AN IMPROVED METHOD OF CONSTRUCTING SHORTENED LIFE-TABLES FOR PUBLIC HEALTH COMPARATIVE STATISTICS.

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As is only too well known by those who have undertaken the task, the work of constructing a Life-Table by an extended method, even when "graphic" means are employed for lightening the labour, is long and wearisome.

When the tables setting forth numerical facts for every separate year of age are at length completed, although they are duly printed, no one, except for the purpose of minute criticism, pays much attention to them.

The practical outcome, both for author and for readers, is that attention is directed to comparative Tables in which the "numbers of survivors" and the "expectation of life" are given for certain ages at intervals of five or ten years. Although corresponding Tables of the fractions expressing the "chances of living a year" at certain ages are also given, these are scarcely necessary, as Tables of mean Death-rates in age-groups are more readily comprehended.

If, therefore, a method can be devised, by which, with but comparatively little labour, from the foundation figures of census enumerations and death records, a series of l_x and E_x values at interval of five years can be obtained, with close approximation to the results which would be obtained by an "extended" method, whether "analytical" or "graphic," all will be gained which is required for the purposes of Public Health.

The present writer has already done some work in this direction in so modifying the original short method of the late Dr Wm. Farr, as to obtain fairly accurate E_x values. A description of the methods has already been published in this *Journal* (see Vol. II, No. 1).

These methods, however, are merely empirical and somewhat crude from a mathematical point of view, and they do not give, especially at the later ages, anything like true l_x values.

It is therefore now proposed to describe how by certain simple applications of exact mathematical principles a series of l_x and E_x values at intervals of five years from age 0 to age 85 may be obtained, with certainty that they will very nearly coincide with the corresponding values of the most laborious and accurate extended method which can be employed.

Instead of having to traverse one by one the successive yearly steps of the stairway of life, after the first five, the steps may be taken five at a time.

In order to limit the scope of this paper and to save needless repetition it is presumed at the outset that the reader is acquainted with, and has access to, papers previously published in this *Journal* in Nos. 1, 2, and 3, Vol. II, and also with the paper by Drs Newsholme and Stevenson in No. 3 of Vol. III.

It is therefore supposed that the preliminary work required in the construction of *all* Life-Tables has been completed, that is, Tables have been compiled of the "lives at risk" or "years of life," and of the deaths in each of the usual age-groups for the decennium being dealt with, and that p_x values have been calculated for each of the first five years of age, and by means of these l_x values from l_0 to l_5 inclusive, and then P_x values from P_0 to P_4 inclusive.

Preliminary remarks relating to the principles on which the methods of calculation to be described are based.

If reference be made to the two diagrams representing the curves of population and deaths which are given in the paper of Drs Newsholme and Stevenson above indicated between pp. 302—303, or to similar diagrams given in Dr Newsholme's *Vital Statistics*, 3rd edition, pp. 266—267, it will be evident that if an exactly corresponding point x be taken anywhere in the base-lines of the two curves, and the two ordinates at point x measured, there will be obtained the numbers of population and deaths respectively belonging to the exact age x (which may be indicated by the letters P and d), and the mean death-rate per unit, during the year of age of which x is the centre, which may be denoted by m_x will be $\frac{d}{P}$, and the chance of living a year, existing at exact age x, denoted by p_x' will be expressed by

$$\frac{2-m_x}{2+m_x} \text{ or by } \frac{2P-d}{2P+d}.$$

It will also be evident that if the original data had been arranged for each age-group in the form of twice population minus deaths and twice population plus deaths, two other curves might have been drawn through parallelograms first constructed, and that the measurement of the two ordinates at a corresponding point x in the base-lines of the two curves would at once give the numerator and denominator of the p_x' fraction, viz. $\frac{2P-d}{2P+d}$.

