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Alfvén wings in nonuniform plasmas: analysis using curvilinear coordinates

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Abstract. The results of a previous work, which describes in the magnetohydrodynamic approximation Alfvén wings in nonuniform plasmas, are extended in order to consider more general variations of the background fields. As mathematical tools we use general curvilinear coordinates and stream functions. We prove the possibility of existence of Alfvén wings when the background fields have cylindrical or helical symmetry. For the former, the wings are cylinders, and for the latter, they have helicoidal form; this last includes the case of uniform background fields. We also obtain the relations among the different physical magnitudes in the wing.

1. Introduction

A conducting source moving uniformly through a magnetized plasma generates, among a variety of perturbations, Alfvén waves. An interesting characteristic of Alfvén waves is that they can build up structures in the plasma similarly to the way in which electromagnetic radiation builds up Cherenkov cones. These structures are the regions, characterized by intense electric currents, where the disturbed fields are different from zero. They are called Alfvén wings, and their shape depends on the source's shape and on the background magnetic field; for a point source, in uniform background fields, they are lines that start on it. The first to appreciate this phenomenon were Drell et al. (1965). After them, many papers have been written analyzing Alfvén wings. References can be found in Neubauer (1980) and McKenzie (1991). In almost every case, a uniform background velocity, magnetic field, density, and plasma pressure are supposed. In a recent work, Sallago and Platzeck (2000) have analyzed Alfvén waves and wings in nonuniform magnetized plasmas using the magnetohydrodynamic (MHD) aproximation; the background fields were supposed to vary in a direction perpendicular to the background velocity and magnetic field, which lie in a plane. They showed the existence, under certain conditions, of nonlinear Alfvén waves and Alfvén wings; the Alfvén group velocity is given by an expression similar to that for Alfvén waves in the uniform-plasma case. For the study of Alfvén wings, the methodology of stream functions has been applied (Tsinganos 1982; Agim and Tataronis 1985; Palumbo and Platzeck 1998), and it has been shown that the total plasma pressure, density, and magnetic field modulus are functions of the magnetic flux.

In the present paper, we extend these results for wings, for more general variations of the background fields. In order to do this, we analyze the problem in general curvilinear coordinates. A brief introduction of stream functions in ideal MHD is given in Sec. 2, and the relations that must be fulfilled in the Alfvén limit by incompressible and adiabatic perturbations are stated. The consequences of such relations on the conditions that the background fields must fulfil, in order to support Alfvén wings, is studied. It is shown that the total pressures (plasma plus magnetic) in the perturbed and unperturbed zones are uniform and equal, and the relations between the components of the velocity and magnetic fields of the perturbation are obtained.

In Sec. 3, the different possible background field structures and the corresponding Alfvén wing shapes are analyzed. We conclude that there exist Alfvén wings in some background fields with cylindrical symmetry, or with helical symmetry. In the former case, the wings are cylinders, while in the latter, they are helicoidal.

Finally, we remark that a conducting source moving in a magnetized plasma generates a variety of perturbations, and for studying some of these, the MHD approximation may not be appropriate.

2. Analysis of Alfvén wings in curvilinear coordinates

A conducting source moving in a magnetized plasma, when the background fields satisfy certain conditions, can generate one or two Alfvén wings. The problem of one wing is a stationary one, with a symmetry, in some reference system. Because of this, if an appropriate coordinate system is chosen, one of the coordinates is ignorable for the physical magnitudes and for the metric tensor. This coordinate system may be a general curvilinear one.

