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A Multi-Station Troposphere Modelling Method Based on Error Compensation Considering the Influence of Height Factor

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One critical issue in network real-time kinematic (NRTK) is the interpolation of atmospheric delay for user stations. Some classic interpolation algorithms, such as linear interpolation method (LIM), ignore the strong correlation between tropospheric delay and height factors, and the interpolation accuracy is poor in areas with large height difference. To solve this problem, a troposphere modelling method based on error compensation, namely ECDIM (Error Compensation-Based DIM), is proposed, and this method can be applied to both conventional single Delaunay triangulated network (DTN) and multi-station scenarios. The results of California Real Time Network (CRTN) with large height difference show that compared with LIM, the overall modelling accuracy with ECDIM has been improved by 50.1% to 67.3%, and especially for low elevation satellites (e.g., 10–20 degree), the accuracy is increased from tens of centimetres to a few centimetres. At user end, the positioning error in up direction with LIM has an obvious systematic deviation, and the fix rate of epoch is relatively low. This situation has been improved significantly after using ECDIM. The results of Tianjin Continuously Operating Reference System (TJCORS) show that in areas with small height difference, both methods have achieved high precision interpolation accuracy, and the positioning accuracy with ECDIM in up direction is improved by 21.2% compared with LIM.

KEYWORDS

1. Network Real-Time Kinematic (NRTK). 2. Troposphere Modelling. 3. Error Compensation. 4. Multi-Station. 5. Height Factor. 6. Positioning Accuracy.

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1. INTRODUCTION.

Network real-time kinematic (NRTK) positioning is currently a popular technique for real-time precise positioning, and has been widely applied in the field of land navigation and intelligent transport systems (Rizos and Han, 2003; Aponte et al., 2008; Meng et al., 2008). Virtual reference station (VRS) is one of the commonly used NRTK data processing technologies, and its critical issue is the interpolation of double-differenced (DD) distance-dependent bias for user stations, such as tropospheric delay (Wanninger, 1999). The Delaunay triangulated network (DTN) is usually adopted in NRTK due to its uniqueness and efficiency, based on the reliable ambiguity resolution (AR) between the reference stations in a network (Li et al., 2014a, 2014b; Pan et al., 2015; Gao et al., 2017b), the atmospheric errors of each baseline are extracted, and the correction of rover stations can be interpolated according to their geographical locations.

In the past few years, several classic algorithms have been proposed successively, such as linear combination method (LCM) (Han and Rizos, 1996), linear interpolation method (LIM) (Wanninger, 1995), distance-based linear interpolation method (DIM) (Gao and Li, 1998), lower-order surface model (LSM) (Fotopoulos, 2000), and least-squares collocation method (Odijk et al., 2000). Dai et al. (2003) conducted a comparative study on the abovementioned interpolation methods, and concluded that their performances are very similar. Wu (2009) used data from GPSnet in Australia to analyse the performance of LIM, DIM and LSM comprehensively, and the results showed that the LIM is slightly better than the DIM, and the DD residuals of LSM are much worse than those of LIM and DIM. Similarly, Al-Shaery et al. (2011) investigated different interpolation models through data from CORSnet-NSW, and a range of 1·9 to 6·5 cm of horizontal positioning accuracy was achieved. Gao et al. (2002) proposed a combined bias interpolation (CBI) method including ionospheric and tropospheric errors. Based on this, Tang et al. (2013) proposed a modified CBI method (MCBI) in sparse NRTK, which decomposed the combined errors into tropospheric delay, ionospheric delay and residual error, respectively. Their results indicated that, with an average inter-station distance of up to 300 km, the positioning performance is consistent with the traditional NRTK with an average inter-station distance of about 50 km. Zou et al. (2018) proposed an undifferenced NRTK augmentation information generation method by properly choosing the additional datum, and the AR fixing rates and positioning accuracy have been significantly improved with dynamic atmospheric constraints.

The abovementioned methods directly model the original tropospheric delay, usually only considering the horizontal variation characteristic of the troposphere, but ignoring the characteristic related to height. Yin et al. (2008) pointed out that the tropospheric zenith delay is strongly correlated with height of stations, and the relationship between them appeared to be an approximately negative exponent. Therefore, the interpolation accuracy of these methods is poor in areas with large height difference. To deal with this problem, Wang et al. (2012) proposed a modified LCM method based on the Niell model and achieved centimetre-level modelling accuracy. Wu et al. (2015) proposed a troposphere interpolation method based on star network, and pointed out that the interpolation accuracy of multi-baselines is better than that of conventional two baselines in single DTN. Further, Shang et al. (2017) proposed a revised elevation linear interpolation model in multi-redundancies NRTK, and the interpolation accuracy of low elevation satellites was significantly improved.

The above studies have achieved certain results in regional troposphere modelling, and aiming at the effect of height difference on the troposphere, some scholars have also done some meaningful research. Based on the relevant conclusions of the above studies, this paper comprehensively considers the correction effect of the prior troposphere model. By modelling and compensating the error of the prior model instead of the original tropospheric delay, an error-compensated troposphere model based on conventional DIM is derived. The proposed method can be applied to both conventional single DTN and multi-station scenarios, and the troposphere modelling accuracy and user positioning performance are better than that of traditional models in both areas with large and small height differences.

The article is structured as follows. Traditional interpolation models are briefly introduced in Section 2. The mathematical model of the proposed method in this paper is elaborated in Section 3. The experimental results and analysis of two sets of data are then detailed in Section 4. A summary of the results is given in Section 5.

2. TRADITIONAL MODEL.

The critical issue for VRS is the generation of high precision virtual observations. The specific expression is as follows:

$$\begin{cases} P_u^s = P_m^s + \Delta\rho_{mu}^s - \left(\Delta\nabla T_{mu}^{is} + \Delta\nabla I_{mu}^{is}\right) \\ L_u^s = L_m^s + \Delta\rho_{mu}^s - \left(\Delta\nabla T_{mu}^{is} - \Delta\nabla I_{mu}^{is}\right) \end{cases} . \tag{1}$$

where u, m and s represent user, main reference station and satellites, respectively; P and L are pseudorange and carrier phase observations in the unit of length; ρ is geometric range between receiver and satellite; T is tropospheric delay; I is ionospheric delay; Δ and ∇ are between-station single-difference (SD) and between-satellite single-difference operator, respectively. The superscript i indicates reference satellite.

In Equation (1), the key to ensure the high accuracy of virtual observations is the interpolation method of DD atmospheric delay between the main station and the user. Over the past few years, in order to represent this spatially-correlated error, several methods have been proposed, such as LCM, LIM, DIM etc. As shown in Figure 1, the DTN where the user is located is usually selected as the optimal triangle and the reference station closest to the user is chosen as the main station. The two baselines connected with the main station can be used to interpolate the atmospheric errors at the user's end. Figure 2 shows the troposphere interpolation results of several commonly used methods at a station in Tianjin, China. In single DTN, both LIM and LCM are two-dimension plane interpolation models, and they are mathematically equivalent, therefore, their interpolation results are equal. It can also be found that their results are similar to that of DIM, with only slight differences. Thus one model, such as LIM, was randomly chosen as a representative for subsequent experimental analysis.

3. ERROR COMPENSATION-BASED DIM.

For a regional reference network, the slant tropospheric delay of each station can be expressed as follows.

$$T_r^s = \tilde{T}_r^s + dT_r^s \tag{2}$$

where r represents reference stations; T_r^s is the true value of the tropospheric delay at reference stations; $\tilde{T}_r^s$ is the tropospheric delay calculated by a prior model, such as Saastamoinen (Saastamoinen, 1972), UNB3 m (Leandro et al., 2008) etc. dT_r^s is the corresponding error of a particular prior model.

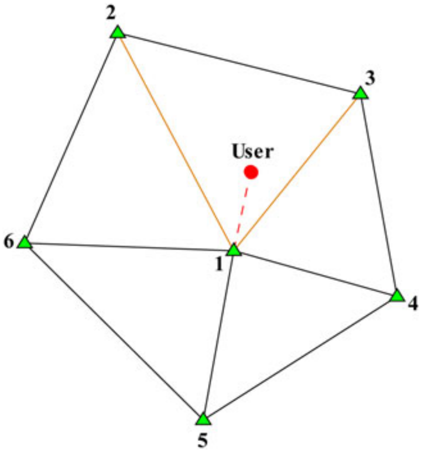


Figure 1. Regional reference stations and user distribution. Green triangles = base stations.

