life prolongation implies a decline in working time. The former dominates the latter, and the amount of educational investment increases.

Next, consider the case of a corner solution. The first-order conditions imply that

$$n_t^{\mathrm{U}} = \frac{\gamma\left(1-e^{-rW}\right)}{z\left(1-e^{-rD}\right)\left[\gamma+(1-\gamma)\frac{1-e^{-rT}}{r}\right]} \tag{13}$$

and

$$C_t^{\mathrm{U}} = \frac{1-e^{-rW}}{r}\frac{(1-\gamma)\,w\underline{h}}{\gamma+(1-\gamma)\frac{1-e^{-rT}}{r}}, \tag{14}$$

where the superscript U represents "uneducated." As in the case of an interior solution, fertility, n_t^{U}, is decreasing in T, increasing in W, and decreasing in D. These comparative-static results are derived using a mechanism identical to that in the case of positive educational investment. It should be noted that in either case, interior or corner, increases in T and W yield opposite effects on fertility, suggesting the importance of distinguishing between retirement and death.

Comparing interior and corner solutions, we obtain the following propositions.

PROPOSITION 1. *There is a threshold level of parental human capital, h^*, above which agents choose an interior solution of investing in education.*

Proof. Denote the indirect utility function of the educated by $V^{\mathrm{E}}(h_{t-1})$ and that of the uneducated by V^{U}. Substituting (10) and (11) into (8), and (13) and (14) into (8), we obtain, respectively,

$$\begin{aligned} V^{\mathrm{E}}\left(h_{t-1}\right) = {} & A-\gamma\ln\left(\gamma+\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right) \\ & +(1-\gamma)\frac{1-e^{-rT}}{r}\ln\frac{r\theta^{\frac{\theta}{1-\theta}}(1-\theta)}{1-e^{-rT}} \\ & \times\left(\frac{1-e^{-rW}}{r}\frac{\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}w\eta}{\gamma+\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}}\right)^{\frac{1}{1-\theta}}h_{t-1}^{\frac{\zeta}{1-\theta}} \end{aligned}$$

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and

$$V^{\mathrm{U}} = A - \gamma \ln\left[\gamma + (1-\gamma)\frac{1-e^{-rT}}{r}\right] + (1-\gamma)\frac{1-e^{-rT}}{r}\ln\frac{1-e^{-rW}}{r}\frac{(1-\gamma)\,w\underline{h}}{\gamma + (1-\gamma)\frac{1-e^{-rT}}{r}},$$

where A is a constant term common to $V^{\mathrm{E}}(h_{t-1})$ and V^{U}. $V^{\mathrm{E}}(h_{t-1})$ is increasing in h_{t-1}, $\lim_{h_{t-1}\to 0} V^{\mathrm{E}}(h_{t-1}) = -\infty$, and $\lim_{h_{t-1}\to\infty} V^{\mathrm{E}}(h_{t-1}) = \infty$, whereas V^{U} does not vary with h_{t-1}. Thus, there exists a unique h^* such that agents with a parental human capital level h^* are indifferent to investing in education and remaining uneducated; that is, $V^{\mathrm{E}}(h^*) = V^{\mathrm{U}}$, $V^{\mathrm{E}}(h_{t-1}) < V^{\mathrm{U}}$ if $h_{t-1} < h^*$, and $V^{\mathrm{E}}(h_{t-1}) > V^{\mathrm{U}}$ if $h_{t-1} > h^*$. ■

The larger the amount of parental human capital, the higher the return on education. Agents whose parents have high levels of human capital are more likely to be educated.

PROPOSITION 2. *(i) The fertility of the educated is lower than that of the uneducated. (ii) The threshold h^* is decreasing in T and W.*

Proof. (i) It follows from (10) and (13) that

$$n_t^{\mathrm{E}} - n_t^{\mathrm{U}} = -\frac{\frac{\gamma}{z}\frac{1-e^{-rW}}{1-e^{-rD}}\frac{\theta}{1-\theta}(1-\gamma)\frac{1-e^{-rT}}{r}}{\left(\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right)\left[\gamma + (1-\gamma)\frac{1-e^{-rT}}{r}\right]} < 0.$$

(ii) Totally differentiating $V^{\mathrm{E}}(h^*) = V^{\mathrm{U}}$, we obtain

$$\frac{dh^*}{dT} = \frac{re^{-rT}}{1-e^{-rT}}\frac{1-\theta}{\zeta}h^*\left[\ln C_t^{\mathrm{U}} - \ln C_t^{\mathrm{E}}\left(h^*\right)\right] < 0$$

and

$$\frac{dh^*}{dW} = -\frac{\theta}{\zeta}\frac{re^{-rW}h^*}{1-e^{-rW}} < 0.$$

■

Result (i) simply comes from the fact that the opportunity costs of child rearing are greater for the educated than for the uneducated. When choosing to be educated, agents face a trade-off between increasing the efficiency units of labor at each time and saving on educational expenditures. The educated with larger efficiency units of labor have a comparative advantage in earning income, whereas the uneducated with smaller efficiency units have a comparative advantage in raising children. Therefore, the utility-maximizing behavior of the educated is characterized by having fewer children. Result (ii) states that as lifetimes and working lives become longer, agents are more likely to be educated: as T rises, agents come to attach importance to consumption relative to children. It then becomes advantageous to acquire education because it gives agents a comparative advantage in earning income for consumption. As such, $dh^*/dT < 0$. The result $dh^*/dW < 0$ reflects the conventional Ben-Porath mechanism: the prolongation of the working life over which investments in human capital are paid off promotes human capital investment.