In the methods of working to be now described, instead of drawing a curve by a "graphic" process, and measuring the ordinates required to be interpolated from this curve, each required ordinate is calculated

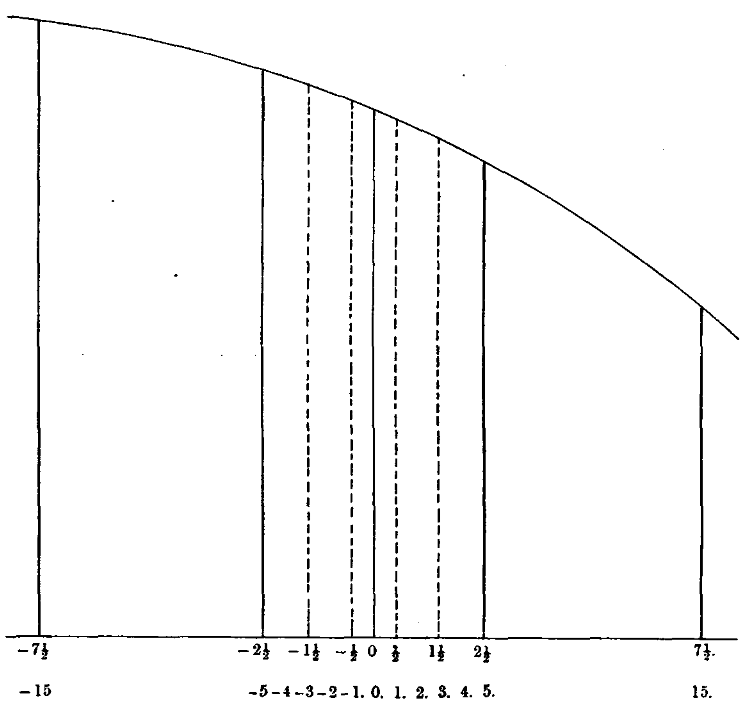

Fig. 1.

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by a formula which expresses its value in terms of the ordinates which are given as the foundation series of the curve. Now when these given ordinates are at equal distances apart, and only four or five are taken in a series (that is, when interpolation is effected by only three or four orders of differences), and when only centrally situated ordinates, or sums of ordinates symmetrically arranged with regard to the central point of the base-line, are required, the formulae can be reduced to extremely simple forms. Thus (see Fig. 1) if four ordinates are given separated by 10 units of measurement (years) in the base-line, which may be denoted respectively by the symbols u_{-15}, u_{-5}, u_5, u_{15}, the central ordinate u_0 is measured by the formula

$$u_0 = \frac{9\,(u_{-5} + u_5) - (u_{-15} + u_{15})}{16}.$$

This simple calculation can certainly be effected in less time than would be required to draw a curve and measure the required ordinate.

In arranging, however, for the interpolation of ordinates, so as to measure by calculation the numbers of population and deaths separately, or combined as $2P - d$ and $2P + d$, as existing at certain *exact ages*, since the foundation figures are given in groups of ages it is necessary to reconstruct these figures in a form giving the numbers at each age and upwards, so that each given ordinate may be a "linear quantity."

Thus in Fig. 2 we have given five equidistant ordinates separated by five-yearly intervals, measuring respectively $2P - d$ or $2P + d$ at age 5 and upwards, at age 10 and upwards, and so on to age 25 and upwards. Now if u_{14} be interpolated, then $u_{14} - u_{15}$ will give the numbers belonging to the year of age 14 to 15, and similarly if u_{16} be interpolated, $u_{15} - u_{16}$ will give the numbers belonging to the year of age 15 to 16, &c.

But it would be very tedious and laborious to calculate $\dfrac{2P-d}{2P+d}$ for every year of age by this method.

There is, however, a simple way of measuring the values of $2P-d$ and $2P+d$ belonging to *exact age* 15 (*i.e.* age 0 in the given series), the formula for which is arrived at by the differential calculus, as it has been applied by Mr A. C. Waters.

