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The Sea Mile and Nautical Mile in Marine Navigation

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The problem of differences between the sea mile and the international nautical mile has been analysed. Algorithms for the calculation of sea miles with applications in different sailing methods on the ellipsoid that can be easily incorporated in modern microprocessor controlled navigational devices are proposed. These algorithms can also be employed on an outfit of large-scale nautical charts with a double scale in sea miles and international nautical miles.

KEYWORDS

1. Marine navigation. 2. Sea mile and nautical mile. 3. Orthodromes and loxodromes. 4. Geodesics. 5. Position fixing.

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1. INTRODUCTION. The nautical mile is a special unit employed for marine and air navigation to express distance. The value 1,852 m was adopted by the First International Extraordinary Hydrographic Conference, Monaco 1929, under the name "International nautical mile" (The International System of Units (SI), 2006). The International Nautical Mile (INM) is used in navigational devices such as logs and radars.

The constant INM is an approximation of the Sea Mile (SM) defined as the length of one minute of arc, measured along the meridian of the Earth, in the latitude of the position. As the Earth is not a perfect sphere and in geodesy is defined as an ellipsoid, its actual length varies with latitude.

Navigational devices such as, for example, logs and radars are calibrated in INMs. On the other hand, distances which can be measured on nautical charts are in SMs which means that two different measures meet together on nautical charts.

This difference is up to $\pm 0{\cdot}5\%$ and such simplification has been necessary and quite justified compared to the accuracy of navigation at the time of the definition of the INM. But the question arises now – are these simplifications still justified in modern navigation?

2. GEODETIC SOLUTIONS. The SM is defined on an ellipsoid and therefore the application of solutions of the problems known in geodesy as the direct and the inverse geodetic problems should be applied.

Lenart (2011) and Lenart (2013) present a set of procedures for calculating distances and azimuths on an ellipsoid of revolution for orthodromes and loxodromes. The orthodrome in this paper (which literally means straight line) is defined as the path of the shortest distance on any surface, for example, a plane, a spheroid or an ellipsoid and the latter is used in these calculations.

In formal notations:

$$S = \mathrm{IGP}(\varphi_1, \lambda_1, \varphi_2, \lambda_2) \tag{1}$$

$$C_{gs}, C_{ge} = \mathrm{IGP}(\varphi_1, \lambda_1, \varphi_2, \lambda_2) \tag{2}$$

$$S_{lx} = \mathrm{LX}(\varphi_1, \lambda_1, \varphi_2, \lambda_2) \tag{3}$$

$$C_{glx} = \mathrm{LX}(\varphi_1, \lambda_1, \varphi_2, \lambda_2) \tag{4}$$

$$\varphi_2, \lambda_2 = \mathrm{DGP}(\varphi_1, \lambda_1, S, C_{gs}) \tag{5}$$

$$C_{ge} = \mathrm{DGP}(\varphi_1, \lambda_1, S, C_{gs}) \tag{6}$$

where $P_1(\varphi_1, \lambda_1)$ and $P_2(\varphi_2, \lambda_2)$ are the departure point and the destination point, respectively, S is the orthodromic distance, C_{gs} is the Course Over the Ground (COG) at the departure point of the orthodrome and C_{ge} is the COG at the destination point of the orthodrome, S_{lx} is the loxodromic distance and C_{glx} is the loxodromic COG.

IGP is the procedure of the Inverse Geodetic Problem solution which calculates orthodromic distance and COG at the departure point from the coordinates of the departure and the destination points. LX is a similar procedure for loxodromic calculations. DGP is the procedure of the Direct Geodetic Problem solution which calculates coordinates and COG of the destination point from coordinates and COG of the departure point and the orthodromic distance.

In this paper these procedures, with results based on full accuracy Sodano's solutions (Sodano, 1958; 1965; 1967) on the WGS-84 (World Geodetic System) reference ellipsoid (as in Lenart (2011) and Lenart (2013)) will be used in general formal form but any other geodetic solutions of comparable or better accuracies can be applied, such as the widely used, in other than navigation applications, algorithms of Vicenty (1975) and Kearney (2013). Sodano's solutions have been selected because they are very satisfactory for marine and air navigation accuracy for any length of geodesics with simple rigorous non-iterative procedures.