2.3. Dynamics of Human Capital

It follows from the results obtained in the preceding that the dynamic equation for human capital is given by

$$h_t = \underline{h} \quad \text{if} \quad h_{t-1} \leq h^* \tag{15}$$

and

$$h_t = \eta \left(\frac{1 - e^{-rW}}{r} \frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-rT}}{r} w\eta\theta}{\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-rT}}{r}} \right)^{\frac{\theta}{1-\theta}} h_{t-1}^{\frac{\zeta}{1-\theta}} \equiv \Omega h_{t-1}^{\frac{\zeta}{1-\theta}} \quad \text{if} \quad h_{t-1} > h^*. \tag{16}$$

Using this dynamic equation, we obtain the following proposition.

PROPOSITION 3. *(i) If $\Omega(h^*)^{\frac{\zeta}{1-\theta}} \leq h^*$, the dynamic system has a unique and stable steady state with no education. If $\Omega(h^*)^{\frac{\zeta}{1-\theta}} > h^*$ and $h^* \geq \underline{h}$, multiple steady states exist: the initial amount of ancestral human capital determines the characteristics of the long-run equilibrium. If $\Omega(h^*)^{\frac{\zeta}{1-\theta}} > h^*$ and $h^* < \underline{h}$, the dynamic system has a unique and stable steady state with positive education. (ii) Increases in T and W increase Ω and decrease h^*, thereby facilitating the growth takeoff.*

Proof. (i) Depending on the parameters, $\Omega(h^*)^{\zeta/(1-\theta)}$ can be larger or smaller than h^* and h^* can be larger or smaller than $\underline{h}$. Thus, it is sufficient to show that $\Omega(h^*)^{\zeta/(1-\theta)} > \underline{h}$. Suppose that $\Omega(h^*)^{\zeta/(1-\theta)} \leq \underline{h}$. Then the relationship $V^{\mathrm{E}}((\underline{h}/\Omega)^{(1-\theta)/\zeta}) \geq V^{\mathrm{E}}(h^*) = V^{\mathrm{U}}$ must hold. However, this relationship cannot hold because $\theta \in (0, 1)$. It follows that $\Omega(h^*)^{\zeta/(1-\theta)} > \underline{h}$.

(ii) It follows from differentiating (16) that $\partial\Omega/\partial T > 0$ and $\partial\Omega/\partial W > 0$. The results $\partial h^*/\partial T < 0$ and $\partial h^*/\partial W < 0$ were obtained in Proposition 2. ∎

Figure 2 depicts the dynamic system of human capital. Depending on the parameter configuration, we can classify the dynamic system into three cases. Panel (a) describes the case in which the only long-run equilibrium is characterized by no education; panel (b) describes the case in which multiple long-run equilibria occur, characterized by no education and positive education; and panel (c) describes the case in which the only long-run equilibrium is characterized by positive education. The result that increases in T and W increase Ω and decrease h^* suggests that as T and W increase, the dynamic system evolves as depicted in panel (d).[24] The dynasties with low longevity are likely to be uneducated and those with high longevity are likely to be educated in the long run, and within an intermediate range of longevity, multiple steady states occur: whether a dynasty

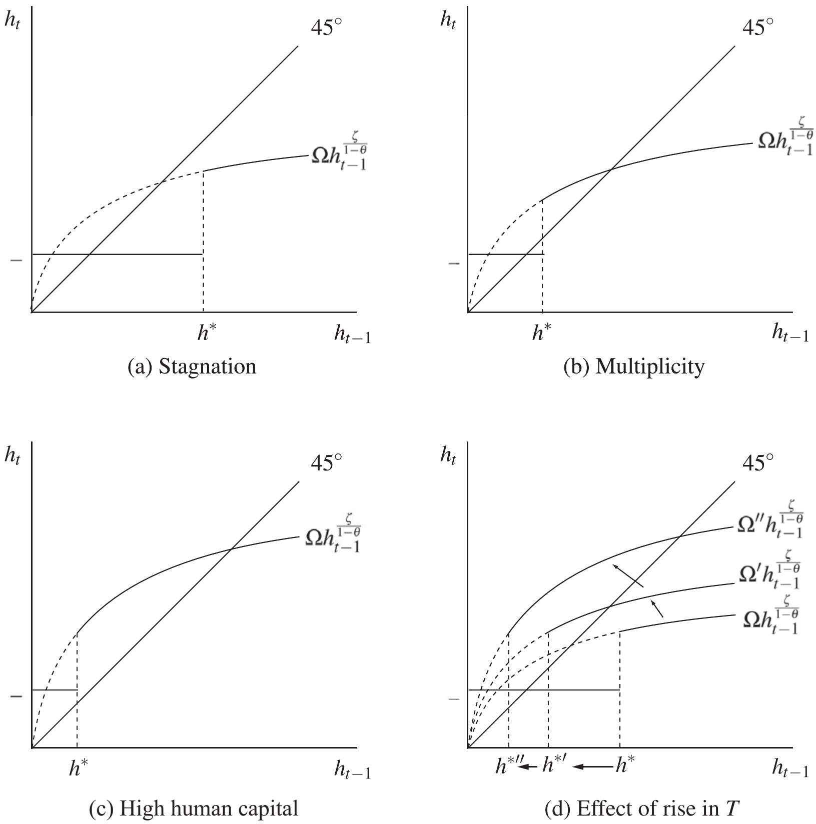

FIGURE 2. Dynamics of human capital.