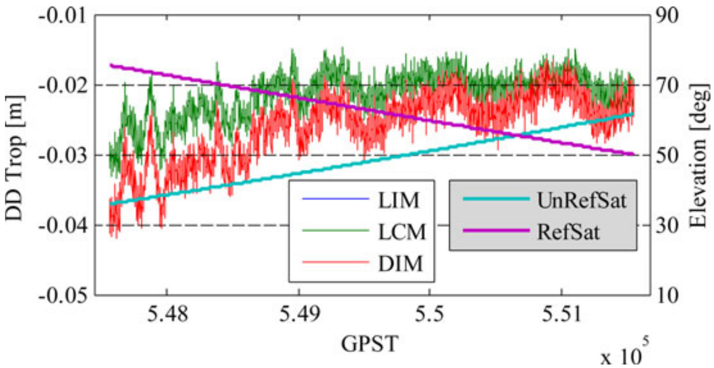


Figure 2. Results of several common troposphere modelling methods: LIM, LCM and DIM, at a station in Tianjin, China. UnRefSat = un-reference satellite; RefSat = reference satellite. Since the results of LIM and LCM are the same, the corresponding two curves coincide.

Considering the spatially-correlated characteristics of the troposphere, the tropospheric delay of the user in the network can be obtained by interpolating that of nearby reference stations.

$$T_u^s = \sum_{k=1}^{n} a_k T_k^s \tag{3}$$

where T_u^s represents the tropospheric delay at user station and a_k represents interpolation coefficients of the model and satisfies the following equation.

$$\begin{cases} \sum_{k=1}^{n} a_k = 1 \\ a_k = \dfrac{1/d_k}{\sum_{k=1}^{n}(1/d_k)} \end{cases} \tag{4}$$

where d_k is the distance between user and reference station k.

The abovementioned zero-difference troposphere modelling method has been widely applied in regional augmented precise point positioning (PPP) (Li et al., 2011; Jiang et al., 2012). Considering that the magnitude of the original tropospheric delay is large, but, in contrast, the magnitude of residual tropospheric delay after correction by a prior model is generally small, since the prior model can mitigate most parts of the tropospheric delay. Compared with directly modelling the original troposphere, modelling the errors of the prior model is expected to further reduce the interpolation errors due to its smaller magnitude. So, the above formula is rewritten as follows:

$$\begin{cases} dT_u^s = \sum_{k=1}^{n} a_k dT_k^s \\ T_u^s = \tilde{T}_u^s + dT_u^s \end{cases} \tag{5}$$

In NRTK the regional reference network can usually track several common-view satellites synchronously. For convenience of description, the symbols i and j are used to indicate reference satellite and un-reference satellite, respectively. The between-satellite SD form of formula (5) is as follows.

$$\begin{cases} \nabla dT_r^{ij} = \nabla T_r^{ij} - \nabla \tilde{T}_r^{ij} \\ \nabla dT_u^{ij} = \sum_{k=1}^{n} a_k \nabla dT_k^{ij} \\ \nabla T_u^{ij} = \nabla \tilde{T}_u^{ij} + \nabla dT_u^{ij} \end{cases} \tag{6}$$

As mentioned in the previous section, one critical issue in NRTK is the interpolation of the distance-dependent errors for user stations, mainly atmospheric delay. As shown in Figure 1, the base station one ($r = 1$) is selected as the main station, and the DD tropospheric delay between main station and user is expressed as follows:

$$\Delta\nabla T_{1u}^{ij} = \nabla T_u^{ij} - \nabla T_1^{ij} \tag{7}$$

Insert Equation (6) into Equation (7), and the following equation is derived.

$$\Delta\nabla T_{1u}^{ij} = \nabla \tilde{T}_u^{ij} + \sum_{k=1}^{n} a_k \left(\nabla T_k^{ij} - \nabla \tilde{T}_k^{ij} \right) - \nabla T_1^{ij} \tag{8}$$

Considering that the coefficient a_k satisfies the relationship in Equation (4), $\nabla \tilde{T}_u^{ij}$ can be equivalently expressed as the following form.

$$\nabla \tilde{T}_u^{ij} = \sum_{k=1}^{n} a_k \nabla \tilde{T}_u^{ij} \tag{9}$$

Further, insert Equation (9) into Equation (8), after simplifying the equation, and the final interpolation model is obtained.

$$\begin{aligned}\Delta\nabla T_{1u}^{ij} &= \sum_{k=1}^{n} a_k \nabla \tilde{T}_u^{ij} + \sum_{k=1}^{n} a_k \left(\nabla T_k^{ij} - \nabla \tilde{T}_k^{ij}\right) - \nabla T_1^{ij} \\ &= \sum_{k=1}^{n} a_k \nabla \tilde{T}_u^{ij} + \sum_{k=1}^{n} a_k \nabla T_k^{ij} - \sum_{k=1}^{n} a_k \nabla \tilde{T}_k^{ij} - \nabla T_1^{ij} \\ &= \left(a_1 \nabla T_1^{ij} - \nabla T_1^{ij}\right) + \sum_{k=2}^{n} a_k \nabla T_k^{ij} + \left(\sum_{k=1}^{n} a_k \nabla \tilde{T}_u^{ij} - \sum_{k=1}^{n} a_k \nabla \tilde{T}_k^{ij}\right) \\ &= \left(\sum_{k=2}^{n} a_k \nabla T_k^{ij} - \sum_{k=2}^{n} a_k \nabla T_1^{ij}\right) + \sum_{k=1}^{n} a_k \Delta\nabla \tilde{T}_{ku}^{ij} \\ &= \sum_{k=2}^{n} a_k \Delta\nabla T_{1k}^{ij} + \sum_{k=1}^{n} a_k \Delta\nabla \tilde{T}_{ku}^{ij}\end{aligned} \tag{10}$$

where the first term on the right side of the equation is the DD tropospheric delay over the baselines formed by main station and other reference stations, which can be retrieved from the residuals of the ionosphere-free observation based on reliable AR between reference stations. The second term is the model value of the DD tropospheric delay on baselines formed by user and nearby reference stations, which can be obtained through empirical models such as UNB3m, GPT2w (Böhm et al., 2015), etc.

The above formula is a general form of expression. Considering the case of conventional single DTN atmosphere interpolation where the number of reference stations is three, the above formula can be simplified as follows:

$$\Delta\nabla T_{1u}^{ij} = a_2 \Delta\nabla T_{12}^{ij} + a_3 \Delta\nabla T_{13}^{ij} + \left(a_1 \Delta\nabla \tilde{T}_{1u}^{ij} + a_2 \Delta\nabla \tilde{T}_{2u}^{ij} + a_3 \Delta\nabla \tilde{T}_{3u}^{ij}\right) \tag{11}$$

The equation of the traditional DIM is as follows:

$$\Delta\nabla T_{1u}^{ij} = a_2 \Delta\nabla T_{12}^{ij} + a_3 \Delta\nabla T_{13}^{ij} \tag{12}$$

By comparing Equations (11) and (12), it is not difficult to find that the method proposed in this paper additionally adds the DD tropospheric correction between user and reference stations on the basis of traditional DIM. More importantly, the proposed method is not only applicable for conventional single DTN, but can also be extended to the case of multi-reference stations.

The model in Equation (10) is based on the idea of modelling and compensating the error of the empirical model, therefore, this method is referred to as ECDIM (error compensation-based DIM) in subsequent sections for convenience of description.

4. DATA AND EXPERIMENTS.

Two experiments were carried out, as listed in Table 1, and the station distribution is shown in Figure 3. The reference station coordinates of the California Real Time Network (CRTN) were obtained from Scripps Orbit and Permanent Array Center (SOPAC) in California, USA (Crowell et al., 2009). The network

Table 1. Details of the experiment data.