3. SEA MILE ANALYSIS.

A simplified formula for the sea mile between latitudes φ and $\varphi + 1'$ (for example, Weintrit, 2015) is widely used:

$$S_{SM} \approx a_0[1 - e^2(1 + 3\cos 2\varphi)/4]\ \mathrm{arc}\ 1' \tag{7}$$

where a_0 is the semi-major axis of the reference ellipsoid and e is the eccentricity of the reference ellipsoid.

More correct and more accurate for the latitude φ is the following procedure (in accordance with Equation (1)):

$$S_{SM} = \mathrm{IGP}(\varphi - 0{\cdot}5', 0, \varphi + 0{\cdot}5', 0) \text{ or for } \varphi = 90^\circ\ S_{SM} = \mathrm{IGP}(89^\circ 59{\cdot}5', 0, 89^\circ 59{\cdot}5', 180^\circ) \tag{8}$$

or similar to Equation (7):

$$S_{SM} = \mathrm{IGP}(\varphi, 0, \varphi + 1', 0) \text{ or for } \varphi = 90^\circ\ S_{SM} = \mathrm{IGP}(90^\circ, 0, 89^\circ 59', 180^\circ) \tag{9}$$

Table 1. Sea mile, comparison of errors and derivative.

φ [°]	Eq. (8) [m]	Eq. (8)–Eq. (9) [m]	Eq. (8)–Eq. (7) [m]	Eq. (8)–Eq. (10) [m]	Eq. (13)
0	1,842·9046	0·0000	0·0000	0·0000	0·0000
5	1,843·0452	−0·0005	−0·0009	−0·0002	3·2351
10	1,843·4627	−0·0009	−0·0036	−0·0028	6·3720
15	1,844·1449	−0·0013	−0·0077	−0·0133	9·3152
20	1,845·0715	−0·0017	−0·0125	−0·0394	11·9754
25	1,846·2147	−0·0021	−0·0174	−0·0882	14·2717
30	1,847·5407	−0·0023	−0·0215	−0·1639	16·1344
35	1,849·0095	−0·0025	−0·0243	−0·2655	17·5068
40	1,850·5772	−0·0027	−0·0251	−0·3852	18·3473
45	1,852·1962	−0·0027	−0·0235	−0·5087	18·6304
50	1,853·8177	−0·0027	−0·0197	−0·6167	18·3473
55	1,855·3921	−0·0026	−0·0137	−0·6881	17·5068
60	1,856·8714	−0·0024	−0·0060	−0·7049	16·1344
65	1,858·2101	−0·0021	0·0026	−0·6563	14·2717
70	1,859·3670	−0·0018	0·0114	−0·5438	11·9754
75	1,860·3063	−0·0014	0·0194	−0·3838	9·3152
80	1,860·9989	−0·0009	0·0257	−0·2081	6·3720
85	1,861·4233	−0·0005	0·0299	−0·0618	3·2351
90	1,861·5663	0·0000	0·0313	−0·0001	0·0000

It can be found that (Weintrit, 2015):

$$S_{SM} \approx M \text{ arc } 1' \tag{10}$$

where M is the radius of curvature in the meridian given by:

$$M = \frac{a_0(1 - e^2)}{\sqrt{(1 - e^2 \sin^2 \varphi)^3}} \tag{11}$$

Table 1 presents results of Equation (8) and errors of Equations (7), (9) and (10) referenced to Equation (8) and these are illustrated in Figures 1–4.

As can be seen from Table 1 and Figures 1–4, errors of Equation (9) are from 0 to −0·27 cm, errors of Equation (7) are from −2·51 cm to +3·13 cm and errors of Equation (11) are from 0 to −70·49 cm. It is evident that between Equation (8) and Equation (9) there is a very small difference, Equation (7) is quite a good approximation of Equation (8) and Equation (7) is simpler and more accurate then Equation (10).

The INM was chosen as the integer number of metres closest to the mean sea mile at latitude 45° hence:

$$S_{SM} = 1852^{+9{\cdot}10}_{-9{\cdot}57} \text{ m} \approx 1852 \text{ m} \pm 0{\cdot}5\% \tag{12}$$

and the biggest differences are at the poles and the equator - the latter is more important in marine navigation because the poles are rather inaccessible for marine navigation (other than underwater navigation at the North Pole).

