Book Review

Michael J. Best, *Portfolio Optimization*, Chapman & Hall/CRC Finance, 2010, 236pp. (hardback), £49.99. ISBN: 9781420085846

This book should really be called 'Quadratic Optimisation with Applications in Portfolio Optimisation'. Readers looking for either any discussion of advanced topics such as tail correlations and skewness, or basic topics such as constructing a vector of expected returns and a variance-covariance matrix given returns data, should look elsewhere. That said, this book does provide an applications focused overview of the relevant parts of optimisation theory, with code for solving problems in MATLAB, and with references to more theoretical optimisation results (though not to more advanced portfolio theory).

If the reader accepts that:

- an optimal portfolio involves a trade off between expected return and variance (i.e. that variance is the investor's definition of risk);

- the expected return of the portfolio can be expressed $\underline{\mu}^T\underline{x}$, and the variance of the portfolio can be expressed as the quadratic form $\frac{1}{2}\underline{x}^T\Sigma\underline{x}$ (where $\underline{\mu}$ is the vector of expected returns of each asset, $\underline{x}$ is the vector of proportions of the portfolio invested in each asset, and Σ is the variance-covariance matrix of the assets); and

- $\underline{\mu}$ and Σ are given;

then an optimal portfolio is one which minimises the quadratic function (of n variables, where n is the number of assets), $f(\underline{x}) = -t\underline{\mu}^T\underline{x} + \frac{1}{2}\underline{x}^T\Sigma\underline{x}$ (where the parameter t is the relative weight given to expected return, when the weight on variance is one, and so is related to degree of risk aversion). Accepting this at the outset, the book goes on to provide some useful algorithms for portfolio optimisation in these circumstances.

The book starts by giving us some basic theory for n variable quadratic minimisation under linear equality constraints. The value of this theory is then demonstrated in the next chapter in which $f(\underline{x})$ is minimised subject to $x_1 + \ldots + x_n = 1$ (the total proportion must be 100%) to derive the efficient frontier. Subsequent chapters introduce a risk free asset, uncorrelated with any of the risky assets, to derive the Capital Market Line, the Security Market Line, and the Sharpe Ratio. In all of this MATLAB programs are provided so that the reader can apply these tools to their own asset market data.

The book then develops (finite) algorithms for solving quadratic optimisation problems with linear inequality constraints: quadratic programming. The applications described here are no short selling ($x_i \geq 0$), limits within particular industries, and limits to the change from an existing portfolio. The algorithms are easy enough to solve simple ($n = 2$) examples on paper while the MATLAB code allows more involved examples to be tackled. In particular, it is shown that a modified efficient frontier can be generated.

This book will be useful to four groups of people (and the last two of these groups have not really been pitched to with the book's title):

- students studying a course similar to the courses taught by the author (though they may also need access to texts on linear algebra, the statistics of constructing their data, and extensions in portfolio theory);

- practitioners who are willing to accept the basic tenets of Markowitz Mean-Variance portfolio optimisation theory and who want access to the tools for applying it, as well as the theory behind these tools (again they may also want to combine this book with texts on linear algebra, the statistics of constructing their data, and extensions in portfolio theory);

- anyone who has access to a computer with MATLAB and who wants some basic programs (that achieve something fairly complex) with which to learn to use this platform;

- anyone who has a quadratic optimisation problem – the methods in this book would generalise to any quadratic optimisation problem, while the finance application described here is really just an application rather than a reason d'etre.

David Comerford

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