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INFLATION AND INEQUALITY IN A GROWING ECONOMY WITH CASH AND CREDIT GOODS

JUIN-JEN CHANG
Academia Sinica

HSIEH-YU LIN
Tunghai University

DAVID W. SAVITSKI
I-Shou University

HSUEH-FANG TSAI
Fu-Jen Catholic University

We develop a dynamic general equilibrium growth model, where households purchase final goods on cash or credit and have different capital and money endowments, to investigate whether inflation affects trends in income and consumption inequality. We show that, under a strong substitutability between cash and credit goods, inflation has a negative relationship with income inequality, but a U-shaped relationship with consumption inequality. The divergence between income and consumption inequality explains several recent empirical observations. This result has important policy implications, as consumption inequality better reflects the welfare distribution whereas income inequality fails to capture consumption disparities resulting from different consumption and asset distributions across households. In the growth model with heterogeneous households, there is a mixed relationship between growth and income inequality, confirming the existence of the Kuznets curve. The inflation-driven asset reallocation might also produce a Mundell–Tobin effect, enhancing growth.

Keywords: Consumption and Asset Reallocations, Income and Consumption Inequality, Inflation, Growth

1. INTRODUCTION

Income inequality has increased in most countries since the 1980s. The United States had the most dramatic increase among developed countries, with the share of income earned by the top 10% expanding from 34.2% in 1980 to 47.0% in 2014. Similar trends also occurred in Organisation for Economic Co-operation

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and Development (OECD) countries, for example, Japan (32.6–41.6%), Germany (28.4–35.0%), Sweden (22.2–27.7%), and the UK (28.6–34.0%).[1]

Increased income inequality has, in turn, led to questions about the corresponding welfare effects. Welfare depends more directly on consumption than on income, although income largely determines the resources available for consumption. Income may not be an appropriate measure of lifetime resources available to households, however, and so its distribution may not be a good measure of welfare distribution (Krueger and Perri (2006)). By contrast, consumption better indicates permanent income and provides a better measure of well-being (Slesnick (1994)).[2] Empirically, there is no consensus on how consumption inequality depends on income inequality.[3] A narrow focus on income inequality thus ignores important changes in consumption inequality and so distorts welfare analysis.[4]

It is particularly relevant as income and consumption inequality may move in the opposite directions. The divergence between income and consumption inequality finds empirical support. In the USA and the UK, income inequality rose while consumption inequality remained stable in the 1980s (Blundell and Preston (1998)). Moreover, income inequality increased while consumption inequality decreased in the USA since 2005 (Fisher et al., 2013), (Meyer and Sullivan, 2013), and (Meyer and Sullivan, 2017), in the UK since 1993 Blundell and Etheridge (2010), and in Sweden during 1988–2005 (Daunfeldt et al., 2010). The divergence between income and consumption inequality thus calls for investigation.

In our analysis, a potential factor looming on the horizon is inflation. While income inequality has increased, most OECD countries have experienced a corresponding decline in inflation since the 1980s (Cabanillas and Ruscher, 2008). In the 1980s, the average inflation rate across OECD countries was 9.45%, versus 1.73% during 2009–2018. Also, inflation rates currently are low in the USA, the UK, and Sweden, comparable to their 1980s levels. Of concern here is whether inflation governs income inequality and changes its relationship with consumption inequality. Such distributional effects of inflation have been largely ignored, particularly in theory (Mersch, 2014). While the empirical evidence is ambiguous, inflation does matter; even moderate inflation leads to income and wealth redistribution ((Doepke and Schneider, 2006) and (Albanesi, 2007)).[5] It does so because its effects are not uniform across wealth levels, asset portfolios, and consumption goods (the main channels in our model).[6] The impact of inflation on inequality depends on household heterogeneity, specifically the different abilities of households to protect against wealth erosion caused by inflation.

We explore how inflation affects consumption inequality and income inequality differently using a dynamic general equilibrium growth model in which households purchase final goods with cash or on credit and have different capital and money endowments. As inflation increases, households reallocate their consumption between cash and credit goods (the consumption reallocation) and reallocate assets between capital and money (the asset reallocation), to hedge against inflation. Both the consumption and asset reallocations give rise to distributional effects. Distributional effects occur because: (i) wealthy (poor) households hold more (less) capital and more (less) money, and have higher (lower) capital–money ratios and (ii) wealthy (poor) households have a stronger (weaker) propensity to consume but a weaker (stronger) propensity to work, receiving more income from capital (labor). Inflation thus differentially impacts income and consumption inequality by operating differently on cash versus credit goods and on capital versus money. These inflation-driven reallocations also have growth effects, which occur because the consumption reallocation affects the consumption-leisure tradeoff and saving, while the asset reallocation affects the asset portfolio and capital accumulation. Inflation thus impacts growth and governs the relationship between growth and inequality.

Given that cash and credit goods are strong substitutes (Dutta and Weale (2001)), we show that income inequality decreases with inflation, while consumption inequality has a U-shaped relationship with inflation, depending on the inflation *status quo*. For initial inflation rates above a certain threshold level, inflation has a negative relationship with income inequality but a positive relationship with consumption inequality. Intuitively, if the inflation *status quo* is relatively high, a further increase in inflation favors wealthy households because of the advantage of their asset portfolio (a higher capital–money ratio), raising their relative wealth. A strong substitutability between cash and credit goods allows wealthy households to increase overall consumption by significantly increasing their credit-good consumption without significantly reducing their cash-good consumption. To support higher consumption, under a cash constraint wealthy households are inclined to hold more money, instead of capital, decreasing the motivation to save in favor of consumption. In contrast, inflation is unfavorable to poor households' relative wealth. To maintain their relative wealth, poor households are inclined to substitute capital for money. With lower money holdings, poor households must substantially decrease the cash-good consumption in exchange for the credit-good consumption, resulting in a reduction in overall consumption. As a consequence of different consumption and asset reallocations, inflation decreases income inequality but increases consumption inequality when the inflation *status quo* is relatively high.

The theoretical divergence between consumption and income inequality provides a plausible explanation for recent empirical observations. Moreover, it provides an insightful implication for welfare, given that consumption measures better reflect long-run resources and social welfare, whereas income measures fail to capture disparities in consumption that result from different consumption and asset distributions across households. Besides, the negative inflation–income inequality relationship supports the empirical evidence in the USA and OECD countries (the average inflation rate of OECD countries was 7.47% during the 1980s and 1990s and 2.31% during the 2000s and 2010s, whereas the corresponding average Gini coefficients were 0.27 and 0.31).

Inflation also has an ambiguous impact on growth. With a high-inflation *status quo*, wealthy households (with a stronger propensity to consume) increase their overall consumption in response to inflation, resulting in a rise in the economy-wide consumption. Given the consumption-leisure tradeoff, higher aggregate consumption implies that, on average, households substitute consumption for leisure, increasing labor supply. Increased labor hours raise the marginal product of capital and, therefore, growth increases with inflation. This produces a so-called Mundell–Tobin effect.[7] In contrast, with a low-inflation *status quo*, inflation decreases total labor hours and, therefore, results in a decrease in growth.

This ambiguous growth–income inequality relationship confirms the existence of the Kuznets curve and provides a plausible explanation for empirically mixed findings.[8] For example, Romer and Romer (1998), Dolmas et al. (2000), Al-Marhubi (2000), and Albanesi (2007) show a positive inflation–income inequality relationship, while Maestri and Roventini (2012) and Coibion et al. (2017) find a negative inflation–income inequality relationship. It is worth noting that, in García-Peñalosa and Turnovsky (2006)'s model, growth and income inequality are positively correlated in the presence of a structural shock (time preference or technology), whereas in our model growth and income inequality can be negatively correlated in the presence of a monetary shock (inflation).

The theoretical literature on the inflation–inequality relationship is relatively limited. Using a shopping-time model with differentiated access to assets, Cysne et al. (2005) provide evidence of a positive link between inflation and income inequality. Albanesi (2007) political economy model similarly shows a positive link. By allowing heterogeneity in capital and labor productivity, Jin (2009) shows that the inflation–income inequality relationship can be negative if the effect of capital heterogeneity dominates that of labor productivity heterogeneity. Erosa and Ventura (2002), Williamson (2009), and Boel and Camera (2009) show that inflation acts as a progressive tax, if money is the only asset, but acts as a regressive tax, if an additional nominal asset exists. None of these papers, however, explores the role of inflation in the divergence between income and consumption inequalities.

An influential paper by Krueger and Perri (2006) develops a pure exchange economy in which two groups of agents own different endowments in terms of labor and capital income and uses different trends in within-group inequality to explain the divergence between consumption and income inequality in the USA. Their analysis focuses on real factors, rather than monetary factors. By shedding light on labor productivity heterogeneity, Camera and Chien (2016) show that inflation reduces wealth inequality but raises consumption inequality if agents self-insure only with money. In our analysis, inflation has different impacts on income and consumption inequalities along different dimensions of heterogeneity, as noted above. Moreover, we spell out the possibly mixed relationship between growth and income inequality.

2. THE MODEL

Our framework combines elements of García-Peñalosa and Turnovsky (2006) income inequality model and Lucas and Stokey (1983), Lucas and Stokey (1987) Cash-in-Advance (CIA) model with credit and cash goods. The model economy consists of households, firms, and the government. Households derive utility from

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consumption and leisure. They purchase goods with cash or credit, which are referred to as cash/credit goods, following Lucas and Stokey (1983), Lucas and Stokey (1987), and Albanesi (2007). Firms operate in two sectors: a perfectly competitive final good sector and a monopolistically competitive intermediate goods sector. The final goods firms use the differentiated intermediate goods as inputs to produce a final good. The intermediate goods firms hire capital and labor to produce their products. The government sets the money growth rate and balances its budget. Time t is continuous and, where unambiguous, time subscripts are omitted.

2.1. Households

There is a unit mass of infinitely lived households in the economy, indexed by j. Households are identical except for their initial (time zero) asset endowments of capital, K_{0j}, and money, M_{0j}. We define the capital share of household j as $k_j = K_j/K$, where K is the aggregate stock of capital. Similarly, we define the money share of household j as $m_j = M_j/M$, where $M(= M'/P)$ is the aggregate stock of real money balances, M' is the nominal money balances, and P is the price of the final good. The relative capital k_j (money holdings m_j) follows the distribution function $D(k_j)$ $(D(m_j))$, with mean $\sum_j k_j = 1$ $(\sum_j m_j = 1)$ and variance σ_k^2 (σ_m^2).

Households have identical utility functions: $u_j = \frac{[C_j(1-l_j)^{\eta}]^{1-\frac{1}{\phi}}}{1-\frac{1}{\phi}}$, where C_j is the household j's overall consumption, l_j is the labor supply (measured by the fraction of hours worked), $\phi(< 1)$ is the intertemporal elasticity of substitution, and η is the elasticity of leisure in utility. The overall consumption is the index composed of credit goods, C_{1j}, and cash goods, C_{2j}, and the corresponding utility follows the constant-elasticity-of-substitution form:

$$C_j = \left[aC_{1j}^{\frac{1+\varepsilon}{\varepsilon}} + (1-a)C_{2j}^{\frac{1+\varepsilon}{\varepsilon}} \right]^{\frac{\varepsilon}{1+\varepsilon}}, \tag{1}$$

where $a \in (0, 1)$ is the distribution factor and $\varepsilon \in (-\infty, 0)$ is the degree of substitutability between C_{1j} and C_{2j}. If $\varepsilon \to 0$, they are perfect complements (the Leontief relationship), while if $\varepsilon \to -\infty$ they are perfect substitutes (the linear relationship). The consumption aggregator follows the Cobb–Douglas functional form when $\varepsilon = -1$.

