Integrated risk management for defined benefit pensions: models and metrics*

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Abstract

The Pension Benefit Guaranty Corporation (PBGC) insures private sector defined benefit (DB) pension plans, when an employer becomes insolvent and is unable to pay the pension liabilities. In principle, the insurance premiums collected by PBGC should be sufficient to cover potential losses; this would ensure that PBGC could pay the insured benefits of terminated pension plan without additional external funding (e.g., from taxpayers). Therefore, the risk exposure of the PBGC from insuring DB pension plans arises from the probability of the employer insolvencies; and the terminating plans' funding status (the excess of the value of the insured plan liabilities over the plan assets). Here we explore only the second component, namely the impact of plan underfunding for the operation of the PBGC. When a DB plan is fully funded, the PBGC's risk exposure for an ongoing plan is low even if the plan sponsor becomes insolvent. Thus the questions most pertinent to the PBGC are: what key risk factors can produce underfunding in a DB plan, and how can these risk factors be quantified? We discuss the key risk factors that produce DB pension underfunding, namely, investment risk and liability risk. These are interrelated and must be considered simultaneously in order to quantify the risk exposure of a DB pension plan. We propose that an integrated risk management model (an Integrated Asset/Liability Model) can help better understand DB pension plan funding risk. We also examine the Pension Insurance Modeling System developed by the PBGC in terms of its own use of some of the building blocks of an integrated risk management model.

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The Pension Benefit Guaranty Corporation (PBGC) is an agency of the U.S. federal government that insures private sector defined benefit (DB) pension plans, when an

employer becomes insolvent and is unable to pay the pension liabilities. When a plan is terminated, the PBGC takes over the plan assets and pays vested employees their future benefits up to certain limits (trusteed plans). For this insurance coverage, the employers of the ongoing plans (non-trusteed) must pay fixed/variable premiums to the PBGC. In principle, the insurance premiums collected by the PBGC should be sufficient to cover the potential losses; this would ensure that PBGC could pay the insured benefits of terminated pension plan without additional external funding (e.g., from the taxpayers). Therefore, the risk exposure of the PBGC from insuring DB pension plans arises from two sources: the probability of employer insolvencies; and the terminating plans' funding status (the excess of the value of the insured plan liabilities over the plan assets).

My comments here focus on only the second component, namely the impact of the plan underfunding for the operation of the PBGC. When a DB plan is fully funded (i.e., the value of plan assets is equal to the value of the plan liability), the PBGC's risk exposure for an ongoing plan is low even if the plan sponsor becomes insolvent. For this reason, the questions most pertinent to the PBGC are: what key risk factors can produce underfunding in a DB pension plan, and how can these risk factors be quantified?

In what follows, we explore the most important risk factors that produce DB pension underfunding, namely, investment risk and liability risk. These are interrelated and must be considered simultaneously in order to quantify the risk exposure of a DB pension plan (Hustead and Mitchell, 2001). Therefore researchers and practitioners have argued that there is a need for an integrated risk management model – also referred as Integrated Asset/Liability Model – to better understand DB pension plan funding risk. We also examine the Pension Insurance Modeling System (PIMS) developed by the PBGC in terms of its own use of some of the building blocks of an integrated risk management model.

The PIMS uses detailed information on liabilities of terminated and ongoing plans. It also implements a wide range of detailed legal rules, i.e., contribution requirements from the Pension Protection Act (PPA), rules from the U.S. tax code, and the way that fixed/variable PBGC-premiums are calculated. It also includes simple management rules on how the assets of the terminated and ongoing plans are invested.[1] Based on seven key stochastic variables,[2] PIMS specifies for each simulation path $i = 1, \ldots, I$ and time period $t = 1, \ldots, T$ for each ongoing plan $m = 1, \ldots, M$ and $n = 1, \ldots, N$ terminated plan[3]:

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1 PBGC assets are invested in a revolving and a trust fund. Plan specific asset returns (stocks and bonds) of ongoing plans are modeled using a set of factor-equations (see equations (5)–(10); PIMS Description, page 5–7) calibrated using historical Form 5,500 information. It should be mentioned that PIMS do not include a time varying asset allocation depending on the funding status of the specific plan. Therefore moral hazard aspects, i.e., a possible increase of the equity exposure in the case of a (very) low funding status are not modeled.