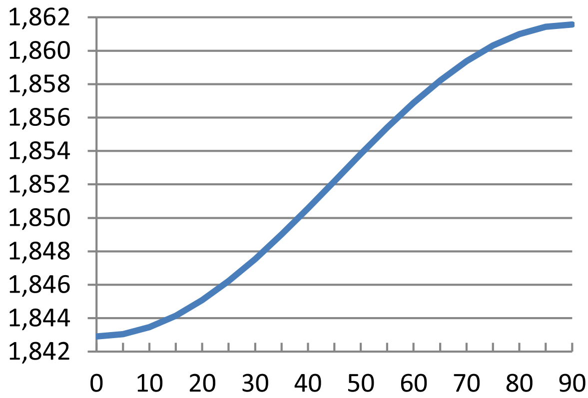

Figure 1. Sea mile – Equation (8) [m].

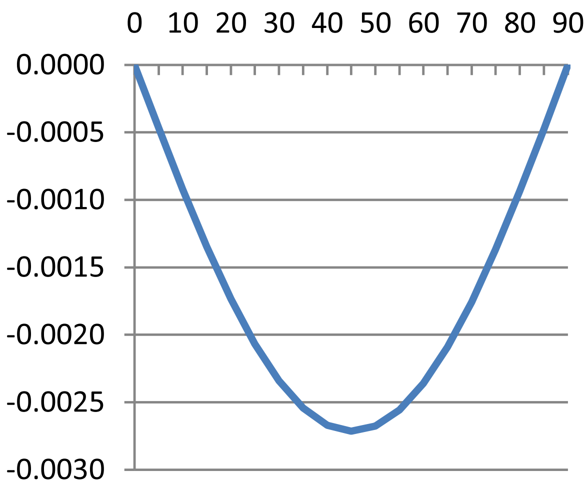

Figure 2. Error – Equation (8)–Equation (9) [m].

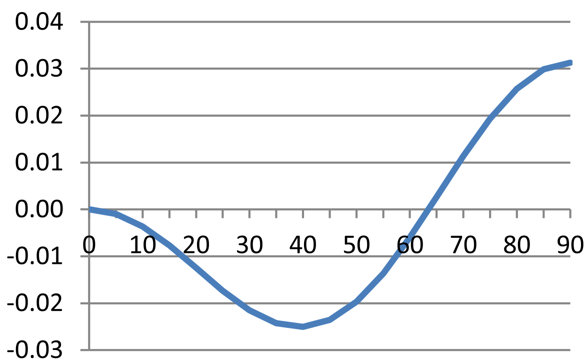

Figure 3. Error – Equation (8)–Equation (7) [m].

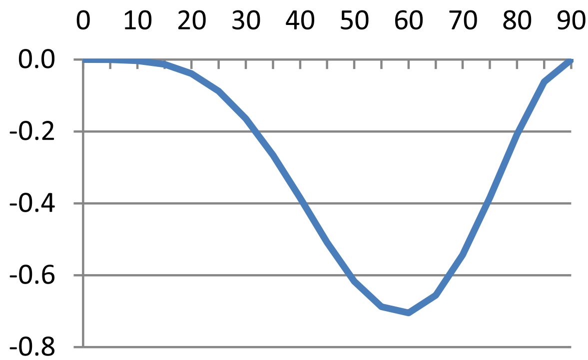

Figure 4. Error – Equation (8)–Equation (10) [m].

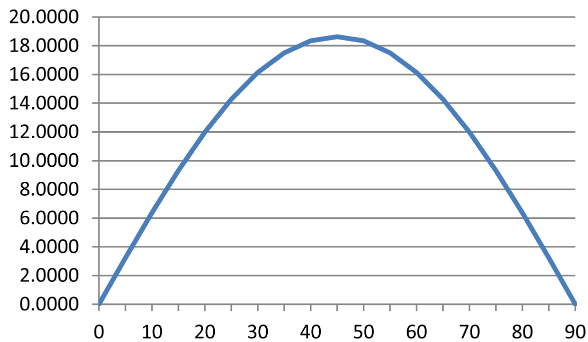

Figure 5. Derivative – Equation (13)