Households are endowed with a unit of time to allocate to work, l_j, or leisure, $1 - l_j$. Given the wage rate w, the interest rate r, and the government's policy, households solve

$$\max_{C_{1j},C_{2j},l_j,I_j,K_j,M_j} \int_0^{\infty} \frac{[C_j(1-l_j)^{\eta}]^{1-\frac{1}{\phi}}}{1-\frac{1}{\phi}} e^{-\beta t} dt, \text{ with } \phi, \eta > 0, \tag{2}$$

$$s.t. \quad \dot{M}_j = wl_j + rK_j + s_j + \tau - (C_{1j} + C_{2j}) - I_j - \pi M_j, \tag{3}$$

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$$\dot{K}_j = I_j - \delta K_j, \text{ and} \tag{4}$$

$$C_{2j} \leq M_j, \tag{5}$$

where $\beta(>0)$ is a constant time preference rate and δ is the capital depreciation rate. Equation (3) is the budget constraint linking real money accumulation to the difference between income (capital rental rK_j, labor income wl_j, firm dividends s_j, and lump-sum government transfers τ) and expenditure (consumption $C_{1j} + C_{2j}$, investments I_j, and seigniorage tax πM_j, where $\pi = \dot{P}/P$ is the inflation rate). Firm dividends are weighted by the household share of the capital stock k_j, i.e., $s_j = s \cdot k_j$, where $s = \int_0^1 \Pi_i di = \Pi$.[9] Equation (4) relates the change in the capital stock over time to investments. Equation (5) indicates that only the cash good C_{2j} is subject to the CIA constraint.[10]

Let λ_{1j}, λ_{2j}, and λ_{3j} be the Lagrangian multipliers associated with household j's budget constraint, capital evolution, and CIA constraint. The first-order conditions for this dynamic optimization problem are

$$C_j^{-(\frac{1}{\varepsilon}+\frac{1}{\phi})}(1-l_j)^{\eta(1-\frac{1}{\phi})} a C_{1j}^{\frac{1}{\varepsilon}} - \lambda_{1j} = 0, \tag{6}$$

$$C_j^{-(\frac{1}{\varepsilon}+\frac{1}{\phi})}(1-l_j)^{\eta(1-\frac{1}{\phi})}(1-a) C_{2j}^{\frac{1}{\varepsilon}} - (\lambda_{1j} + \lambda_{3j}) = 0, \tag{7}$$

$$-\eta C_j^{1-\frac{1}{\phi}}(1-l_j)^{\eta(1-\frac{1}{\phi})-1} + \lambda_{1j} w = 0, \tag{8}$$

$$-\lambda_{1j} + \lambda_{2j} = 0, \tag{9}$$

$$\frac{\dot{\lambda}_{2j}}{\lambda_{2j}} = \beta - r + \delta, \text{ and} \tag{10}$$

$$\frac{\dot{\lambda}_{1j}}{\lambda_{1j}} = \beta + \pi - \frac{\lambda_{3j}}{\lambda_{1j}}, \tag{11}$$

plus the transversality conditions of M_j and K_j:

$$\lim_{t\to\infty} \lambda_{1j} M_j e^{-\beta t} = \lim_{t\to\infty} \lambda_{2j} K_j e^{-\beta t} = 0.$$

Equations (9)–(11) imply the no-arbitrage condition between physical capital and real money balances

$$R \equiv \frac{\lambda_{3j}}{\lambda_{1j}} = \pi + r - \delta. \tag{12}$$

Equation (12) is the Fisher equation: the nominal interest rate R equals the real interest rate r plus the inflation rate π. Dividing (6) by (7) yields the ratio of the credit to cash goods:

$$\frac{C_{1j}}{C_{2j}} = \left[\left(\frac{1-a}{a}\right)\left(\frac{\lambda_{1j}}{\lambda_{1j} + \lambda_{3j}}\right)\right]^{\varepsilon} = \left\{\left(\frac{1-a}{a}\right)\left[\frac{1}{1+R}\right]\right\}^{\varepsilon}. \tag{13}$$

It is clear from (13) that credit goods provide a hedge against inflation in the sense that if inflation is higher (hence a higher nominal interest rate R), households will demand more credit goods but purchase fewer cash goods, resulting in a higher ratio of $\frac{C_{1j}}{C_{2j}}$ (given that $\varepsilon \in (-\infty, 0)$). We can then use (7) and (8) to obtain the optimal tradeoff between consumption and leisure:

$$\frac{\left(1-l_j\right)}{\eta C_j}=\frac{\left[a^{-\varepsilon}+(1-a)^{-\varepsilon}\,(1+R)^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}}{w}. \tag{14}$$

This implies that a higher wage raises the cost of leisure, while a higher nominal interest rate raises the cost of holding money and, hence, increases the cost of purchasing cash goods.

2.2. Firms

The specification of production follows Benhabib and Farmer (1994). The homogeneous final good Y is produced with a range of differentiated intermediate inputs y_i and $i \in [0, 1]$ and technology:

$$Y=\left(\int_0^1 y_i^{\rho}\,di\right)^{\frac{1}{\rho}}, \quad \rho \in (0, 1]. \tag{15}$$

The representative final-good firm solves the profit maximization problem,

$$\max_{y_i} \quad \Pi_f = P\left(\int_0^1 y_i^{\rho}\,di\right)^{\frac{1}{\rho}} - \int_0^1 p_i y_i di, \tag{16}$$

where p_i and P are the prices of the ith intermediate good (y_i) and the final good (Y). The solution implies that the inverse demand function for the ith intermediate good is

$$p_i = P\left(\frac{y_i}{Y}\right)^{\rho-1}, \tag{17}$$

which is characterized by constant price elasticity $1/(1-\rho)$. When $\rho \to 1$, intermediate goods are perfect substitutes in the production of the final good and, hence, the intermediate goods market is perfectly competitive. If $0<\rho<1$, the intermediate goods firms face a downward-sloping demand curve, and $1/\rho$ measures the market power of these firms. Because the final good market is competitive, the equilibrium price is determined by the zero-profit condition $P=(\int_0^1 p_i^{\frac{\rho}{\rho-1}}\,di)^{\frac{\rho-1}{\rho}}$.

In the intermediate goods market, producer i hires capital K_i and labor l_i, with the output sold to final good producers at the profit-maximizing price. The production technology for intermediate good i is specified as:

$$y_i = AK_i^{\alpha}\,(l_i K)^{1-\alpha}, \quad 0<\alpha<1, \tag{18}$$

where A is a technology parameter, and the economy-wide stock of capital K is a positive production externality that captures knowledge spillovers. This production externality generates perpetual growth. Given the demand function (17) and the production function (18), the optimization problem of intermediate-good producer i is

$$\max_{l_i,k_i} \quad \Pi_i = \left(\frac{p_i}{P}\right) y_i - wl_i - rK_i, \tag{19}$$

$$s.t. \quad p_i = P\left(\frac{y_i}{Y}\right)^{\rho-1} \quad \text{and} \quad y_i = AK_i^{\alpha}\,(l_iK)^{1-\alpha}\,.$$

The first-order conditions for l_i and k_i are

$$w = (1-\alpha)\rho\left(\frac{p_i}{P}\right)\left(\frac{y_i}{l_i}\right) \quad \text{and} \quad r = \alpha\rho\left(\frac{p_i}{P}\right)\left(\frac{y_i}{K_i}\right). \tag{20}$$

Substituting (20) into (19) yields the profit of the ith intermediate-goods producer:

$$\Pi_i = (1-\rho)\left(\frac{p_i}{P}\right) y_i. \tag{21}$$

We confine the analysis to a symmetric equilibrium, where $l_i = l$, $K_i = K$, and $p_i = p$ for all $i \in [0, 1]$. Accordingly, from (17) and (18), we have $p = P$, $y_i = y = AKl^{1-\alpha}$, and $\Pi_i = \Pi = (1-\rho)y\ \forall i$. Thus, in the symmetric equilibrium, the wage and rental rates can be rewritten as

$$w = (1-\alpha)\rho\frac{Y}{l} \quad \text{and} \quad r = \alpha\rho\frac{Y}{K}, \tag{22}$$

where $Y = \int_0^1 y di = y = AKl^{1-\alpha}$. Equation (22) implies that, under imperfect competition ($0 < \rho < 1$), the factor prices w and r are lower than the marginal products $(1-\alpha)Y/l$ and $\alpha Y/K$.

2.3. Government

The government determines the nominal money growth rate, $\mu \equiv \dot{M}'/M'$. Real money balances then grow by

$$\frac{\dot{M}}{M} = \mu - \pi. \tag{23}$$

To balance its budget, the government levies a seigniorage tax μM to finance the lump-sum transfers τ, that is,

$$\tau = \mu M. \tag{24}$$

3. BALANCED-GROWTH-PATH EQUILIBRIUM AND INEQUALITY

A *dynamic competitive equilibrium* (DCE) is a set of prices $\{w, r, P, \pi\}$, resource allocations $\{C_{1j}, C_{2j}, I_j, \ell_j, K_j, M_j\}$, and policy variables $\{\tau, \mu\}$, such that

(i) household j maximizes lifetime utility (2), subject to constraints (3)–(5), that is, the optimizing conditions (6)–(11) hold;

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(ii) the final- and intermediate-good firms maximize profits, that is, the optimizing conditions (17) and (20) hold; and

(iii) the real money balances evolution equation and the government budget constraint hold, that is, (23) and (24) are met.

In the DCE, the aggregate-consistency conditions $\int_0^1 l_j d_j = l = \int_0^1 l_i d_i$, $\int_0^1 K_j d_j = K = \int_0^1 K_i d_i$, and $M = \int_0^1 M_j d_j$ must be satisfied.

3.1. Balanced-Growth-Path Equilibrium

To simplify notation, define $Z = \left[a^{-\varepsilon} + (1-a)^{-\varepsilon}(1+R)^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$, which is a function of the nominal interest rate, that is, $Z = Z(R)$. With $\frac{\dot{\lambda}_{1j}}{\lambda_{1j}} = \frac{\dot{\lambda}_{2j}}{\lambda_{2j}} = \beta - r + \delta$ from (9) and (10), taking the time derivative of (8) and (14) yields

$$\left(1 - \frac{1}{\phi}\right)\frac{\dot{C}_j}{C_j} + \left[\eta\left(1 - \frac{1}{\phi}\right) - 1\right]\frac{(\dot{1 - l_j})}{1 - l_j} = \beta - r + \delta + \frac{\dot{w}}{w}, \text{ and} \quad (25)$$

$$\frac{(\dot{1 - l_j})}{1 - l_j} - \frac{\dot{C}_j}{C_j} = \frac{\dot{Z}}{Z} - \frac{\dot{w}}{w}, \quad (26)$$

where $\frac{\dot{Z}}{Z} = (\frac{1+R}{1-a})^{\varepsilon} Z^{-(1+\varepsilon)} \cdot \dot{R}$. It follows from (25) and (26) that all households choose the same growth rates for consumption and leisure, independent of their capital and money endowments, K_{0j} and M_{0j}, since they face the same time preference rate β, real wage w, real interest rate r, and nominal interest rate R. That is, the economy-wide (average) consumption C $(= \sum_j C_j)$ and leisure $1 - l$ $(= \sum_j (1 - l_j))$ grow at their common rates, independent of asset endowments:

$$\frac{\dot{C}_j}{C_j} = \frac{\dot{C}}{C} \text{ and } \frac{(\dot{1 - l_j})}{1 - l_j} = \frac{(\dot{1 - l})}{1 - l}, \ \forall j. \quad (27)$$

Let the average money holding, credit-good consumption, and cash-good consumption be $M = \sum_j M_j$, $C_1 = \sum_j C_{1j}$, and $C_2 = \sum_j C_{2j}$. We can see from (6) and (7) that for household j, the credit-good consumption and cash-good consumption are proportional to the aggregate consumption, that is, $\frac{C_{1j}}{C_j} = (aZ(R))^{-\varepsilon}$ and $\frac{C_{2j}}{C_j} = [\frac{(1+R)}{(1-a)Z(R)}]^{\varepsilon}$. Thus, (27) implies that

$$\frac{\dot{C}_{1j}}{C_{1j}} = \frac{\dot{C}_1}{C_1} \text{ and } \frac{\dot{C}_{2j}}{C_{2j}} = \frac{\dot{C}_2}{C_2}, \ \forall j. \quad (28)$$

Furthermore, the CIA constraint (5) and the real money balances evolution equation (23) imply that

$$\frac{\dot{C}_{2j}}{C_{2j}} = \frac{\dot{C}_2}{C_2} = \frac{\dot{M}_j}{M_j} = \frac{\dot{M}}{M} = \mu - \pi, \ \forall j. \quad (29)$$

That is, under the CIA constraint, all households choose the same growth rate for money holdings to purchasing cash goods. To derive the aggregate variables, given (22), summing (14) over households yields the consumption-capital ratio:

$$\frac{C}{K} = \frac{(1-l)(1-\alpha)\rho A l^{-\alpha}}{\eta Z(R)}. \tag{30}$$

Similarly, given (22), (23), (24) and $\Pi = (1-\rho)Y$, summing (3) and (4) over households yields the aggregate resource constraint:

$$\dot{K} = Y - C_1 - C_2 - \delta K \Longrightarrow \frac{\dot{K}}{K} = A l^{1-\alpha} - \frac{C}{K}\left(\frac{C_1}{C} + \frac{C_2}{C}\right) - \delta. \tag{31}$$

Recall that $R = r + \pi - \delta = \alpha\rho A l^{1-\alpha} + \pi - \delta$, $\frac{C_2}{C} = [\frac{(1+R)}{(1-a)Z(R)}]^{\varepsilon}$, and $w = (1-\alpha)\rho A K l^{-\alpha}$. Differentiating these three relations with respect to time yields[11]

$$\dot{R} = \alpha(1-\alpha)\rho A l^{-\alpha}\dot{l} + \dot{\pi}, \text{ and}$$

$$\frac{\dot{C}}{C} = \mu - \pi - \Psi_1 \dot{R}, \text{ and } \frac{\dot{w}}{w} = \Psi_2 - \alpha\frac{\dot{l}}{l},$$

where $\Psi_1 = \frac{\varepsilon}{1+R}\frac{a^{-\varepsilon}}{a^{-\varepsilon} + (1-a)^{-\varepsilon}(1+R)^{1+\varepsilon}} < 0$ and $\Psi_2 = A l^{1-\alpha} - \frac{(1-l)(1-\alpha)\rho A l^{-\alpha}(aZ(R))^{-\varepsilon}}{\eta Z(R)}$ $\{1 + [(1+R)\frac{a}{1-a}]^{\varepsilon}\} - \delta > 0$. Substituting these relations into (25) and (26) and rearranging yields

$$\left(1-\frac{1}{\phi}\right)\left\{\mu - \pi - \Psi_1\left[\dot{\pi} + (1-\alpha)r\frac{\dot{l}}{l}\right]\right\} + \left[1 - \eta\left(1 - \frac{1}{\phi}\right)\right]\frac{\dot{l}}{1-l}$$
$$= \beta - \rho\alpha A l^{1-\alpha} + \delta + \Psi_2 - \alpha\frac{\dot{l}}{l}, \text{ and} \tag{32}$$

$$\frac{\dot{l}}{1-l} + \mu - \pi + \left[\left(\frac{1+R}{1-a}\right)^{\varepsilon} Z(R)^{-(1+\varepsilon)} - \Psi_1\right]\left[\dot{\pi} + \alpha(1-\alpha)\rho A l^{1-\alpha}\frac{\dot{l}}{l}\right]$$
$$= \Psi_2 - \alpha\frac{\dot{l}}{l}. \tag{33}$$