is stuck in a poverty trap or takes off depends on the initial amount of ancestral human capital.[25] Multiplicity of steady states is based on the effect of parental human capital on returns on education. The larger the parental human capital, the greater the investment in the offspring's education is, and thus the difference in the amount of ancestral human capital is inherited by the following generation. However, even if a dynasty is initially caught in a poverty trap, a sufficiently large increase in life expectancy can liberate the dynasty from poverty.

We conclude this subsection by investigating the effect of simultaneous changes in T and W. To see whether such changes promote education and growth, we need only examine the effect on x_t^{E}:

$$\begin{aligned} dx_t^{\mathrm{E}} &= \frac{\partial x_t^{\mathrm{E}}}{\partial W} dW + \frac{\partial x_t^{\mathrm{E}}}{\partial T} dT \\ &= \frac{(1-\gamma)\, w\theta\Omega h_{t-1}^{\frac{\zeta}{1-\theta}}}{(1-\theta)^2 \left(\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right)} \left(\frac{1-e^{-rT}}{r} e^{-rW} dW + \gamma \frac{1-e^{-rW}}{r} e^{-rT} dT \right). \end{aligned}$$

Early retirement implies that $dW < 0$ and increased life expectancy implies that $dT > 0$. Even if early retirement has a negative effect on education through the conventional Ben-Porath mechanism, the total effect on education can be positive if life expectancy increases sufficiently. According to Lee (2001), over the last 150 years, the length of the post-retirement period expected by 20-year-old male workers in the United States increased considerably, whereas their expected working life length has exhibited little change. Our model predicts that agents increase educational investment in response to such changes.

3. EXTENSIONS AND DISCUSSION

3.1. Endogenous Retirement

Although life expectancy, which relies on medical knowledge and technological development, is largely exogenous for individuals [see, for example, Soares (2005)], retirement is usually an individual choice. Here, we extend the basic model by endogenizing the retirement date. We reformulate the utility function of agents so that they receive disutility from work:

$$\gamma \ln n_t + (1-\gamma) \int_0^T e^{-r\tau} \ln C_t d\tau - \int_0^{W_t} e^{-r\tau} f(\tau)\, d\tau, \tag{17}$$

where $f(\tau)$ represents disutility from work at adult age τ and W_t is the retirement date, which is determined by individual choice. Note that the constant consumption path is incorporated into this expression. We assume the following instantaneous disutility function:

$$f(\tau) = \begin{cases} 0 & \text{if} \quad \tau \leq D, \\ e^{\lambda\tau}\sigma & \text{if} \quad \tau > D, \end{cases} \tag{18}$$

where $\lambda > r$ and $\sigma > 0$. This function indicates that work becomes increasingly hard with age.[26] The assumption that $f(\tau) = 0$ for $\tau \leq D$ ensures that agents do not retire before completing child care, which is employed for simplification. All the other assumptions are the same as in the basic model.

The agent maximizes (17) with respect to n_t, C_t, x_t, and W_t, subject to equations (2), (9), and (18). We concentrate on interior solutions for retirement choice; that is, we assume that λ and σ are sufficiently large so that agents retire before death.

As in the basic model, the solution can either be interior, $x_t > 0$, or at a corner, $x_t = 0$. First, consider the case of an interior solution. The first-order conditions with respect to n_t and W_t are given by, respectively,

$$n_t^{\mathrm{E}} = \frac{\gamma\left(1 - e^{-rW_t^{\mathrm{E}}}\right)}{z\left(1 - e^{-rD}\right)\left(\gamma + \frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right)} \tag{19}$$

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and

$$\frac{\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}e^{-rW_t^{\mathrm{E}}}}{\frac{1-e^{-rD}}{r}\left(1-zn_t^{\mathrm{E}}\right)+\frac{e^{-rD}-e^{-rW_t^{\mathrm{E}}}}{r}}=e^{(\lambda-r)W_t^{\mathrm{E}}}\sigma. \tag{20}$$

Equation (20) states that agents choose their retirement date so that the disutility from work at retirement (right-hand side) is equal to the marginal cost of retiring, measured in terms of the loss in utility from foregone consumption (left-hand side). Solving (19) and (20), we obtain

$$e^{-\lambda W_t^{\mathrm{E}}}\left(\gamma+\frac{1-\gamma}{1-\theta}\frac{1-e^{-rT}}{r}\right)=\frac{1-e^{-rW_t^{\mathrm{E}}}}{r}\sigma, \tag{21}$$

which defines the retirement date, W_t^{E}. The other variables, C_t^{E}, x_t^{E}, and n_t^{E}, are given by (11), (12), and (19). We see that other than the retirement date being endogenously determined by (21), the individual decision is the same as in the basic model.

