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RANDOM MATCHING AND MONEY IN THE NEOCLASSICAL GROWTH MODEL: SOME ANALYTICAL RESULTS

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I use the monetary version of the neoclassical growth model developed by Aruoba, Waller, and Wright [*Journal of Monetary Economics* (2011)] to study the properties of the model when there is exogenous growth. I first consider the planner's problem, and then the equilibrium outcome in a monetary economy. I do so by first using proportional bargaining to determine the terms of trade and then considering competitive price taking. I obtain closed-form solutions for all variables along the balanced growth path in all cases. I then derive closed-form solutions for the transition paths under the assumption of full depreciation and, in the monetary economy, a particular nonstationary interest rate policy. The key result is that inflation is damaging to per capita income levels along the balanced growth path and to short-run growth of the economy.

Keywords: Growth, Money, Search

1. INTRODUCTION

The effect of inflation on economic growth is a classic issue in monetary economics. Early contributions by Tobin (1965) and Sidrauski (1967a, 1967b) gave us insights as to how inflation could deter (or stimulate) economic growth. The RBC literature revived the neoclassical growth model and made it the workhorse of modern macroeconomics. This gave rise to a renewed interest in studying the effects of inflation on growth, with notable work being done by Cooley and Hansen (1989), Gomme (1993), and Ireland (1994). In all of these models, money is "forced" into the neoclassical growth model via the assumption of cash in advance. Thus, although the real side of these models has well-understood microfoundations, the monetary side does not.

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During this same time period, tremendous progress was made in understanding the microfoundations of money. Starting with the seminal work of Kiyotaki and Wright (1989, 1993), search-theoretic models of money provided deep insights into the role of money as a medium of exchange. These models aided us in understanding how the value of money is affected by information frictions, matching frictions and pricing protocols such as bargaining—features that are absent from the standard neoclassical growth model. As a result, substantial work has been done trying to integrate modern monetary theory with mainstream macroeconomics so that we would have a better understanding of how inflation affects capital accumulation and/or growth. Research along these lines has been done by Shi (1999), Aruoba and Wright (2003), Menner (2006) Berentsen et al. (2009), Aruoba and Chugh (2010), and Aruoba et al. (2011).

My objective here is to contribute to this growing literature. I do so by providing analytical results on steady-state growth and transitional dynamics in the Aruoba et al. (AWW) (2011) model of money and capital. Whereas AWW focus mainly on the quantitative aspects of inflation on capital accumulation and growth, in this paper I focus on analytical properties of the model, in particular the necessary conditions for balanced growth and how key features such as search frictions and bargaining affect steady-state ratios and transitional dynamics.[1]

The AWW framework embeds a monetary search sector into the neoclassical growth model. However, the AWW paper does not have growth, nor does it address the conditions needed for balanced growth. Thus, in this paper, I add exogenous labor-enhancing technological change to the AWW model and determine the necessary conditions for balanced growth in this model. I then obtain closed-form analytical solutions for the steady state capital-to-labor ratio for (1) the planner allocation, (2) the monetary equilibrium with proportional bargaining, and (3) the monetary equilibrium with price-taking. I then study the transition dynamics of the model under the assumption of full depreciation of capital. For the planner allocation, the saving rate is constant, the capital–labor ratio converges monotonically to its steady state value, and hours are constant along the transition path. For the monetary economy, given a constant–interest rate policy, this is not the case—hours vary along the transition path, which makes the saving rate vary as well. I then consider a particular nonstationary policy for the nominal interest rate. Under this policy, the nominal interest adjusts to the growth rate of real wages—if wage growth is excessively high, the nominal interest is below its steady state value. This policy makes the opportunity cost of holding money constant. With this policy, I am able to obtain closed-form solutions for the transition paths, under both pricing mechanisms. These solutions involve a constant saving rate, constant hours along the transition path, and monotone convergence of the capital–labor ratio to its steady state value.

The key findings of this analysis are as follows. First, inflation lowers the capital–labor ratio along the balanced growth path for both bargaining and price-taking. Hence, inflation lowers the level of per capita income along the balanced growth path. Second, inflation lowers the growth rate of the capital–labor ratio along the transition path, implying that inflation lowers short-term growth. Third, with bargaining, the hold-up problem on capital lowers the capital–output ratio along the balanced growth and lowers the growth rate of the economy along the transition path. These results provide support for the view that inflation is bad for both income per capita and short-term economic growth.

2. ENVIRONMENT

The environment is essentially that of AWW, which builds on the basic Lagos–Wright (2005) monetary model, denoted LW hereafter. A [0, 1] continuum of agents live forever in discrete time. Following LW, trade occurs in two separate subperiods. In the first subperiod trade occurs in a decentralized market, or DM for short, whereas in subperiod 2, trade occurs in a perfect competitive centralized market, denoted CM. In the DM, there is a double coincidence problem and private trading histories are private information; i.e., agents are anonymous.

As in AWW, there are two assets available to households, capital and money. Capital is assumed to be nonportable in the DM, so buyers must search for sellers. So capital cannot be used as a medium of exchange and claims to such capital can be costlessly counterfeited just as IOUs can be counterfeited. Thus, money has a role even when capital is a storable factor of production.

In the CM there is a general good produced using labor H and capital K that can be used for consumption or investment. Production occurs according to the aggregate production function $Y_t = F(K_t, Z_t H_t)$, where F is the technology and Z_t is a labor/effort-augmenting technology factor that evolves according to the process $Z_t = (1+\mu) Z_{t-1}$. We also have $Y_t/Z_t = F(K_t/Z_t, H_t)$. Capital is assumed to depreciate at a rate $0 \leq \delta \leq 1$.

In the DM, each period with probability σ an agent can consume but not produce, whereas with the symmetric probability he or she can produce but not consume. With probability $1-2\sigma$ he or she is a *nontrader*—he neither produces nor consumes and gets a utility payoff of zero. Due to symmetry in the measure of buyers and sellers, I assume that there is a matching technology that randomly assigns one buyer to one seller. Sellers in the DM can produce output q_t using their own effort e and capital k using a CRTS technology $f(k_t, Z_t e_t)$. Sellers produce where their capital is located, so they have access to their capital, even though buyers do not. We then have $q_t/Z_t = f(k_t/Z_t, e_t)$.