Thus $2P - d$, or $2P + d$, at exact age 0

$$= -\frac{8\,(u_{-5} - u_5) - (u_{-10} - u_{10})}{60},$$

and p_0' (that is, the chance of living a year which exists at exact age 0)

$$= \frac{2P-d}{2P+d} \text{ at exact age } 0.$$

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This formula is true when the ordinates represent *numbers*, but as will now be explained it has to be modified when the ordinates in Fig. 2 represent the *logarithms* of $2P-d$ and $2P+d$ at age x and upwards.

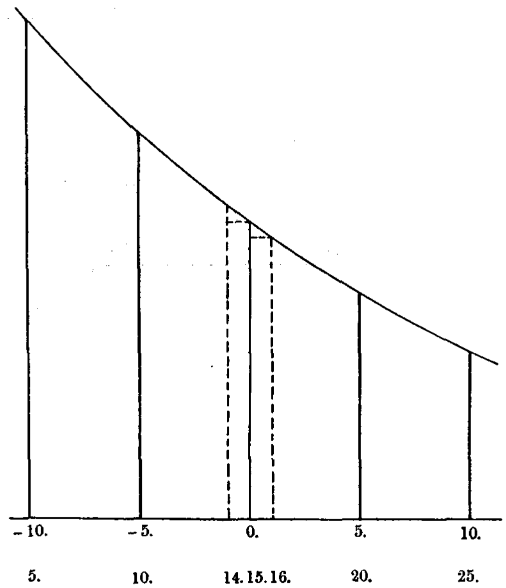

Fig. 2.

When this is the case, as is required in actual working, if the symbol u_x be used to denote the log of $2P-d$ at age x and upwards, and the symbol U_x to denote the log of $2P+d$ at age x and upwards, then it can be shown that

$$\log p_0' = \{u_0 + \log[8(u_{-5} - u_5) - (u_{-10} - u_{10})]\}$$
$$- \{U_0 + \log[8(U_{-5} - U_5) - (U_{-10} - U_{10})]\}.$$

If the foundation series of u_x and U_x values be completed at five-yearly intervals by simple formulae of interpolation, it is then possible to readily obtain by the formula just given a complete series of $\log p_x'$ values at five-yearly intervals.

These values may be considered as ordinates of a curve (the p_x curve), and the series may be completed at yearly intervals by formulae of interpolation. In No. 2 of Vol. II. of this *Journal* it has also been shown how by a graphic process applied to the given $\log p_x'$ values, the intermediate values may be obtained by measurement. Thus if (see Fig. 1) in the series at five-yearly intervals, viz. $-7\frac{1}{2}$, $-2\frac{1}{2}$, $2\frac{1}{2}$ and $7\frac{1}{2}$, we obtain the sum of the logs at $-2, -1, 0, 1$ and 2, we can pass by one step of addition from $\log l_{-2\frac{1}{2}}$ to $\log l_{2\frac{1}{2}}$.

However, it is possible with still greater ease to obtain a truer value by a single calculation. Since any number of ordinates may be interpolated between $-2\frac{1}{2}$ and $2\frac{1}{2}$, each having as good a claim as the other to represent the true $\log p_x'$ value *at its own point*, it is obvious that if we wish to obtain the true mean $\log p_x$ value between ages $-2\frac{1}{2}$ and $2\frac{1}{2}$ the greater the number of intermediate ordinates interpolated the nearer to the true value will be the resulting mean. Thus

(1) If (as shown by the dotted lines in Fig. 1) *four* intermediate equidistant ordinates are interpolated by calculation at yearly intervals, the mean of the six ordinates $-2\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}$, and $2\frac{1}{2}$, can be shown to be equal to

$$\frac{12{\cdot}8\,(u_{-2\frac{1}{2}} + u_{2\frac{1}{2}}) - {\cdot}8\,(u_{-7\frac{1}{2}} + u_{7\frac{1}{2}})}{24}.$$

(In this and the succeeding formulae u_x means $\log p_x'$.)

