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LABOR SHARE AND GROWTH IN THE LONG RUN

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This paper establishes some stylized facts of the long-run relationship between growth and labor shares using historical data for the USA (1898–2010), the United Kingdom (1856–2010), and France (1896–2010). Performing individual country time–frequency analysis, we demonstrate the existence of long-term cycles in labor share of 30–50 years explaining a major part of the variance in the data. Further, the impact of labor share on growth changes sign with the frequency considered from negative at high frequencies to positive at low frequencies. Finally, the positive coefficient associated with the labor share at low frequencies increases over time.

Keywords: Labor Share, Growth, Income Distribution, Wavelet Analysis

1. INTRODUCTION

Interest in the labor share of income has a long lineage in economics. For the three major classical economists—Smith, Ricardo, and Marx—how national income was divided between the owners of capital and labor services was a fundamental issue in political economy. Similarly, the labor share of income has been discussed at length in the modern era, albeit from different standpoints: for example, Kalecki (1938), Keynes (1939), Solow (1958), Elsby et al. (2013), and Karabarbounis and Neiman (2014).[1] Recently, there has been a particular revival of interest reflecting to some extent the impact of authors such as Piketty and Saez (2003), Piketty (2014), Atkinson (2015), Milanovic (2018), and others. A notable aspect of their work being the provision of long-dated historical data.

We view the availability of such long-run data as a chance to re-assess the nature of the labor share (and of inequality) in the long (and short) run and its

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connection to growth.[2] Long-run data also allow us to address what is arguably a disconnect between economic theory and economic data regarding the long run.[3] Typically we assume that, aside from normal business cycle fluctuations, in the long run an economy is characterized by balanced growth and thus that factor shares are ultimately constant—this arises either via the assumption that production is Cobb Douglas or that technical progress is labor saving [e.g., Uzawa (1961), Acemoglu (2003), and Jones (2005)]. Thus, there should effectively be no relationship between the labor share and the growth rate of the economy. This can be seen at its most stark under a unitary elasticity of aggregate factor substitution since the income share of labor is a technological constant (the exponent in the production function).

Turning to the empirical literature, however, we know that for many economies the case for a unitary elasticity is weak [e.g., Antràs (2004), Klump et al. (2007), Chirinko (2008), and León-Ledesma et al. (2010)], as is the presumption that technical progress is neutral [e.g., McAdam and Willman (2013) and León-Ledesma and Satchi (2018)]. However, much of our understanding of such issues relies on relatively short samples of data. The presence of long samples that are now available allows us to revisit the stylized factors of labor shares and growth, and the two key issues arising:

1. Are factor income shares stable over long-run samples?
2. What (if any) is the long-run relationship between growth and labor income?

To address the first question, we use historical data that goes sufficiently far back to capture long-term fluctuations for the United States (USA) from 1898 to 2010, the United Kingdom (UK) from 1856 to 2010, and France from 1896 to 2010. This contrasts markedly with many existing papers, which mainly rely on post-war data.[4]

Second, we perform time–frequency analysis using wavelet analysis to differentiate between short- and medium-to-long-term fluctuations in the labor share, and determine the dominant frequency and its phase relationship with growth. As far as we are aware, ours is the first application of wavelet techniques to uncover the nature of the labor share and its links to growth. And yet it is very well suited to this.[5] Moreover, existing empirical studies as well as theoretical models are strongly suggestive of the possibility that the labor share has different characteristics depending on the frequency considered. For example, in the short run the labor share may be expected to be impacted by business cycles, transitory shocks, labor hoarding, variations in mark-ups, product and labor market rigidities, and search frictions, etc. [Bertola et al. (2005) and Schneider (2011)]. In the long run, per-capita growth is deemed to be ultimately determined by technology and institutional quality [Acemoglu (2009), Acemoglu and Autor (2011), and Cantore et al. (2014)]. Consequently, it would be surprising if the strength and sign of the relationship between the labor share and growth would be constant across these frequency ranges.