#	Time	Interval	Location	Ref. No.	Rover No.	Distance
I	Doy274, 2019	30 s	California, USA	15	16	78 km
II	Doy294, 2017	1 s	Tianjin, China	5	1	58 km

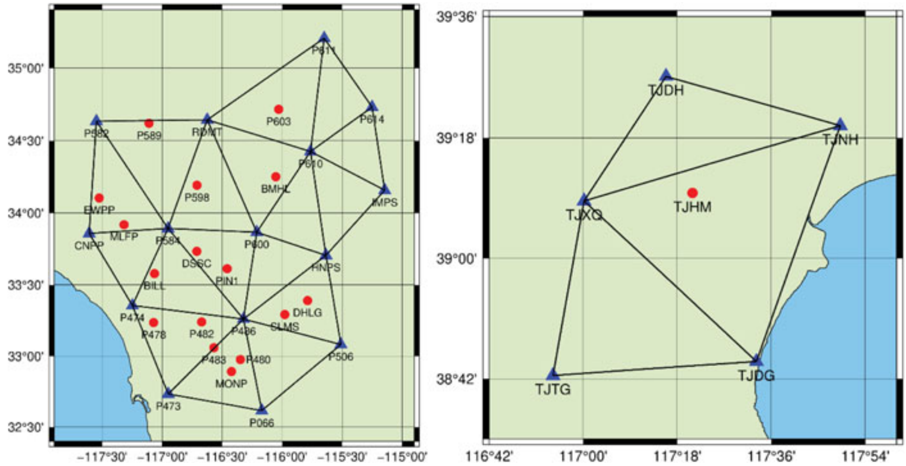


Figure 3. Distribution of reference network and rover stations for CRTN (left) and TJCORS (right). *Blue triangles* denote reference stations; *red dots* denote rover stations.

solution from the GAMIT-GLOBK (version 10·5) software package with an accuracy of millimetres was used as the reference coordinates of stations in the Tianjin Continuously Operating Reference System (TJCORS) (Herring et al., 2010). The overall data processing is divided into two parts: server and user. At the server end, through reliably AR between reference stations, the data is processed to extract the atmospheric errors of each baseline and further generate virtual observation according to the user's approximate position. At the user end, the terminal obtains its own positon by ultra-short baseline resolution with virtual observation from the server. In data processing, the least-square ambiguity decorrelation adjustment (LAMBDA) method is used to fix DD ambiguities (Teunissen, 1995). To ensure that ambiguities are fixed correctly, the thresholds for ratio and bootstrapping success rate are set to 3·0 and 0·999, respectively (Teunissen, 1998; Teunissen and Verhagen, 2009). In addition, partial ambiguity resolution for a subset of ambiguities selected according to the successively increased elevation and continuous tracking number is applied (Li et al., 2014a; Gao et al., 2017a). In the troposphere interpolation process of ECDIM, all baselines connected with the main station are used, and UNB3 m is selected as the prior model. In order to compare the performance of the traditional model and the model proposed in this paper, both the troposphere modelling errors and positioning accuracy at the user end are evaluated respectively. The true value of tropospheric delay is obtained in advance through baseline resolution between main station and rover stations, and the position of the user is estimated as white noise with variance of 50^2 m^2.

4.1. *Results of CRTN.* Figure 4 shows the station height distribution of CRTN. From the figure, it can be seen that the height difference between stations is very large, the

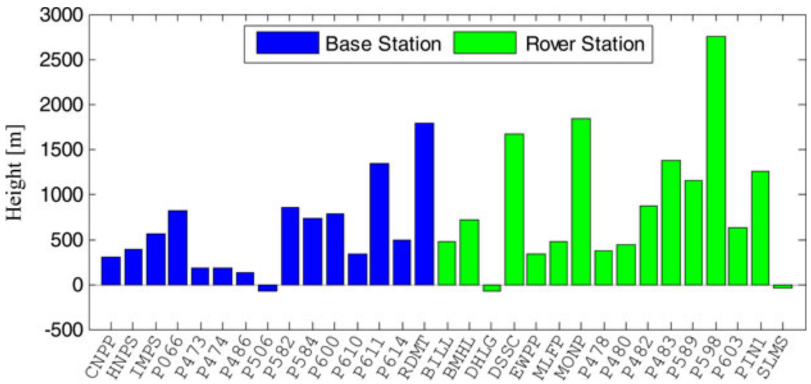


Figure 4. Station height distribution of CRTN.

maximum exceeds 2,000 m, which may significantly affect the accuracy of troposphere modelling, and further affect the positioning performance at the user end.

4.1.1. *Results of troposphere modelling.* In order to analyse the troposphere modelling accuracy specifically with different height differences, the six user stations listed in Table 2 are selected as an example to analyse the troposphere interpolation accuracy of satellites at different elevation angles. Figures 5 and 6 show the troposphere interpolation results of several specific satellite pairs in a certain period. It is not difficult to find from the figures that, for satellites with different elevations, the troposphere interpolation results of LIM and DIM are similar, and the results for $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ are also very similar. For low elevation angle satellites, the interpolated troposphere delay obtained from the traditional LIM and DIM is far from the true value, sometimes the deviation even reaches tens of centimetres, while the model value calculated by $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ is in good agreement with the true value. Table 3 gives detailed statistics for each station. According to these statistics, the average modelling accuracy of low elevation satellites for LIM and DIM is 0·130 m and 0·125 m respectively, and the corresponding results for $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ are 0·024 m and 0·026 m respectively. There is only a difference of millimetre level. Compared with LIM, the accuracy of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ is improved by 81·5% and 80·0%, respectively. For medium elevation satellites, the magnitude of the DD tropospheric delay is significantly smaller than that of low elevation satellites, and the average modelling accuracies of LIM and DIM are both 0·033 m. Although LIM and DIM achieved an accuracy of a few centimetres, the trends are still quite different from that of the true value. The interpolated results of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ are still consistent with the trend of the true value, and the average modelling accuracies are both 0·008 m. Compared with LIM and DIM, the accuracy of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ is improved by 75·8%. For high elevation angle satellites, the magnitude of DD tropospheric delay is only a few centimetres, and all the four methods can achieve good interpolation results. However, in terms of change trend, the results of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ match the true value better than LIM and DIM, as shown in Figures 5 and 6. The average modelling accuracies of LIM and DIM are both 0·009 m, and the corresponding accuracies of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ are both 0·004 m. Compared with LIM and DIM, the accuracies of $\mathrm{ECDIM_{tri}}$ and $\mathrm{ECDIM_{star}}$ are improved by 55·6%.

Table 2. Detailed information of the selected user stations.

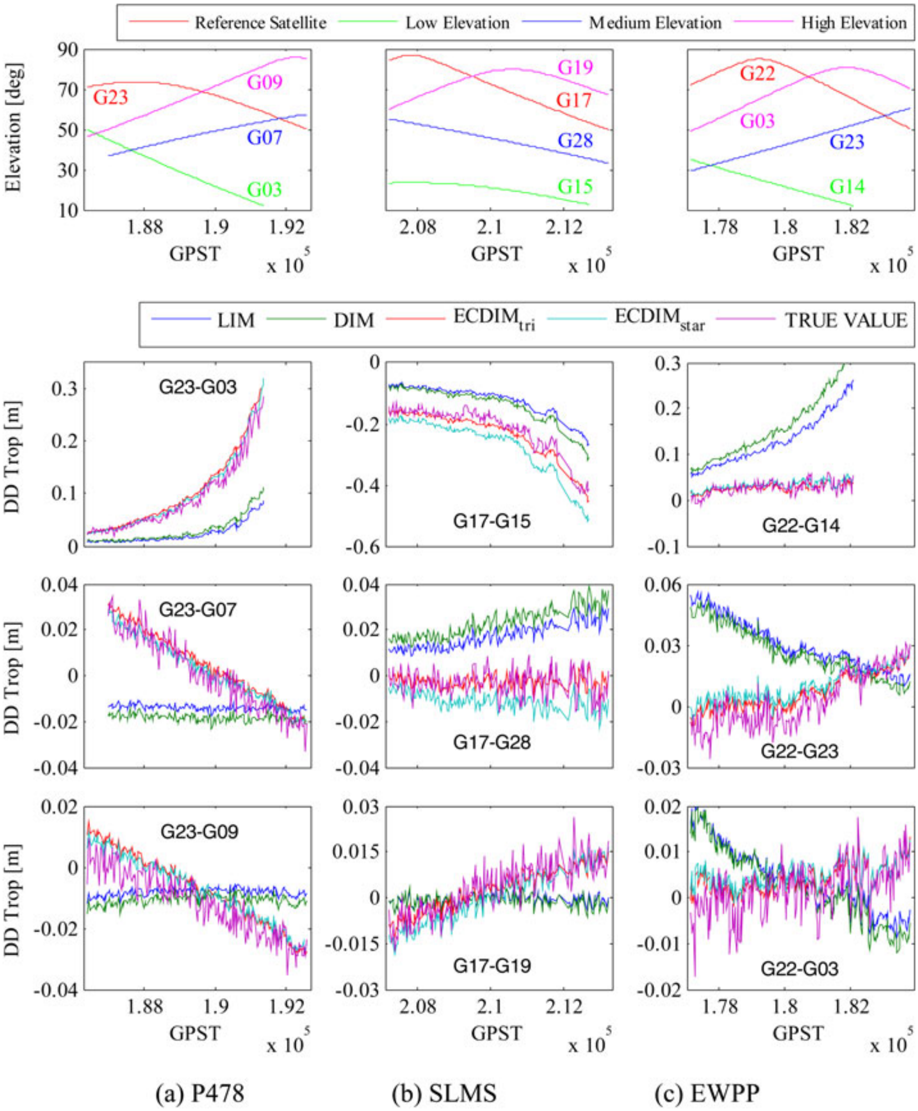
Station	Main station	Location cell	Height difference (m)	# of baseline used LIM/DIM	$\text{ECDIM}_{\text{tri}}$	$\text{ECDIM}_{\text{star}}$
P478	P474	P474 → P473 → P486	245	2	2	4
SLMS	P486	P486 → P506 → HNPS	477	2	2	7
EWPP	CNPP	CNPP → P584 → P582	548	2	2	3
P480	P486	P486 → P473 → P066	697	2	2	7
PIN1	P600	P600 → P584 → P486	1130	2	2	5
P603	P610	P610 → P611 → RDMT	1439	2	2	6