Instantaneous utility for everyone in the CM is $U(x) - Ah$, where x is consumption and h labor. Preferences are separable in consumption and leisure. In the DM, with probability σ you are a buyer and enjoy utility $u(q)$, and with probability σ you are a seller and get disutility $\ell(e)$, where q is consumption and e labor. The utility functions u and U have the usual monotonicity and curvature properties and $u(0) = 0$. Solving $q_t/Z_t = f(k_t Z_t, e_t)$ for $e_t = f^{-1}(q_t/Z_t, k_t/Z_t)$, we get the utility cost of producing q given k – $\ell(e) = \ell[f^{-1}(q_t/Z_t, k_t/Z_t)] \equiv c(q_t/Z_t, k_t/Z_t)$. Monotonicity and convexity imply this latter function has the properties $c_q, c_{qq} > 0$, $c_k < 0$, $c_{kk} > 0$, and

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$c_{qk} < 0$, because $f_k f_{ee} < f_e f_{ek}$ holds when k is a normal input. Agents discount across periods at rate $\beta = (1+\rho)^{-1}$, where ρ is the time rate of discount.

The money stock is given by M_t and evolves according to the process $M_t = \gamma M_{t-1}$. Agents receive a lump-sum transfer of cash, τM, in the CM. In an earlier version of this paper, I included exogenously determined government spending and taxes; they are excluded here to minimize clutter and focus on how trading frictions and bargaining affect the steady-state allocation and dynamics. For notational simplicity, period $t+1$ is denoted $+1$, and so. Agents discount between the CM and DM at a rate β but not between the DM and CM.

3. PLANNER ALLOCATION

Consider the planner's problem in this economy, where agents are treated symmetrically and the planner can dictate quantities traded. The planner's problem is

$$J(K) = \max_{q,X,H,K_{+1}} \left[\sigma u(q) - \sigma c\left(\frac{q}{Z}, \frac{K}{Z}\right) + U(X) - AH + \beta J(K_{+1}) \right], \quad \textbf{(1)}$$
$$\text{s.t. } X = F(K, ZH) + (1-\delta)K - K_{+1}.$$

Eliminating X and differentiating, the first-order conditions are

$$q: \qquad u'(q) = c_q\left(\frac{q}{Z}, \frac{K}{Z}\right)\frac{1}{Z}, \quad \textbf{(2)}$$

$$H: \qquad A = U'(X)F_H(K, ZH)Z,$$
$$K_{+1}: \quad U'(X) = \beta J'(K_{+1}).$$

Also, using $J'(K) = U'(X)[F_K(K, ZH) + 1 - \delta] - \sigma c_k(q/Z, k/Z)\,1/Z$, we have

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1}, Z_{+1}H_{+1}) + 1 - \delta] - \beta\sigma c_k\left(\frac{q_{+1}}{Z_{+1}}, \frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}. \quad \textbf{(3)}$$

So the equilibrium allocation solves

$$u'(q) = c_q\left(\frac{q}{Z}, \frac{K}{Z}\right)\frac{1}{Z}, \quad \textbf{(4)}$$

$$A = U'(X)F_H(K/Z, H)Z, \quad \textbf{(5)}$$

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1}/Z_{+1}, H_{+1}) + 1 - \delta]$$
$$- \beta\sigma c_k\left(\frac{q_{+1}}{Z_{+1}}, \frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}, \quad \textbf{(6)}$$

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$$X = ZF(K/Z, H) + (1-\delta)K - K_{+1}. \tag{7}$$

Two comments are in order. First, if $\sigma = 0$, then the DM shuts down and the model collapses to the standard neoclassical growth model. Second, if capital is not productive in the DM, then the model dichotomizes as in Aruoba and Wright (2003)—the steady evolution of K, X, H, and Y can be determined independently using (5)–(7), whereas (4) determines q/Z.

Consider the following functional forms:

$$\begin{aligned}
F(K, ZH) &= K^{\alpha}(ZH)^{1-\alpha}, \quad 0<\alpha<1,\\
U(X) &= B\frac{X^{1-\varepsilon}-1}{1-\varepsilon}, \quad \varepsilon \neq 1,\ \text{or}\, U(X) = B\ln X \text{ for } \varepsilon = 1,\\
u(q) &= \frac{(q+b)^{1-\eta}-b^{1-\eta}}{1-\eta}, \quad \eta \neq 1,\ \text{or}\, u(q) = \ln\left(\frac{q+b}{b}\right) \text{ for } \eta = 1,\\
c\left(\frac{q}{Z},\frac{k}{Z}\right) &= \left(\frac{q}{Z}\right)^{\psi}\left(\frac{k}{Z}\right)^{1-\psi}, \quad \psi \geq 1\\
&\Rightarrow c_q\left(\frac{q}{Z},\frac{k}{Z}\right)\frac{1}{Z} = \frac{\psi}{Z}\left(\frac{q}{k}\right)^{\psi-1}\\
&\Rightarrow c_k\left(\frac{q}{Z},\frac{k}{Z}\right)\frac{1}{Z} = -\frac{(\psi-1)}{Z}\left(\frac{q}{k}\right)^{\psi}.
\end{aligned}$$

Having DM utility depend on the parameter $b > 0$ is done to force utility through the origin. This only matters for bargaining, because it prevents buyers from suffering infinite disutility if bargaining were to break down. However, we can allow $b \to 0$ asymptotically to eliminate its influence on equilibrium outcomes. Without bargaining, we do not need $u(q)$ to go through the origin, so $b = 0$ in this case. Hence, (4)–(7) become

$$X^{\varepsilon} = \frac{(1-\alpha)B}{A}\left(\frac{K}{ZH}\right)^{\alpha} Z, \tag{8}$$

$$q^{-\eta} = \frac{\psi}{Z}\left(\frac{K}{q}\right)^{1-\psi}, \tag{9}$$

$$\begin{aligned}
\left(\frac{X_{+1}}{X}\right)^{\varepsilon} &= \beta\left[\alpha\left(\frac{K_{+1}}{Z_{+1}H_{+1}}\right)^{\alpha-1} + 1 - \delta\right]\\
&+ \beta\sigma(\psi-1)\frac{X_{+1}^{\varepsilon}}{BZ_{+1}}\left(\frac{q_{+1}}{K_{+1}}\right)^{\psi},
\end{aligned} \tag{10}$$