(2) If *nine* intermediate ordinates, at intervals of half a year, be interpolated, the mean of the 11 ordinates

$$= \frac{12{\cdot}9\,(u_{-2\frac{1}{2}} + u_{2\frac{1}{2}}) - {\cdot}9\,(u_{-7\frac{1}{2}} + u_{7\frac{1}{2}})}{24}.$$

(3) If *nineteen* intermediate ordinates, at intervals of $\frac{1}{4}$ year, be interpolated, the mean of the 21 ordinates

$$= \frac{12{\cdot}95\,(u_{-2\frac{1}{2}} + u_{2\frac{1}{2}}) - {\cdot}95\,(u_{-7\frac{1}{2}} + u_{7\frac{1}{2}})}{24}.$$

(4) On increasing the number of intermediate ordinates more and more, the mean value would be found to approximate more and more closely to

$$\frac{13\,(u_{-2\frac{1}{2}} + u_{2\frac{1}{2}}) - 1\,(u_{-7\frac{1}{2}} + u_{7\frac{1}{2}})}{24}.$$

On multiplying the above value by 5, corresponding to the five separate yearly units of interval, the coefficients in the numerator become 65 and -5, and then, multiplying both the numerator and the denominator of the expression by 2, the result is reduced to the convenient working formula

$$\frac{130\,(u_{-2\frac{1}{2}} + u_{2\frac{1}{2}}) - 10\,(u_{-7\frac{1}{2}} + u_{7\frac{1}{2}})}{48}.$$

The above formula is *one* application of a general formula worked out by the integral calculus, viz.

$$\int_{-n}^{n} u_x dx = \frac{n}{3}\left[\frac{13(u_{-1}+u_1)-(u_{-3}+u_3)}{4}\right].$$

It is thus possible in this extremely simple way to obtain the logs which by successive steps of addition enable us to pass from $\log l_x$ to $\log l_{x+5}$.

After having obtained the complete series of $\log l_x$ values and taken out their corresponding numerical values, these are also to be considered ordinates of a continuous curve (the l_x curve), and (see Fig. 1), taking u_x as denoting l_x, the sum of the years of life lived by $l_{-2\frac{1}{2}}$ persons in the interval from age $-2\frac{1}{2}$ to age $2\frac{1}{2}$ is obtained by the formula

$$\frac{130(u_{-2\frac{1}{2}}+u_{2\frac{1}{2}})-10(u_{-7\frac{1}{2}}+u_{7\frac{1}{2}})}{48}.$$

Construction of the Shortened Life-Table.

In the actual process a working-sheet with successive series of columns will be used. It is proposed to explain in order how the respective columns are to be constructed.

Columns 1 *and* 2, *headed* u_x *and* U_x.

It is first of all necessary to put the foundation figures of "lives at risk" and deaths into the form of $2P-d$ and $2P+d$ at age x and upwards. The lowest age-group being at age 85 and upwards, the addition of $2P-d$ and $2P+d$ for ages 75—85 will give the number for age 75 and upwards, and so on until the figures for age 5 and upwards are arrived at.

From the *numbers* are then to be derived the corresponding *logarithms* so that No. 1 column will contain the logs of $2P-d$ at age x and upwards, the given series being for ages 5, 10, 15, 20, 25, 35, 45, 55, 65, 75, and 85. This is headed u_x. Similarly the second column U_x is obtained by setting down for the same series of ages the logs of $2P+d$ at age x and upwards.