2 See Tables 2–14 and calibrated equations (5)–(4) to (5)–(10) in PIMS Description, (2010): Nominal interest rate, real stock return, equity-to-debt ratio, employment, cash-flow-to assets, firm equity, plan asset returns. Important deterministic user inputs are the real interest rate and productivity growth.

3 Typical values are $I = 500$ runs, $T = 20$ years.

1. the funding status (value of plan assets minus actuarial value of plan liabilities),
2. the cash flows from assets, the premiums paid to PBGC from ongoing plans, and the benefits paid to retirees of terminated plans,
3. the probability of insolvency for ongoing plans.

Consequently, PIMS considers both pure and parameter risk in the simulation procedure. The resulting simulations produce a wide range of potential future outcomes of claims. These outcomes are reported in various summary statistics; for example, as distributions over time (using quantiles and averages) and/or as a stochastic present value of PBGC net financial position using the riskless interest rate to discount (on each path) future payments.

Overall, we can conclude that the PBGC has developed a complex, reasonably sensible, and defensible stochastic model to predict the probability distribution of potential future losses for PBGC from both terminated and ongoing DB plans. Additionally, we outline two key areas where extensions of PIMS may be valuable in the future to incorporate systematic mortality risk in the liability part of the model, and to model the complete term structure of interest rate instead of only a single interest rate (30-year government bonds). Besides these two areas, the model should be expanded to include not only equity and bonds, but also real estate and alternative investments. And finally, the Conditional Value-at-Risk (CVaR) metric may be a useful way to communicate the model outcomes.

Liability risk

DB-plan liabilities evolve as a function of various demographic assumptions, economic assumptions, and assumptions about future benefit flows. The actuarial profession has established well-developed standards on how to set and work with such assumptions when evaluating the current liabilities of a pension plan. Yet the future realizations of these factors may differ from expectations, and/or the assumption may change which results in fluctuations in the value of the plan liabilities (liability risk). For example, actuaries use a specific mortality table to specify the present value of future benefits. But realized mortality rates for a specific pension plan could be higher or lower than those of the assumed mortality table. Such uncertainty is referred as idiosyncratic mortality risk. In addition, mortality can also change systematically because of unexpected improvements in survival probabilities (systematic mortality risk). Research shows that especially for long-term liabilities like pension benefits, systematic mortality risk is an important and often underestimated risk factor.[4] Figure 1 illustrates the impact of systematic mortality risk,[5] indicating that the probability of living to advanced ages exhibits significant dispersion. For example, the probability of attaining age 80 spans the range from 45 to 65%, with a median of 56%. Thus the range of the expected lifetime remaining from age 20 varies between 53.1 and 67.2 years, with a median of 59.5 years. Hence a stochastic mortality model shows

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4 See Cairns *et al.* (2010).
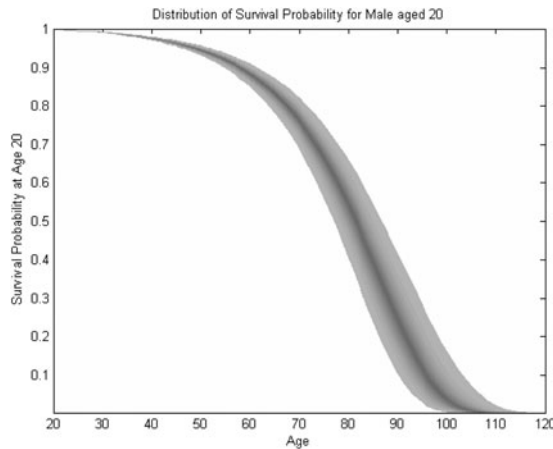5 This section including Figure 2 is based on Maurer *et al.* (2013).