The fields with zero divergence, in a curvilinear system (α, β, γ) with γ ignorable, can be derived from stream functions. Since $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \cdot (\rho \mathbf{V}) = 0$, two different stream functions $\psi(\alpha, \beta)$, the magnetic flux, and $\chi(\alpha, \beta)$ are defined in such a way that the following contravariant components B^α, B^β, ρV^α, and ρV^β result (Agim and Tataronis 1985):

$$B^\alpha = \frac{1}{\sqrt{g}} \frac{\partial \psi}{\partial \beta}, \tag{1}$$

$$B^\beta = -\frac{1}{\sqrt{g}} \frac{\partial \psi}{\partial \alpha}, \tag{2}$$

$$\rho V^\alpha = \frac{1}{\sqrt{g}} \frac{\partial \chi}{\partial \beta}, \tag{3}$$

$$\rho V^\beta = -\frac{1}{\sqrt{g}} \frac{\partial \chi}{\partial \alpha}, \tag{4}$$

where g is the determinant of the metric tensor.

Stationary problems in MHD, when an ignorable coordinate is present, have been analyzed by several authors (Tsinganos 1982; Agim and Tataronis 1985; Palumbo and Platzeck 1998). Using (1)–(4), from the MHD equations, the following relations can be derived (Palumbo and Platzeck 1998):

$$\chi = \chi(\psi), \tag{5}$$

$$-\chi' V_\gamma + \frac{B_\gamma}{4\pi} = F_1(\psi), \tag{6}$$

$$-V^\gamma + \frac{\chi' B^\gamma}{\rho} = F_2(\psi), \tag{7}$$

where V_γ and B_γ are covariant components, $\chi' = d\chi/d\psi$, and $F_1(\psi)$ and $F_2(\psi)$ are arbitrary functions of the magnetic flux.

From (6) and (7), using the relation between co- and contravariant components (Santaló 1961; Jackson 1999), we arrive at

$$\left(1 - \frac{4\pi\chi'^2}{\rho}\right) B_\gamma = 4\pi[F_1(\psi) - g_{\gamma\gamma}\chi' F_2(\psi)]. \tag{8}$$

This equation allows us to express B_γ as a function of ψ, $g_{\gamma\gamma}$, and ρ, except in the Alfvén limit given by the condition

$$\chi' = \mp\sqrt{\frac{\rho}{4\pi}}. \tag{9}$$

Instead, in the Alfvén wings, (8) gives the following relation between the functions F_1 and F_2:

$$F_1(\psi) = \pm g_{\gamma\gamma}\sqrt{\frac{\rho}{4\pi}} F_2(\psi). \tag{10}$$

In order that the condition (9) be satisfied along the wing, the density ρ must also be a function of ψ:

$$\rho = \rho(\psi). \tag{11}$$

This condition is equivalent to the incompressibility condition (Palumbo and Platzeck 1998).

In a similar way, the adiabaticity condition implies that

$$p = p(\psi). \tag{12}$$

If Alfvén wings grow in a plasma, the background fields $\mathbf{B}_0$, $\mathbf{V}_0$, ρ_0, and p_0 must be invariant in γ. Let us suppose that the background fields depend on only one variable: α or β, or a function of α and β, $\eta = \eta(\alpha, \beta)$. This means that $\mathbf{B}_0$ and $\rho_0\mathbf{V}_0$ can be derived from stream functions ψ_0 and χ_0, respectively, with these stream functions depending on only one variable; ψ_0 and χ_0 must satisfy (5), $\chi_0 = \chi(\psi_0)$, and (9) for one of the signs. As a consequence, the contravariant components α and β of the Alfvén velocity in a moving plasma given by

$$\mathbf{V}'_A = \mathbf{V}_0 \pm \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}, \tag{13}$$

are zero (see (1)–(4)). Therefore, the only nonzero contravariant component of $\mathbf{V}'_A$ is (see (7) and (9))

$$V_A'^{\gamma} = -F_2(\psi_0). \tag{14}$$

The upper (lower) sign in (9) corresponds to the upper (lower) sign in (13); in what follows, we choose the upper sign.