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$$\frac{X}{ZH} = \left(\frac{K}{ZH}\right)^{\alpha} + (1-\delta)\frac{K}{ZH} - \frac{Z_{+1}H_{+1}}{ZH}\frac{K_{+1}}{Z_{+1}H_{+1}}. \tag{11}$$

3.1. Balanced Growth

Conjecture that there is a balanced growth path with constant aggregate hours $H_{+1} = H$ for all t. This implies that we have a constant value of capital per efficiency labor unit, $\hat{K} = K/ZH$, and all real variables grow at the rate $1+\mu$.

Using (8) and (11) yields

$$K = [(1-\alpha)BA^{-1}]^{1/\varepsilon} \frac{\hat{K}^{1-\alpha+\alpha/\varepsilon}}{1-(\delta+\mu)\hat{K}^{1-\alpha}} Z^{1/\varepsilon},$$

where $K > 0$ if $(\delta+\mu)^{-1/(1-\alpha)} > \hat{K}$. This implies that K grows at a gross rate $(1+\mu)^{1/\varepsilon}$. With constant hours and Z growing at rate $1+\mu$ we need $\varepsilon = 1$ or log utility in the CM. This is standard in the neoclassical growth model when preferences are separable over consumption and leisure. Thus, I will impose $\varepsilon = 1$ for the remainder of this section. Steady-state hours and consumption are then given by

$$H = \frac{(1-\alpha)\,BA^{-1}}{1-(\delta+\mu)\,\hat{K}^{1-\alpha}},$$

$$X = (1-\alpha)\,BA^{-1}\hat{K}^{\alpha}Z.$$

From (9) we obtain

$$q = \left\{\frac{1}{\psi}[(1-\alpha)\,BA^{-1}]^{\psi-1}\left[\frac{\hat{K}}{1-(\delta+\mu)\,\hat{K}^{1-\alpha}}\right]^{\psi-1} Z^{\psi}\right\}^{\frac{1}{\psi+\eta-1}},$$

$$\frac{q_{+1}}{q} = (1+\mu)^{\frac{\psi}{\psi+\eta-1}},$$

$$\frac{q_{+1}}{K_{+1}} = \left\{\frac{1}{\psi}\left[\frac{1}{(1-\alpha)\,BA^{-1}}\right]^{\eta}\left[\frac{1-(\delta+\mu)\,\hat{K}^{1-\alpha}}{\hat{K}}\right]^{\eta} Z^{1-\eta}\right\}^{\frac{1}{\psi+\eta-1}}.$$

The growth rate of q equals $1+\mu$ when $\eta = 1$, which also makes q_{+1}/K_{+1} constant in the steady state. Hence, we need log preferences in both the DM and the CM to have balanced growth. Set $\eta = 1$ for the rest of this section to simplify expression. Note that $dq/d\hat{K} > 0$.

Finally, using (8), (10), and (11) with $\varepsilon = \eta = 1$, we obtain the planner's choice of $\hat{K}$ and H:

$$\hat{K}_p = \left\{ \frac{\alpha\beta + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)}{1 + \mu - \beta(1-\delta) + (\delta + \mu)\sigma\beta\left(\frac{\psi - 1}{\psi B}\right)} \right\}^{\frac{1}{1-\alpha}}, \tag{12}$$

$$H_p = (1-\alpha) B A^{-1} \left[\frac{1 + \mu - \beta(1-\delta) + (\delta + \mu)\sigma\beta\left(\frac{\psi - 1}{\psi B}\right)}{1 + \mu - \beta(1-\delta) - (\delta + \mu)\alpha\beta} \right], \tag{13}$$

$$\hat{q}_p \equiv \frac{q}{Z} = \left(\frac{1}{\psi}\right)^{\frac{1}{\psi}} \left[(1-\alpha) B A^{-1}\right]^{\frac{\psi-1}{\psi}} \left[\frac{\hat{K}_p}{1 - (\delta + \mu)\hat{K}_p^{1-\alpha}} \right]^{\frac{\psi-1}{\psi}}. \tag{14}$$

So we have a balanced growth path with K, X, and q all growing at a gross rate $1 + \mu$. For $\sigma > 0$ and $\psi > 1$, capital has additional value for producing in the DM, so the steady-state capital per efficiency unit of labor is higher than in the standard neoclassical growth model and hours are higher.

3.2. Dynamics

To obtain some analytical results on the transitional dynamics, let $\delta = 1$. Clearly this is a severe restriction that is violated in the data, but it allows us to obtain analytical results and insights into how the model works. From (9),

$$\frac{q}{K} = \left(\frac{Z}{\psi K}\right)^{1/\psi},$$

whereas (8) and (11) yield

$$K_{+1} = \left[1 - \frac{(1-\alpha) B A^{-1}}{H}\right] K^{\alpha} (ZH)^{1-\alpha}. \tag{15}$$

We can then write the Euler equation as

$$K_{+1} = \frac{1}{H}\left[\alpha\beta\frac{H_{+1}}{H} + \sigma\beta\left(\frac{\psi - 1}{\psi}\right)(1-\alpha) A^{-1}\right] K^{\alpha} Z^{1-\alpha} H^{1-\alpha}. \tag{16}$$

Conjecture that hours are constant for all t along the transition path. Combining (15) and (16) gives us the planner's choice of hours,

$$H_p = \frac{(1-\alpha) B A^{-1}}{1 - \beta\alpha}\left[1 + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)\right].$$