Figure 1. Systematic mortality risk: Distribution of
survival probability: Age-20 male.
Notes: The simulated distribution of age-20 male
t-period survival probabilities (99%:1%) is based on
Cairns *et al.* (2006*b*) mortality model. Darker areas
represent higher probability mass.
Source: Author's elaboration.

that future survival rates and years of life remaining are likely to substantially exceed
those derived from the deterministic scenario.

To indicate the economic cost of stochastic mortality, one can calculate the age-
dependent implied loadings for pre-selected confidence levels that an insurance com-
pany or a pension fund must charge for providing lifelong annuity benefits. For each
initial age, these are derived by sampling 10,000,000 realizations of the stochastic
mortality process for a deferred annuity (until 67), and then by calculating
the required premium with respect to a certain solvency confidence level (see
Maurer *et al.* 2013).

Figure 2 shows that premium increases which take into account the stochastic mor-
tality can be substantial, especially for younger annuitants. For example, providers
must charge 20-year-old females loadings of about 32% on the top of the actuarially
fair premium, to maintain a 99.99% solvency confidence level. At the 90% confidence
level, loadings must amount to over 10% for this age group. At later ages, the disper-
sion of possible future mortality outcome decreases and so does the implied loading.
Nevertheless, at age 66, the loadings that are required to meet the 99.99% (90%) confi-
dence level still amount to about 14% (5%).

Assumptions in the PIMS model

Overall, the liability section of PIMS relies on professional actuarial standards for
evaluating the current liabilities of the DB pension plans they insure. In addition,
the PIMS model provides a methodology to project the uncertainty of the future
liabilities by varying population dynamics (firm employment), interest rates, and
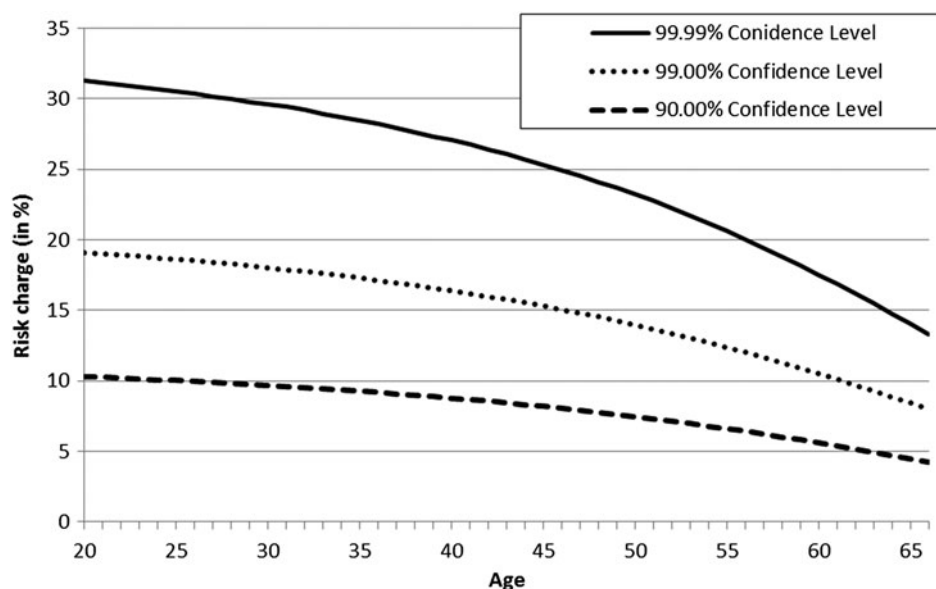
Figure 2. Stochastic mortality and risk charges for annuity benefits.
Notes: The implied loadings represent additional premiums relative to the actuarial fair annuity-premium. *Source*: Maurer *et al.* (2013) with author's modifications.

benefit growth.[6] To discount future liabilities, PIMS uses the yield for corporate bonds. The corporate bond yield is a function of the (simulated) nominal interest rate for government bond plus a spread and an adjustment factor. Owing to their long-term character, pension liabilities have a long duration and therefore are sensitive to the discount rate selected. There is an ongoing debate, whether a (higher) discount rate other than the yield for (quasi) risk-free government bonds is appropriate[7].