Next, consider a corner solution. The first-order conditions with respect to n_t and W_t are, respectively, given by

$$n_t^{\mathrm{U}}=\frac{\gamma\left(1-e^{-rW_t^{\mathrm{U}}}\right)}{z\left(1-e^{-rD}\right)\left[\gamma+(1-\gamma)\frac{1-e^{-rT}}{r}\right]} \tag{22}$$

and

$$\frac{(1-\gamma)\frac{1-e^{-rT}}{r}e^{-rW_t^{\mathrm{U}}}}{\frac{1-e^{-rD}}{r}\left(1-zn_t^{\mathrm{U}}\right)+\frac{e^{-rD}-e^{-rW_t^{\mathrm{U}}}}{r}}=e^{(\lambda-r)W_t^{\mathrm{U}}}\sigma. \tag{23}$$

The interpretation of (23) is the same as that of (20). Solving (22) and (23), we obtain

$$e^{-\lambda W_t^{\mathrm{U}}}\left[\gamma+(1-\gamma)\frac{1-e^{-rT}}{r}\right]=\frac{1-e^{-rW_t^{\mathrm{U}}}}{r}\sigma, \tag{24}$$

which defines the retirement date, W_t^{U}. The other variables, C_t^{U} and n_t^{U}, are given by (14) and (22). We see that other than the retirement date being endogenously determined by (24), the individual decision is the same as in the basic model.

Totally differentiating (19) and (20), (22) and (23), (21), and (24), we have $dn_t^{\mathrm{E}}/dT<0$, $dn_t^{\mathrm{U}}/dT<0$, $dW_t^{\mathrm{E}}/dT>0$, and $dW_t^{\mathrm{U}}/dT>0$, respectively. In either case, interior or corner, fertility decreases and working life length increases with lifetime length. Let us elaborate on the interaction between fertility and retirement decisions by exploiting the first-order conditions. We proceed by focusing on an interior solution because similar reasoning is applicable to the corner case.

Equation (19) indicates the positive relationship between n_t^{E} and W_t^{E}, reflecting that, as mentioned in Section 2, working life prolongation has positive income effects on fertility because agents have already been liberated from child care upon retirement. Equation (20) indicates the positive association between n_t^{E} and

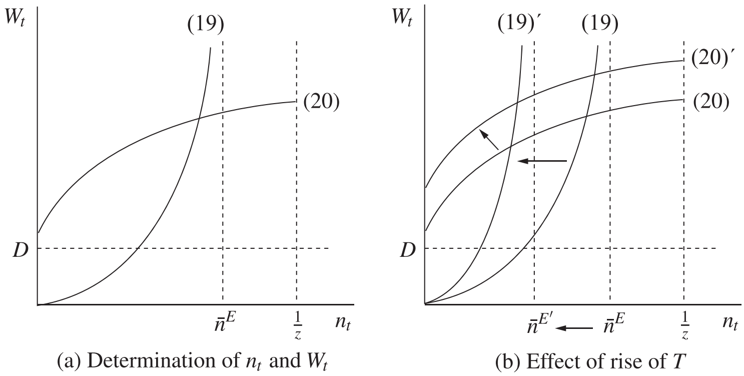

FIGURE 3. Individual choice of n_t and W_t. $\bar{n}^{\mathrm{E}}$ is defined by $\lim_{W_t^{\mathrm{E}} \to \infty} n_t^{\mathrm{E}}$ in (19).

W_t^{E}, reflecting that increases in fertility imply decreases in working time and earnings, thereby increasing the loss of utility from foregone consumption because of diminishing marginal utility and inducing agents to work longer. As depicted in Figure 3, individual choices on fertility and retirement are characterized by the intersection of these two equations.[27]

Consider the effect of a rise in T. Equation (19) indicates that the rise in T decreases n_t for any given W_t. Lifetime prolongation stimulates the demand for consumption relative to the demand for children, inducing agents to have fewer children. Equation (20) indicates that the rise in T increases W_t for any given n_t. The rise in T implies the prolongation of the lifetimes over which agents must consume goods, and thus greater losses in utility from foregone consumption, thereby extending working life length. This effect is what Kalemli-Ozcan and Weil (2010) call the "horizon effect," through which gains in adult longevity delay retirement. In the diagram, the total effects on fertility and retirement are ambiguous. As mentioned previously, however, the calculations establish that $dn_t^{\mathrm{E}}/dT < 0$ and $dW_t^{\mathrm{E}}/dT > 0$.

Many factors affect individual retirement decisions. For example, the establishment of a public pension system, income effects of rising wages, and technological progress obsoleting the skills of the elderly are well-known possible causes of early retirement. It is natural that our model that excludes these factors cannot explain the declining trends in the labor force participation of the elderly. What is more important here is that endogenizing fertility mitigates the horizon effect of increased adult longevity. Decreasing fertility, that is, increasing labor supply, enables agents to earn and save more when they are young adults and retire early through the income effect. This mechanism possibly induces early retirement, although this has not been identified in the literature. As will be shown, this fertility effect allows us to generate a salient feature of the process of economic growth: simultaneous increases in education and decreases in fertility and labor force participation by the elderly.