Equations (32) and (33) form a 2×2 dynamic system in l and π. Let the symbol "~" denote the stationary values of variables in the steady state. Thus, the steady-state values for labor $\tilde{l}$ and inflation $\tilde{\pi}$ are determined by (32) and (33), with $\dot{l} = \dot{\pi} = 0$. Once the steady-state values $\tilde{l}$ and $\tilde{\pi}$ are determined, (30) and (31) show that the aggregate consumption, capital, and total output (Gross Domestic Product) share the same growth rate γ (i.e., the balanced-growth rate), which is determined by (29), that is,

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{C}_1}{C_1} = \frac{\dot{C}_2}{C_2} = \frac{\dot{M}}{M} = \mu - \tilde{\pi} = \gamma. \tag{34}$$

Accordingly, we define a non-degenerate balanced-growth-path (BGP) equilibrium to be a tuple of paths such that the perpetually growing variables $\{C(t), C_1(t), C_2(t), K(t), M(t), w(t)\}_{t=0}^{\infty}$ grow at the constant rate γ, while the other variables $\{l(t), r(t), R(t), \pi(t)\}_{t=0}^{\infty}$ are constants. Note that the aggregate consumption $C_j(t)$, credit-good consumption $C_{1j}(t)$, cash-good consumption $C_{2j}(t)$, and money holdings $M_j(t)$ for all households also grow at the common rate, independent of their asset endowments.

Given that $\varepsilon \in (-\infty, 0)$, we impose

Condition E. $\varepsilon > \varepsilon^* \equiv -(\frac{1+R}{R})\{1 + [a(\frac{1+R}{1-a})]^{\varepsilon} + \frac{\eta}{(1-l)\rho(1-\alpha)Al^{-\alpha}z^{-2(1+\varepsilon)}a^{-\varepsilon}(\frac{1+R}{1-a})^{\varepsilon}}\}$.

This sufficient (but not necessary) condition guarantees the existence and uniqueness of a non-degenerate BGP equilibrium. We have ensured the existence of the critical value ε^* in the Appendix. This condition then establishes Proposition 1:

PROPOSITION 1. *(Local existence and uniqueness of the BGP equilibrium) Under Condition E, there exists a unique BGP equilibrium of money, which is locally determinate.*

Proof. See Appendix A. ∎

Given that l and π are both jump variables, a local determinacy requires that there are two roots with real positive parts in the dynamic system. Accordingly, Proposition 1 indicates that if the elasticity of substitution between credit- and cash-good consumption satisfies Condition E, the dynamic system has two positive eigenvalues and, as a result, the steady-state BGP equilibrium is unique and determinate (see the Appendix). This implies that there is no transitional dynamics in the model; the economy when shocked will jump immediately to the BGP equilibrium.

3.2. Inequality

We now investigate the distribution of households, focusing on income and consumption.

Given that K and l are the economy-wide (average) capital and labor, we use (3)–(5) and (23) to obtain

$$\frac{\dot{K}_j}{K} - \frac{\dot{K}}{K} = \frac{w}{K}(l_j - l) + \left(r - \delta + \frac{s}{K}\right)(k_j - 1) - (1+\mu)\frac{M}{K}(m_j - 1) - \frac{C_1}{K}\left(\frac{C_{1j}}{C_1} - 1\right).$$

Given that $\frac{\dot{K}}{K} = \mu - \pi$ (from (34)), $\frac{M}{K} = \frac{C_2}{K} = \frac{C}{K}\frac{C_2}{C} = \frac{(1-l)(1-\alpha)\rho Al^{-\alpha}}{\eta Z(R)}[\frac{(1+R)}{(1-a)Z(R)}]^{\varepsilon}$ (from (30) and the proportion of cash-good consumption), $\frac{s}{K} = (1-\rho)Al^{1-\alpha}$ (from (21) and (24)), $w = \rho(1-\alpha)Al^{-\alpha}K$, $r = \rho\alpha Al^{1-\alpha}$ (from (22)), $(\frac{C_{1j}}{C_1} - 1) = (\frac{C_{2j}}{C_2} - 1) = (\frac{C_j}{C} - 1) = (m_j - 1)$ (from (5) and the proportion of credit-good consumption), and $l_j - l = -(1-l)(m_j - 1)$ (from (14) and (22)), under the BGP equilibrium, we can further obtain

$$\dot{k}_j = \Omega_k(k_j - 1) - \Omega_m(m_j - 1), \tag{35}$$

where[12]

$$\Omega_k = \pi + r - \delta - \mu + \frac{s}{K} = (1 - \rho + \rho\alpha)Al^{1-\alpha} - (\mu - \pi) - \delta > 0, \text{ and} \quad (36)$$

$$\Omega_m = \Omega_k + \frac{w}{K} + \mu\frac{M}{K} = \left[(1+\mu) + \left(\frac{a(1+R)}{1-a}\right)^{-\varepsilon} + \frac{\eta(1-a)^{\varepsilon}Z(R)^{(1+\varepsilon)}}{(1+R)^{\varepsilon}}\right] \frac{(1-l)(1-\alpha)\rho Al^{-\alpha}\left[\frac{(1+R)}{(1-a)Z(R)}\right]^{\varepsilon}}{\eta Z(R)} > 0.$$

Equation (35) shows that the relative capital changes over time based on the jth household's capital holdings relative to the average capital and its money holdings relative to the average money in the economy.

The relative money holdings, however, are constant over time (in the dynamic adjustment) in the BGP equilibrium, that is,

$$\frac{\dot{m}_j}{m_j} = 0, \quad (37)$$

since all households choose the same growth rate for money holdings under the CIA constraint, $\frac{\dot{M}_j}{M_j} = \frac{\dot{M}}{M} = \mu - \pi$, as shown in (29). This is characterized by a knife-edge condition in the sense that, to meet (37) for all time, m_j will jump to its long-run equilibrium value immediately after a shock hits the economy and will remain constant at this value along the transition path.[13] Moreover, the transversality condition requires a positive eigenvalue for the relative capital k_j in (35) and, accordingly, the relative capital k_j when shocked also immediately jumps to its long-run equilibrium value. Therefore, there must be an instantaneous adjustment between capital and money holdings once the initial V_{j0} is determined and remains at its long-run equilibrium value. See, for example, Mankiw (1987) and Sen and Turnovsky (1989).

The once-and-for-all adjustment between capital and money holdings, however, is not arbitrary; it will be endogenously determined in the equilibrium based on initial conditions. To the end, we assume that the endowments of household j and the economy are

$$V_{j0} = K_{j0} + M_{j0} \quad \text{and} \quad V_0 = K_0 + M_0, \quad (38)$$

where, again, the subscript "0" represents the initial values of capital and money holdings. By denoting household j's initial relative wealth as $v_{j0} \equiv \frac{V_{j0}}{V_0}$, we have

$$v_{j0} - 1 = \frac{K_0}{V_0}(k_j - 1) + \frac{M_0}{V_0}(m_j - 1), \quad (39)$$

recalling that, by definition, $M_0 = M'_0/P_0$. Under the knife-edge situation, a change in the money growth rate (inflation) leads the initial price P_0 to jump to its long-run equilibrium value, which pins down the initial wealth V_{j0} and share v_{j0}. Once V_{j0} and v_{j0} are determined by the initial conditions, households will engage in an upon-impact adjustment between K_{j0} and M'_{j0} to a shock and, accordingly,

k_j and m_j immediately jump to their long-run equilibrium values, that is, $k_{j0} = \tilde{k}_j$ and $m_{j0} = \tilde{m}_j$. See Appendix B for more details.

Thus, we can solve the steady-state relative capital holdings $\tilde{k}_j$ and money holdings $\tilde{m}_j$ by (35) with $\dot{k}_j = 0$ and (39):

$$\tilde{k}_j - 1 = \frac{\Omega_m}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}}(v_{j0} - 1) \text{ and } \tilde{m}_j - 1 = \frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}}(v_{j0} - 1). \quad \mathbf{(40)}$$

Define relative consumption as $c_j = \frac{C_j}{C}$, where C is the average consumption. With the steady-state relative capital holdings $\tilde{k}_j$ and money holdings $\tilde{m}_j$, (5), (14), (22), and (40) enable us to derive the relative consumption and working hours:

$$\tilde{c}_j - 1 = \frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}}(v_{j0} - 1) \text{ and } \tilde{l}_j - \tilde{l} = \frac{-(1-\tilde{l})\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}}(v_{j0} - 1). \quad \mathbf{(41)}$$

Moreover, define relative income as $\hat{y}_j = \hat{Y}_j/\hat{Y}$, where household j's total income (before the government's intervention) is $\hat{Y}_j = wl_j + rK_j + s_j$ and average income is $\hat{Y} = wl + rK + s$.[14] From (21), (22), (40), and (41), we thus have the steady-state relative income:

$$\begin{aligned} \tilde{y}_j - 1 &= \frac{\rho(1-\alpha)}{\tilde{l}}(\tilde{l}_j - \tilde{l}) + (1 - \rho + \alpha\rho)(\tilde{k}_j - 1) \qquad \mathbf{(42)} \\ &= \left[(1-\rho+\alpha\rho)\frac{\Omega_m}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} - \frac{\rho(1-\alpha)(1-\tilde{l})}{\tilde{l}} \frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}}\right](v_{j0}-1). \end{aligned}$$

Equations (40)–(42) lead to Proposition 2:

PROPOSITION 2 (Wealth, consumption, and propensity to work). *In the BGP equilibrium,*

(i) households with more total assets ($v_{j0} > 1$) hold more capital ($\tilde{k}_j > 1$) and more money balances ($\tilde{m}_j > 1$), and have higher capital–money ratios ($\frac{\tilde{k}_j}{\tilde{m}_j} > 1$), compared to the average.

(ii) households with more total assets have a stronger propensity to consume ($\tilde{c}_j > 1$) and have a weaker propensity to work ($\tilde{l}_j < l$), compared to the average, implying that asset-rich households receive more income from capital than income from labor.

Proposition 2(i) indicates that wealthy households (that own more assets) hold more capital and money, compared to the average. More importantly, the capital–money ratio is larger (less) than one $\frac{\tilde{k}_j}{\tilde{m}_j} > 1$ ($\frac{\tilde{k}_j}{\tilde{m}_j} < 1$) for asset-rich households, $v_{j0} > 1$ (asset-poor households, $v_{j0} < 1$). Empirically, Wolff (2017) shows that during 1983–2016 the top 20% of American households (as ranked by wealth) on average held 63% of their wealth in the form of investment assets (real estate, businesses, corporate stock, and financial securities) and only 11% in the form of

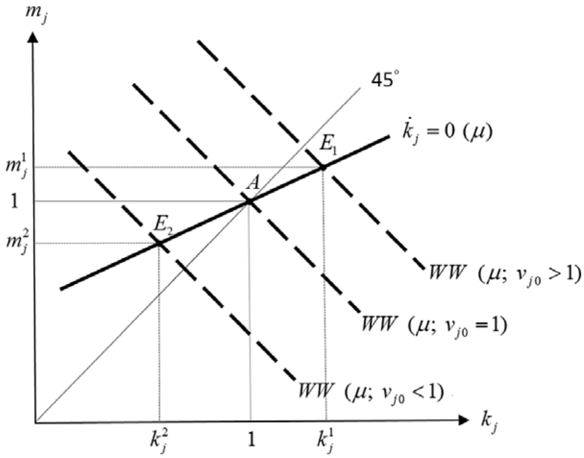

FIGURE 1. The asset allocation.

liquid assets. In contrast, for the remaining 80%, investment assets accounted for only 13% of their total wealth, while liquid assets accounted for 15%. Moreover, it is clear from (41) that asset-rich households are inclined to consume more ($\tilde{c}_j > 1$) but work less ($\tilde{l}_j < l$) than the average. These results are also supported by the empirical studies of Holtz-Eakin et al. (1993) and Algan et al. (2003). Recent evidence shows that households in the top 1% of the wealth distribution received 66% of their income from assets (such as capital and business) and 30% from wages. Households in the bottom 20% of the wealth distribution, however, received 2% of their income from assets and 79% from wages (see Nakajima (2015)).