From Equation (7) the derivative is given by:

$$\frac{dS_{SM}}{d\varphi} = \frac{3}{2} a_0 e^2 \sin 2\varphi \cdot \text{arc } 1' \tag{13}$$

which reaches an extreme at $\varphi = 45^\circ$ (Table 1, Figure 5):

$$\frac{dS_{SM}}{d\varphi} \max = \frac{3}{2} a_0 e^2 \text{ arc } 1' \tag{14}$$

and consequently:

$$\frac{dS_{SM}}{d\varphi} \max(1') = \frac{dS_{SM}}{d\varphi} \max \cdot \text{arc } 1' = 0{\cdot}54\,\text{cm}/1' \tag{15}$$

$$\frac{dS_{SM}}{d\varphi} \max(1^\circ) = \frac{dS_{SM}}{d\varphi} \max \cdot \text{arc } 1^\circ = 32{\cdot}52\,\text{cm}/1^\circ \tag{16}$$

4. POSITION FIXING. As has been stated in Section 1, navigational devices such as logs and radars are calibrated in INMs. On the other hand, distances which can be measured on nautical charts are in SMs – two different measures meet together. Thus errors in manual position fixing, such as in dead reckoning or radar position fixes, of to $\pm 0{\cdot}5\%$

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of the distance, can be expected. Manual position fixing is still required as a backup for electronic position fixing.

Modern navigational devices are more accurate than simple mechanical or electromechanical devices in terms of the definition of the INM. What is more, these devices are microprocessor controlled and optionally switched calibration in sea miles is possible.

In logs, current latitude should be entered from, for instance a Global Positioning System (GPS) receiver via a NMEA input (National Marine Electronics Association – NMEA is a specification commonly used for communication between marine navigation devices) and current S_{SM} can be calculated with regard to Equation (7) or from the table of S_{SM} as a function of latitude taking into consideration Equations (14)–(16). Fortunately, the extreme of the derivative is at the longitudes near the smallest difference between the INM and the SM.

Radars and Automatic Radar Plotting Aids (ARPAs) are often connected with position fixing devices and for manual radar position fixes only the distance from a Variable Range Marker (VRM) should be optionally calculated in SM according to:

$$D_{SM} = \frac{D_{INM} \cdot 1852}{S_{SM}} \tag{17}$$

where D_{SM} is the distance measured in sea miles, D_{INM} is the distance measured in INM and S_{SM} is from Equation (7) or from the table as in logs.

It is also possible to produce an equivalent graphical solution on future large-scale paper nautical charts by printing a double scale in SM and INM.

5. ORTHODROMIC DISTANCE IN SEA MILES. Orthodromic distances on the ellipsoid (for example from Equation (1)) are calculated in metres (since the ellipsoid WGS-84 is defined in metres) and results - where necessary - are converted to and from INM.

For orthodromic distances in SM the following iterative procedure can be used:

$$\begin{aligned}
&S = \mathrm{IGP}(\varphi_1, \lambda_1, \varphi_2, \lambda_2)\\
&C_{gsi} = \mathrm{IGP}(\varphi_1, \lambda_1, \varphi_2, \lambda_2)\\
&D_{SM} = 0\\
&S_i = 0\\
&\varphi_i = \varphi_1; \lambda_i = \lambda_1\\
&\mathrm{DO}\\
&\quad \varphi_{2i}, \lambda_{2i} = \mathrm{DGP}(\varphi_i, \lambda_i, S_{SM}(\varphi_i), C_{gsi})\\
&\quad C_{gei} = \mathrm{DGP}(\varphi_i, \lambda_i, S_{SM}(\varphi_i), C_{gsi})\\
&\quad S_i = S_i + S_{SM}(\varphi_i)\\
&\quad D_{SM} = D_{SM} + 1\\
&\quad \varphi_i = \varphi_{2i}; \lambda_i = \lambda_{2i}\\
&\quad C_{gsi} = C_{gei}\\
&\mathrm{LOOP\ UNTIL}\ S_i > S\\
&D_{SM} = D_{SM} + (S - S_i)/S_{SM}(\varphi_i)
\end{aligned} \tag{18}$$

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Table 2. Errors of the procedure for orthodromes in sea miles

φ_1 [°]	φ_2 [°]	D′ [′]	D_{SM} [SM]	$D' - D_{SM}$ [SM]	S [INM]
30	50	1,200·000	1,199·986	0·014	1,199·100
70	90	1,200·000	1,200·016	−0·016	1,205·715
0	45	2,700·000	2,699·982	0·018	2,691·655
45	90	2,700·000	2,700·011	−0·011	2,708·975
0	90	5,400·000	5,399·993	0·007	5,400·629

where D_{SM} is the distance in SMs and $S_{SM}(\varphi_i)$ is the length of the SM from Equation (7) at latitude φ_i.