These columns have then to be completed at five-yearly intervals by interpolations, the formulae for which are as follows, beginning at the top of the series:

$$u_0 = 5(u_5 - u_{20}) + u_{25} - 10(u_{10} - u_{15}),$$

$$u_{30} = \frac{5(u_{15} + 9u_{25} + 3u_{35}) - (24u_{20} + u_{45})}{40},$$

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$$u_{40} = \frac{9(u_{35}+u_{45})-(u_{25}+u_{55})}{16} = \frac{10(u_{35}+u_{45})-(u_{25}+u_{35}+u_{45}+u_{55})}{16},$$

$$u_{50} = \frac{9(u_{45}+u_{55})-(u_{35}+u_{65})}{16},$$

$$u_{60} = \frac{9(u_{55}+u_{65})-(u_{45}+u_{75})}{16},$$

$$u_{70} = \frac{3(u_{45}+30u_{65}+20u_{75})-5(4u_{55}+u_{85})}{128},$$

$$u_{80} = \frac{5(3u_{65}+9u_{75}+u_{85})-(u_{55}+40u_{70})}{24},$$

$$u_{90} = u_{65}+10(u_{75}-u_{80})-5(u_{70}-u_{85}),$$

$$u_{95} = u_{70}+10(u_{80}-u_{85})-5(u_{75}-u_{90}).$$

(Similar formulae of course apply to the U_x values in column 2.)

It is desirable to check the interpolations of u_{70}, u_{80}, u_{90} and u_{95} by finding that the last term coincides with the value obtained by

$$u_{95} = u_{45}+10(u_{65}-u_{75})-5(u_{55}-u_{85}).$$

The series from u_{65} to u_{95} inclusive should have a constant fourth difference.

Column 3, headed log p_x'.

From the logs in columns 1 and 2 it is now possible to obtain a series of log p_x' values from log p_{10}' to log p_{85}' inclusive by the formula previously given.

Each log p_x' value is to be derived from a series of five equidistant u_x and U_x values of which the central term in each series is taken as u_0 and U_0 respectively.

Thus, to take as an illustration the calculation of log p_{10}' from the data of the Brighton Life-Table for 1898—1900 (males).

	x	u_x	U_x
−10	0	6·1011849	6·1072497
− 5	5	6·0543709	6·0604667
0	10	5·9956540	6·0024205
5	15	5·9296871	5·9374090
10	20	5·8601732	5·8689494

$8(u_5-u_{15})-(u_0-u_{20})=0·7564587$
log ·7564587 = $\bar{1}$·8787852
$8(U_5-U_{15})-(U_0-U_{20})=0·7461613$
log ·7461613 = $\bar{1}$·8728328
5·9956540 + $\bar{1}$·8787852 = 5·8744392
6·0024205 + $\bar{1}$·8728328 = 5·8752533
5·8744392 − 5·8752533 = $\bar{1}$·9991859 = log p_{10}'.

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This column has to be completed by differencing the series of log p_x' values for ages 65, 70, 75, 80 and 85, and by carrying down the differences values may be obtained for ages 90, 95, 100, 105, and 110.

Column 4, headed $\int_x^{x+5} log\, p_x$.

In the first place it must be noted that for the age-periods 5—10 and 10—15 the required values are to be simply obtained from the "lives at risk" and total deaths for the respective age-groups 5—10 and 10—15 by the fraction $\left(\frac{2P-d}{2P+d}\right)^5$, that is the log is obtained by

$$[(\log 2P-d)-(\log 2P+d)]\times 5.$$

It has been found by repeated trials involving much more complicated calculations than those set forth in this paper that the values obtained by the above indicated simple method are the best for the present purpose. However, the value for the age-period 5—10 thus obtained is a little less than the true value.

These values may be therefore obtained fiıst and set down in column 4.

The first value to be obtained by the method of "integration" which has been already described, is the log p_x value between ages 15 and 20.

To take an illustrative case again from the Brighton Life-Table.

	Ages	log p_x'
$-7\frac{1}{2}$	10	$\bar{1}\cdot9991859$
$-2\frac{1}{2}$	15	$\bar{1}\cdot9988287$
$2\frac{1}{2}$	20	$\bar{1}\cdot9980144$
$7\frac{1}{2}$	25	$\bar{1}\cdot9974963$

The formula being $\dfrac{130\,(u_{-2\frac{1}{2}}+u_{2\frac{1}{2}})-10\,(u_{-7\frac{1}{2}}+u_{7\frac{1}{2}})}{48}$.