The distributional characteristics of assets in Proposition 2 is illustrated in Figure 1. Under the BGP equilibrium, the asset distribution $(\tilde{k}_j, \tilde{m}_j)$ satisfies the differential equation of relative capital (35) with $\dot{k}_j = 0$ and the initial condition (39), given that (37) is a knife-edge condition. In the $(\tilde{k}_j, \tilde{m}_j)$ space, the slope of the $\dot{k}_j = 0$ locus is positive but less than one; specifically, $1 > \frac{\partial m_j}{\partial k_j}|_{\dot{k}_j=0} = \frac{\Omega_k}{\Omega_m} = \frac{\pi+r-\delta-\mu+\frac{s}{K}}{\Omega_k+\frac{w}{K}+\mu\frac{M}{K}} > 0$. Because the mean of relative capital and of money holding are one, the $\dot{k}_j = 0$ locus, as shown in Figure 1, must intersect the 45^o line at Point A where $k_j = 1$ and $m_j = 1$. Moreover, the initial condition (39), defined as the *Relative Wealth* (WW) locus, is downward sloping, that is, $\frac{\partial m_j}{\partial k_j}|_{WW} = -\frac{K_0}{M_0} < 0$. The WW locus can be viewed as an iso-wealth line which measures household j's wealth level. Any combination of capital k_j and money m_j on a specific WW locus refers to the same level of wealth, while the WW locus moves to the right to represent a higher level of wealth. It follows from Figure 1 that, for the households with more assets, say Point E_1 associated with $WW(\mu_0; v_{j0} > 1)$, they will hold more

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capital ($\tilde{k}_j > 1$) and more money balances ($\tilde{m}_j > 1$), and have a higher capital–money ratio ($\frac{\tilde{k}_j}{\tilde{m}_j} > 1$) than the average. In contrast, for the households with less assets, say Point E_2 associated with $WW(\mu_0; v_{j0} < 1)$, they will hold less capital ($\tilde{k}_j < 1$) and less money balances ($\tilde{m}_j < 1$), and have a lower capital–money ratio ($\frac{\tilde{k}_j}{\tilde{m}_j} < 1$) than the average. A change in inflation (caused by monetary policy μ), as we will see later, will influence the relative capital $\tilde{k}_j$ and money $\tilde{m}_j$ (and, hence, the relative wealth) which further govern income and consumption inequality.

Let the standard deviations of capital and money be σ_k and σ_m. From (40), we derive the coefficients of capital variation and money variation as follows:

$$\sigma_k = \frac{\Omega_m}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} \sigma_v, \text{ and} \tag{43}$$

$$\sigma_m = \frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} \sigma_v. \tag{44}$$

Moreover, from (41), we have the coefficients of consumption variation, σ_c, and labor variation, σ_l:

$$\sigma_c = \left(\frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} \right) \sigma_v, \text{ and} \tag{45}$$

$$\sigma_l = \left(1 - \tilde{l}\right) \sigma_c. \tag{46}$$

It is intuitive from (46) that, since the time endowment is allocatable to both work and leisure, the standard deviations of labor and leisure are the same, that is, $\sigma_{1-l} = \sigma_l = (1 - l)\, \sigma_c$. Finally, we use (42) to obtain the coefficient of income variation, σ_y:

$$\begin{aligned} \sigma_y &= (1 - \rho + \alpha\rho)\sigma_k - \frac{\rho(1-\alpha)}{\tilde{l}} \sigma_l \qquad (47) \\ &= \left[(1 - \rho + \alpha\rho) \frac{\Omega_m}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} - \frac{\rho(1-\alpha)(1-\tilde{l})}{\tilde{l}} \frac{\Omega_k}{\Omega_k \frac{M_0}{V_0} + \Omega_m \frac{K_0}{V_0}} \right] \sigma_v. \end{aligned}$$

Equation (47) measures income inequality, which depends on disparities in capital, labor, and total assets.

4. NUMERICAL ANALYSIS

In Section 3, we characterized the steady-state BGP equilibrium and the distribution of income and consumption. It is difficult (if not impossible), however, to continue the comparative statics analytically. In this section, therefore, we numerically conduct the comparative statics exercises based on a reasonable parametrization of the economy developed above.

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4.1. Calibration and Parameterization

In line with Hansen and Wright (1992), the time preference rate is set at $\beta = 0.01$ per quarter, which implies 4 percent annually. The capital income share of output is set at $\alpha = 0.4$, within the reasonable range of 0.25–0.43 estimated by Christiano (1988). The quarterly depreciation rate for capital is set at $\delta = 0.025$, consistent with commonly used values in the literature (e.g., Smets and Wouters (2003)). To be consistent with the average estimates of Dutta and Weale (2001), the degree of substitutability between credit and cash goods is set at $\varepsilon = -30$, and the credit-good share of aggregate consumption is set at $a = 0.52$. For simplicity, the technology scale parameter A is normalized to one.

Four parameters (μ, ρ, ϕ, η) are chosen to meet four empirical targets (π, γ, l, r). First, we rewrite (22) and (34) as:

$$\tilde{r} = \alpha\rho A \tilde{l}^{1-\alpha}, \tag{48}$$

$$\mu - \tilde{\pi} = \tilde{\gamma}. \tag{49}$$

Next, given that $\frac{C_1}{C} = (aZ(R))^{-\varepsilon}$, $\frac{C_2}{C} = [\frac{(1+R)}{(1-a)Z(R)}]^{\varepsilon}$, and $R = r + \pi - \delta$ in the BGP equilibrium, manipulating (30) and (31) yields

$$\tilde{\gamma} = A\tilde{l}^{1-\alpha} - \frac{(1-\tilde{l})(1-\alpha)\rho A\tilde{l}^{-\alpha}}{\eta Z}\left\{(aZ)^{-\varepsilon} + \left[\frac{(1+\tilde{r}+\tilde{\pi}-\delta)}{(1-a)Z}\right]^{\varepsilon}\right\} - \delta, \tag{50}$$

where $Z = \left[a^{-\varepsilon} + (1-a)^{-\varepsilon}(1+\tilde{r}+\tilde{\pi}-\delta)^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$. From (32), in the steady state, we have

$$\rho\alpha A\tilde{l}^{1-\alpha} - \frac{1}{\phi}(\mu - \tilde{\pi}) = \beta + \delta. \tag{51}$$

Based on (48)–(51), we can calibrate $\mu = 7.4\%$, $\rho = 0.26$, $\phi = 0.96$, and $\eta = 0.32$, so that the steady-state inflation rate is $\tilde{\pi} = 5\%$, the balanced-growth rate is $\tilde{\gamma} = 2.4\%$, the steady-state hours worked is $\tilde{l} = 0.35$, and the steady-state return rate of capital is $\tilde{r} = 5.5\%$. The inflation rate and growth rate are consistent with the sample means of OECD countries between 1980Q1 and 2018Q4 from the World Bank National Accounts and the OECD National Accounts. The steady-state working hours imply that households spend approximately one-third of their discretionary time in market work, which is consistent with time-use studies, such as Juster and Stafford (1991). The rental rate of capital is also within the plausible range of the real returns on capital: in the USA, the rental rate of capital varies around from 4 to 7% between 1980 and 2012 (Caballero et al. (2017)).

The asset allocation between money and capital is calculated by using the Distributional Financial Accounts (DFA) of the USA. Based on the money of zero maturity (MZM), which is used on a daily basis to buy goods and services, money holdings are defined as the sum of currency and coins, checking accounts, savings accounts, and money market accounts.[15] Capital holdings are defined as non-financial assets, assets with physical values. Accordingly, wealth

TABLE 1. Benchmark values of parameters

Value	Steady-state target
$\beta = 0.01$	Literature (Hansen and Wright (1992))
$\alpha = 0.4$	Literature (Christiano (1988))
$\delta = 0.025$	Literature (Smets and Wouters (2003))
$a = 0.52$	Literature (Dutta and Weale (2001))
$\varepsilon = -30$	Literature (Dutta and Weale (2001))
$A = 1$	Normalization
$M'_0 = 6,512,436$	Calibrated (target to $\frac{M_0}{V_0}$)
$v_{j0} \in [0.24, 1.76]$	Calibrated (target to unit mean and σ_v)
$\mu = 7.4\%$	Calibrated (target to π, γ, l, and r)
$\phi = 0.96$	Calibrated (target to π, γ, l, and r)
$\rho = 0.26$	Calibrated (target to π, γ, l, and r)
$\eta = 0.32$	Calibrated (target to π, γ, l, and r)

is the sum of the defined money and capital holdings. From (38), we thus can calibrate $M'_0 = 6,512,436$, so that the money holding share $\frac{M_0}{V_0} = 0.24$ (and hence, $\frac{K_0}{V_0} = 0.76$), consistent with the average share over the period from 1989 to 2018 in the DFA.[16] To shed light on the initial adjustment between capital and money holdings, we assume that the relative wealth v_{j0} is distributed uniformly in the interval [0.24, 1.76] with a unit mean for a sample of 10,000 households.[17] This specification implies a standard deviation of relative wealth of $\tilde{\sigma}_v = 0.44$, which is the average standard deviation of relative wealth in the USA since 1989 (calculated from the DFA). With $\tilde{\sigma}_v = 0.44$, we obtain from (47) the initial standard deviation of relative income, $\tilde{\sigma}_y = 0.29$.[18]

Finally, by using (43), (44), (45), and (46), the initial standard deviation of capital is $\tilde{\sigma}_k = 0.442$, the initial standard deviation of money is $\tilde{\sigma}_m = 0.09$, the initial standard deviation of consumption is $\tilde{\sigma}_c = 0.09$, and the initial standard deviation of labor (leisure) $\tilde{\sigma}_l = \tilde{\sigma}_{1-l} = 0.06$. Consistent with common observations, the standard deviation of consumption is less than that of income ($\tilde{\sigma}_c < \tilde{\sigma}_y$) and the standard deviation of income is less than that of wealth ($\tilde{\sigma}_y < \tilde{\sigma}_v$). Table 1 summarizes the benchmark parameter values.

4.2. Effects of Inflation (Money Growth)

The macroeconomic and distributional effects of inflation caused by an expansionary money growth are summarized in Result 1:

Result 1. (Steady-state effects) *An expansion in money growth μ raises the inflation rate $\tilde{\pi}$. Accordingly,*

(i) (Consumption and asset reallocation) *the average cash–credit goods ratio $\frac{\tilde{C}_2}{\tilde{C}_1}$ and money–capital ratio $\frac{\tilde{M}}{\tilde{K}}$ decline;*

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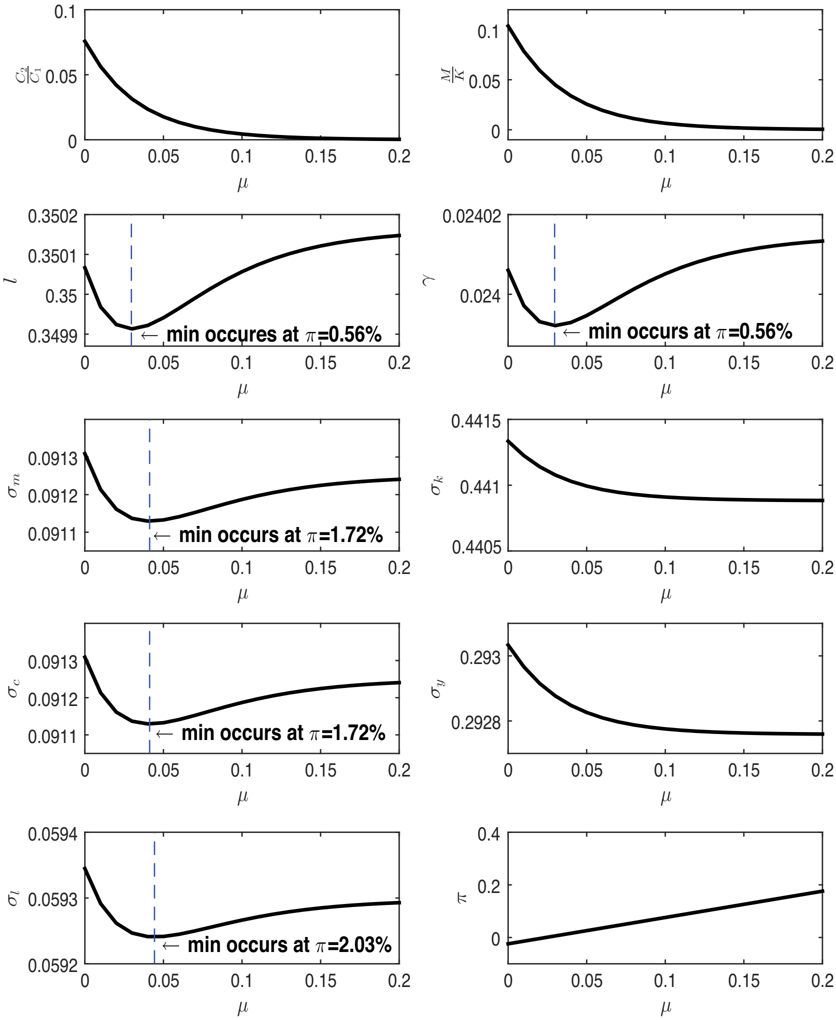

FIGURE 2. Effects of money growth (inflation): Benchmark parameterization. *Y*-axis is a value axis. *X*-axis is the money growth rate.