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With full depreciation, the planner keeps hours at the steady state value. For $\sigma > 0$ and $\psi > 1$, hours are higher along the transition path than in the standard neoclassical growth model. It also implies that investment (CM consumption) is a higher (lower) fraction of output with transitional dynamic paths given by

$$K_{+1} = \left[\frac{\alpha\beta + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)}{1 + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)} \right] K^{\alpha} (ZH)^{1-\alpha},$$

$$X = \left[\frac{1 - \alpha\beta}{1 + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)} \right] K^{\alpha} (ZH)^{1-\alpha},$$

and the transition path for $\hat{K}$ is given by

$$\hat{K}_{+1} = \left(\frac{1}{1+\mu}\right) \left[\frac{\alpha\beta + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)}{1 + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)} \right] \hat{K}^{\alpha},$$

so the corresponding growth rate of capital is

$$\frac{\hat{K}_{+1}}{\hat{K}} = \left(\frac{1}{1+\mu}\right) \left[\frac{\alpha\beta + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)}{1 + \sigma\beta\left(\frac{\psi - 1}{\psi B}\right)} \right] \hat{K}^{\alpha-1}.$$

If $\sigma = 0$ or $\psi = 1$, we have the standard transition path for capital in the Cass–Koopmans model. For $\sigma > 0$ and $\psi > 1$, capital has additional productivity in the DM, which implies that, starting from the same value $\hat{K}$, capital is also accumulated at a faster rate than in the standard neoclassical growth model.

4. MONETARY ECONOMY

In the monetary economy, firms hire labor and capital to produce output, which is sold in the CM at the monetary price p. Goods and input markets are perfectly competitive. Profit maximization implies that $r_t = F_K(K_t/Z_t, H_t)$ and $w_t = F(K_t/Z_t, H_t)Z_t$, where r is the rental rate, and w is the real wage. Constant returns implies equilibrium profits are 0. Firms do not operate in the DM, but agents can use their capital and effort to produce output.

Let $W(m, k, Z)$ and $V(m, k, Z)$ be the value functions of agents in the CM and DM, respectively, when they hold m units of money and k units of capital given

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the aggregate state Z. Beginning with the CM, we have

$$W(m,k,Z) = \max_{x,h,m_{+1},k_{+1}} \{U(x) - Ah + \beta V(m_{+1},k_{+1},Z_{+1})\},$$

$$\text{s.t. } x = wh + (1+r-\delta)\,k - k_{+1} + \tau M + \frac{m-m_{+1}}{p}.$$

Eliminating h using the budget equation, we have the first-order conditions

$$x : U'(x) = \frac{A}{w}, \tag{17}$$

$$m_{+1} : \frac{A}{pw} = \beta V_m\,(m_{+1},k_{+1},Z_{+1}),$$

$$k_{+1} : \frac{A}{w} = \beta V_k\,(m_{+1},k_{+1},Z_{+1})$$

and the envelope conditions

$$W_m(m,k,Z) = \frac{A}{pw}, \tag{18}$$

$$W_k(m,k,Z) = \frac{A\,(1+r-\delta)}{w}. \tag{19}$$

In the DM, we have

$$V(m,k,Z) = \sigma V_b(m,k,Z) + \sigma V_s(m,k,Z) + (1-2\sigma)W(m,k,Z) \tag{20}$$

with

$$V_b(m,k,Z) = u(q_b) + W\,(m-d_b,k,Z), \tag{21}$$

$$V_s(m,k,Z) = -c\left(\frac{q_s}{Z},\frac{k}{Z}\right) + W\,(m+d_s,k,Z), \tag{22}$$

where q_b and d_b are the quantities of goods acquired and money spent by buyers in the DM, whereas q_s and d_s are the quantities of goods produced and money earned by sellers.

Using (18), we have

$$V(m,k,Z) = \sigma\left[u(q_b) - d_b\frac{A}{pw} - c\left(\frac{q_s}{Z},\frac{k}{Z}\right) + d_s\frac{A}{pw}\right] + W(m,k,Z).$$

Differentiating yields

$$V_m(m,k,Z) = \sigma\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw}\frac{\partial d_b}{\partial m}\right] + \sigma\left[-c_q\frac{\partial q_s}{\partial m} + \frac{A}{pw}\frac{\partial d_s}{\partial m}\right] + \frac{A}{pw},$$

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$$V_k(m,k,Z) = \sigma\left[-\frac{c_q}{Z}\frac{\partial q_s}{\partial k} - \frac{c_k}{Z} + \frac{A(1+r-\delta)}{w}\frac{\partial d_s}{\partial k}\right] + \sigma\left[u'\frac{\partial q_b}{\partial k} - \frac{A(1+r-\delta)}{w}\frac{\partial d_b}{\partial k}\right] + \frac{A(1+r-\delta)}{w}.$$

To solve (18), we must evaluate these derivatives. To do that, we need to describe how the terms of trade are determined in the DM. One possibility is price-taking. Another is bargaining.

4.1. Proportional Bargaining

Suppose agents are randomly matched in a bilateral fashion in the DM, with each buyer being randomly paired with a seller. In the search-theoretic models of money, bargaining has traditionally been used to determine the terms of trade in bilateral trades, with Nash bargaining being the standard. However, as Aruoba et al. (2007) emphasize, in the LW framework, Nash bargaining generates nonmonotonic surpluses for buyers. Thus inefficiencies occurring under the Friedman rule are due to this property of the bargaining solution rather than a holdup problem, as suggested by LW.