	$u_{-2\frac{1}{2}}+u_{2\frac{1}{2}}$	$u_{-7\frac{1}{2}}+u_{7\frac{1}{2}}$
	$\bar{1}\cdot9988287$	$\bar{1}\cdot9991859$
	$+\bar{1}\cdot9980144$	$+\bar{1}\cdot9974963$
	$\bar{1}\cdot9968431$	$\bar{1}\cdot9976822$
taking the differences from 0, or the co-logs ...	$0\cdot0031569$	$0\cdot0033178$
	$\times 130$	
	947070	
	31569	
	$\cdot4103970$	
	$-\ 331780$	

$0\cdot3772190 \div 48 = 0\cdot0078587$

$\therefore$ required $\log = \bar{1}\cdot9921413$

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This series of calculations is to be carried on until the value between ages 100 and 105 is obtained.

Column 5, *headed log* l_x.

Commencing with $\log l_5$ by successive addition of the logs in the preceding column, the values are obtained for this column from $\log l_{10}$ to $\log l_{105}$.

Column 6, *headed* l_x.

This column is simply obtained by taking out the numerical values of the logs in the preceding column. An additional term l_0 must be interpolated at the top of the column by the formula

$$l_0 = 4\,(l_5 + l_{15}) - (6l_{10} + l_{20}).$$

This, of course, has no relation to the *true* l_0 value, but is merely required for the purpose of calculating the value of $P_{5 \text{ to } 10}$ for the succeeding column.

Column 7, *headed* $P_{x \text{ to } x+5}$, or $Q_x - Q_{x+5}$.

In this column the years of life lived by l_x persons in the interval from age x to age $x+5$ are set down.

The simple formula required is identical with that already used for column 4,

$$\frac{130\,(l_{-2\frac{1}{2}} + l_{2\frac{1}{2}}) - 10\,(l_{-7\frac{1}{2}} + l_{7\frac{1}{2}})}{48}.$$

Taking an illustrative case from the Brighton Life-Table.

	x	l_x
$-7\frac{1}{2}$	0	78541
$-2\frac{1}{2}$	5	75970
$2\frac{1}{2}$	10	74549
$7\frac{1}{2}$	15	73784

75790	78541
+74549	+73784
150339	152325
×130	
4510170	
150339	
19544070	
− 1523250	
18020820 ÷ 48 = 375434	

This formula has to be used as far as the interval between age 90 and age 95.

The years of life lived after age 95 are to be obtained by the simpler integration formula

$$\frac{5\,(l_{95}+4l_{100}+l_{105})}{3}.$$

Column 8, *headed* Q_x.

The value obtained by the last given formula is to be set down as Q_{95}, then by successive additions from below upwards the Q_x values are obtained as far as Q_5.

In order to obtain Q_0 it is simply necessary to add to Q_5 the sum of P_0, P_1, P_2, P_3 and P_4 already obtained.

Column 9, *headed* E_x.

This is to be obtained from age 0 to age 85 by the formula $E_x=\frac{Q_x}{l_x}$.

Column 10, *headed* $E_{x \text{ to } x+n}$.

It is desirable for the sake of being able to obtain those useful and interesting applications of a Life-Table which are related to the term "Life-capital" to have values expressing the mean expectation of life in age-groups. This can be readily accomplished—

(1) By successively adding to the Q_5 of the shortened Life-Table the values of P_4, P_3, P_2, P_1 and P_0 the corresponding Q_x values are obtained, then

$$E_{0-5}=\frac{Q_0+Q_1+Q_2+Q_3+Q_4}{P_0+P_1+P_2+P_3+P_4}-\tfrac{1}{2}.$$

Before proceeding to the next value E_{5-10} it is necessary to calculate a hypothetical Q_0 value (which must not be confounded with the *true* Q_0 value) by the formula $Q_0=4\,(Q_5+Q_{15})-(6Q_{10}+Q_{20})$.