(ii) (Labor and growth) *the aggregate labor hours* $\tilde{l}$ *and the balanced-growth rate* $\tilde{\gamma}$ *have a U-shaped response to inflation; and*

(iii) (Inequality) *inequality in income* $\tilde{\sigma}_y$ *and capital* $\tilde{\sigma}_k$ *decrease, while inequality in consumption* $\tilde{\sigma}_c$*, money* $\tilde{\sigma}_m$*, and labor hours* $\tilde{\sigma}_l$ *(leisure* $\tilde{\sigma}_{1-l}$*) have a U-shaped relationship with inflation.*

We first focus on the macroeconomic effects and then turn to the distributional effects. Figure 2 shows that an increase in the money growth rate μ raises the inflation rate $\tilde{\pi}$, which increases the cost of holding money. Inflation thus acts like a tax, decreasing real money balances. To hedge against the inflation tax,

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households adjust their consumption compositions between credit and cash goods and their asset portfolios between money and capital. Households engage in "consumption reallocation" by decreasing the cash-good consumption (because cash good purchases are subject to the CIA constraint) and increasing the credit-good consumption (because credit goods hedge against inflation). Thus, the *economy-wide* cash–credit goods ratio $\frac{\tilde{C}_2}{\tilde{C}_1}$ declines, as shown in Figure 2. Households also engage in "asset reallocation" by holding less money (because nominal assets depreciate with inflation) but more capital (because real assets preserve purchasing power). Thus, the economy-wide money–capital ratio $\frac{\tilde{M}}{\tilde{K}}$ also declines.

In the presence of a high elasticity of substitution between credit and cash goods ($\varepsilon = -30$ in our benchmark parameterization), inflation could either increase or decrease the *economy-wide* consumption $\tilde{C}$, depending on the inflation *status quo*. If the inflation *status quo* is relatively high (low), and money depreciates significantly (insignificantly), the consumption and asset reallocations are more (less) pronounced. When households tend to adjust their consumption compositions more intensely, a strong substitutability between credit and cash goods motivates them to increase overall consumption by substantially increasing the credit-good consumption without significantly reducing the cash-good consumption. Meanwhile, a more pronounced asset reallocation is associated with more capital holding. Thus, *economy-wide* consumption increases with inflation under a relatively high inflation *status quo*. By contrast, under a relatively low inflation *status quo* , the economy-wide consumption decreases with inflation as the consumption and asset reallocations are not significant.

An increase (a decrease) in economy-wide consumption implies that, on average, households with more (less) capital holding substitute consumption (leisure) for leisure (consumption), increasing (decreasing) the labor supply. Therefore, Figure 2 shows that, in the BGP equilibrium, *aggregate* labor hours $\tilde{l}$ exhibit a U-shaped response to inflation. As aggregate labor hours $\tilde{l}$ increase (decrease), the marginal product of capital rises (falls), giving rise to a positive (negative) growth effect. As a result, the balanced-growth rate $\tilde{\gamma}$ also has a U-shaped relationship with inflation.

The distributional effects are more complicated, since wealthy ($v_{j0} > 1$) and poor ($v_{j0} < 1$) households have quite different consumption and asset reallocations in response to inflation. Wealthy households differ in their ability to hedge against inflation from poor households, because (i) wealthy households hold more assets in both capital ($\tilde{k}_j > 1$) and money ($\tilde{m}_j > 1$) and have higher capital–money ratios ($\frac{k_j}{m_j} > 1$) and (ii) wealthy households are inclined to consume more, regardless of cash goods or credit goods, but work less and rely more on capital than on labor income. Due to this asymmetry between wealthy and poor households, Figure 2 shows that income inequality has an unambiguously negative relationship with inflation, but consumption inequality has a U-shaped relationship with inflation.

Two effects, the *wealth effect* and the *relative factor price effect*, govern households' consumption and asset reallocations. A graphical analysis is helpful to

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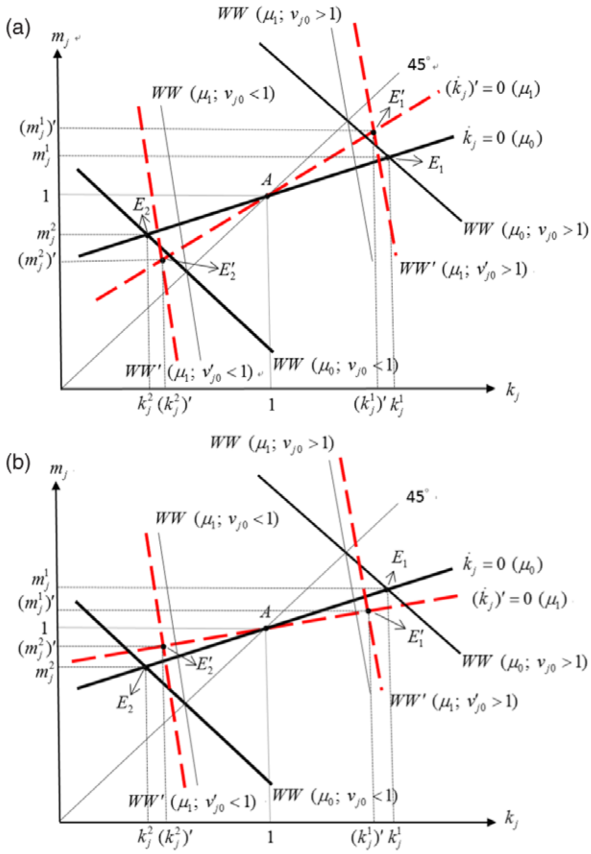

FIGURE 3. Asset effect of inflation: (a) High-status quo levels of inflation (b) Low status quo levels of inflation.

illustrate our results. In Figure 3, Point A represents the average household's asset status ($v_{j0} = k_j = m_j = 1$), while Points E_1 and E_2 represent wealthy ($v_{j0} > 1$) and poor ($v_{j0} < 1$) households' asset status. Figure 3 describes the asset effect when the inflation *status quo* is relatively high, while Figure 4 describes the asset effect when the inflation *status quo* is relatively low.

First, the *wealth effect* reflects changes in the iso-wealth WW locus (which measures household j's relative wealth level). As shown in Figures 3 and 4, in response to an expansionary money growth from μ_0 to μ_1, the iso-wealth WW locus becomes steeper, changing from $WW(\mu_0; v_{j0} > 1)$ to $WW(\mu_1; v_{j0} > 1)$ for wealthy households and from $WW(\mu_0; v_{j0} < 1)$ to $WW(\mu_1; v_{j0} < 1)$ for poor households.[19] A steeper WW locus implies that an increase in inflation harms wealthy (poor) households' *nominal assets* more (less) significantly, comparable to the average, with $v_{j0} = k_j = m_j = 1$, because wealthy (poor) households hold more (less) money. Relative to the initial point E_1 (E_2), there is an unfavorable

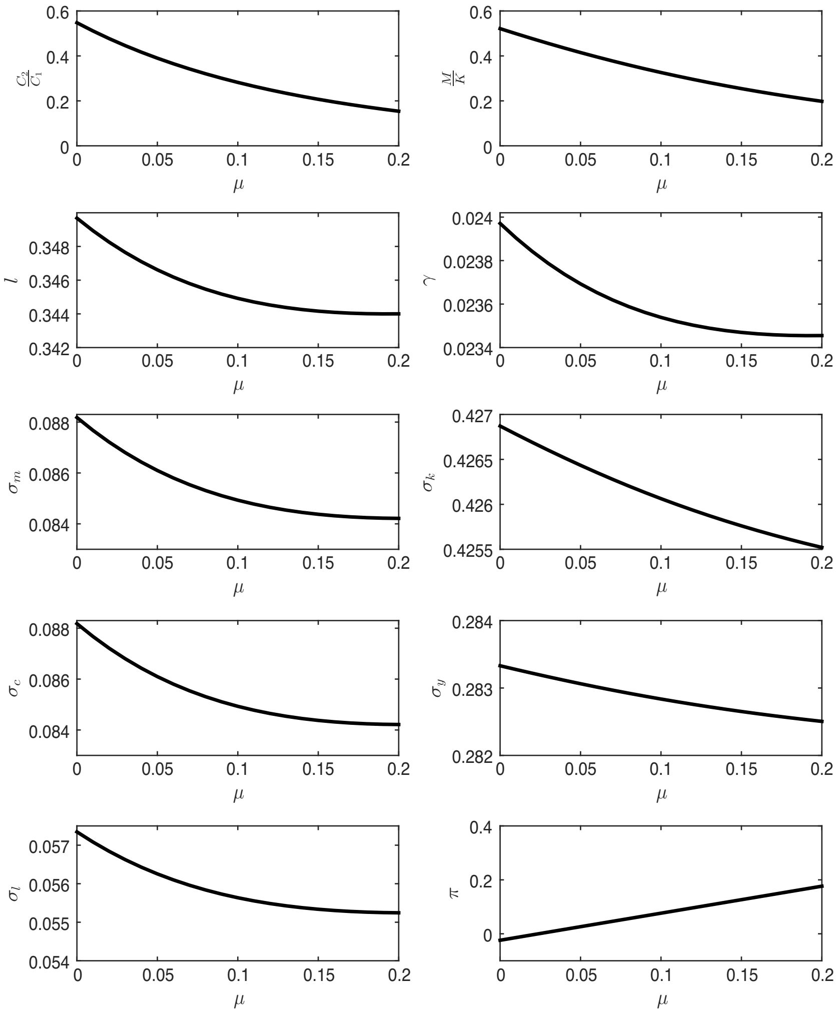

FIGURE 4. Weak substitution between credit and cash goods ($\epsilon = -7$). Y-axis is a value axis. X-axis is the money growth rate.

(favorable) impact on wealthy (poor) households' nominal assets. Nevertheless, inflation favors wealthy households' *real assets*. It follows from (39) that inflation raises the price of the final good, which changes the relative real wealth endowment v_{j0}, that is,

$$\frac{\partial v_{j0}}{\partial P_0} = \frac{K_0 M_0}{V_0(M_0^{'} + P_0 K_0)} \left(k_{j0} - m_{j0}\right). \qquad \textbf{(52)}$$

Because wealthy households, with $v_{j0} > 1$ (poor households, with $v_{j0} < 1$), have a higher proportion of assets in capital, $\frac{k_j}{m_j} > 1$ (money, $\frac{k_j}{m_j} < 1$), wealthy

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households have better inflation-hedging asset portfolios than poor households. Thus, (52) shows that inflation increases the relative real wealth for wealthy households ($k_{j0} - m_{j0} > 0$) but decreases that for poor households ($k_{j0} - m_{j0} < 0$). Accordingly, as shown in Figures 3 and 4, the $WW(\mu_1; v_{j0} > 1)$ locus shifts rightward to $WW(\mu_1; v'_{j0} > 1)$, raising the relative real wealth endowment for wealthy households, whereas the $WW(\mu_1; v_{j0} < 1)$ locus shifts leftward to $WW(\mu_1; v'_{j0} < 1)$, lowering the relative wealth endowment for poor households. A positive (negative) relative real wealth effect induces wealthy (poor) households to increase (decrease) both capital k_j and money m_j. Overall, the wealth effect (both nominal and real asset changes) can increase or decrease households' capital and money holdings, depending on how much their total assets and asset portfolios are affected by inflation.

Second, the *relative factor price* effect (the interest to wage rate ratio, $\frac{r}{w}$) reflects changes in the $\dot{k}_j = 0$ locus, which is related to the *status quo* rate of inflation. If the inflation *status quo* is relatively high (low), an increase in inflation increases (decreases) total labor hours $\tilde{l}$ (Result 1(ii)). An increase (a decrease) in labor supply, in turn, lowers (raises) the marginal product of labor, pushing down (up) the wage rate w and raises (lowers) the marginal product of capital, pushing up (down) the return to capital r. Given that the slope of the $\dot{k}_j = 0$ locus is $\frac{\partial m_j}{\partial k_j}|_{\dot{k}_j=0} = \frac{\pi + r - \delta - \mu + \frac{s}{K}}{\Omega_k + \frac{w}{K} + \mu \frac{M}{K}}$, Figure 3 (4) shows that a higher (lower) $\frac{r}{w}$ ratio, associated with a steeper (flatter) $\dot{k}_j = 0$ locus, is favorable (unfavorable) to wealthy households who rely more on capital income but unfavorable (favorable) to poor households who rely more on labor income. When the inflation *status quo* is relatively high (Figure 3), in the face of a higher $\frac{r}{w}$ ratio, their asset portfolio advantage allows wealthy households to decrease capital k_j in exchange for money m_j (in order to purchase cash goods), while their real relative assets remain unchanged. Their asset portfolio disadvantage, however, leads poor households to decrease money m_j in exchange for capital k_j in order to maintain their real relative assets. By contrast, when the inflation *status quo* is relatively high (Figure 4), the $\frac{r}{w}$ ratio decreases, rather than increases. As a result, wealthy households increase capital in exchange for money, but poor households increase money in exchange for capital.

The wealth effect and the relative factor price effect jointly affect the asset and consumption reallocations, which further determines the distributional effects of inflation. When the inflation *status quo* is relatively high, Figure 3 shows that, under our parameterization, an increase in inflation changes the equilibrium point from E_1 to E'_1 for wealthy households (with $v'_{j0} > 1$), while it changes the equilibrium point from E_2 to E'_2 for poor households (with $v''_{j0} < 1$). Thus, for the wealthy, the relative capital decreases from k_j^1 to $(k_j^1)'$ but the relative money increases from m_j^1 to $(m_j^1)'$. For the poor, in contrast, the relative capital increases from k_j^2 to $(k_j^2)'$ but relative money decreases from m_j^2 to $(m_j^2)'$. Given that the wealthy are endowed with more money and capital, Figure 2 shows that money inequality $\tilde{\sigma}_m$ increases but capital inequality $\tilde{\sigma}_k$ decreases.