To avoid this problem, I will consider proportional bargaining as the way in which terms of trade are determined. Under proportional bargaining, the buyer's gains from trade are a fixed share, θ, of the trade surplus:

$$u(q) - \frac{A}{pw}d = \theta\left[u(q) - c\left(\frac{q}{Z},\frac{k}{Z}\right)\right].$$

Imposing $d = m$, we have

$$\frac{A}{pw}m = (1-\theta)\,u(q) + \theta c\left(\frac{q}{Z},\frac{k}{Z}\right)$$

and

$$\frac{\partial q}{\partial m} = \frac{1}{(1-\theta)\,u'(q) + \theta c_q\left(\frac{q}{Z},\frac{k}{Z}\right)\frac{1}{Z}}\frac{A}{pw} > 0,$$

$$\frac{\partial q}{\partial k} = \frac{-\theta c_k\left(\frac{q}{Z},\frac{k}{Z}\right)\frac{1}{Z}}{(1-\theta)\,u'(q) + \theta c_q\left(\frac{q}{Z},\frac{k}{Z}\right)\frac{1}{Z}} > 0.$$

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We have

$$V_m(m,k,Z) = \sigma \frac{A}{pw} \frac{\theta \left[u'(q) - c_q\left(\frac{q}{Z}, \frac{k}{Z}\right)\frac{1}{Z}\right]}{(1-\theta)\,u'(q) + \theta c_q\left(\frac{q}{Z}, \frac{k}{Z}\right)\frac{1}{Z}} + \frac{A}{pw},$$

$$V_k(m,k,Z) = -\sigma c_k\left(\frac{q}{Z}, \frac{k}{Z}\right)\frac{1}{Z} \frac{(1-\theta)\,u'(q)}{(1-\theta)\,u'(q) + \theta c_q\left(\frac{q}{Z}, \frac{k}{Z}\right)\frac{1}{Z}} + \frac{A\,(1+r-\delta)}{w}.$$

An equilibrium allocation solves

$$U'(X) = \frac{A}{F_H(K, ZH)}, \tag{23}$$

$$\frac{A}{pw} = \beta \frac{A}{p_{+1}w_{+1}} \left[\frac{\sigma\theta \left[u'(q_{+1}) - c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{k_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}\right]}{(1-\theta)\,u'(q_{+1}) + \theta c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{k_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}} + 1 \right], \tag{24}$$

$$U'(X) = \beta U'(X_{+1})\,[1 + F_K(K_{+1}, Z_{+1}H_{+1}) - \delta] - \beta\sigma c_k\left(\frac{q_{+1}}{Z_{+1}}, \frac{k_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}} \frac{(1-\theta)\,u'(q_{+1})}{(1-\theta)\,u'(q_{+1}) + \theta c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{k_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}}, \tag{25}$$

$$X = F(K, ZH) + (1-\delta)K - K_{+1}. \tag{26}$$

Balanced growth. Along the balanced growth path, hours are constant and X, K_{+1}, and q grow at a rate $1+\mu$. Conjecture that real balances M/p also grow at the rate $1+\mu$, implying that

$$1 + \tau = (1+\pi)\,(1+\mu)\,.$$

It then follows that the nominal interest satisfies

$$1 + i = (1+\pi)\,(1+\mu)\,(1+\rho)\,.$$

Using the functional forms above, conjecture there is a constant value of $\hat{K} = K/ZH$ along the balanced growth path. Then (23) and (26) yield

$$X = (1-\alpha)\,BA^{-1}\hat{K}^{\alpha}Z,$$

$$K = (1-\alpha)\,BA^{-1}\left[\frac{\hat{K}}{1-(\delta+\mu)\,\hat{K}^{1-\alpha}}\right]Z,$$

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$$H = \frac{(1-\alpha)\, B A^{-1}}{1-(\delta+\mu)\,\hat{K}^{1-\alpha}}.$$

With $\hat{K} = K/ZH$ and letting $b \to 0$, (24) yields

$$i = \frac{\sigma\theta\left[u'\left(q_{+1}\right) - c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}\right]}{(1-\theta)\, u'\left(q_{+1}\right) + \theta c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}}.$$

Note that if $i = 0$, then for any $0 < \theta \leq 1$ the numerator satisfies

$$u'\left(q_{+1}\right) = c_q\left(\frac{q_{+1}}{Z_{+1}}, \frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}},$$

which is the efficient quantity *given* the current capital stock K_{+1}. This is consistent with the results in Aruoba et al. (2007)—under the Friedman rule, proportional bargaining generates the efficient quantity of goods traded in the DM even though buyers do not get the entire trade surplus. In short, there is no holdup problem on *buyers* under the Friedman rule. Note that, even though q_{+1} is efficient, it is not equal to the planner's choice of q_{+1} unless K_{+1} is the same as the planner's choice. As we show below, this is not the case due to the hold-up problem on capital discussed in AWW.

Using the functional forms above, imposing $b \to 0$ and solving for q_{+1} yields

$$q_{+1} = \left\{\frac{\sigma\theta - (1-\theta)\, i}{\theta\,(i+\sigma)}\right\}^{\frac{1}{\psi+\eta-1}} \left\{\frac{(1-\alpha)\, B\hat{K}}{A[1-(\delta+\mu)\,\hat{K}^{1-\alpha}]}\right\}^{\frac{\psi-1}{\psi+\eta-1}} Z_{+1}^{\frac{\psi}{\psi+\eta-1}}.$$

Again, q grows at $1+\mu$ when $\eta = 1$; i.e., utility is log in the DM. Also note that for $q_{+1} > 0$, we need

$$\frac{\sigma\theta}{1-\theta} > i.$$

For a given value of θ, $q_{+1} = 0$ at a finite inflation rate. In short, for sufficiently high inflation rates, the monetary equilibrium collapses. This is typically not the case in most monetary models that have Inada conditions, but it is a typical result when agents use proportional bargaining. In what follows, I consider only inflation rates that satisfy this condition.

The steady state has

$$\hat{K}_b = \left[\frac{\alpha\beta + (1-\theta)^2\,\beta\left(\frac{\psi-1}{\psi B\theta}\right)\left(\frac{\sigma\theta}{1-\theta} - i\right)}{1+\mu-\beta\,(1-\delta) + (1-\theta)^2\,(\delta+\mu)\beta\left(\frac{\psi-1}{\psi B\theta}\right)\left(\frac{\sigma\theta}{1-\theta} - i\right)}\right]^{\frac{1}{1-\alpha}},$$

$$H_b = \frac{(1-\alpha)\,B\left[1+\mu-\beta\,(1-\delta)+(\delta+\mu)\,(1-\theta)^2\,\beta\left(\frac{\psi-1}{\psi B\theta}\right)\left(\frac{\sigma\theta}{1-\theta}-i\right)\right]}{A\,[1+\mu-\beta\,(1-\delta)-(\delta+\mu)\,\alpha\beta]},$$

$$\hat{q}_b \equiv \left[\frac{\sigma\theta-(1-\theta)\,i}{\theta\,(i+\sigma)}\right]^{\frac{1}{\psi}}\left[\frac{(1-\alpha)\,BA^{-1}\hat{K}_b}{1-(\delta+\mu)\,\hat{K}_b^{1-\alpha}}\right]^{\frac{\psi-1}{\psi}}.$$

Notice that an increase in the nominal interest rate (1) lowers the capital–labor ratio, (2) lowers hours worked, (3) reduces DM consumption/production, and (4) lowers per capita income. Hence, as occurs in CIA models, inflation has negative consequences for capital accumulation, output, and per capita incomes.