Note: $\text{ECDIM}_{\text{tri}}$ indicates that the proposed method used the same two baselines as the LIM and DIM in traditional single DTN. $\text{ECDIM}_{\text{star}}$ indicates that the proposed method used all baselines connected with the main station in a star network.

Since LIM and DIM usually consider the horizontal variation characteristic of the troposphere, the magnitude of the two methods is approximately the same, which can also be seen from Figure 2, with only slight differences. So the subsequent analysis takes the LIM as a representative. In addition, from the above statistics and analysis, it can be seen that the interpolation results of $\text{ECDIM}_{\text{tri}}$ and $\text{ECDIM}_{\text{star}}$ are similar, and even for low elevation satellites, only millimetre-level difference is observed. Therefore, $\text{ECDIM}_{\text{star}}$ is used for subsequent analysis, and for convenience, ECDIM is used to denote $\text{ECDIM}_{\text{star}}$ unless otherwise stated.

In order to analyse the troposphere modelling accuracy of all satellites at different elevation angles during the entire observation period of the selected stations, Figure 7 shows the variation curves of the modelling error of all satellites with elevation angle and the RMS statistics of different elevation angle intervals. As shown in Figure 7, in general, the accuracy of troposphere modelling increases with rising elevation angle. For different elevation angle intervals, the interpolation accuracy of ECDIM is always better than that of LIM, and the difference between the two methods decreases with increasing elevation angle. For LIM, the interpolation error of the high elevation angle satellites is small, but the interpolation accuracy of the low elevation angle satellites is very poor; taking station PIN1 as an example, the deviation from the true value can even reach 0·8 m. The mean values of the interpolation error at the selected six stations are −0·037 m, 0·034 m, 0·040 m, −0·024 m, −0·150 and 0·080 m respectively, and the corresponding standard deviations are 0·040 m, 0·037 m, 0·043 m, 0·027 m, 0·159 m and 0·085 m respectively. The interpolation accuracy has improved significantly with ECDIM, especially for low elevation angle satellites. The mean values for different stations are 0·008 m, −0·020 m, 0·001 m, −0·007 m, 0·024 and −0·003 m respectively, and the corresponding standard deviations are 0·012 m, 0·022 m, 0·010 m, 0·015 m, 0·026 m and 0·008 m, respectively. Compared with LIM, the mean value of the ECDIM is closer to 0, and the standard deviations of the selected six stations are reduced by 70·0%, 40·5%, 76·7%, 44·4%, 83·6%, and 90·6%, respectively.

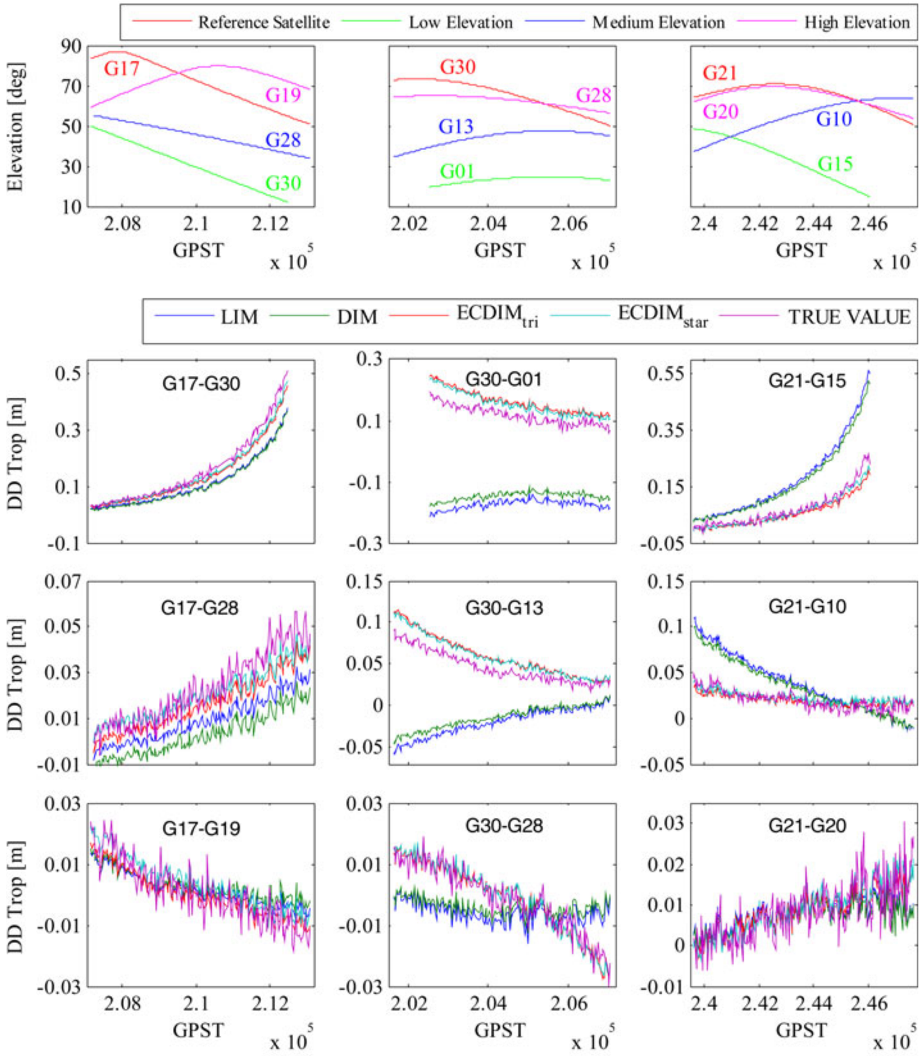
The height difference between the above selected stations ranges from about 200 m to more than 1,400 m. In this case, the conventional LIM does not consider the effect of height factor, so the modelling accuracy is poor, especially for low elevation angle satellites. Taking station PIN1 and P603 as an example, the height difference of the cell exceeds 1,000 m, and the deviation between the tropospheric delay of the low elevation angle interpolated by LIM and the true value is close to 0·5 m and 0·8 m respectively. In addition, similar conditions occurred in the other four stations, and the modelling error of



(a) P478 (b) SLMS (c) EWPP

Figure 5. Troposphere interpolation results for satellites with different elevation angles at stations P478 (left), SLMS (centre) and EWPP (right).

the low elevation angle satellites is close to 0·2–0·3 m. To some extent, this indicates that the height difference has a significant impact on the accuracy of the troposphere modelling, but it does not mean that the larger the height difference, the larger the modelling error of conventional methods, such as LIM. A special case is the station BMHL, located in cell P610 → P600 → RDMT, and the maximum height difference between stations is nearly 1,400 m. However, the troposphere delay obtained by LIM and the true value do not show a significant systematic deviation, and the interpolation results of LIM and ECDIM are similar, as shown in Figure 8. With LIM, the mean and standard deviation of modelling error



(a) P480 (b) PIN1 (c) P603

Figure 6. Troposphere interpolation results for satellites with different elevation angles at stations P480 (left), PIN1 (centre) and P603(right).

is −0·0026 m and 0·0085 m respectively and the mean and standard deviation obtained from ECDIM is 0·0025 m and 0·00076 m respectively. Only minor difference is observed. According to the RMS statistics, the two methods differ only when the satellite elevation angle is low, and both methods achieve consistent modelling results.

