INTERPRETATION OF SOLAR RADIO-FREQUENCY DISK BRIGHTNESS DISTRIBUTIONS DERIVED FROM OBSERVATIONS WITH AERIALS EXTENDED IN ONE DIMENSION

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Several recent papers have dealt with observations of brightness distributions over the solar disk, which were derived either from two-aerial interferometer observations at various spacings and orientations (e.g. O'Brien, 1953) [1], or from multiple-element interferometer fan-beam observations at various orientations (e.g. Christiansen and Warburton, 1954) [2]. In each a two-dimensional distribution is derived from a number of essentially one-dimensional observations by a Fourier synthesis method described by O'Brien. The detail given by these methods must be limited by the finite resolution of the individual observations (limited by the maximum aperture of the aerial system), but the form of the limitation is not obvious, though its knowledge is required when relating the observations to a solar model.

We have found that the 'derived' distribution is identical with that which would be obtained on scanning the true distribution point by point with a hypothetical pencil-beam aerial. This hypothetical aerial has a circular beam such that a strip scan of the power response along any diameter gives the power response of the observing aerial system in its direction of high resolution. This result may be obtained by the following reasoning:

Consider a two-dimensional brightness distribution $f(x, y)$ having small angular extensions, so that a system of *rectangular* co-ordinates (x, y) may be used to specify positions on the celestial sphere. When such a distribution is scanned across the x-direction by a fan-beam parallel to the y-axis and with power response $A(x)$, the instrument registers the profile

$$\iint A(x-x')\, f(x, y)\; dx\, dy \qquad (1)$$

(the integrals here and subsequently extending over the entire distribution). When many such profiles, obtained by scanning the distribution in

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different directions, are combined to derive a two-dimensional distribution (e.g. following the method used by O'Brien), we obtain not the true distribution $f(x, y)$, but a smoothed distribution $g(x, y)$. This distribution must be such that its line integral in any direction corresponds to the observed profile. Thus, considering the x-direction (which is chosen arbitrarily in the first place) we have

$$\int g(x', y')\, dy' = \iint A(x-x')\, f(x, y)\, dx\, dy. \tag{2}$$

We now consider the distribution g to be formed from the true distribution, f, by smoothing the latter point by point with a pencil beam whose power response is given by $B(x, y)$. By symmetry, the latter must be a circular pattern. So B, f and g are related by

$$g(x', y') = \iint B(x-x', y-y')\, f(x, y)\, dx\, dy. \tag{3}$$

Substituting (3) in (2), we obtain

$$\iiint B(x-x', y-y')\, f(x, y)\, dx\, dy\, dy' = \iint A(x-x')\, f(x, y)\, dx\, dy.$$

This is satisfied if

$$\int B(x-x', y-y')\, dy' = A(x-x')$$

i.e. if

$$\int B(x, y)\, dy = A(x),$$

giving the result stated above.

As $B(x, y)$ depends on x and y only through the function $r=\sqrt{x^2+y^2}$, the practical interest is centred on finding the function $B(r)$ from the integral equation:

$$2\int_x^\infty \frac{B(r)\, r\, dr}{\sqrt{r^2-x^2}} = A(x).$$

The application of this theorem to two cases of particular importance is shown in Figs. 1 *a* and 1 *b*. In either figure the dashed curve represents the power response function of the strip instrument, $A(x)$, and the solid curve represents the radial distribution of the circular equivalent, $B(r)$. Both x and r are measured in units of λ/l radians. Fig. 1 *a* refers to a uniform one-dimensional aerial of length l, so that $A(x)$ is proportional to $\theta^{-2}\sin^2(l\pi\theta/\lambda)$. Fig. 1 *b* refers to the aerial pattern appropriate to observations made with a two-element interferometer, in which the information obtained using a succession of different aerial spacings between 0 and l is appropriately combined (Stanier, 1950) [3]. Here $A(x)$ is proportional to $\theta^{-1}\sin(2l\pi\theta/\lambda)$. In both cases it is seen that the side lobes of the equivalent circular pattern are negative as well as positive, and are of somewhat smaller amplitude than those of the observing linear aerial.