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As a thought experiment, consider a technological change prejudicial to the elderly.[28] In our model, such a change can be expressed as a rise in λ, which represents the pace at which the disutility from work increases with age.[29] The rise in λ shifts (20) so that W_t decreases for any given n_t, but does not shift (19). Hence, by increasing T and λ appropriately, we can decrease n_t and W_t simultaneously. Note that fertility and retirement affect returns on education only by affecting the present discounted value of the labor supply, $\int_0^D e^{-r\tau}(1 - zn_t)d\tau + \int_D^{W_t} e^{-r\tau}d\tau$, and thus simultaneous changes in n_t and W_t that do not change this value do not change the expenditure on education.[30] Suppose that such a change is caused by simultaneous increases in T and λ. This change then leads to decreases in n_t and W_t but does not change x_t. Then suppose there is an infinitesimal increase in T. As a result, x_t increases, whereas n_t and W_t are still lower than their levels before the initial changes in T and λ. It follows that we obtain the following proposition.

PROPOSITION 4. *There are changes in T and λ such that rises in schooling, declines in fertility, and declines in retirement age are simultaneously generated.*

Finally, we explore the implications of endogenizing the retirement date for the dynamics of human capital. As in the basic model, there is a threshold level of parental human capital, above which agents choose an interior solution for investing in education. The dynamic system is still given by (15) and (16), except that W_t in (16) is endogenously determined by (21). Result (i) of Proposition 3 holds entirely even in the case of endogenous retirement. Thus, the dynamic system is still classified into three cases: stagnation, multiple long-run equilibria, and sustained growth. It follows from (12) that x_t^{E} positively depends on T and W_t^{E}, and thus, rises in T have the effects of increasing x_t^{E} directly and by raising W_t^{E}. This implies that result (ii) of Proposition 3 (rises in T facilitate the growth takeoff) also holds.

3.2. Quantity–Quality Model

It is standard in the literature to include in the utility of parents not only the number of children, but also some measure of their quality, the so-called quantity–quality model à la Becker (1960). In this subsection, we extend the model so that parents care about the amount of education for their children, and we explore the robustness of our model to this extension. The utility function and the production function of human capital are modified as, respectively,

$$\gamma \ln n_t + \gamma\phi \ln b_t + (1-\gamma)\int_0^T e^{-r\tau} \ln c_t(\tau)\, d\tau$$

and

$$h_t = \max\left\{\eta x_t^{\theta} b_{t-1}^{\xi} h_{t-1}^{\zeta}, \underline{h}\right\}, \qquad \textbf{(25)}$$

where b_t is the amount of education for each child and $\phi \in (0,1)$ and $\xi \in (0, 1-\theta-\zeta]$ are parameters.[31,32]

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The agent's problem can be rewritten as

$$\max_{n_t, C_t, x_t, b_t} \gamma \ln n_t + \gamma\phi \ln b_t + (1-\gamma)\int_0^T e^{-r\tau} \ln C_t d\tau,$$

$$\text{s.t.} \quad \int_0^T e^{-r\tau} C_t d\tau + x_t + b_t n_t = \int_0^D e^{-r\tau}(1 - z n_t) w h_t d\tau + \int_D^W e^{-r\tau} w h_t d\tau,$$
$$\text{and } \; h_t = \max\left\{\eta x_t^{\theta} b_{t-1}^{\xi} h_{t-1}^{\zeta}, \underline{h}\right\}. \tag{26}$$

The product of the amount of per-child education and the number of children, $b_t n_t$ in the left-hand side of (26), represents the quantity–quality trade-off. The greater the amount of education, the higher the cost of children; the larger the number of children, the higher the cost of education.

As in the basic model, the solution can either be interior, $x_t > 0$, or at a corner, $x_t = 0$. First, consider an interior solution. The first-order conditions imply that

$$n_t^{\mathrm{E}} = \frac{\gamma\left(1 - e^{-rW}\right)}{z\left(1 - e^{-rD}\right)\left[\gamma + \frac{\gamma\phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)}\frac{1-e^{-rT}}{r}\right]}, \tag{27}$$

$$x_t^{\mathrm{E}} = \left\{\frac{1 - e^{-rW}}{r} \frac{\left[\frac{\gamma\phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)}\frac{1-e^{-rT}}{r}\right] w\eta\theta}{\gamma + \frac{\gamma\phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)}\frac{1-e^{-rT}}{r}} b_{t-1}^{\xi} h_{t-1}^{\zeta}\right\}^{\frac{1}{1-\theta}}, \tag{28}$$

and

$$b_t^{\mathrm{E}} = \frac{1 - e^{-rD}}{r}\frac{\phi z w h_t}{1-\phi} = \frac{1 - e^{-rD}}{r}\frac{\phi z w}{1-\phi}\eta\left(x_t^{\mathrm{E}}\right)^{\theta} b_{t-1}^{\xi} h_{t-1}^{\zeta}. \tag{29}$$

Next, consider a corner solution. The first-order conditions imply that

$$n_t^{\mathrm{U}} = \frac{\gamma(1-\phi)\left(1 - e^{-rW}\right)}{z\left(1 - e^{-rD}\right)\left[\gamma + (1-\gamma)\frac{1-e^{-rT}}{r}\right]}$$

and

$$b_t^{\mathrm{U}} = \frac{1 - e^{-rD}}{r}\frac{\phi z w \underline{h}}{1-\phi}. \tag{30}$$