In order to analyse further the troposphere modelling accuracy in the coverage area of the base station network, Figures 9–12 show the troposphere modelling accuracy of each user station corresponding to different elevation angle intervals. As can be seen

Table 3. Troposphere interpolation accuracy for satellites with different elevation angles at stations P478, SLMS, EWPP, P480, PIN1 and P603.

		Interpolation accuracy/m						
Elevation angle	Model	P478	SLMS	EWPP	P480	PIN1	P603	Average
Low elevation	LIM	0·084	0·099	0·114	0·059	0·295	0·129	0·130
	DIM	0·075	0·081	0·148	0·062	0·268	0·114	0·125
	$\mathrm{ECDIM_{tri}}$	0·018	0·023	0·009	0·026	0·046	0·023	0·024
	$\mathrm{ECDIM_{star}}$	0·013	0·057	0·011	0·020	0·038	0·015	0·026
Medium elevation	LIM	0·021	0·022	0·036	0·015	0·074	0·029	0·033
	DIM	0·024	0·027	0·033	0·021	0·068	0·025	0·033
	$\mathrm{ECDIM_{tri}}$	0·006	0·005	0·007	0·009	0·015	0·006	0·008
	$\mathrm{ECDIM_{star}}$	0·005	0·010	0·009	0·006	0·014	0·006	0·008
High elevation	LIM	0·011	0·009	0·011	0·006	0·011	0·005	0·009
	DIM	0·010	0·009	0·011	0·006	0·010	0·005	0·009
	$\mathrm{ECDIM_{tri}}$	0·006	0·004	0·004	0·004	0·003	0·004	0·004
	$\mathrm{ECDIM_{star}}$	0·006	0·005	0·005	0·005	0·003	0·003	0·004

from the figure, compared with the LIM, the ECDIM improves the troposphere modelling accuracy of different elevation intervals in all stations except BILL and BMHL by more than 20%. Through statistics of all stations, corresponding to 10–20 degree, 20–30 degree, 30–50 degree, 50–90 degree elevation angle interval, the average modelling accuracy of LIM is 0·344 m, 0·178 m, 0·072 m and 0·018 m, respectively. The modelling accuracy increases with rising elevation angle. The average modelling accuracy of ECDIM is 0·063 m, 0·034 m, 0·015 m and 0·005 m respectively. Compared with LIM, the average improvement corresponding to different elevation angle interval is 67·3%, 64·7%, 62·5% and 50·1% respectively. It is obvious that, as the elevation angle increases, the degree of improvement in accuracy gradually decreases.

4.1.2. *Results of user positioning.* Figure 13 shows the positioning error distributions of stations P478, SLMS, EWPP, P480, PIN1, and P603 using LIM and ECDIM, respectively. As can be seen, for station P478, SLMS, and EWPP, the results of the two methods are similar in terms of the distribution of horizontal positioning errors, and in terms of positioning deviation in the up direction, the error distribution obtained by the LIM has a systematic deviation. However, this phenomenon does not exist in the error distribution obtained by the ECDIM. For stations P480, PIN1, and P603, the positioning error obtained by the LIM shows a certain systematic deviation in both horizontal and up directions, and the positioning deviation in up direction especially shows very large fluctuations. It can also be found that with LIM the fixed rate of epoch is obviously lower than that of ECDIM. Table 4 gives the specific statistical results. It is worth mentioning that, because the accuracy of float solutions is uncontrollable, the results in the table only count the fixed solutions.

In terms of positioning accuracy, for stations P478, SLMS, EWPP and P480, the positioning accuracy in horizontal direction is similar, within range of 1–4 cm. Compared with LIM, the positioning accuracy with ECDIM has been improved slightly, ranging from 4·0% to 34·6%, except for the case where the statistical accuracy of the two methods at station SLMS in the north direction is the same. For positioning accuracy in the up direction, the accuracy with ECDIM is significantly better than that of LIM, and the accuracy has improved from a dozen centimetres to several centimetres, with improvement of 66·1%,

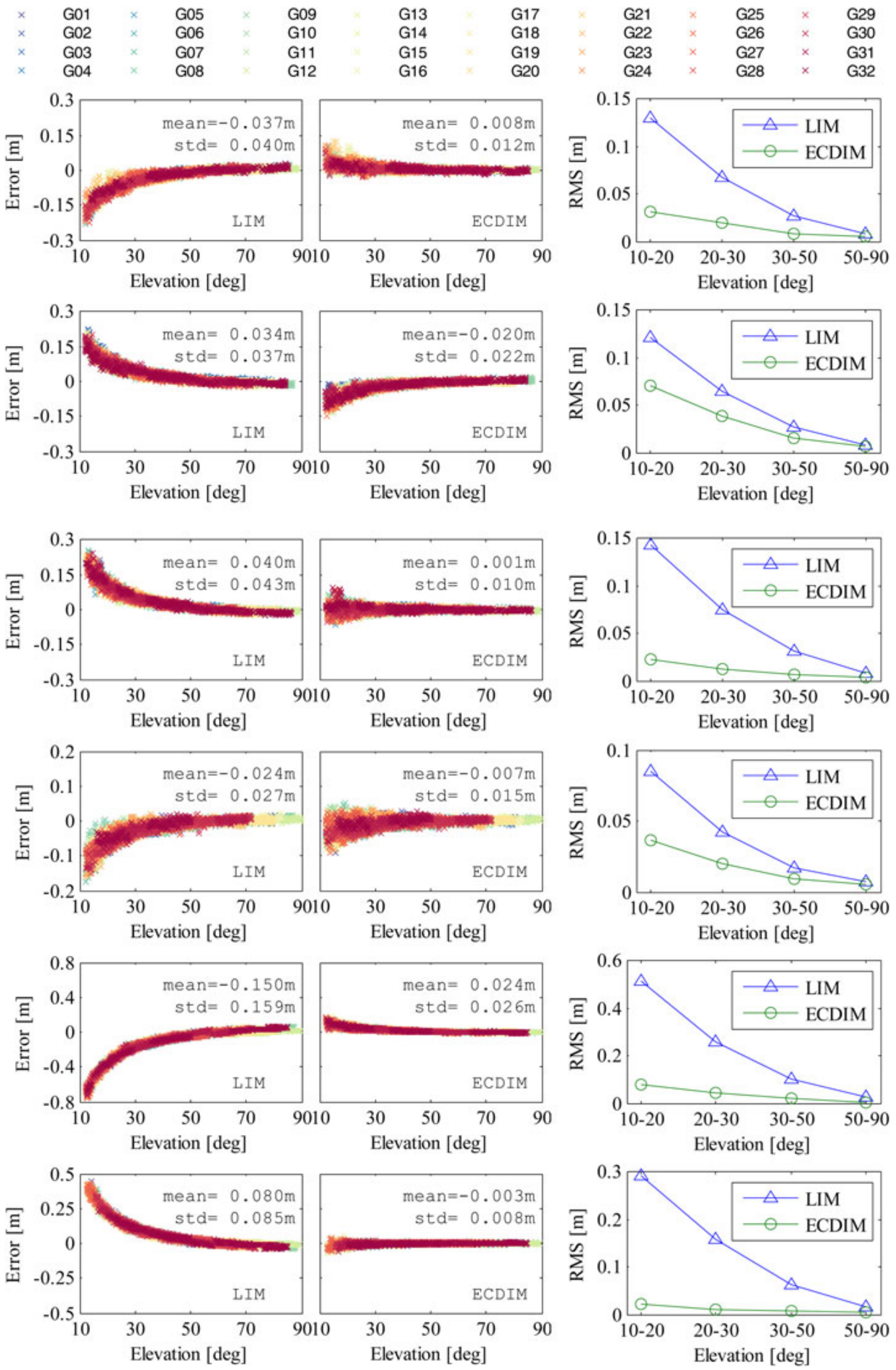


Figure 7. Troposphere interpolation error (left and centre) and RMS (right) with satellite elevation angle at different user stations (from top to bottom): P478, SLMS, EWPP, P480, PIN1 and P603.