The values of the unperturbed fields also determine the functions F_1, ρ, and p (see (6), (11) and (12)):

$$\rho_0 = \rho(\psi_0), \tag{15}$$

$$p_0 = p(\psi_0), \tag{16}$$

$$F_1(\psi_0) = -\chi' V_{0\gamma} + \frac{B_{0\gamma}}{4\pi}. \tag{17}$$

On the other hand, in the Alfvén limit, the velocity and total magnetic field satisfy

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the relations (see (1)–(4), (5), (7), and (9))

$$V^{\alpha} = -\frac{B^{\alpha}}{\sqrt{4\pi\rho}}, \tag{18}$$

$$V^{\beta} = -\frac{B^{\beta}}{\sqrt{4\pi\rho}}, \tag{19}$$

$$V^{\gamma} = -\frac{B^{\gamma}}{\sqrt{4\pi\rho}} - F_2(\psi). \tag{20}$$

The respective relations for the perturbations are

$$V_1^{\alpha} = -\frac{B_1^{\alpha}}{\sqrt{4\pi\rho}} + B_0^{\alpha}\left(\frac{1}{\sqrt{4\pi\rho_0}} - \frac{1}{\sqrt{4\pi\rho}}\right), \tag{21}$$

$$V_1^{\beta} = -\frac{B_1^{\beta}}{\sqrt{4\pi\rho}} + B_0^{\beta}\left(\frac{1}{\sqrt{4\pi\rho_0}} - \frac{1}{\sqrt{4\pi\rho}}\right), \tag{22}$$

$$V_1^{\gamma} = -\frac{B_1^{\gamma}}{\sqrt{4\pi\rho}} + B_0^{\gamma}\left(\frac{1}{\sqrt{4\pi\rho_0}} - \frac{1}{\sqrt{4\pi\rho}}\right) + F_2(\psi_0) - F_2(\psi), \tag{23}$$

which are similar to those that are valid when the background fields vary in a rectilinear direction (Sallago and Platzeck 2000).

Equation (6) takes into account only the γ component of the equation of motion (Palumbo and Platzeck 1998). This equation in the stationary case can be written as

$$\rho(\mathbf{V}\cdot\boldsymbol{\nabla})\mathbf{V} = -\boldsymbol{\nabla}p - \boldsymbol{\nabla}\frac{|\mathbf{B}|^2}{8\pi} + \left(\frac{\mathbf{B}}{4\pi}\cdot\boldsymbol{\nabla}\right)\mathbf{B}. \tag{24}$$

From the invariance in γ, the relations (18) and (19), and the fact that χ' is a function of ψ, the following equation results:

$$\boldsymbol{\nabla}P = (\mathbf{B}\cdot\boldsymbol{\nabla})\left(\frac{\mathbf{B}}{4\pi} - \chi'\mathbf{V}\right), \tag{25}$$

where P is the total pressure (plasma plus magnetic):

$$P = p + \frac{|\mathbf{B}|^2}{8\pi}. \tag{26}$$

The covariant γ component of $\boldsymbol{\nabla}P$ is zero because of the invariance in γ. From (18), (19), and (25), the contravariant α and β components also vanish. Then, in the Alfvén limit,

$$\boldsymbol{\nabla}P = 0. \tag{27}$$

Therefore, for the perturbed and the unperturbed regions, the total pressure remains constant:

$$p + \frac{|\mathbf{B}|^2}{8\pi} = p_0 + \frac{|\mathbf{B}_0|^2}{8\pi} = \text{const.} \tag{28}$$

Taking into account that from the adiabaticity condition $p = p(\psi)$ (see (12)), it results that

$$|\mathbf{B}|^2 = |\mathbf{B}|^2(\psi). \tag{29}$$

In order to determine the magnetic flux ψ, we take into account that the value of the contravariant component J^γ determines a differential equation for the stream function ψ (Palumbo and Platzeck 1998):

$$\frac{D^2\psi}{g_{\gamma\gamma}} - G(B_\gamma) = -\frac{4\pi}{c}J^\gamma, \tag{30}$$