Although the calculation is a bit more complex than in the basic model, the results concerning the household's decision remain qualitatively unchanged. There is a threshold level of the return on own education, which is determined by h_{t-1} and b_{t-1}, and agents faced with returns higher than the threshold invest in own education. The comparative-static results are $\partial n_t^{\mathrm{E}}/\partial T < 0$, $\partial n_t^{\mathrm{E}}/\partial W > 0$, $\partial x_t^{\mathrm{E}}/\partial T > 0$, $\partial x_t^{\mathrm{E}}/\partial W > 0$, $\partial n_t^{\mathrm{U}}/\partial T < 0$, $\partial n_t^{\mathrm{U}}/\partial W > 0$, and $n_t^{\mathrm{E}} < n_t^{\mathrm{U}}$, all of which have the same inequality sign as in the basic model. The quantity of education for each child, b_t, is increasing in T in the case of positive own education (through the increase in x_t^{E}) and constant with respect to T otherwise.

When adult longevity, T, increases, despite the decreased weight given to children, b_t does not decrease. This is because there exists a quantity–quality interaction: because increased adult longevity decreases the number of children, as in the basic model, the price of education for children falls and parents are induced to educate children, which offsets the effect of decreased weight given to children on b_t.[33]

Consider the dynamics of human capital. If initial values of h_t and b_t are given, then in one period, the relationship between h_t and b_t becomes linear by household choices, (29) and (30). Thereafter, the economy (the dynasty) evolves according to a dynamic system similar to that of the basic model: substituting (28) and (29) into (25), we find that h_t is increasing and concave with respect to h_{t-1} in the case of $x_t > 0$, and $h_t = \underline{h}$ in the case of $x_t = 0$.

3.3. Hump-Shaped Fertility Transition

The model presented in the preceding predicts a negative, monotonic relationship between adult longevity and fertility: It follows from (10) and (13) that $\partial n_t^{\mathrm{E}}/\partial T < 0$, $\partial n_t^{\mathrm{U}}/\partial T < 0$ and $n_t^{\mathrm{E}} < n_t^{\mathrm{U}}$. Although such a model may be a good description of the dynamics of fertility in countries that have already experienced the onset of the demographic transition, for example, in the United Kingdom or the United States over the last 150 years, it is not universal. It is widely known that many Western countries experienced upward fertility trends before the onset of the demographic transition. In this subsection, we discuss the possibility that our model provides a novel explanation of the reversal in the fertility transition.

This paper is motivated by the view that increased life expectancy has led to the prolongation of retirement rather than working life. Lee (2001) and Hazan (2009) support this view for the United States in the nineteenth and twentieth centuries, but this might not be the case for earlier stages of development. According to Lee (2001), the expected length of the retirement period for U.S. male workers born in 1830 was 2.65 years. Because there is a lower bound on the length of retirement, 0, it is likely that working life length was closely linked with adult longevity in earlier times. As the simplest case of such environments, now consider the case where $W = T$, that is, rises in adult longevity lengthen working life by exactly the same length. It follows from (10) and (13) that $\partial n_t^{\mathrm{E}}/\partial T > 0$ and $\partial n_t^{\mathrm{U}}/\partial T > 0$. As mentioned in the paragraph after equation (10), rises in T and W have opposing effects on fertility. In the case we are now considering, the positive income effect of working life prolongation dominates, and thus increased adult longevity raises fertility.

Our model suggests that the association between fertility and adult longevity is positive when rises in T are accompanied by rises in W, and negative otherwise. The result of Lee (2001) indicates that in the nineteenth-century United States, expected working life length stopped being linked to adult longevity because of retirement. That the U.S. fertility rate started to decline in the early nineteenth century may not be a coincidence.

4. CONCLUDING REMARKS

There is ample evidence that life expectancy is positively correlated with economic development and growth. Motivated by this, many researchers have developed growth models in which gains in life expectancy promote education and growth via the Ben-Porath mechanism, which offers a clear explanation for the positive association between life expectancy and education. However, since the mid-nineteenth century, working life length has not increased substantially because of falls in the retirement age, which casts doubt on the role of the Ben-Porath mechanism in the growth process. This paper presents an overlapping-generations model in which agents make educational and fertility decisions under a life-cycle consideration and retirement from work is explicitly distinguished from death. We reconcile the theory that gains in life expectancy promote education with the observation that increased life expectancy has predominantly resulted in the prolongation of retirement rather than working life. Despite its simplicity, the model makes fairly rich predictions about the links among life expectancy, retirement, education, fertility, and growth.

The model does not consider several factors relevant to economic growth. A prominent example is the effect of technological progress. As initially noted by Galor and Weil (1999, 2000), technological progress must have played an important role in increasing the demand for human capital and raising education during the transition from stagnation to growth. If such a demand shift occurs, the amount of educational investment can increase even without increases in working time, which in our model is caused by declines in child-rearing time. This argument might render moot our endeavor to establish the mechanism through which higher longevity promotes education without the prolongation of working life. Historical evidence, however, suggests that the return on human capital did not increase during the period when European countries transitioned into modern growth regimes, indicating that the rise in education over this period cannot be explained by the demand shift alone [see Clark (2005) and Galor (2005b) for this discussion]. This paper provides an explanation for the supply shift in terms of changes in individual life-cycle behavior without contradicting the observation that the length of working life has not been substantially prolonged because of retirement. Our model is more likely to complement than to contradict the models focusing on demand shifts, such as that of Galor and Weil (2000). Models explicitly incorporating the life-cycle consideration, such as ours, add new insight to the literature.