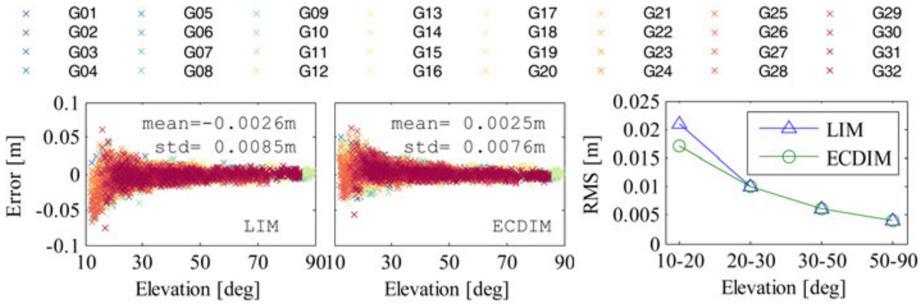


Figure 8. Troposphere interpolation error (left and centre) and RMS (right) with satellite elevation angle at station BMHL.

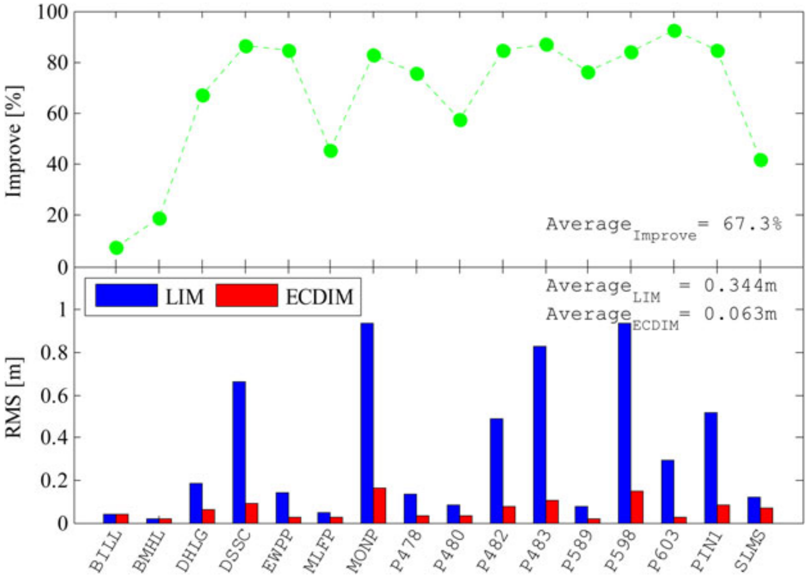


Figure 9. Comparison of troposphere modelling accuracy of LIM and ECDIM at each station within 10 to 20 degrees elevation angle.

77·9%, 71·9%, and 53·8% for the four stations respectively. For station PIN1 and P603, the positioning accuracy of both the horizontal and up direction with LIM is significantly worse than that obtained by ECDIM. In addition, a certain proportion of ambiguity is incorrectly fixed in the positioning results. According to the statistics, the RMS of station PIN1 in north, east, up directions are 0·092 m, 0·092 m and 0·650 m, respectively. In comparison, corresponding RMS with ECDIM are 0·027 m, 0·024 m and 0·062 m, and the accuracy in each direction is increased by 70·7%, 73·9% and 90·5%, respectively. The statistical results of station P603 are similar. Compared with LIM, the positioning accuracy with ECDIM in north, east, and up directions is improved from 0·072 m, 0·050 m, and 0·312 m to 0·033 m, 0·026 m, and 0·051 m, with an improvement of 54·2%, 48·0% and 83·7%, respectively. In terms of fix rate of epoch, using LIM, the fix rates of the selected six stations are 98·2%, 98·5%, 98·9%, 62·0%, 46·4% and 52·8%, respectively. And the corresponding

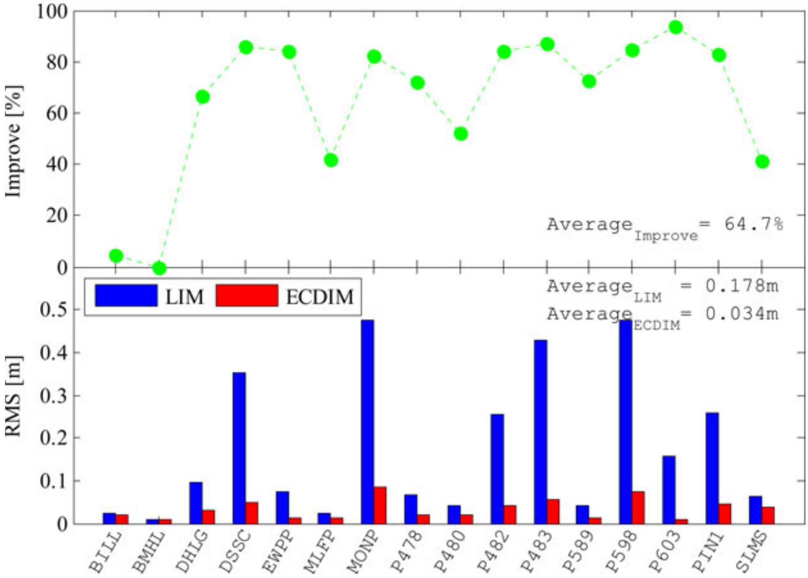


Figure 10. Comparison of troposphere modelling accuracy of LIM and ECDIM at each station within 20 to 30 degrees elevation angle.

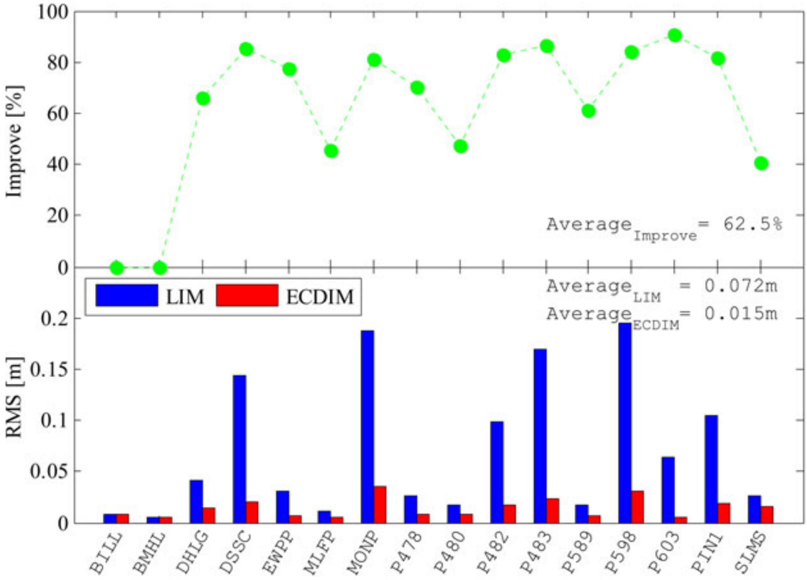


Figure 11. Comparison of troposphere modelling accuracy of LIM and ECDIM at each station within 30 to 50 degrees elevation angle.

fix rates with ECDIM are increased to 99·2%, 98·7%, 99·2%, 81·1%, 93·5% and 97·5%, respectively.