Systematic mortality risk is still not incorporated in the PIMS model. Yet a large literature, developed over the last two decades, explores the uncertain evolution of human mortality rates.[8] In their seminal work, Lee and Carter, (1992) introduced a discrete-time one-factor model for the central death rate. Although this model was able to trace the time trend in mortality, the one-factor approach implied perfect correlation of mortality innovations over all ages, an implication contradicted by empirical evidence (see Maurer *et al.* 2013, p. 651). Subsequent stochastic mortality models try to overcome this drawback by employing a multi-factor approach (e.g., Renshaw and Haberman 2003; Cairns *et al.* 2006*b*). Currently, the parsimonious two-factor

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6 Note: the stochastic evolution of benefit growth depends (indirectly) on the evolution of interest rates.

7 This debate is also referred as the difference between an actuarial versus an economic valuation of pension liabilities. Traditionally, actuaries choose the discount rate which reflects a reasonable projected expected return of the asset backing the pension liabilities. If the pension assets are partly invested in equities, then the discount rate includes also an equity risk premium, which is from an *ex ante* perspective not realized. By contrast, many economists argue that the relevant number for discounting future pension payments is the riskless rate of interest rate reflecting the financing cost of the plan sponsor to build up a replicating portfolio. See Blake (2006, p. 77) and Maurer *et al.* (2009).

8 See Pitacco *et al.* (2009), Cairns *et al.* (2006*a*), as well as Cairns *et al.* (2010).

model[9] by Cairns, Blake, and Dowd is widely used as a model for stochastic mortality.

Investment risk

General aspects

The pension plan assets are invested in various categories, e.g., stocks, fixed income instruments, real estate, and alternative investments. The future returns of these assets typically fluctuate over time and could be lower than the assumed interest rate used to discount future benefits (investment risk). Fluctuating interest rates (a spot rate curve) are particularly important, because they simultaneously affect the development of the plan assets and the evaluation of plan liabilities. Therefore it is necessary to model the long run stochastic dynamics of interest rates and future returns on the various assets accumulated in the pension plan. Various models are documented in the literature,[10] which also notes that the modeling of illiquid assets (e.g., real estate or alternative assets) represents a big challenge.[11]

Specific comments on the PIMS model

Currently PIMS uses a Value-at-Risk (VAR) approach to model two asset classes: risky stocks and bonds. The returns for the bonds (interest and capital gains) depend on the development of the nominal interest rate and a duration adjustment for capital gains. Stock returns are modeled independent and identically distributed. The return on corporate bonds is assumed to depend on the interest rate level adjusted for a possible (but certain) spread. The model parameters are calibrated using U.S. data. Additional assets such as real estate or alternative assets are not included in the model. The range of asset holdings for the specific pension plans is approximated using historical Form 5,500 information. PIMS also allows plans to smooth asset values within the context of the minimum PPA funding rules. Plan assets of PPGC are invested first in long-term bonds (and *T*-bills) to immunize the liabilities and for the trust fund equity investments, they are allowed within limits.

Overall, the PIMS investment model is a reasonable approach, providing a good compromise between the tractability of the model (length, time of running the code, and calibration of model parameters) and the need to use plan specific information. Useful extensions would include additional asset classes (e.g., real estate), the modeling of uncertainty of the spread for corporate bonds, and (most important) a complete term structure model for nominal interest rates.