Note that even if the FR holds $i = 0$, we do not replicate the planner allocation, because $1-\theta \leq 1$. The reason is that $1-\theta$ appears in front of the second term of the numerator and denominator on the RHS. This is capturing the holdup problem on capital. Thus, although the FR eliminates the holdup problem on money, there is still a holdup problem on capital. This distortion is the result of the search and matching frictions, which are absent in a standard CIA model. Consequently, the trading frictions that naturally give rise to the need for a medium of exchange have an additional effect on capital accumulation via the existence of a holdup problem.[2]

Dynamics. To obtain analytical results, again assume $\delta = 1$. We then have

$$\frac{q_{+1}}{Z_{+1}} = \left\{\left(\frac{1}{\theta\psi}\right)\left[\frac{1-\theta+\sigma\theta-(1+i)\,(1-\theta)\left(\hat{K}_{+1}^{\alpha}/\hat{K}^{\alpha}\right)}{(1+i)\left(\hat{K}_{+1}^{\alpha}/\hat{K}^{\alpha}\right)-1+\sigma}\right]\right\}^{1/\psi}\left(\frac{K_{+1}}{Z_{+1}}\right)^{\frac{\psi-1}{\psi}}$$

and

$$\hat{K}_{+1} = \frac{1}{1+\mu}\left\{\alpha\beta+\frac{(1-\alpha)}{AH_{+1}}\beta\left(\frac{\psi-1}{\psi}\right)\left(\frac{1-\theta}{\theta}\right)\right.$$
$$\left.\times\left[1-\theta\,(1-\sigma)-(1+i)\,(1-\theta)\,\frac{\hat{K}_{+1}^{\alpha}}{\hat{K}^{\alpha}}\right]\right\}\hat{K}^{\alpha},$$

$$\hat{K}_{+1} = \frac{1}{1+\mu}\left[\frac{H}{H_{+1}}-\frac{(1-\alpha)\,B}{AH_{+1}}\right]\hat{K}^{\alpha}.$$

These two equations can be combined to obtain a nonlinear equation for H_{+1} as a function of H and $\hat{K}$.

Noting that

$$\frac{\hat{K}_{+1}^{\alpha}}{\hat{K}^{\alpha}} = \frac{1}{1+\mu}\frac{w_{+1}}{w},$$

consider a nonstationary interest rate policy given by

$$1+i=\left[1+\frac{\sigma\theta-\lambda}{1-\theta}\right](1+\mu)\,\frac{w}{w_{+1}},$$

where λ is an arbitrary constant and satisfies $\sigma\theta \geq \lambda$. When wages grow at the balanced path growth rate, we have $i = (\sigma\theta - \lambda)/(1-\theta) \geq 0$. So this policy would be one where for some (unmodeled) reason, the monetary authority wants a nonzero nominal interest rate along the balanced growth path. This could be because running the Friedman rule requires lump-sum taxation of money balances, which is often argued to be implausible for a central bank or may violate some participation constraints.[3] Note that setting $\sigma\theta = \lambda$ corresponds to the Friedman rule. Note that a lower value of λ implies a higher nominal interest rate and inflation rate along the balanced growth path.

What does this nonstationary policy do? It adjusts the interest rate so that the cost of acquiring money in t and $t+1$ is unaffected by the transition to the steady state. Suppose hours were at their balanced growth level and real wages were growing faster than $1+\mu$. Then it would take less labor to acquire a unit of money in $t+1$ than working in t to acquire a unit of money and carry it over to $t+1$. Hence the demand for money would fall, along with its real value. This would also put downward pressure on hours worked in the CM. To counter this, the policy above lowers i to raise the value of money and counter the effect on hours worked. When wage growth is too low relative to $1+\mu$, the opposite occurs. As I will show shortly, this policy has the effect of keeping hours worked in the CM constant along the transition path, just as the planner would choose. One way of thinking about this policy is that it aims at employment stability.

It then follows that the transition paths for $\hat{K}_{+1}$ and H_{+1} are given by

$$\hat{K}_{+1}=\frac{1}{1+\mu}\left[\alpha\beta+\frac{(1-\alpha)}{AH_{+1}}\beta\left(\frac{\psi-1}{\psi}\right)\left(\frac{1-\theta}{\theta}\right)\lambda\right]\hat{K}^{\alpha},$$

$$\hat{K}_{+1}=\frac{1}{1+\mu}\left[\frac{H}{H_{+1}}-\frac{(1-\alpha)\,B}{AH_{+1}}\right]\hat{K}^{\alpha}.$$

Conjecture that hours are constant along the transition path. Then we have

$$\hat{K}_{+1}=\frac{1}{1+\mu}\left[\frac{\alpha\beta+\beta\left(\frac{\psi-1}{\psi B}\right)\left(\frac{1-\theta}{\theta}\right)\lambda}{1+\beta\left(\frac{\psi-1}{\psi B}\right)\left(\frac{1-\theta}{\theta}\right)\lambda}\right]\hat{K}^{\alpha},$$

$$H=\frac{(1-\alpha)\,BA^{-1}\left[1+\beta\left(\frac{\psi-1}{\psi B}\right)\left(\frac{1-\theta}{\theta}\right)\lambda\right]}{1-\alpha\beta},$$

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$$\frac{q}{Z} = \left\{ \left(\frac{1}{\theta\psi} \right) \left[\frac{\lambda(1-\theta)}{\sigma - \lambda} \right] \right\}^{1/\psi} (\hat{K}H)^{\frac{\psi-1}{\psi}}.$$

Under this policy the transition path for $\hat{K}_{+1}$ is monotone. Note that even at the Friedman rule $\lambda = \sigma\theta$, the transition paths do not mimic the planner allocation, due to the holdup problem on capital. Thus, the holdup problem on capital leads to a lower steady state $\hat{K}$, lower investment along the transition path, and thus a lower growth rate of the economy for $\hat{K} < \hat{K}_b$.