With more money holdings M_j', wealthy households increase their overall consumption C_j by significantly increasing the credit-good consumption C_{1j} without significantly reducing the cash-good consumption C_{2j} (the consumption reallocation) under strong credit–cash goods substitutability. Notice that although wealthy households increase their nominal money balances M_j', real money balances are decreased by higher price (i.e., the real money balances $M_j = M_j'/P$), resulting in a slight decrease in the cash-good consumption C_{2j}. In other words, under the CIA constraint, wealthy households are inclined to hold more money, instead of capital, decreasing the motivation to save in favor of increasing overall consumption. Poor households, with less money holdings M_j', decrease their overall consumption C_j by substantially decreasing the cash-good consumption C_{2j} in exchange for a slight increase in the credit-good consumption C_{1j}. Moreover, from (14), $\frac{C_j}{C} = \frac{1-l_j}{1-l}$ implies that household j with higher overall consumption is inclined to supply less labor, given the fact that all households face the same wage rate and interest rate. Therefore, wealthy households decrease their labor supply but poor households increase their labor supply. Because the wealthy are endowed with higher consumption and the poor are endowed with higher labor supply, Figure 2 shows that inequality in both consumption $\tilde{\sigma}_c$ and labor $\tilde{\sigma}_l$ increases when the inflation *status quo* is relatively high (the inflation thresholds of consumption and labor are $\hat{\pi}_{\sigma_c} = 1.72\%$ and $\hat{\pi}_{\sigma_l} = 2.03\%$).

In addition, (47) indicates that inequality in overall income $\tilde{\sigma}_y$ is positively related to the capital disparity $\tilde{\sigma}_k$ and negatively related to labor disparity $\tilde{\sigma}_l$. Because capital inequality decreases while labor inequality increases, income inequality $\tilde{\sigma}_y$ falls in response to higher inflation.

When the inflation *status quo* is relatively low, Figure 4 shows that, in response to higher inflation, the equilibrium point E_1 changes to E_1' for wealthy households (with $v_{j0}' > 1$) and E_2 changes to E_2' for poor households (with $v_{j0}' < 1$). As a result, relative capital k_j and money m_j decrease for wealthy households, but increase for poor households. Given that the wealthy are endowed with more money and capital, both capital $\tilde{\sigma}_k$ and money $\tilde{\sigma}_m$ inequalities, as shown in Figure 2, decrease when inflation increases.

Under the CIA constraint, a decrease in money holdings M_j' leads wealthy households to substantially decrease their cash-good consumption, resulting in a reduction in overall consumption C_j. In contrast, due to an increase in money holdings, poor households are able to increase overall consumption by significantly increasing credit-good consumption without significantly reducing cash-good consumption. With a negative relationship between overall consumption and labor supply (under the condition $\frac{C_j}{C} = \frac{1-l_j}{1-l}$), labor hours increase for wealthy households but decrease for poor households.[20] Given that wealthy households are endowed with higher consumption and poor households are endowed with higher labor supply, inequality in both consumption $\tilde{\sigma}_c$ and labor $\tilde{\sigma}_l$ decrease if the inflation *status quo* is relatively low, as shown in Figure 2. And, since the decrease in capital inequality is larger than that in labor inequality, inequality in overall income $\tilde{\sigma}_y$ unambiguously falls in response to higher inflation.

Result 1 has important implications for inequality and growth. First, income and consumption inequality can move in opposite directions, provided that the inflation *status quo* is relatively high: $\pi > 1.72\%$ (Figure 2 shows that the consumption inequality is minimized when inflation is $\pi = 1.72\%$). The divergence between consumption and income inequality provides a convincing theoretical explanation to the noted empirical observations (Krueger and Perri (2006), Daunfeldt et al. (2010), Fisher et al. (2013), Meyer and Sullivan (2013), and Meyer and Sullivan (2017)). It also has different implications for welfare, as income inequality fails to capture consumption disparities resulting from different consumption and asset distributions across households. Given the relationship $\sigma_{1-l} = \left(1 - \tilde{l}\right) \sigma_c$, from (46), Figure 2 shows that consumption inequality and leisure inequality may also move in opposite directions as inflation increases (when $1.72\% < \pi < 2.03\%$ in our parameterization). This indicates that including both leisure and consumption, as opposed to just consumption, might yield different implications for welfare.

Second, the negative inflation–income inequality relationship in our analysis is plausible for OECD countries. The average inflation rate of OECD countries was 7.47% during the 1980s and 1990s and 2.31% during the 2000s and 2010s. By contrast, the corresponding average Gini coefficients were 0.279 and 0.31. It is also valid for the USA. While the US Gini coefficients had the most dramatic increase among developed countries, US inflation decreased from 4.33% in 1984 to 2.41% in 2018.[21] In addition, the positive consumer credit–income inequality relationship is also supported empirically evidence during 1965–2005 (Krueger and Perri (2006)).

Third, the growth–inequality relationship could be mixed, consistent with the Kuznets curve. When the inflation *status quo* is relatively high, the asset reallocation implies a decrease in the money–capital ratio $\frac{\tilde{M}}{\tilde{K}}$, a "Mundell–Tobin effect", and stimulates growth $\tilde{\gamma}$. When the inflation *status quo* is relatively low, the negative impact of inflation on labor hours is strong for poor households (who rely more on labor income), which lowers the overall marginal product of capital and, hence, growth. Thus, the Mundell–Tobin effect is not observed. Our results provide a plausible explanation for the empirically mixed findings (see Romer and Romer (1998), Dolmas et al. (2000), Al-Marhubi (2000), and Albanesi (2007), for a positive inflation–inequality relationship, versus Maestri and Roventini (2012) and Coibion et al. (2017), for a negative inflation–inequality relationship). Note that, in García-Peñalosa and Turnovsky (2006) model, growth and income inequality are positively correlated in the presence of a structural shock (the time preference or technology) through factor price changes. In our model, growth and income inequality, however, can be negatively correlated in the presence of a monetary shock (inflation) through both factor price changes and quantitative adjustments.[22]

4.3. Credit–Cash Goods Substitution and Credit-Good Consumption Share

In this section, we perform a sensitivity analysis to investigate the role played by the elasticity of substitution between credit and cash goods, ε, and by the credit-good share of aggregate consumption, a, in terms of inequality and growth.

Result 2. (Substitution between credit and cash goods) *If cash and credit goods are weakly substitutable ($\varepsilon = -7$),*

- (i) (Labor and growth) *inflation unambiguously decreases the aggregate labor hours and the balanced-growth rate and*
- (ii) (Inequality) *inflation has a monotonically negative relationship with income and consumption inequality.*

In response to higher inflation, the economy-wide cash–credit goods ratio, $\frac{\tilde{C}_2}{\tilde{C}_1}$, and the money–capital ratio, $\frac{\tilde{M}}{\tilde{K}}$, decline, as in the baseline case above. However, in the presence of a substantially low elasticity of substitution between cash and credit goods (decreasing to $\varepsilon = -7$ from the benchmark value $\varepsilon = -30$), it becomes more difficult for households to increase overall consumption by reallocating consumption from cash to credit goods. Thus, on average, households tend to substitute leisure for consumption in response to higher inflation, resulting in an unambiguous reduction in both the aggregate consumption $\tilde{C}$ and labor hours $\tilde{l}$. With lower labor hours, Figure 5 shows that growth has a monotonically negative relationship with inflation.

In terms of distributional effects, a decrease in the aggregate labor hours raises the marginal product of labor, pushing up the wage rate w, but lowers the marginal product of capital, pushing down the return to capital r. Thus, similar to Figure 4, a lower $\frac{r}{w}$ ratio makes the $\dot{k}_j = 0$ locus flatter, which is unfavorable to wealthy households who rely more on capital income but favorable to poor households who rely more on labor income. As a result, in response to an increase in inflation, the relative capital k_j and money m_j decrease for the wealthy, but increase for the poor, decreasing both capital $\tilde{\sigma}_k$ and money $\tilde{\sigma}_m$ inequalities, as shown in Figure 5.

With lower money holdings, a weak substitution (or strong complementarity) between cash and credit goods requires wealthy households to decrease their overall consumption since the credit-good consumption is unable to greatly increase to compensate for the decrease in cash-good consumption. Lower consumption, as noted above, implies higher working hours. Thus, like the case where the inflation *status quo* is relatively low, wealthy households consume less but work more, while poor households consume more but work less. Therefore, both consumption and labor inequalities decrease and, similar to Figure 4, income inequality $\tilde{\sigma}_y$ decreases in response to higher inflation.[23] It is clear from Figure 5 that, in the presence of a substantially low substitution between cash and credit goods,

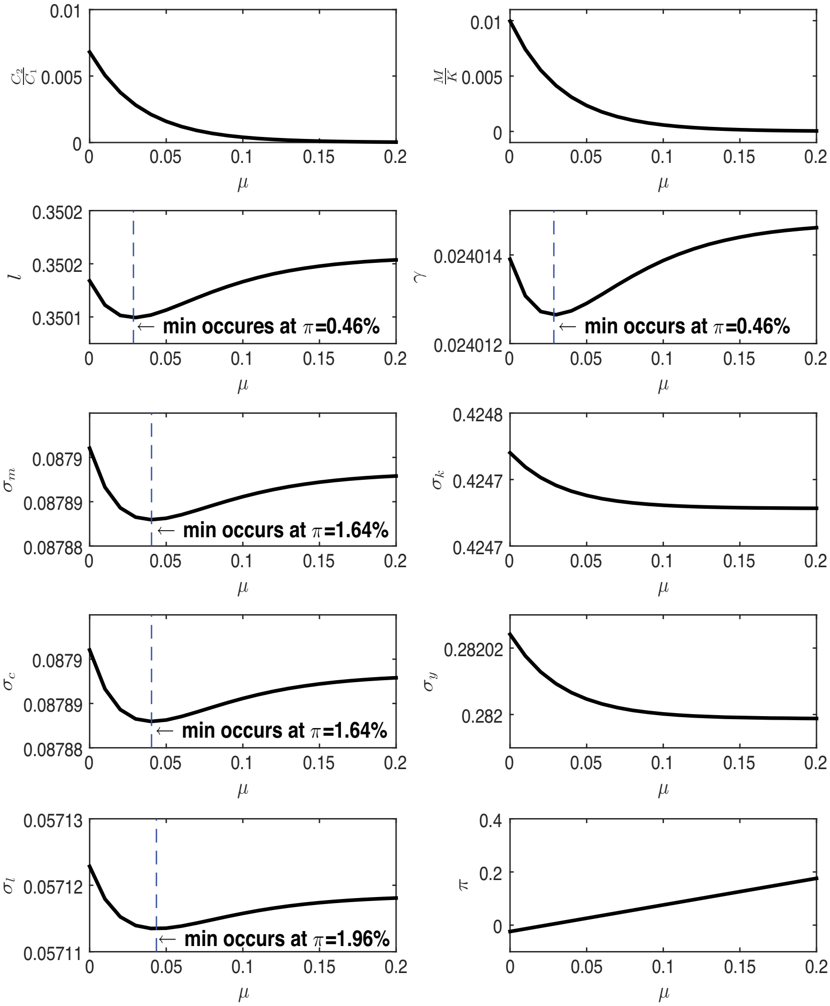

FIGURE 5. High credit-good consumption share ($a = 0.54$). Y-axis is a value axis. X-axis is the money growth rate.

the divergence between consumption and income inequality disappears and the growth–inequality relationship is always positive.

Next, Figure 5 shows that the effects of inflation, in general, are robust as the credit-good share of consumption, a, is higher.

Result 3. (Credit-good consumption share) *In the presence of a higher credit-good consumption share* ($a = 0.54$)*, inflation is more likely to increase consumption inequality and stimulate economic growth.*

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As inflation increases, households reallocate their consumption by decreasing cash goods purchases and increasing credit goods purchases. A higher credit-good consumption share, a (a lower cash-good consumption share $1-a$), implies that the marginal utility gain from increasing the credit-good consumption becomes higher, and the marginal utility loss from decreasing the cash-good consumption becomes lower. Therefore, the consumption reallocation of households becomes more pronounced. As shown in Figure 5, the economy-wide cash–credit goods consumption ratios become lower relative to the baseline case. Under a CIA constraint, the money–capital ratios are also lower. Under a strong substitution between cash and credit goods, credit-good consumption increases more and, therefore, inflation becomes more likely to increase overall consumption, $\tilde{C}$. This implies that households replace more leisure with consumption, reinforcing the positive effect on labor supply. As a result, the balanced-growth rate is more likely to increase in response to higher inflation, compared to the baseline case (the inflation threshold, $\hat{\pi}_\gamma$, reduces to 0.46% from the benchmark level of 0.56%).

A strongly positive labor effect amplifies the relative factor price effect so that the interest to wage rate ratio $\frac{r}{w}$ is more likely to increase, making the $\dot{k}_j = 0$ locus steeper. Similar to Figure 3, k_j decreases and m_j increases for the wealthy, but k_j increases and m_j decreases for the poor. Thus, capital inequality $\tilde{\sigma}_k$ decreases and money inequality $\tilde{\sigma}_m$ increases. More (less) money holdings, M_j', lead the wealthy (poor) to increase (decrease) their overall consumption, C_j. Therefore, consumption inequality, $\tilde{\sigma}_c$, becomes more likely to increase with inflation (the inflation threshold, $\hat{\pi}_{\sigma_c}$, decreases to 1.64% from the benchmark level of 1.72%).