Then (the l_0 value being the previously *calculated* one),

$$(2)\qquad E_{5-10}=\frac{13\,(Q_5+Q_{10})-(Q_0+Q_{15})}{13\,(l_5+l_{10})-(l_0+l_{15})}.$$

A similar formula is to be used for the values of E_{10-15}, E_{15-20} and E_{20-25}.

$$(3)\qquad E_{25-35}=\frac{Q_{25}+4Q_{30}+Q_{35}}{l_{25}+4l_{30}+l_{35}}.$$

A similar formula will give the values as far as E_{75-85}.

$$(4)\qquad E_{85-105}=\frac{(Q_{85}+4Q_{90}+Q_{95})+(l_{95}+4l_{100}+l_{105})}{(l_{85}+4l_{90}+l_{95})+(l_{95}+4l_{100}+l_{105})}.$$

Comparison of the results obtained by the shortened method just described, with the corresponding figures of four extended Life-Tables.

The methods of calculation which have been described have been applied to the data of four extended Life-Tables:

(1) The London Life-Table (for males) based on the experience of 1891—1900.

This was calculated by a very laborious extended method which had been previously suggested by the present writer in a paper contributed to the *Journal of the Royal Statistical Society*, Vol. LXII. Parts 3 and 4.

By elaborate processes of interpolation values of $2P-d$ and $2P+d$ were calculated for every separate year of age, and then the yearly p_x values by $\frac{2P-d}{2P+d}$.

(2) England and Wales (males) 1891—1900.

No official Life-Table has as yet been issued, but in addition to other Life-Tables which have been published by the writer, another has been specially prepared for the purpose of this paper, by a method not quite identical with that of the London Life-Table, but essentially similar, in that values of $2P-d$ and $2P+d$ have been interpolated all throughout in series with five orders of differences. The exact details of the method are given in a lecture by the writer, which forms one of a series of advanced Public Health Lectures given in 1903-4 under the auspices of the Victoria University of Manchester, and published by Sherratt and Hughes at the University Press, Manchester (see pp. 16—19, "Method IV.").

(3) The second Life-Table for Brighton of Dr Newsholme for 1891—1900 (males).

This was constructed by means of the "graphic" method which Dr Newsholme has so ably expounded and advocated.

(4) A Life-Table for Scotland (males) based on the experience of 1891—1900 by Mr T. Adam, M.A., &c., published in the *Journal of the Royal Statistical Society*, Vol. LXVII., Part 3, Sept. 1904.

This also was calculated by the "graphic" method as described by Dr Newsholme.

In order to economise space it is only proposed to give two tables setting forth the differences from the values of the respective extended Life-Tables of the corresponding values obtained by the shortened method.

(1) *Differences of l_x values*—100,000 *at birth.*

Males.

Age	England and Wales	London	Brighton	Scotland
0	0	0	0	0
5	0	0	0	0
10	− 2	−20	− 1	− 8
15	− 2	−19	− 1	− 7
20	+36	−18	+ 31	+ 39
25	+50	+47	+ 51	+ 70
30	+76	−37	+ 64	+ 17
35	− 2	+ 1	+ 16	+ 10
40	+14	+56	+ 32	+ 19
45	+22	+31	− 2	+ 12
50	+14	+27	+ 19	− 14
55	+23	+35	− 14	− 15
60	+52	+55	+ 10	+140
65	+71	+85	+ 44	− 31
70	+57	+68	+128	−167
75	+32	+28	− 15	+ 63
80	+ 7	+ 5	− 27	+203
85	− 2	+ 2	+100	−146

(2) *Differences of E_x values.*

Males.