This model is simple and can be extended in several ways to explain more observations. For instance, introducing endogenous physical capital accumulation into a closed-economy framework would generate some interesting predictions. Such an extension would allow the model to capture the diluting effect of fertility and longevity on per capita capital, and possibly improve our understanding of the results in the recent empirical literature on the relationship between life expectancy and growth [e.g., Acemoglu and Johnson (2007), Lorentzen et al. (2008), and

Cervellati and Sunde (2011)]. Furthermore, our model can be applied to the analysis of public policy, such as the effects of improvements in public pension systems and public education.

NOTES

1. Life expectancy generally reflects both child mortality and adult longevity. This study is focused on adult longevity, and "life expectancy" is used to mean adult longevity unless otherwise noted.

2. According to Lee (2001), the expected retirement period length at age 20 rose from 2.65 years for the cohort born in 1830 to 5.50 years for that born in 1880 and 13.13 years for that born in 1930, whereas the expected working life lengths for these cohorts were 41.05, 41.79, and 41.89 years, respectively.

3. According to Hazan (2009), in the United States, expected retirement life length increased more than working life length in the nineteenth and twentieth centuries: 20-year-old men from the cohort born in 1840 were expected to live for another 43.2 years and work for 37.23 years, whereas their counterparts born in 1930 were expected to live for another 53.01 years and work for 41.73 years.

4. Although the studies mentioned are based on U.S. data, there is little doubt that most developed countries have experienced rises in life expectancy and declines in labor force participation by the elderly. See the World Bank's World Development Indicators for life expectancy and Kalemli-Ozcan and Weil (2010) and the references therein for labor force participation by the elderly.

5. Regarding the controversy on the Ben-Porath channel, see Sheshinski (2009), Cervellati and Sunde (2010), and Cai and Lau (2011). These studies aim to overcome the challenge raised by Hazan (2009) by generalizing survival law. In contrast, we focus on fertility choice while retaining the rectangularity of the survival function as in Hazan (2009).

6. Empirical analyses in Bloom et al. (2003) and Zhang and Zhang (2005) indicate that life expectancy has positive effects on savings rates.

7. Bar and Leukhina (2010) present a growth model in which reductions in adult mortality trigger a growth takeoff by improving knowledge transmission and encouraging innovation activity. Their mechanism also works without rises in the working life length.

8. Ferreira and Pessôa (2007) have a similar motivation. They attempt to reconcile the increases in education and retirement by incorporating labor–leisure choice, not fertility choice, into a life-cycle model. In their model, to reconcile the two facts, agents must work more intensively at the expense of leisure when they are young. According to Ramey and Francis (2009), however, leisure time did not decrease for all age groups in the twentieth-century United States. Furthermore, it should be noted that there is more to incorporating fertility choice, not labor–leisure choice, than data consistency. Under the conventional assumption that the utility from leisure is discounted at the same rate as that from consumption, increased longevity does not alter the relative marginal utility of consumption compared to leisure, and thus the effect proposed in this paper does not exist [see also the discussion following equation (1) in Section 2].

9. Although our theory, as well as that of Soares (2005), concentrates on exogenous reductions in mortality as the main driving force behind demographic transition and growth takeoff, we do not deny the possibility that individuals can invest in their own health to reduce mortality. However, several studies support the view that a large fraction of mortality declines are unrelated to economic conditions and exogenous to individuals and countries [see, for instance, Preston (1975, 1980); Becker et al. (2005)].

10. Many data, including those presented in Soares (2005), suggest a strong correlation between life expectancy and economic development and growth. See, for instance, Shastry and Weil (2003) for the cross-country relationship between adult longevity and per capita income. Jayachandran and Lleras-Muney (2009) find that life expectancy positively affects individual educational attainment, using data from Sri Lanka. However, the causal interpretation of this relationship is still controversial.

See, for instance, Acemoglu and Johnson (2007), Lorentzen et al. (2008), and Cervellati and Sunde (2011).

11. This prediction implies that if the initial average level of human capital is above the threshold, then a larger fraction of dynasties converge to a high-education and low-fertility equilibrium in a more equal economy. In this respect, our paper, like Hazan and Berdugo (2002) and Moav (2005), is related to the literature on the effect of income inequality on economic development and growth [e.g., Banerjee and Newman (1993); Galor and Zeira (1993)].

12. Boucekkine et al. (2002) distinguish retirement from death, but assume exogenous fertility; de la Croix and Licandro (2013) endogenize fertility, but do not distinguish retirement from death. Although Cervellati and Sunde (2013) endogenize fertility and distinguish retirement from death, they do not consider the endogeneity of the retirement period.

13. See Kalemli-Ozcan and Weil (2010) for details on such works. Kalemli-Ozcan and Weil (2010) present a novel explanation for the rise in retirement, which they call the "uncertainty effect": as mortality falls, the risk of dying before consuming retirement savings falls, and it becomes optimal to save for retirement.