where

$$\begin{aligned} D^2\psi = \frac{g_{\gamma\gamma}}{\sqrt{g}}\bigg[&\frac{\partial}{\partial\alpha}\left(\frac{g^{\alpha\alpha}\sqrt{g}}{g_{\gamma\gamma}}\frac{\partial\psi}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{g^{\beta\beta}\sqrt{g}}{g_{\gamma\gamma}}\frac{\partial\psi}{\partial\beta}\right) \\ &+\frac{\partial}{\partial\alpha}\left(\frac{g^{\alpha\beta}\sqrt{g}}{g_{\gamma\gamma}}\frac{\partial\psi}{\partial\beta}\right) + \frac{\partial}{\partial\beta}\left(\frac{g^{\alpha\beta}\sqrt{g}}{g_{\gamma\gamma}}\frac{\partial\psi}{\partial\alpha}\right)\bigg], \end{aligned} \tag{31}$$

$$G(B_\gamma) = \frac{\Gamma B_\gamma}{g_{\gamma\gamma}} + \frac{1}{g_{\gamma\gamma}\sqrt{g}}\left(g_{\beta\gamma}\frac{\partial B_\gamma}{\partial\alpha} - g_{\alpha\gamma}\frac{\partial B_\gamma}{\partial\beta}\right), \tag{32}$$

with

$$\Gamma = \frac{g_{\gamma\gamma}}{\sqrt{g}}\left[\frac{\partial}{\partial\alpha}\left(\frac{g_{\beta\gamma}}{g_{\gamma\gamma}}\right) - \frac{\partial}{\partial\beta}\left(\frac{g_{\alpha\gamma}}{g_{\gamma\gamma}}\right)\right]. \tag{33}$$

Moreover, since

$$\rho\mathbf{V}\cdot\boldsymbol{\nabla}\psi = \frac{\partial\chi}{\partial\alpha}\frac{\partial\psi}{\partial\beta} - \frac{\partial\chi}{\partial\beta}\frac{\partial\psi}{\partial\alpha} = 0, \tag{34}$$

the convective derivative of ψ is zero; therefore the value of ψ for a given plasma element is the same before and after entering the wing. As a consequence, the same is true for ρ, p, and $|\mathbf{B}|^2$ (see (11), (12), and (29)). This implies also that J^γ cannot take arbitrary values across all of the wing.

It is important to note that (10) imposes restrictions on the possible functional dependences of $g_{\gamma\gamma}$, F_1, and F_2 in order to construct Alfvén wings. Since $g_{\gamma\gamma}$ and ψ are not related, (10) can only be satisfied in two cases:

(i) when the $g_{\gamma\gamma}$ component of the metric tensor is a constant;

(ii) when $F_1(\psi)$ and $F_2(\psi)$ are both zero functions.

This implies a restriction on the kind of spatial dependence of the background fields in which Alfvén wings can be generated. The shape of the wings is closely related to the structure of the background fields.

3. Structure of the background fields and the wing shapes

In this section, we analyze the structure of the different background fields that can support Alfvén wings and the corresponding wing shapes.

3.1. Constant $g_{\gamma\gamma}$

If one imposes that $g_{\gamma\gamma}$ be a constant, there are essentially two kinds of coordinate systems: Cartesian and cylindrical coordinates. The situation in Cartesian coordinates has been discussed in a previous paper (Sallago and Platzeck 2000). In cylindrical coordinates, in order that $g_{\gamma\gamma}$ be constant, z must be the ignorable coordinate; therefore the group velocity $\mathbf{V}_A'$ is in the direction of the cylinder axis, and then the Alfvén wing shape is a z-axis cylinder whose section depends on the source shape. The conducting source, in this case, can be at rest or moving uniformly along

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an axis parallel to the z axis. The problem may be consider as stationary with a symmetry in a subspace that contains the wing and does not contain the source.

Different configurations are possible according to the variable, r, φ, or $\eta(r,\varphi)$, on which depends the stream function ψ_0 of the background field $\mathbf{B}_0$, and, in consequence, ρ_0, p_0, and $|\mathbf{B}_0|^2$.