 9 Renshaw and Haberman, (2003) examine the forecasting performance of various stochastic mortality models and show that the CBD model provides a good fit, especially for age 50 onwards; it is somewhat less accurate for younger ages.
10 Many popular model use a first-order vector autoregressive process (VAR 1); see Campbell *et al.* (2003) and Hoevenaars *et al.* (2008). More recent developments use a Markov-Regime Switching approach (see Guidolin and Timmermann, 2007). While these models capture the short term (e.g., daily, weekly, and monthly) probability distribution of risky assets reasonable (i.e., skewness, excess-kurtosis), for long-term returns it is not clear if the substantially higher complexity to calibrate such models is justified.
11 For possible ways to model real-estate investments see Fischer *et al.* (1994), Hoesli and McGregor (2000), Hoevenaars *et al.* (2008), and Maurer *et al.* (2009).

Integrating assets and liabilities: the key importance of the interest rate

Besides the budget equations and the correlations of (the residuals of) the stochastic variables, the nominal interest rate is of key importance for PIMS. The nominal interest rate enters in the following components in the PIMS model in several ways:

1. actuarial valuation of liabilities of terminated/ongoing plans,[12]
2. minimum contribution rules according to PPA rules,
3. specifying the return on plan assets for ongoing plans (stock and bonds)
4. specifying the return PBGC assets for terminated plans (stock and bonds),
5. yield for corporate bonds = $(i + \text{spread})*\text{adj. factor}$
6. inflation rate = $(1 + i)/(1 + \text{fix real interest rate}) - 1$,
7. nominal benefit (wage) growths (since inflation is related to the nominal interest rate),
8. calculation of the stochastic present value of PBGC claims.

In the current version of PIMS, the nominal interest is modeled and calibrated as a single stochastic variable for U.S. government bonds with a maturity of 30 years. Such an approach does not allow for any variation in the shape (curvature) of the term structure, nor does it include any uncertainty in credit spreads. This is not the state of the art in Asset-Liability-Modeling (ALM), nor it is consistent with empirical fluctuations of the term structure. For example, the current term structure is very steep (0% for *T*-bills and 3% for 30 maturities). Given the key importance of the interest rate for PIMS using only one single random variable may be too parsimonious. One way to extend PIMS would model the complete term structure of interest rates. Such a term structure can be used to discount future benefits[13] and also to model the development of fixed income assets in terminated and ongoing plans. Various (arbitrage free) term structure models (including procedure to calibrate the parameters) are available[14].

Communication of outcomes: CVaR

One outcome from the PIMS model is the distribution of the stochastic present values of the agency's future net positions using a 10-year horizon (except for the 20-year probability of PBGC insolvency). One way to communicate this distribution by a risk measure is to use the CVaR of pension cost at a certain confidence level x (e.g., 95%). The x%-CVaR is defined as the expected present value of total future net liabilities under the condition that its ralization exceeds the VAR for that confidence level. The CVaR risk metric is in many ways superior to the more widely-used VaR measure, i.e., the costs that will not be exceeded with a given

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12 'The interest assumption for the annul valuation of ongoing plans is currently based on the 24 months average of the corporate bond rates; captured via a single effective interest rate. Alternatively, the three segment rates can be used for PIMS valuation. During the simulation, the underlying bond rates are assumed to change by a stochastic adjustment process" (PIMS System Desc. 3–13)'

13 In the sense of a replication bond portfolio to match future liabilities.

14 See Balduzzi *et al.* (1996) or the book by Cairns (2004). The Federal Reserve Bank uses a 3-factor model for the U.S. term structure: http://www.newyorkfed.org/research/staff_reports/research_papers/9619. html
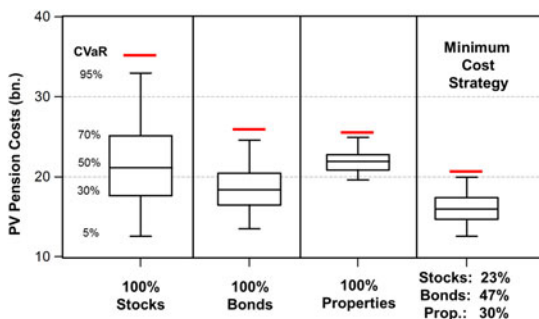
Figure 3. (colour online) Illustration of CVaR to communicate the range of pension costs.