Finally, our results are also robust to various values of the intertemporal substitution elasticity, ϕ. The impacts of inflation still hold in the presence of a lower elasticity of intertemporal substitution, for example, $1/3$, as in García-Peñalosa and Turnovsky (2006). A lower elasticity of intertemporal substitution, as they show, leads to a decrease in income inequality.[24]

5. CONCLUDING REMARKS

We have developed a dynamic general equilibrium growth model, where households purchase final goods on cash or credit with different capital and money endowments. When faced with higher inflation, households engage in reallocations in consumption (between cash goods and credit goods) and in assets (between money and capital). The consumption and asset reallocations govern the distributional effects on income and consumption and the growth effects of inflation.

We have provided analytical and numerical analyses to shed light on empirical observations on inequality. While income inequality decreases with inflation, consumption inequality has a U-shaped relationship with inflation. The theoretical divergence between consumption and income inequality provides not only a plausible explanation for recent empirical observations, but also an insightful implication for welfare analysis, because consumption measures better reflect long-run resources and social welfare, whereas income measures fail to capture disparities in consumption that result from different consumption and asset distributions across households. Moreover, the negative inflation–income inequality in our analysis is consistent with the empirical evidence in the USA and OECD countries.

We have also provided important implications for growth. It has been shown that if cash and credit goods are strongly substitutable, inflation can increase the balanced-growth rate, which resembles the Mundell–Tobin effect. Moreover, higher growth can be associated with either lower or higher income inequality in response to inflation. This ambiguous growth–income inequality relationship provides a plausible explanation for empirically mixed findings and provides evidence of the Kuznets curve.

NOTES

1. Commonly cited drivers of growing inequality include globalization (Goldberg and Pavcnik (2007)), technology (Acemoglu and Autor (2011)), and deunionization (International Labour Office (ILO) (2008)).
2. Generally speaking, consumption depends on life-cycle earnings or permanent income, with short-run fluctuations in income more likely to impact savings rather than consumption. Consumption thus depends on wealth, which varies across households with similar incomes. Blundell et al. (2008) show that the cross-sectional distribution of consumption may be a sufficient statistic for cross-sectional welfare.
3. Attanasio and Pistaferri (2016) provide an overview of studies that compares trends in income inequality with those in consumption inequality.
4. See, for example, Piketty (2015) and a corresponding review by McCloskey (2014), for contrasting views on the appropriateness of focusing narrowly on income inequality in welfare analyses.
5. The empirical effects of inflation on income inequality could be either positive (e.g., Romer and Romer (1998), Dolmas et al. (2000), Al-Marhubi (2000), and Albanesi (2007)) or negative (e.g., Maestri and Roventini (2012) and Coibion et al. (2017)).
6. Bell et al. (2012) and Amaral (2017) categorize different channels of monetary transmission more generally.
7. Mundell (1965) and Tobin (1965) argue that nominal interest rates will rise less than one for one with inflation (as the Fisher effect states) because inflation induces the public to hold less money balances and more real assets, such as capital, which will drive interest rates down. As a result, the classical dichotomy is broken and money is no longer neutral.
8. The Kuznets curve shows that the tradeoff between income inequality and economic growth depends on different stages of economic development (see, e.g., Kuznets (1955) and Barro (2000)). The reader can also refer to Townsend (2008) (a special issue in *Macroeconomic Dynamics*, 2008) for a comprehensive survey.
9. The weight k_j can be thought of as the household's shareholding divided by the stock market value of the firm.
10. While the existing studies use the CIA constraint to analyze the effects of inflation on income inequality (e.g., İmrohoroğlu (1992), Erosa and Ventura (2002), Camera and Chien (2016)), they do not distinguish between cash goods and credit goods.

11. Note that we have $\frac{\dot{C}_2}{C_2} = \frac{\dot{M}}{M} = \mu - \pi$ from (28) and $\Psi_2 = \frac{\dot{K}}{K} = Al^{1-\alpha} - \frac{(1-l)(1-\alpha)\rho A l^{-\alpha}(aZ(R))^{-\varepsilon}}{\eta Z(R)}\{1 + [(1+R)\frac{a}{1-a}]^{\varepsilon}\} - \delta$ from (30) and (31).

12. The transversality condition $\lim_{t\to\infty} \lambda_{1t} M_t e^{-\beta t} = \lim_{t\to\infty} \lambda_{10} e^{(\beta - r + \delta)t} M_0 e^{(\mu-\pi)t} e^{-\beta t} = 0$ requires that, in the BGP equilibrium, the rate of net return on capital $r - \delta = \rho\alpha A\tilde{l}^{1-\alpha} - \delta$ exceeds the balanced-growth rate $\gamma = \mu - \tilde{\pi}$. That is, $R = \pi + r - \delta > \mu$. Thus, we have $\Omega_k > 0$ and $\Omega_m > 0$.

13. See Turnovsky (2002) for the discussion regarding knife-edge conditions in macroeconomic models.

14. In line with Chatterjee and Turnovsky (2012) and Turnovsky (2015), income measures ignore the distributional impacts of lump-sum transfers, given that in the model, transfers are available to all households, regardless of income.

15. MZM is a measure of the liquid money supply within an economy. It has become one of the preferred measures of money supply because it better represents money readily available within the economy for spending and consumption. In addition, the Federal Reserve relies heavily on MZM data, given the fact that its velocity is a proven indicator of inflation.

16. The relevant DFA data for the USA are only available since 1989.

17. Our results are robust for various sample sizes.

18. García-Peñalosa and Turnovsky (2015) point out that relative deviations are dimensionally equivalent to the widely used Gini coefficients.

19. Given that the slope of the *WW* locus is $\frac{\partial m_j}{\partial k_j}|_{WW} = -\frac{K_0}{M_0'/P_0} < 0$, an increase in inflation increases the price, making the *WW* locus steeper.

20. Proposition 2 indicates that poor households have a stronger propensity to work. Thus, Figure 2 shows that, if the inflation *status quo* is relatively high (low), the labor increase (decrease) of poor households is larger than the labor decrease (increase) of wealthy households, resulting in higher (lower) overall labor hours.

21. Since monetary policy in the early 1980s underwent a transition to regain price stability, we start the sample in 1984.

22. Generally speaking, household income includes labor income (the return rate to labor (the wage rate) times working hours) and capital income (the return rate to capital (the interest rate net of depreciation plus corporate dividends) times the amount of capital). Households face the same wage and interest rates, but they provide different working hours and hold different amounts of capital. In our monetary model, inflation has a disproportional drag on the incomes of wealthy and poor households *via* quantity adjustments (in cash goods versus credit goods, labor versus capital, and money versus capital), and price changes (in the ratio of interest-wage rate). In García-Peñalosa and Turnovsky (2006) study, price changes are more important than quantity adjustments in the presence of a *real* technology shock.

23. Because wealthy households have a stronger propensity to consume, the consumption decrease of wealthy households is thus larger than the consumption increase of poor households so that the aggregate consumption $\tilde{C}$ unambiguously falls. Because the poor have a stronger propensity to work, the labor increase of wealthy households is smaller than the labor decrease of poor households, so that total labor hours $\tilde{l}$ unambiguously decrease as well.

24. The relevant robustness analysis is available on request.

REFERENCES

Acemoglu, D. and D. Autor (2011) Skills, tasks and technologies: Implications for employment and earnings. In: O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, Vol. 4B, Chapter 12, pp. 1043–1171. Amsterdam: Elsevier-North Holland.

Albanesi, S. (2007) Inflation and inequality. *Journal of Monetary Economics* 54, 1088–1114.

Algan, Y., A. Cheron, J. O. Hairault and F. Langot (2003) "Wealth effect on labor market transitions. *Review of Economic Dynamics* 6, 156–178.

Al-Marhubi, F. A. (2000) Income inequality and inflation: The cross evidence. *Contemporary Economic Policy* 18, 428–439.

Amaral, P. (2017) Monetary policy and inequality. *Economic Commentary*, Federal Reserve Bank of Cleveland, issue January.

Attanasio, O. and L. Pistaferri (2016) Consumption inequality. *Journal of Economic Perspectives* 30, 3–28.

Barro, R. J. (2000) Inequality and growth in a panel of countries. *Journal of Economic Growth* 5, 5–32.

Bell, V., M. Joyce, Z. Liu and C. Young (2012) The distributional effects of asset purchases. *Bank of England Quarterly Bulletin* 52, 254–266.

Benhabib, J. and R. E. Farmer (1994) Indeterminacy and increasing returns. *Journal of Economic Theory* 63, 19–41.

Blundell, R. and B. Etheridge (2010) Consumption, income and earnings inequality in Britain. *Review of Economic Dynamics* 13, 76–102.

Blundell, R., L. Pistaferri and I. Preston (2008) "Consumption inequality and partial insurance. *American Economic Review* 98, 1887–1921.

Blundell, R. and I. Preston (1998) Consumption inequality and income uncertainty. *Quarterly Journal of Economics* 113, 603–640.

Boel, P. and G. Camera (2009) Financial sophistication and the distribution of the welfare cost of inflation. *Journal of Monetary Economics* 56, 968–978.

Caballero, R. J., E. Farhi and P. O. Gourinchas (2017) Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. *American Economic Review* 107, 614–620.

Cabanillas, L. G. and E. Ruscher (2008) "The great moderation in the euro area: What role have macroeconomic policies played? *European Economy Economic Papers* No. 331, June (Brussels: Directorate General for Economic and Financial Affairs, European Commission).

Camera, G. and Y. Chien (2016) Two monetary models with alternating markets. *Journal of Money, Credit and Banking* 48, 1051–1064.

Chatterjee, S. and S. J. Turnovsky (2012) Infrastructure and inequality. *European Economic Review* 56, 1730–1745.

Christiano, L. J. (1988) Why does inventory investment fluctuate so much? *Journal of Monetary Economics* 21, 247–280.

Coibion, O., Y. Gorodnichenko, L. Kueng and J. Silvia (2017) Innocent bystanders? Monetary policy and inequality in the US. *Journal of Monetary Economics* 88, 70–89.

Cysne, R. P., W. L. Maldonado and P. K. Monteiro (2005) Inflation and income inequality: A shopping-time approach. *Journal of Development Economics* 78, 516–528.

Daunfeldt, S. O., S. Fölster and P. Hortlund (2010) Consumption and Income Inequality in Sweden: A Different Story. No. 39, HUI Research.

Doepke, M. and M. Schneider (2006) Inflation and the redistribution of nominal wealth. *Journal of Political Economy* 114, 1069–1097.

Dolmas, J., G. W. Huffman and M. A. Wynne (2000) Inequality, inflation, and central bank independence. *Canadian Journal of Economics/Revue canadienne d'éonomique* 33, 271–287.

Dutta, J., and M. Weale (2001) Consumption and the means of payment: An empirical analysis for the United Kingdom. *Economica* 68, 293–316.

Erosa, A., and G. Ventura (2002) On Inflation as a regressive consumption tax. *Journal of Monetary Economics* 49, 761–795.

Fisher, J. D., D. S. Johnson and T. M. Smeeding (2013) Measuring the trends in inequality of individuals and families: Income and consumption. *American Economic Review* 103, 184–188.

García-Peñalosa, C. and S. J. Turnovsky, 2006. Growth and income inequality: A canonical model. *Economic Theory* 28, 25–49.

García-Peñalosa, C. and S. J. Turnovsky (2015) Income inequality, mobility, and the accumulation of capital. *Macroeconomic Dynamics* 19, 1332–1357.

Goldberg, P. K. and N. Pavcnik (2007) Distributional effects of globalization in developing countries. *Journal of Economic Literature* 45, 39–82.

Hansen, G. and R. Wright (1992) The labor market in real business cycle theory. In: *Federal Reserve Bank of Minneapolis Quarterly Review*, pp. 2–12, Spring.

Holtz-Eakin, D., D. Joulfaian and H. S. Rosen (1993) The Carnegie conjecture: Some empirical evidence. *Quarterly Journal of Economics* 108, 413–435.

İmrohoroğlu, A. (1992) The welfare cost of inflation under imperfect insurance. *Journal of Economic Dynamics and Control* 16, 79–91.

International Labour Office (ILO) (2008) Labour institutions and inequality. In: *World of Work Report*, pp. 71–114. Geneva: International Labour Office.

Jin, Y. (2009) A note on inflation, economic growth, and income inequality. *Macroeconomic Dynamics* 13, 138–147.

Juster, F. T. and F. P. Stafford (1991) The allocation of time: Empirical findings, behavioral models, and problems of measurement. *Journal of Economic Literature* 29, 471–522.

Krueger, D. and F. Perri (2006) Does income inequality lead to consumption inequality? Evidence and theory. *Review of Economic Studies* 73, 163–193.

Kuznets, S. (1955) Economic growth and income inequality. *American Economic Review* 45, 1–28.

Lucas Jr, R. E. and N. L. Stokey (1983) Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12, 55–93.

Lucas Jr, R. E. and N. L. Stokey (1987) Money and interest in a cash-in-advance economy. *Econometrica* 55, 491–513.

Maestri, V. and A. Roventini (2012) Inequality and macroeconomic factors: A time-series analysis for a set of OECD countries. LEM Papers Series 2012/21, Laboratory of Economics and Management (LEM), Sant'Anna School of Advanced Studies, Pisa.