4.2. *Results of TJCORS.* Figure 14 shows the station height distribution of TJCORS, located in the plane area. The maximum height difference between stations does not exceed

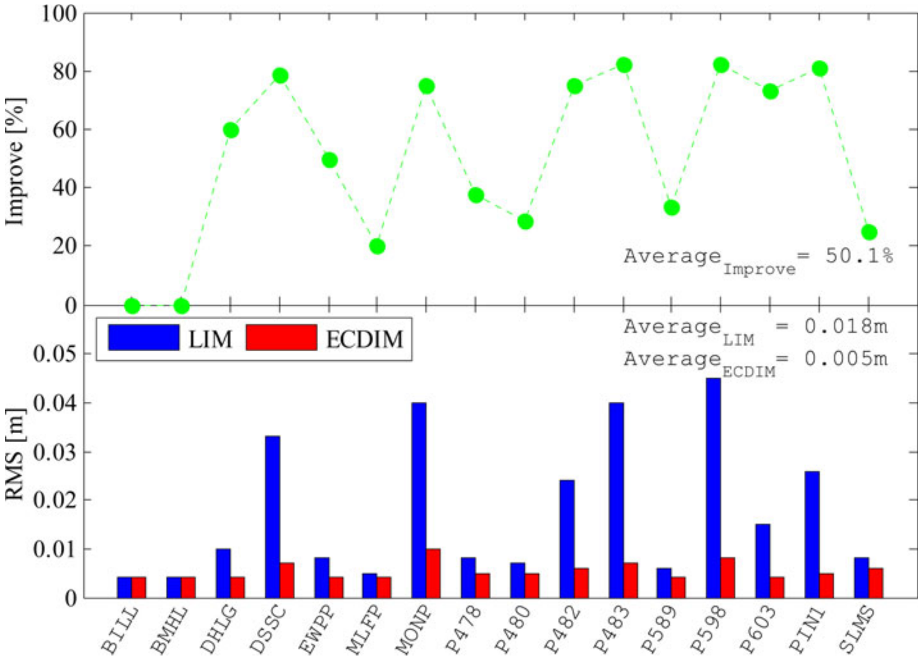


Figure 12. Comparison of troposphere modelling accuracy of LIM and ECDIM at each station within 50 to 90 degrees elevation angle.

40 m. The following compares the performance of LIM and ECDIM in terms of accuracy of troposphere modelling and terminal positioning respectively.

4.2.1. *Results of troposphere modelling.* As shown in Figure 3, for station TJHM, the main reference station is TJXQ. When using the LIM method, the atmosphere information of two baselines (TJXQ → TJNH and TJXQ → TJDG) is used for troposphere modelling. And two additional baselines (TJXQ → TJDH and TJXQ → TJTG) are used in ECDIM. Figure 15 shows the troposphere modelling results of several satellites with different elevation angles. It is not difficult to find that the interpolation results of the two methods are similar and consistent with the trend of true value. Using LIM, the troposphere modelling RMS of G21, G10 and G18 satellites are 0·008 m, 0·007 m and 0·004 m, respectively, and the corresponding RMS with ECDIM are 0·008 m, 0·004 m, and 0·003 m, respectively. Both LIM and ECDIM have achieved high precision modelling accuracy.

Further, the troposphere modelling error of all satellites over the entire observation period is shown in Figure 16. As the difference in height between stations is not obvious, the conventional LIM can also achieve good interpolation results for low elevation angle satellites, and the mean and standard deviation of the modelling error are 0·005 m and 0·010 m respectively. For ECDIM, corresponding mean and standard deviation are −0·001 m and 0·008 m respectively. The results of the two methods are very close. According to the RMS within different elevation angles, compared with LIM, the RMS with ECDIM, corresponding to 10–20 degree, 20–30 degree, 30–50 degree, and 50–90 degree elevation interval, is improved from 0·022 m, 0·013 m, 0·007 m and 0·004 m to 0·016 m, 0·009 m, 0·005 m and

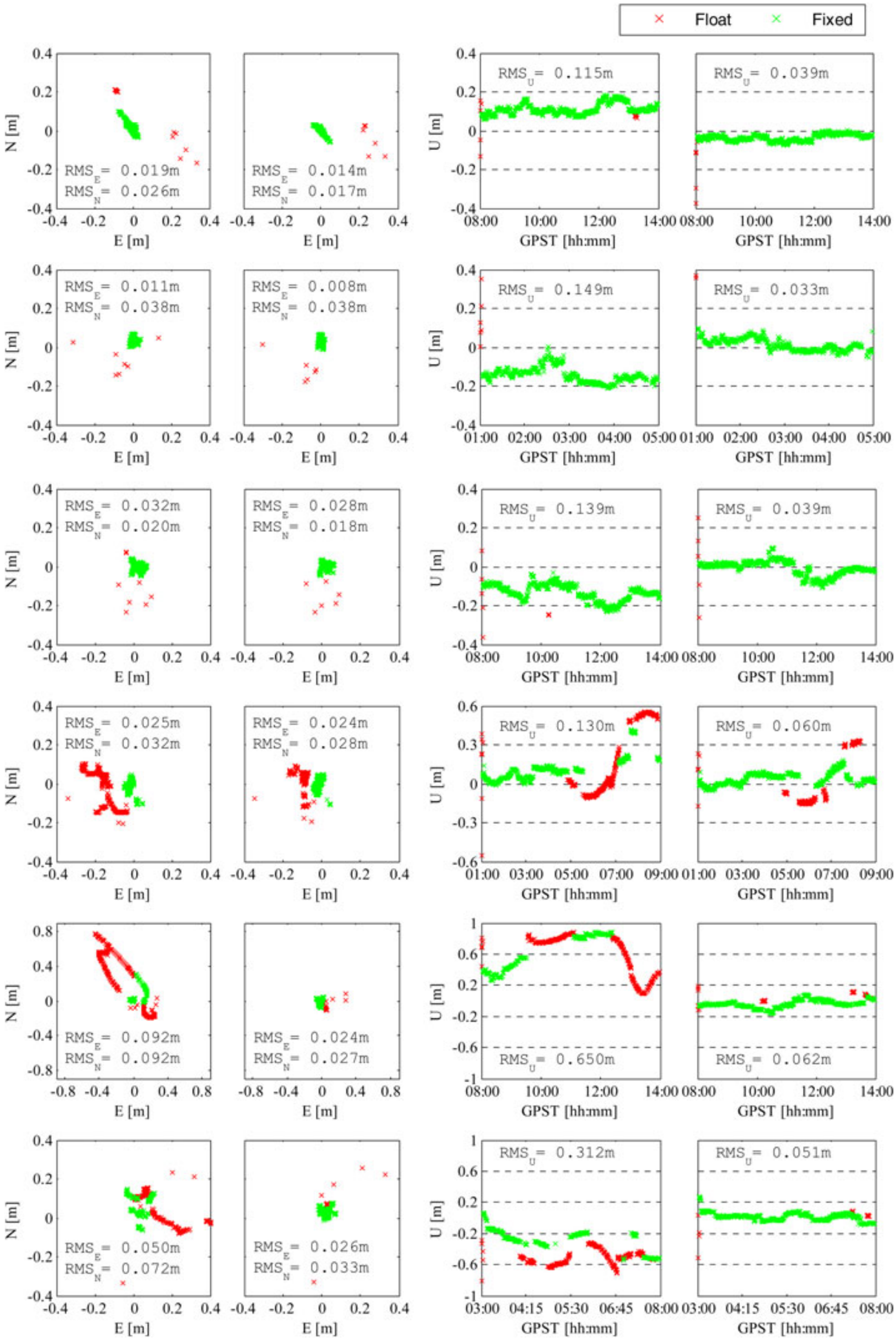


Figure 13. Comparison of positioning error of LIM and ECDIM at user stations (from top to bottom): P478, SLMS, EWPP, P480, PIN1 and P603. From left to right: horizontal error of LIM, horizontal error of ECDIM, up direction error of LIM, up direction error of ECDIM.

Table 4. Detailed statistics for selected user stations.