Notes: Total pension costs is defined as the net of regular and supplementary contributions using 3% discount rate. Annotations refer to the respective percentiles of total pension cost distributions for various asset allocations.

Source: Maurer *et al.* (2009) with author's modifications.

probability. In particular, the CVaR focuses attention on a given percentile of a loss distribution, and it also accounts for the magnitude of losses in the distributional tails beyond this percentile (cf Maurer *et al.* 2009).[15] The CVaR risk metric could be a promising way to communicate risk to policymakers. In addition, this risk measure is consistent with important theoretical properties (e.g., it is a coherent risk measure).

Figure 3 illustrates the CVaR of the (stochastic) present value of costs for different investment strategies that might be used to fund the benefits of a DB pension plan.[16] The range of the optimal portfolio is substantially smaller than for pure equity or bond investments, whereas investing only in real estate would result in an even smaller range. The overall level of costs resulting from following the optimal strategy is also substantially lower, compared to the pure real estate investment case. Figure 3 illustrates the benefit of diversification to minimize the worst-case risk of pension costs represented with the CVaR.

Conclusions

Overall, the PIMS model is an important tool for modeling the liability risk of pension plans insured by PBGC. There is, to my knowledge, no other documented model available that can do a comparable job. Nevertheless, there are several opportunities for extensions:

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15 For a detailed discussion see, Artzner *et al.* (1999).
16 Results are based on an ALM-study of a large (German) public DB pension plan; see Maurer *et al.* 2009. Here, pension costs are defined as the stochastic present value of regular and supplementary (additional required in an underfunding situation) into the pension plan by the plan sponsors using a 50-year horizon). An illustration of a general ALM model appears in Appendix Figure A.1.

a) incorporating systematic mortality risk,
b) incorporation of a stochastic investment model for the major asset classes beyond stocks and bonds (including real estate),
c) modeling of the full term structure of interest rate.

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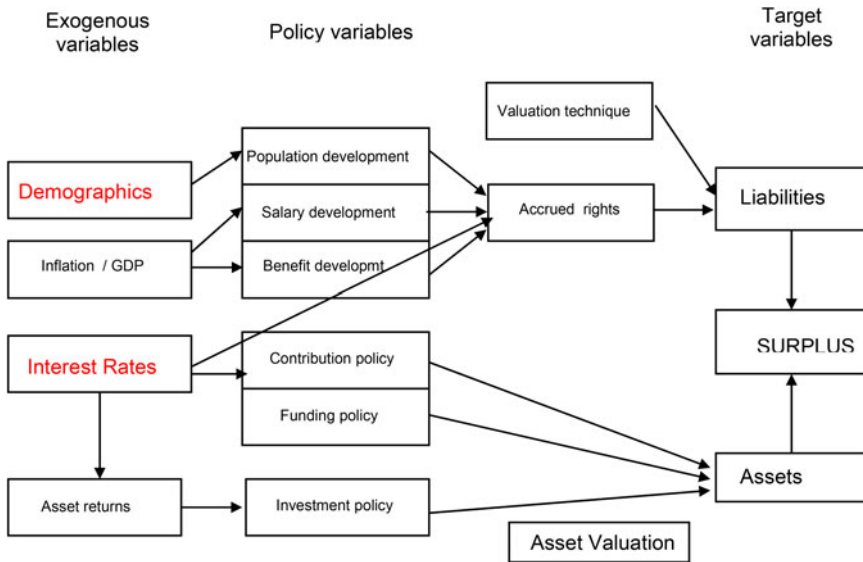
Appendix

Figure A.1. (colour online) General structure of integrated ALM models.
Source: Author's elaboration.