3.1.1. Dependence on r. The contraviariant r components of the magnetic induction and velocity fields are zero (see (1)–(4)). The other contravariant components B_0^φ, B_0^z, V_0^φ, and V_0^z depend only on r; therefore, the field lines are helices. Notice that the Alfvén wing axis does not coincide, in general, with the axis of the helices; this depends on the position of the source.

3.1.2. Dependence on φ. In this case the non-zero components of the magnetic induction field are

$$B_0^r = \frac{1}{r}\frac{d\psi_0}{d\varphi}, \tag{35}$$

$$B_0^z = \sqrt{|\boldsymbol{B}_0|^2(\varphi) - \frac{1}{r^2}\left(\frac{d\psi_0}{d\varphi}\right)^2}. \tag{36}$$

Using (3), (5), and (7), for the velocity field we have

$$V_0^r = -\frac{1}{\sqrt{4\pi\rho_0(\varphi)}r}\frac{d\psi_0}{d\varphi}, \tag{37}$$

$$V_0^z = -\frac{B_0^z}{\sqrt{4\pi\rho_0(\varphi)}} - F_2[\psi(\varphi)]. \tag{38}$$

The field lines of the background fields lie on constant φ-planes; the relations (35)–(38) allow a great variety for their shapes.

In order to avoid the singularity at $r = 0$, it is necessary to divide the space into two regions: $r < R$ and $r > R$, with an appropriate value for R. If the Alfvén wing exists, it will be located in the region $r > R$.

3.1.3. Dependence on $\eta(r,\varphi)$. In the most general case, one can define a variable $\eta = \eta(r,\varphi)$. If the magnetic stream function ψ_0 depends on η, then the contravariant components of the background magnetic field are

$$B_0^r = \frac{1}{r}\frac{\partial\eta}{\partial\varphi}\frac{d\psi_0}{d\eta}, \tag{39}$$

$$B_0^\varphi = -\frac{1}{r}\frac{\partial\eta}{\partial r}\frac{d\psi_0}{d\eta}, \tag{40}$$

$$B_0^z = \sqrt{|\mathbf{B}_0|^2(\eta) - \left[\frac{1}{r^2}\left(\frac{\partial\eta}{\partial\varphi}\right)^2 + \left(\frac{\partial\eta}{\partial r}\right)^2\right]\left(\frac{d\psi_0}{d\eta}\right)^2}. \tag{41}$$

For the components $\rho_0 V_0^r$ and $\rho_0 V_0^\varphi$, we have relations similar to (39) and (40), changing ψ_0 to χ_0; and the expression for V_0^z is similar to (38), where F_2 and ρ_0 are now functions of η. The magnetic field lines, and the velocity lines also, lie on surfaces $\eta = \text{const}$.

Depending on the functional dependence of η, a singularity may appear at $r = 0$. If this happens, it is solved as in Sec. 3.1.2.

3.2. $F_1(\psi)$ and $F_2(\psi)$ zero

As discussed above, the other possibility for the fulfilment of (10) is that $F_1(\psi)$ and $F_2(\psi)$ both be zero functions. This means that $\mathbf{V}_A'$ is zero in the reference system K in which the problem is a stationary one. In this case, since the restriction $g_{\gamma\gamma} = \text{const}$ is not necessary, it is possible to choose helicoidal coordinates $r = r$, $\xi = a\varphi - z$, and $\gamma = z$, where (r, φ, z) are the cylindrical coordinates. Since the Alfvén wing is invariant in γ, its 'axis' is along a line $r = \text{const}$, $\xi = \text{const}$. This is a helix whose axis is the z axis.

The conducting source is moving in a circular motion in a plane perpendicular to the z axis, in a reference system K_* that is moving with a constant velocity $v_* = (\omega/2\pi)h$ along z, as viewed from K, where h is the pitch of the helical wing and ω is the angular velocity of the source. In this system, the problem is not a stationary one, $\mathbf{V}_{A*}' = -\mathbf{v}_*$, and the helicoidal wing appears to be moving with this velocity.