Station	Mode	RMS/m N	RMS/m E	RMS/m U	Fix Rate/%
P478	LIM	0·019	0·026	0·115	98·2
	ECDIM	0·014	0·017	0·039	99·2
	Improve/%	26·3	34·6	66·1	–
SLMS	LIM	0·038	0·011	0·149	98·5
	ECDIM	0·038	0·008	0·033	98·7
	Improve/%	0·0	27·3	77·9	–
EWPP	LIM	0·020	0·032	0·139	98·9
	ECDIM	0·018	0·028	0·039	99·2
	Improve/%	10·0	12·5	71·9	–
P480	LIM	0·032	0·025	0·130	62·0
	ECDIM	0·028	0·024	0·060	81·1
	Improve/%	12·5	4·0	53·8	–
PIN1	LIM	0·092	0·092	0·650	46·4
	ECDIM	0·027	0·024	0·062	93·5
	Improve/%	70·7	73·9	90·5	–
P603	LIM	0·072	0·050	0·312	52·8
	ECDIM	0·033	0·026	0·051	97·5
	Improve/%	54·2	48·0	83·7	–

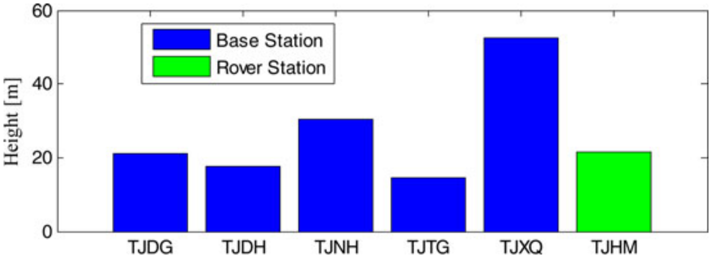


Figure 14. Station height distribution of TJCORS.

0·003 m, with an improvement of 27·3%, 30·8%, 28·6% and 25·0%, respectively. For stations in TJCORS, both LIM and ECDIM can generally achieve an accuracy of 2–3 cm in troposphere modelling.

4.2.2. *Results of user positioning.* Figure 17 shows the positioning errors of the station TJHM in a certain period. Since both LIM and ECDIM can obtain high precision troposphere modelling results, the corresponding positioning error distribution is relatively close. Compared with the LIM method, the positioning accuracy is only slightly improved in the up direction. With the ECDIM method, the positioning accuracy in the up direction is improved from 0·066 m to 0·052 m, with an improvement of 21·2%. In addition, the fix rates of epoch for both methods are 99·6%.

In summary, for networks with small height difference between reference stations, conventional LIM can achieve good troposphere modelling accuracy and guarantee the positioning accuracy at the user end, and the ECDIM is applicable to both areas with large and small height differences.

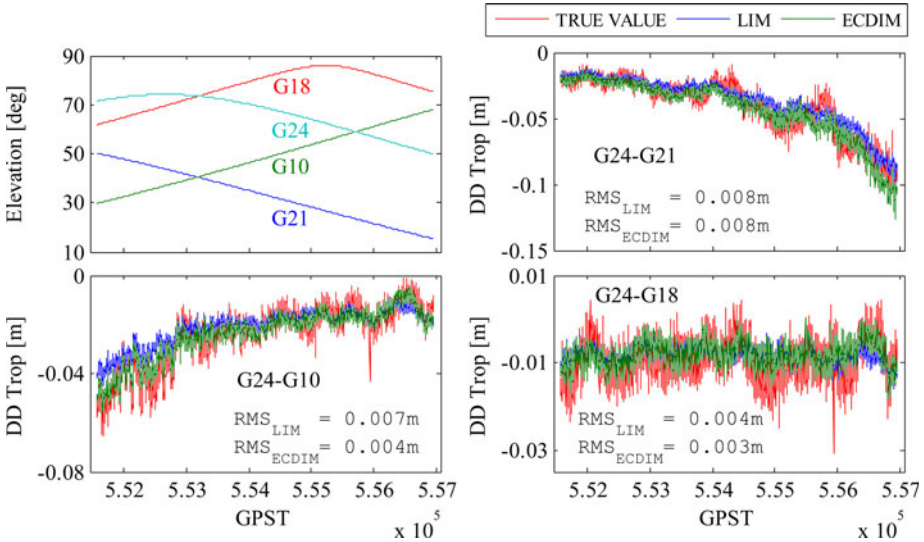


Figure 15. Troposphere interpolation results for satellites with different elevation angles at station TJHM.

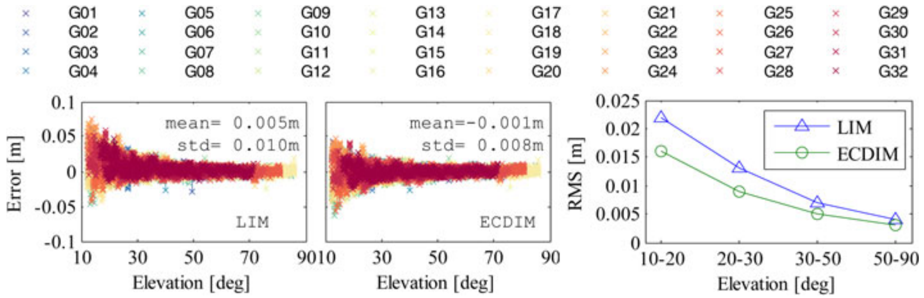


Figure 16. Troposphere interpolation error (left and centre) and RMS (right) with satellite elevation angle at station TJHM.

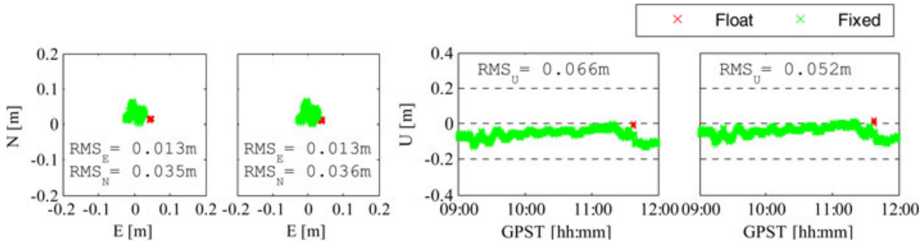


Figure 17. Comparison of positioning error of LIM and ECDIM at station TJHM. From left to right: horizontal error of LIM, horizontal error of ECDIM, up direction error of LIM, up direction error of ECDIM.

5. CONCLUSION. This paper focuses on investigating high precision troposphere modelling method in NRTK. Based on the idea of modelling and compensating the errors of the prior model, an error compensation-based troposphere model, namely ECDIM, is

proposed. On the basis of conventional DIM, the proposed method additionally adds correction of DD troposphere model value between user and reference stations. This method not only improves the accuracy of interpolated troposphere in areas with large height differences, but also can be applied to both conventional single DTN and multi-station scenarios. Further, experimental verification was performed with data from CRTN and TJCORS.

The station height difference of CRTN is large, ranging from 200 m to nearly 2,000 m. The troposphere modelling accuracy of conventional LIM is very poor, especially for low elevation angle satellites, and the deviation between the interpolated value and the true value can even reach 0·8 m. In contrast, the results of ECDIM can maintain high consistency with the true value. Within the coverage of the network, compared with LIM, the overall troposphere modelling accuracy of ECDIM can be improved by more than 20%. Corresponding to 10–20 degree, 20–30 degree, 30–50 degree and 50–90 degree elevation angle interval, compared with LIM, the accuracy with ECDIM is increased from 0·344 m, 0·178 m, 0·072 m and 0·018 m to 0·063 m, 0·034 m, 0·015 m and 0·005 m, with an improvement of 67·3%, 64·7%, 62·5% and 50·1%, respectively. As the satellite elevation angle increases, the degree of improvement gradually decreases. In the positioning experiment at user end, the error in up direction with LIM has an obvious systematic deviation, and the fix rate of epoch is relatively low. This situation has improved significantly with ECDIM. The positioning accuracy in north, east, and up directions has all achieved centimetre-level accuracy, and the fixed rate of the epoch has also been improved to varying degrees.

In the TJCORS experiment, the heights of stations are similar, and the modelling results of the two methods are close. Both have achieved consistent high precision troposphere interpolation accuracy. Corresponding to different elevation interval, compared with LIM, the modelling accuracy with ECDIM is increased from 0·022 m, 0·013 m, 0·007 m and 0·004 m to 0·016 m, 0·009 m, 0·005 m and 0·003 m, with an improvement of 27·3%, 30·8%, 28·6% and 25·0%, respectively. In the terminal positioning experiment, both methods achieved similar horizontal positioning accuracy with several centimetres, and the accuracy in the up direction with ECDIM has improved by 21·2%, compared with LIM.

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