We can also see that lowering λ leads to a higher nominal interest rate and inflation rate. Therefore, a higher steady-state inflation rate lowers the capital–labor ratio along the balanced growth path and also lowers the growth rate of the economy along the transition path. Consequently, inflation is a deterrent to short-run growth and long-run income per capita.

4.2. Price-Taking

As shown in AWW, price-taking eliminates the holdup problems on both buyers and sellers. This leaves the time cost of holding money as the only remaining friction. In this section, I consider price-taking in order to see how the model behaves in the absence of holdup problems. Assume that agents trade anonymously in a competitive market in the DM and take the market price $\tilde{p}$ parametrically. The buyer's problem is

$$V_b(m, k, Z) = \max_{q_b, d} u(q_b) + W(m - d, k, Z),$$
$$\text{s.t. } \tilde{p}q_b = d \text{ and } d \leq m,$$

whereas the seller's problem becomes

$$V_s(m, k, Z) = \max_{q_s} -c\left(\frac{q_s}{Z}, \frac{k}{Z}\right) + W\left[m + \tilde{p}q, k, Z\right].$$

It is easy to show that the buyer's constraint $d \leq m$ is binding in equilibrium, and so $q = M/\tilde{p}$. The seller's choice satisfies $c_q\,(q_s/Z, k/Z)\,1/Z = \tilde{p}A/pw$. At equilibrium we have

$$\frac{\partial q_s}{\partial m} = \frac{\partial d_s}{\partial m} = \frac{\partial q_b}{\partial k} = \frac{\partial d_b}{\partial k} = \frac{\partial d_s}{\partial k} = 0,$$
$$\frac{\partial q_b}{\partial m} = \frac{1}{\tilde{p}}, \frac{\partial d_b}{\partial m} = 1.$$

So

$$V_m(m, k, Z) = \sigma\frac{u'(q)}{\tilde{p}} + (1 - \sigma)\frac{A}{pw},$$

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$$V_k(m,k,Z) = -\sigma c_k\left(\frac{q_s}{Z},\frac{k}{Z}\right)\frac{1}{Z} + \frac{A\left[1+(r-\delta)\right]}{w}.$$

We now have

$$\frac{A}{pw} = \beta\frac{A}{w_{+1}p_{+1}}\left[\sigma\frac{u'(q_{+1})}{c_q\left(\frac{q_{+1}}{Z_{+1}},\frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}} + 1 - \sigma\right], \quad \textbf{(27)}$$

$$U'(X) = \beta U'(X_{+1})\left[F_K(K_{+1},H_{+1}) + 1 - \delta\right] - \sigma\beta c_k\left(\frac{q_{+1}}{Z_{+1}},\frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}},$$

$$U'(X) = \frac{A}{ZF_H(K,ZH)}, \quad \textbf{(28)}$$

$$X + K_{+1} = F(K,ZH) + (1-\delta)K. \quad \textbf{(29)}$$

A monetary equilibrium is a sequence of quantities $\{X, K_{+1}, H, q\}$ solving (27)–(29) given an initial capital stock K_0 and money stock M_0.

Balanced growth. As before, money growth satisfies

$$1+\tau = (1+\pi)(1+\mu),$$

whereas the nominal interest satisfies

$$1+i = (1+\pi)(1+\mu)(1+\rho).$$

As with the planner, (28) and (29) yield

$$X = (1-\alpha)BA^{-1}\hat{K}^{\alpha}Z, \quad \textbf{(30)}$$

$$K = \frac{(1-\alpha)BA^{-1}\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}Z, \quad \textbf{(31)}$$

$$H = \frac{(1-\alpha)BA^{-1}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}. \quad \textbf{(32)}$$

From (27) we have

$$\frac{i}{\sigma} = \frac{u'(q_{+1}) - c_q\left(\frac{q_{+1}}{Z_{+1}},\frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}}{c_q\left(\frac{q_{+1}}{Z_{+1}},\frac{K_{+1}}{Z_{+1}}\right)\frac{1}{Z_{+1}}}.$$

So $i = 0$ generates the efficient quantity of goods in the DM for a given stock of capital. I now show that at $i = 0$ we replicate the planner's allocation and achieve the first-best. Rewriting this expression, we get

$$\frac{q}{Z} = \left[\frac{\sigma}{\psi(i+\sigma)}\right]^{\frac{1}{\psi}} \left[\frac{(1-\alpha)\,BA^{-1}\hat{K}}{1-(\delta+\mu)\,\hat{K}^{1-\alpha}}\right]^{\frac{\psi-1}{\psi}}.$$

Using this expression as well as (28) in the Euler equation, we obtain the equilibrium values of $\hat{K}$ and H in the monetary economy with price-taking:

$$\hat{K}_m = \left[\frac{\alpha\beta\left(1+\frac{i}{\sigma}\right)+\sigma\beta\left(\frac{\psi-1}{\psi B}\right)}{[1+\mu-\beta(1-\delta)]\left(1+\frac{i}{\sigma}\right)+(\delta+\mu)\,\sigma\beta\left(\frac{\psi-1}{\psi B}\right)}\right]^{\frac{1}{1-\alpha}},$$

$$H_m = (1-\alpha)\,BA^{-1}\left[\frac{1+\mu-\beta(1-\delta)+(\delta+\mu)\,\sigma\beta\left(\frac{\psi-1}{\psi B}\right)\frac{\sigma}{i+\sigma}}{1+\mu-\beta(1-\delta)-(\delta+\mu)\,\alpha\beta}\right].$$

Compared to the planner allocation, (12) and (13), we have $\hat{K}_m < \hat{K}_p$ and $H_m < H_p$ for any $i > 0$. Furthermore, we have $d\hat{K}_m/di < 0$ and $dH_m/di < 0$. We have $\hat{K}_p = \hat{K}_m$ if $i = 0$ or

$$\pi = \frac{1}{(1+\mu)(1+\rho)} - 1.$$

So at the Friedman rule, deflation must be greater than the time rate of discount—it must also account for growth in the real return to capital.