14. In this section, we assume that D, W, and R are exogenously fixed. In Section 3, as an extension, we endogenize the length of working life, W, under the assumption that T is exogenously fixed.

15. As suggested by many studies on human capital, such as Bils and Klenow (2000), opportunity cost constitutes a major part of educational costs in the real world. The assumption that education is goods-consuming is made for analytical convenience; we can introduce time-consuming education in place of goods-consuming education without affecting the qualitative results. A possible approach is to assume that agents choose how much time to spend in schooling and work in childhood; human capital in childhood is exogenously given and schooling in childhood increases human capital used in adulthood.

16. The result that lifetime prolongation decreases fertility does not depend on the assumption that fertility involves time costs. Even if fertility entails only pecuniary costs, increased longevity reduces fertility because it decreases the relative weight given to children in the utility function. The assumption of time costs is crucial for producing fertility declines in line with economic growth in most of the literature, whereas the mechanism presented in this paper does not require such an assumption.

17. A specification such as (1) is only one extreme, as is the specification that the term on children is integrated from 0 to T and discounted at rate r; the reality might be somewhere between these two specifications. The effect proposed in this paper does not vanish as long as the latter extreme is not assumed.

18. The complementarity between education and parental human capital in human capital formation is consistent with a number of studies [e.g., Glomm and Ravikumar (1992), Benabou (1996), de la Croix and Doepke (2003, 2004), Kalemli-Ozcan (2003), Lagerlöf (2003), Soares (2005), Zhang and Zhang (2005), and Soares and Falcão (2008)].

19. This equation is compatible with endogenous growth for $\zeta = 1 - \theta$, and with exogenous growth otherwise.

20. As will become apparent, the number of children is exclusively determined by the parameters (see n_t^{E} given by (10) and n_t^{U} given by (13)). Because $n_t^{\mathrm{E}} < n_t^{\mathrm{U}}$ (see Proposition 2), we can exclude the corner case by concentrating on the parameter configurations under which $zn_t^{\mathrm{U}} < 1$.

21. In contrast to fertility choice, we cannot exclude the corner solutions for educational choice by imposing restrictions on the parameters, except for limiting cases (e.g., the case where $\theta \to 0$). Whether the agent chooses an interior or a corner solution depends on the amount of parental human capital, h_{t-1}, which is the state variable of this economy.

22. The result that lifetime prolongation decreases fertility does not depend on the assumption that fertility involves a time cost. Unlike most preceding models of fertility, our model can generate fertility declines without assuming a time cost.

23. Because n_t^{E} is constant with respect to h_{t-1} and decreasing in T, the model predicts that the amount of resources allocated to children decreases as the economy develops and adult mortality declines. One might think that this prediction is unreasonable, but such a criticism is not crucial. If we incorporate educational investment for children, the model predicts that the amount of resources

allocated to children is increasing in h_{t-1} and not always decreasing in T [see equations (27), (28, and (29) in Section 3.2].

24. Whether $\Omega(h^*)^{\zeta/(1-\theta)}$ increases with T and W is ambiguous.

25. Some have argued that increased life expectancy cannot result in growth taking of,f based on the observation that life expectancy in today's developing countries is comparable to that during the onset of the takeoff of growth in today's developed countries, but they remain poor and their growth seems not to have taken off. Our model provides an explanation for this observation without denying the role of life expectancy as the driving force of growth. According to our model, increased life expectancy alone is not sufficient for an economy to take off from stagnation if the dynamic system is characterized by multiple steady states; human capital is also necessary. Since the late nineteenth century, universal education systems have been established in many developed countries. Our theory suggests that such education reforms, combined with increased adult longevity, may have put these countries on a growth path [Galor and Moav (2006) provide a concise survey on the history of education reforms in the Western world].

26. One interpretation is that the disutility of work (and the relative utility of leisure) is higher when health is poorer. This is a convenient and occasionally implemented mechanism to model the retirement motive. See, for instance, Bloom et al. (2007) and Hazan (2009).

27. The uniqueness can be easily verified from (21). We also have $d^2n_t^{\mathrm{E}}/dW_t^{\mathrm{E}2} < 0$ from (19) and $d^2W_t^{\mathrm{E}}/dn_t^{\mathrm{E}2} < 0$ from (20).

28. Introducing a transfer that promotes retirement permits us to conduct a similar thought experiment.

29. Such a formulation of the effect of technological change can be interpreted as meaning that technological progress promotes skill obsolescence and calls for increased efforts to continue working.

30. There exists an upward isoquant curve of the present discounted value of labor supply in the (n_t, W_t) space.

31. We assume that parents value the wealth, in the form of education, that they pass on to their descendants. This motive is much more tractable than other bequest motives and is often used in the development and growth literature, for example, Banerjee and Newman (1993) and Galor and Zeira (1993).

32. The assumption that $\phi \in (0, 1)$ is conventional in the literature. This assumption guarantees that the second-order condition for maximization is satisfied in the household's maximization problem: if $\phi \geq 1$, households obtain infinite utility by reducing the number of children close to zero and thereby increasing their education level close to infinity.

33. Although including the quality choice does not qualitatively change most of the results obtained in the basic model, it might matter for the time-series relationship between longevity and education. If education is self-financed, it is one's own longevity that affects one's own education. If education is parent-financed, it is the parent's longevity that matters.

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