Age	England and Wales	London	Brighton	Scotland
0	+0·03	+0·03	+0·03	+0·01
5	+0·04	+0·04	+0·06	+0·01
10	+0·04	+0·04	+0·07	+0·02
15	+0·04	+0·05	+0·07	+0·02
20	+0·02	−0·01	+0·05	±0·00
25	+0·01	+0·01	+0·03	−0·02
30	±0·00	+0·05	+0·03	±0·00
35	+0·03	+0·03	+0·05	+0·01
40	+0·02	+0·01	+0·04	±0·00
45	+0·01	+0·02	+0·06	+0·01
50	+0·02	+0·02	+0·06	+0·02
55	+0·02	+0·01	+0·08	+0·02
60	+0·01	+0·01	+0·08	−0·04
65	+0·01	−0·01	+0·08	+0·01
70	±0·00	−0·01	+0·06	+0·08
75	+0·01	±0·00	+0·15	+0·03
80	±0·00	±0·00	+0·30	−0·12
85	±0·00	−0·02	+0·58	+0·12

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The results given in the above tables may almost be left to speak for themselves.

When the results of the short method are compared with those of the two elaborately constructed extended Life-Tables for England and Wales and for London the closeness of approximation is remarkable both as regards l_x and E_x values, but especially the latter, and even the differences of the l_x numbers, to any one who has by experiment found out how wide is the range of differences in the l_x values obtainable from the same data by different extended methods, will appear relatively insignificant.

Such differences, small as they are, as do exist in the E_x values are practically wholly due to differences in the p_x values and therefore in the l_x values, because it has been found in both instances that the simple method of integrating l_x values given at five-yearly intervals *when applied to the numbers of the respective extended Life-Tables* gives coincident E_x values, with the exception of a few differences of $\pm$ 0·01 or $\pm$ 0·02.

The comparison of the results of the short method with those of the two extended Life-Tables constructed by the "graphic" method is also striking as regards the degree of closeness of approximation until the later ages are arrived at.

(1) One inference which may be drawn is that certainly until about the age of 65 the results of the graphic method correspond closely with those obtainable by the most minutely accurate analytical method.

(2) Another possible inference which the writer would suggest is only an extension of what has been already admitted by the advocates of the "graphic" method, viz. that the unreliability and want of accuracy in the results of this method which exist at ages *after* 85, really begin *before* this age is reached, and that it might be better to commence to use the method of differencing the logs of p_x values (*not* the numerical values of the logs) after age 65, certainly not later than age 75.

A simple method of effecting this has been described in this *Journal*, Vol. III., No. 3, pp. 348, 349.

The discrepancies in the E_x results of the Brighton Life-Table are due to the fact that the value of log p_{85} (see Vol. III., No. 3, p. 308 of this *Journal*) is much lower than it is found to be by the more exact application of an analytical method. This has meant that the succeeding values of p_x are too low and the value of E_{85} has been made 3 years instead of about $3\frac{1}{2}$ years. The deficiency in Q_{85} has been enough to lower the E_x values above, even to making E_0 too little by 0·02.

If this correction were made the comparative Table of E_x values would be as close as the results of the other instances.

(3) Another inference which may perhaps be drawn by some readers is that the results obtainable by the shortened method, which it has been the object of this paper to describe, are sufficiently accurate to render it a reliable instrument of statistical work, and to dispense with the trouble of using any extended method whether analytical or graphic.

In conclusion it may be stated that the simplicity and ease of the method have been only arrived at by devious wanderings in the mazes of methods much more complex and difficult.

Corresponding tables for females have also been worked out in all four instances with results equally satisfactory.

However, in the case of England and Wales the comparison has had to be made not with the results of the extended method as used for males, but with the results of a more elaborate shortened method, involving the use of five or six orders of differences, which in the case of males had been found to give results almost coinciding with those of the extended method.

As the object of this paper has been to simplify to the utmost possible extent, the more elaborate shortened method has been abandoned in favour of the simpler method as above described, seeing that the results obtained by this method are sufficiently accurate for all practical purposes.