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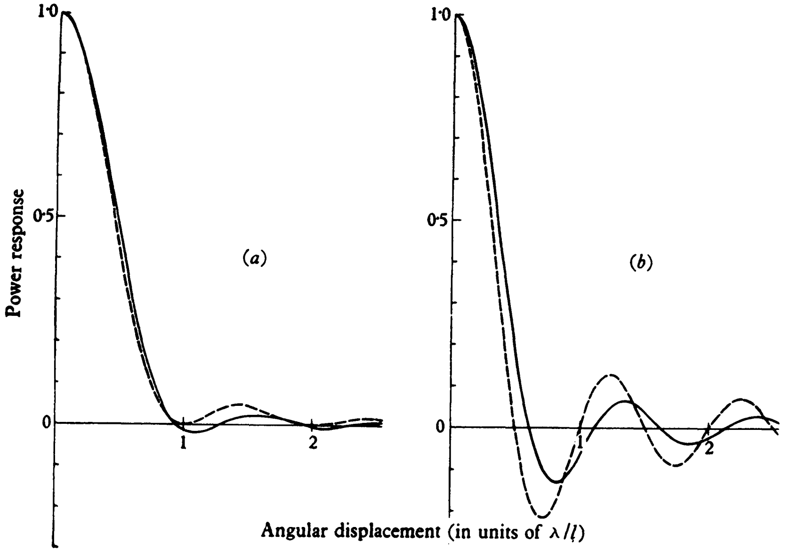

Fig. 1. The power response of aerials having high resolution in only one dimension (dashed curve) and the cross-section of their circular equivalents (full curve).

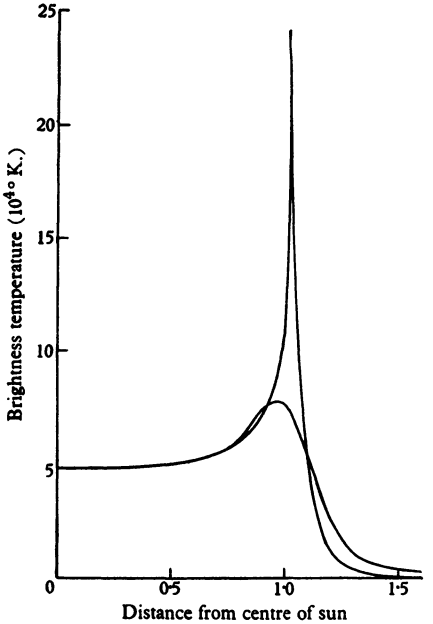

Fig. 2. The thin line shows the radial cross-section of the brightness distribution at $\lambda = 21$ cm. derived from a spherically symmetrical model of the solar atmosphere. The thick line shows the radial cross-section of the distribution which would be derived from strip scans in different directions using a uniform strip-shaped aerial having a beam width of 4 ft. 3 in. between point of half-power.

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Using this theorem to compare solar brightness distributions derived from observations with those derived from model solar atmospheres, we find that the finite resolving power of the instrument tends to introduce the following effects:

(1) Broadening, lowering, and shifting towards adjacent bright regions of peaks in the true distribution (e.g. a peak at the limb).

(2) An artificial extension of the outer fringes of the distribution.

Observed values of the central brightness, however, are practically correct.

Numerical examples (e.g. Fig. 2) show effects (1) and (2) to be significant and it seems that in comparing a theoretical with an observed distribution the only reliable procedure is to degrade the theoretical one by smoothing it with the appropriate aerial beam. Current results suggest that it will not be necessary to depart from present ideas on electron densities, etc., in the solar atmosphere in explaining the radio observations which apply near sunspot minimum.

REFERENCES

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[3] Stanier, H. M. *Nature*, **165**, 354, 1950.