In this procedure, with the direct geodetic problem solution, the consecutive points on the orthodrome in steps of the length of the SM at the current latitude are calculated and concurrently the distance from the departure point is counted in metres as S_i and the number of SM is counted as D_{SM}. The procedure ends when S_i is bigger than S – the orthodromic distance - and finally D_{SM} is corrected for the last step.

Equation (7) is simplified and errors of this simplification in the above procedure are integrated. These errors can be easily revealed for meridional orthodromes because for these orthodromes the correct value is equal to the difference of latitudes in minutes. As can be seen from Table 2 these errors are less then ±0·02 SM even when the first two positions in this table are for latitudes for which the error Equation (8)–Equation (7) has extremes (Figure 3).

6. LOXODROMIC DISTANCE IN SEA MILES.

The loxodromic distance in Equation (3) is calculated from:

$$S_{lx} = \left| \frac{S_M(\varphi_1, \varphi_2)}{\cos C_{glx}} \right| \tag{19}$$

where $S_M(\varphi_1, \varphi_2)$ is the meridian distance between latitudes φ_1 and φ_2 and this distance in SM can be calculated (as in Section 5) as the difference of latitudes in minutes. Therefore, the procedure for the loxodromic distance is as follows:

$$\begin{aligned}
&\text{IF } \varphi_1 = \varphi_2 \text{ THEN} \\
&\quad S_{lx}\,[\text{SM}] = \frac{S_{lx}\,[\text{m}]}{S_{SM}(\varphi_1)} \\
&\text{ELSE} \\
&\quad S_{lx}\,[\text{SM}] = \left| \frac{(\varphi_2 - \varphi_1) \cdot 60}{\cos C_{glx}} \right| \\
&\text{ENDIF}
\end{aligned} \tag{20}$$

In the case $\varphi_1 = \varphi_2$, the loxodrome is latitudinal at a constant latitude.

7. EXEMPLARY REAL ROUTES.

The first exemplary route is from Cape Horn (S 55°59′, W 67°17′) to Sydney (S 33°50′, E 151°17′). This route is intentionally at higher

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latitudes to magnify all differences. For this route:

$$S = 5{,}077{\cdot}7 \text{ INM} = 5{,}064{\cdot}9 \text{ SM} - \text{difference } -0{\cdot}25\%$$

$$S_{lx} = 6{,}063{\cdot}8 \text{ INM} = 6{,}063{\cdot}2 \text{ SM} - \text{difference } -0{\cdot}01\%$$

Although the maximum latitude for this orthodrome is high for marine navigation (73·1°), the mean latitude for this loxodrome is near 45° hence the small difference between the length of the loxodrome in INM and SM.

The second exemplary route - intentionally at lower latitudes - is from Manta in Ecuador (S 00°56′, W 80°43′) to Jayapura on New Guinea (S 02°32′, E 140°43′). The results are:

$$S = 8{,}320{\cdot}8 \text{ INM} = 8{,}361{\cdot}5 \text{ SM} - \text{difference } 0{\cdot}47\%$$

$$S_{lx} = 8{,}325{\cdot}4 \text{ INM} = 8{,}366{\cdot}4 \text{ SM} - \text{difference } 0{\cdot}49\%$$

8. CONCLUSIONS.

Simplification of the sea mile dependent on latitude to the constant international nautical mile was justified in 1929 when it was defined. Modern navigation devices are more accurate and more powerful and consequently this simplification is no longer necessary. The proposed algorithms for the calculation of sea miles in different sailing methods can be easily incorporated in modern microprocessor controlled navigational devices, so that both miles can exist in the same device. An equivalent graphical solution on selected nautical charts is also proposed.

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