Mankiw, N. G. (1987) Government purchases and real interest rates. *Journal of Political Economy* 95, 407–419.

McCloskey, D. N. (2014) Measured, unmeasured, mismeasured, and unjustified pessimism: A review essay of Thomas Piketty's capital in the twenty-first century. *Erasmus Journal for Philosophy and Economics* 7, 73–115.

Mersch, Y. (2014) Monetary policy and economic inequality. In: *Keynote Speech, Corporate Credit Conference*, October 17, 2014.

Meyer, B. D. and J. X. Sullivan (2013) Consumption and income inequality and the great recession. *American Economic Review* 103, 178–183.

Meyer, B. D. and J. X. Sullivan (2017) Consumption and Income Inequality in the US Since the 1960s. NBER Working Paper: No. 23655.

Mundell, R. A. (1965) Growth, stability, and inflationary finance. *Journal of Political Economy* 73, 97–109.

Nakajima, M. (2015) The redistributive consequences of monetary policy. *Business Review* Q2, 9–16.

Piketty, T. (2015) About capital in the twenty-first century. *American Economic Review* 105, 48–53.

Romer, C. D. and D. H. Romer (1998) Monetary policy and the well-being of the poor. In: *Income Inequality: Issues and Policy Options*, pp. 159–201. Kansas City: Fed. Reserve Bank Kansas.

Sen, P., and S. J. Turnovsky (1989) Deterioration of the terms of trade and capital accumulation: A re-examination of the Laursen-Metzler effect. *Journal of International Economics* 26, 227–250.

Slesnick, D. T. (1994) Consumption, needs and inequality. *International Economic Review* 35, 677–703.

Smets, F. and R. Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1, 1123–1175.

Tobin, J. (1965) Money and economic growth. *Econometrica* 33, 671–684.

Townsend, R. M. (2008) Introduction to macroeconomic dynamics special issue: Inequality. *Macroeconomic Dynamics* 12, 149–153.

Turnovsky, S. J. (2002) Knife-Edge conditions and the macrodynamics of small open economies. *Macroeconomic Dynamics* 6, 307–335.

Turnovsky, S. J. (2015) Economic growth and inequality: The role of public investment. *Journal of Economic Dynamics and Control* 61, 204–221.
Williamson, S. D. (2009) Transactions, credit, and central banking in a model of segmented markets. *Review of Economic Dynamics* 12, 344–362.
Wolff, E. N. (2017) Household Wealth Trends in the United States, 1962 to 2016: Has Middle Class Wealth Recovered?" National Bureau of Economic Research Working Paper: No. 24085.

APPENDIX A

Proof of Proposition 1. First of all, we manipulate (32) and (33) as:

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \dot{\pi} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\phi}-1\right)(\mu-\pi)+\beta-\rho\alpha A l^{1-\alpha}+\delta+\Psi_2 \\ \Psi_2-\mu+\pi \end{bmatrix}, \tag{A1}$$

where $b_{11}=(\frac{1}{\phi}-1)\Psi_1<0$, $b_{12}=\frac{b_{11}(1-\alpha)r+\alpha}{l}+\frac{1-\eta(1-\frac{1}{\phi})}{1-l}\gtrless 0$, $b_{21}=(\frac{1+R}{1-a})^{\varepsilon}Z(R)^{-(1+\varepsilon)}-\Psi_1>0$, $b_{22}=\frac{b_{21}(1-\alpha)r+\alpha}{l}+\frac{1}{1-l}>0$ and recall that $\Psi_1=\frac{\varepsilon}{1+R}\frac{a^{-\varepsilon}}{a^{-\varepsilon}+(1-a)^{-\varepsilon}(1+R)^{1+\varepsilon}}<0$, $\Psi_2=Al^{1-\alpha}-\frac{(1-l)(1-\alpha)\rho A l^{-\alpha}(aZ(R))^{-\varepsilon}}{\eta Z(R)}\{1+[(1+R)\frac{a}{1-a}]^{\varepsilon}\}-\delta>0$, and the intertemporal elasticity of substitution is less than one ($\phi<1$). Define $\Lambda=b_{11}b_{22}-b_{12}b_{21}=b_{11}(\frac{1}{1-l}+\frac{\alpha}{l})-b_{21}\{[1-\eta(1-\frac{1}{\phi})]\frac{1}{1-l}+\frac{\alpha}{l}\}<0$. Thus, by taking a first-order Taylor expansion of (A1) around the steady-state $\tilde{\pi}$ and $\tilde{l}$, we can further rewrite the dynamic system as follows:

$$\begin{bmatrix} \dot{\pi} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} \Upsilon_\pi & \Upsilon_l \\ \Gamma_\pi & \Gamma_l \end{bmatrix} \begin{bmatrix} \pi-\tilde{\pi}\, l-\tilde{l}+ \end{bmatrix} \Upsilon_\mu \Gamma_\mu \left[\mu-\mu_0\right], \tag{A2}$$

where μ_0 is the initial money growth rate and

$\Upsilon_\pi=\frac{1}{\Lambda}[(b_{22}-b_{12})\Phi-b_{22}\frac{1}{\phi}]$,
$\Upsilon_l=\frac{1}{\Lambda}[(b_{22}-b_{12})(\Phi\frac{(1-\alpha)r}{l}+\theta)-b_{22}\rho\alpha(1-\alpha)Al^{-\alpha}]$,
$\Upsilon_\mu=\frac{1}{\Lambda}[b_{22}\frac{1}{\phi}-(b_{22}-b_{12})]$,
$\Gamma_\pi=\frac{1}{\Lambda}[(b_{11}-b_{21})\Phi+b_{21}\frac{1}{\phi}]$,
$\Gamma_l=\frac{1}{\Lambda}[(b_{11}-b_{21})(\Phi\frac{(1-\alpha)r}{l}+\theta)+b_{21}\rho\alpha(1-\alpha)Al^{-\alpha}]$,
$\Gamma_\mu=\frac{-1}{\Lambda}[b_{21}\frac{1}{\phi}+(b_{11}-b_{21})]$,
$\Phi=1+\frac{\partial\Psi_2}{\partial\pi}=1+(1-l)\rho(1-\alpha)Al^{-\alpha}\frac{a^{-\varepsilon}Z(R)^{-2(1+\varepsilon)}(\frac{1+R}{1-a})^{\varepsilon}}{\eta}\{\varepsilon\frac{R}{1+R}+1+[\frac{a(1+R)}{1-a}]^{\varepsilon}\}\gtrless 0$,
$\theta=A(1-\alpha)l^{-\alpha}(1-\alpha\rho)+\rho(1-\alpha)Al^{-\alpha}[1+\frac{\alpha(1-l)}{l}]\frac{a^{-\varepsilon}Z(R)^{-(1+\varepsilon)}\{1+[\frac{a(1+R)}{1-a}]^{\varepsilon}\}}{\eta}>0$.

Assume that ξ_1 and ξ_2 are the eigenvalues of the dynamic system. It follows from the Jacobian matrix of (A2) that the trace and determinant are given by

$$\begin{aligned} Tr(J) &= \xi_1+\xi_2=\Upsilon_\pi+\Gamma_l \\ &= \frac{1}{\Lambda}\left\{\left(1-\frac{1}{\phi}\right)\frac{(1-\alpha)r}{l}b_{21}-(b_{21}-b_{11})\theta-\frac{1}{\phi}\left(\frac{\alpha}{l}+\frac{1}{1-l}\right)\right. \\ &\qquad \left. +\eta(1-\frac{1}{\phi})\frac{\Phi}{1-l}\right\}, \end{aligned} \tag{A3}$$

$$\begin{aligned} Det(J) &= \xi_1\xi_2=\Upsilon_\pi\Gamma_l-\Upsilon_l\Gamma_\pi \\ &= \frac{-1}{\Lambda}\left[\left(\frac{1}{\phi}-1\right)\frac{(1-\alpha)r}{l}\Phi+\frac{1}{\phi}\theta\right]. \end{aligned} \tag{A4}$$

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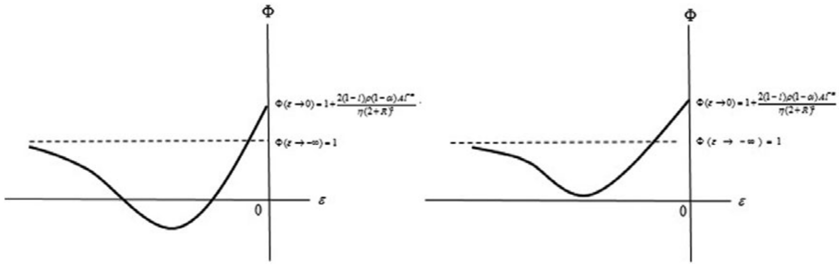

FIGURE A1. Existence and uniqueness of the BGP Equilibrium.

As noted in the context, since both π and l are jump variables, the steady-state equilibrium is locally determinate if there are two roots with positive real parts ($\xi_1 > 0$ and $\xi_2 > 0$). That is, $Tr(J) > 0$ and $Det(J) > 0$.

If Condition E holds ($\varepsilon > \varepsilon^* \equiv -(\frac{1+R}{R})\{1 + [a(\frac{1+R}{1-a})]^{\varepsilon} + \frac{\eta}{(1-l)\rho(1-\alpha)Al^{-\alpha}z(R)^{-2(1+\varepsilon)}a^{-\varepsilon}(\frac{1+R}{1-a})^{\varepsilon}}\}$), we can infer that $\Phi = 1 + \frac{\partial \Psi_2}{\partial \pi} > 0$. Accordingly, we can see from (A3) and (A4) that the trace and determinant of the Jacobian matrix must be positive, that is, $Tr(J) = \xi_1 + \xi_2 > 0$ and $Det(J) = \xi_1 \xi_2 > 0$. Note that while it is not possible to explicitly solve the critical value of ε^*, we can prove its existence (when it is necessary). Given that Φ is a function of ε ($\Phi(\varepsilon) = 1 + \frac{\partial \Psi_2}{\partial \pi}$), we can derive that this function intercepts the vertical axis at $\Phi(\varepsilon \to 0) = 1 + \frac{2(1-l)\rho(1-\alpha)Al^{-\alpha}}{\eta(2+R)^2} > 0$ and $\Phi(\varepsilon \to -\infty) = 1$ if $\frac{a(1+R)}{1-a} > 1$ (or $\Phi(\varepsilon \to -\infty) = 1 + \frac{(1-l)\rho(1-\alpha)Al^{-\alpha}}{\eta(1+R)^2} > 0$ if $\frac{a(1+R)}{1-a} < 1$). Thus, there are two possible cases, as shown in Figure A1 (which only focuses on the scenario where $\frac{a(1+R)}{1-a} > 1$). One case (the left panel) requires a critical value of ε^* (i.e., Condition E) which ensures that $\Phi = 1 + \frac{\partial \Psi_2}{\partial \pi} > 0$ such that $Tr(J) > 0$ and $Det(J) > 0$. In the other one (the right panel), $\Phi > 0$ automatically holds and, consequently, $Tr(J) > 0$ and $Det(J) > 0$ are true without any condition. ■

APPENDIX B

(The determination of initial wealth): Recall that, in the steady state,

$$\frac{\tilde{M}}{\tilde{C}} = \frac{\tilde{C}_2}{\tilde{C}} = \left[\frac{(1 + \tilde{r} + \tilde{\pi} - \delta)}{(1-a)Z}\right]^{\varepsilon}, \tag{A5}$$

$$\frac{\tilde{C}}{\tilde{K}} = \frac{(1-\tilde{l})(1-\alpha)\rho A\tilde{l}^{-\alpha}}{\eta Z}, \tag{A6}$$

where $Z = \left[a^{-\varepsilon} + (1-a)^{-\varepsilon}(1 + \tilde{r} + \tilde{\pi} - \delta)^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$. From (A5) and (A6), we have the steady-state money–capital ratio as follows:

$$\frac{\tilde{M}}{\tilde{K}} = \frac{\tilde{M}}{\tilde{C}}\frac{\tilde{C}}{\tilde{K}} = [\frac{(1 + \tilde{r} + \tilde{\pi} - \delta)}{(1-a)Z}]^{\varepsilon} \cdot \frac{(1-\tilde{l})(1-\alpha)\rho A\tilde{l}^{-\alpha}}{\eta Z}. \tag{A7}$$

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Given the definition of real money balances $M = M'/P$, we have the following initial condition:

$$\frac{M_0}{K_0} = \frac{M_0'/P_0}{K_0} = [\frac{(1+\tilde{r}+\tilde{\pi}-\delta)}{(1-a)Z}]^{\varepsilon} \cdot \frac{(1-\tilde{l})(1-\alpha)\rho A\tilde{l}^{-\alpha}}{\eta Z}. \qquad \textbf{(A8)}$$

Because M_0' and K_0 are exogenously given, (A8) allows us to pin down the initial level of price P_0 as the money growth rate μ changes. With the price P_0, we can endogenously determine $V_0 = K_0 + M_0$ (and $V_{j0} = K_{j0} + M_{j0}$, as well). Given the household's initial wealth V_{j0} (and hence v_{j0}), households will engage in an upon-impact adjustment between K_{j0} and M_{j0}' to a shock, and, accordingly, k_j and m_j immediately jump to their long-run equilibrium values, that is, $k_{j0} = \tilde{k}_j$ and $m_{j0} = \tilde{m}_j$.