If we suppose helical symmetry for the background fields, the variety of possible structures for them is very large. The simplest situation is when the background fields $\mathbf{V}_0$ and $\mathbf{B}_0$ have only z components. Let us analyze the dependence of ψ_0 in the reference system in which the wing is stationary. Since helicoidal coordinates are not orthogonal, it is convenient to write down the relations between the contravariant components of a vector in this system and the corresponding components in cylindrical coordinates:

$$B^r_{\text{cyl}} = B^r, \tag{42}$$

$$B^\varphi_{\text{cyl}} = \frac{B^\xi + B^\gamma}{|a|}, \tag{43}$$

$$B^z_{\text{cyl}} = B^\gamma. \tag{44}$$

Thus, taking into account that $\sqrt{g} = r/|a|$, the cylindrical components of $\mathbf{B}$, using the current function $\psi(r, \xi)$, can be written as (see (1) and (2))

$$B^r_{\text{cyl}} = \frac{|a|}{r}\frac{\partial\psi}{\partial\xi}, \tag{45}$$

$$B^\varphi_{\text{cyl}} = \frac{1}{|a|}\left(-\frac{|a|}{r}\frac{\partial\psi}{\partial r} + B^\gamma\right). \tag{46}$$

In order that the background field $\mathbf{B}_0$ have only z component, $B^r_{0\,\text{cyl}}$ and $B^\varphi_{0\,\text{cyl}}$ must be zero. Therefore, ψ_0 must depend only on r, and

$$B^\gamma_0(r) = \frac{|a|}{r}\frac{d\psi_0}{dr}; \tag{47}$$

as a consequence, $B^z_{0\,\text{cyl}}$ depends only on r. From (15), the density ρ_0 also depends only on r; since $\mathbf{V}_A'$ is zero, the only non-zero z component of the plasma unperturbed velocity is (see (13))

$$V^z_{0\,\text{cyl}}(r) = -\frac{B^z_{0\,\text{cyl}}}{\sqrt{4\pi\rho_0}}. \tag{48}$$

Although this seems to be a restriction on the background plasma velocity, it need be fulfilled only in the reference system K.

Helical Alfvén wings can also exist in general axisymmetric background fields that do not depend on z in order to be invariant in γ. The non-zero components of the background magnetic field are $B^{\varphi}_{0\,\mathrm{cyl}}$ and $B^{z}_{0\,\mathrm{cyl}}$, and one can obtain the value of $d\psi_0/dr$ from (46).

Note that we have used helicoidal coordinates, but the background fields can be uniform; any conducting source moving circularly in a plane perpendicular to $\mathbf{B}_0$ generates an Alfvén wing. This looks like a large curled object whose section depends on the source's shape.

4. Conclusions

Extending the results of a previous paper, we have proved that Alfvén wings are solutions of the MHD equations for a magnetized plasma with nonuniform background fields. The spatial dependence for background fields $\mathbf{B}_0$, $\mathbf{V}_0$, ρ_0, and p_0 is quite general, but a symmetry is needed. We have applied the methodology of stream functions in ideal MHD. We have considered cylindrical and helicoidal symmetries. For the former, the Alfvén wings are cylinders whose sections depend on the conducting source. For the latter, the wings have helicoidal form; this includes the case in which the background fields are uniform and the source moves in a circular motion. In a similar way that happens when the background fields vary in a rectilinear direction, the plasma pressure, the density, and the magnetic field modulus are functions of the magnetic flux ψ; therefore, their values for a given plasma element are the same before and after entering the wing. The total pressure (magnetic plus plasma) is uniform, so it is equal in the perturbed and unperturbed regions; the velocity and magnetic field components of the perturbation are related, and this relation depends on the background fields.

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