Finally, we have

$$\hat{q}_m = \frac{q_m}{Z} = \left[\frac{\sigma}{\psi\,(i+\sigma)}\right]^{\frac{1}{\psi}} \left[\frac{(1-\alpha)\,BA^{-1}\hat{K}_m}{1-(\delta+\mu)\,\hat{K}_m^{1-\alpha}}\right]^{\psi-1}.$$

Because $\hat{K}_m = \hat{K}_p$ at $i = 0$, we have $\hat{q}_m = \hat{q}_p$ at the Friedman rule. So the Friedman rule replicates the first-best allocation. Because price-taking eliminates the holdup problem, the only remaining distortion is the inflation tax, which reduces the return on money below the time rate of discount adjusted for growth. Thus at the Friedman rule this distortion is eliminated and the first-best allocation occurs.

Dynamics. As before, set $\delta = 1$. The Euler equation and intratemporal condition are given by

$$\hat{K}_{+1} = \frac{1}{(1+\mu)}\left[\beta\alpha + \sigma^2\beta\left(\frac{\psi-1}{\psi}\right)\frac{1}{\frac{\hat{K}^{\alpha}_{+1}}{\hat{K}^{\alpha}}(1+i) - 1 + \sigma}\frac{(1-\alpha)}{AH_{+1}}\right]\hat{K}^{\alpha},$$

$$\hat{K}_{+1} = \frac{1}{H_{+1}}\left[\frac{AH-(1-\alpha)B}{A(1+\mu)}\right]\hat{K}^{\alpha}.$$

Combining these two equations gives us a nonlinear dynamic equation in H_{+1} in terms of H and $\hat{K}^{\alpha}$. So the dynamical system

$$[AH-(1-\alpha)B]\left[\frac{AH-(1-\alpha)B}{A(1+\mu)}\right]^{\alpha}\hat{K}^{\alpha}(1+i)$$
$$= \left[\sigma^2\beta\left(\frac{\psi-1}{\psi}\right)(1-\alpha) + [AH-(1-\alpha)B](1-\sigma)\right]H^{\alpha}_{+1}$$
$$+\alpha\beta AH_{+1}\left[\frac{AH-(1-\alpha)B}{A(1+\mu)}\right]^{\alpha}\hat{K}^{\alpha}(1+i) - (1-\sigma)\alpha\beta AH^{\alpha+1}_{+1},$$
$$\hat{K}_{+1} = \frac{1}{H_{+1}}\left[\frac{AH-(1-\alpha)B}{A(1+\mu)}\right]\hat{K}^{\alpha}$$

determines the paths of H_{+1} and $\hat{K}_{+1}$ as a function of current H and $\hat{K}$.

Consider a nonstationary monetary policy along the transition path. One such policy is

$$1+i = (\lambda+1-\sigma)\frac{\hat{K}^{\alpha}}{\hat{K}^{\alpha}_{+1}},$$

where $\lambda \geq \sigma$ is some arbitrary constant. Manipulate this expression to write it in terms of real wages,

$$1+i = (\lambda+1-\sigma)(1+\mu)\frac{w}{w_{+1}}.$$

If real wages converge to the balanced growth rate, then this policy rule converges to the value $i = \lambda - \sigma$. If $\lambda = \sigma$, this policy rule generates the Friedman rule along the balanced growth path. As shown above, such a policy keeps hours constant along the transition path, just as the planner would choose.

Under this nonstationary interest rate policy, we have

$$\hat{K}_{+1} = \frac{1}{(1+\mu)}\left[\frac{\alpha\beta + \sigma\beta\left(\frac{\psi-1}{\psi B}\right)\frac{\sigma}{\lambda}}{1+\sigma\beta\left(\frac{\psi-1}{\psi B}\right)\frac{\sigma}{\lambda}}\right]\hat{K}^{\alpha},$$

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$$H = \frac{(1-\alpha)\,BA^{-1}}{1-\alpha\beta}\left[1+\sigma\beta\left(\frac{\psi-1}{\psi B}\right)\frac{\sigma}{\lambda}\right],$$

$$\frac{q}{Z} = \left(\frac{\sigma}{\psi\lambda}\right)^{1/\psi}(\hat{K}H)^{\frac{\psi-1}{\psi}}.$$

Using this policy rule, the transition path for $\hat{K}$ is monotone. It mimics the planner's transition path, but at a lower growth rate, when $\lambda > \sigma$. So the higher λ is, the higher is the steady-state nominal interest rate. A higher steady-state nominal interest rate has two effects. First, it lowers the capital–labor ratio along the balanced growth path. Second, for a given value of $\hat{K}$, the growth rate of the economy will be lower.

5. CONCLUSION

This paper contributes to our analytical understanding of the effect of matching frictions, bargaining, and money on growth dynamics. Whereas AWW focus on numerical analysis, I am able to derive analytical results that provide additional insight for the numerical results obtained in AWW. The benefit of this analysis is that it provides clear and simple intuition for how bargaining, random matching, and changes in the nominal interest rate affect the capital–labor ratio along the balanced growth path, as well as short-run growth rates of the economy.

NOTES

1. These conditions have been shown by Aruoba and Schorfheide (in press) to be critical in estimating the AWW model, as opposed to calibrating it.
2. A holdup problem requires three elements: (1) irreversibility of investment, (2) the inability to contract ex ante with one's trading partner, and (3) ex post bargaining over the terms of trade. The search and matching frictions give rise to the last two elements and thus the holdup problem.
3. See Andolfatto (2010) or Berentsen and Waller (in press) for more on this point.

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