We demonstrate that the labor share can be characterized by long-term cycles of 30–50 years. These long fluctuations in the labor share account for half the variance of the series. This indicates that the appropriate frequency with which to analyze movements in the labor share is the long term rather than the business cycle (despite the widespread modeling of factor share movements at business cycle frequencies).

Additionally, we demonstrate that the impact of labor share on growth changes sign with the frequency considered. Labor share impacts growth negatively in the short term, but positively in the long term. The positive long-run coefficient is robust to the inclusion of control variables and to different lag specification in a regression-based analysis. Estimations were also made taking into account the potential endogeneity between labor share and growth. The phase difference analysis suggests that labor share *leads* growth in the long term reducing the risk that the positive coefficient estimated captures simultaneous causality.[6] To account for the major transformations experienced by the three countries considered over the 20th century, we also explore the changing relationship over time by performing rolling regressions using different windows. The finding that labor share positively impacts growth in the long term is reinforced when restricting the data to the post-World War II period. These results are also robust to the use of alternative filters such as the Christiano–Fitzgerald bandpass filter as well as to the use of alternative labor share definitions.

These results stand in contrast with two pillars of conventional wisdom: the stability of factor income shares and the assumption of no relationship or a negative relationship between labor share and growth. Although potentially controversial, our results are a first step toward revisiting this topic in the light of newly available historical data and statistical techniques, and thus building on existing findings in the field .

The paper is organized as follows. In Section 2, we provide some brief review of the link between labor share (and inequality) and growth. Section 3 sets out the data source and definitions and then analyzes the cycles in the labor share. Section 4 outlines the co-variance between labor share and growth over time and across frequencies. This section also contains scale-by-scale regressions to test whether the sign of the relationship changes across frequencies and is robust to the inclusion of third factors. This section also looks at the changing relationship over time. Finally, we conclude.

2. GROWTH, THE LABOR SHARE, AND INEQUALITY

In this paper, we use measures of the labor share as a proxy for inequality, reflecting our interest in historical data. There is a close though necessarily imperfect relationship between both series [see Atkinson et al. (2011) and Atkinson (2015)]. The literature on this topic is substantial and growing [Milanovic (2018)]. To motivate our analysis, we attempt to overview that literature here (albeit it in a necessarily selective manner).

As already mentioned, in the short run the labor share may be impacted by business cycles, transitory shocks, technical change and technical biases, labor hoarding, variations in mark-ups, product and labor market rigidities, and search

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frictions [Bertola et al. (2005), Mendes (2011), and Schneider (2011)]. In the long run, per-capita growth is deemed to be ultimately determined by technology and institutional factors [Acemoglu (2009), Acemoglu and Autor (2011), and Cantore et al. (2014)]. Consequently, it would be surprising if the strength and sign of the relationship between the labor share and growth would be unchanged across these frequency ranges.

2.1. Short-Run Factors

Regarding the short run, there is much empirical support [e.g., EC (2007)] for a (albeit sometimes weak and possibly declining) negative co-movement between growth and the labor share—that is, labor share rises (falls) during recessions (recoveries). This is often rationalized by the presence of insurance mechanisms in the wage bargaining process. Risk-averse workers attempt to insure themselves against future income and unemployment risk, and the presence of labor hoarding incentives makes the firm be willing to insure themselves against downturns.

In this study, we also find a negative correlation between output and labor share in the short run, but the issue (from both a modeling and theoretical standpoint) has attracted much controversy [e.g., Schneider (2011)].

2.2. Beyond the Business Cycle

In terms of functional income, in the longer run, movements in the labor share are often associated with production technologies, factor demands, and factor availabilities. In neoclassical growth theory, factors are paid their marginal products and income shares are determined accordingly. Indeed, if production is Cobb Douglas (i.e., a unitary elasticity of substitution), there is no link between growth and distribution over any frequency.[7] Otherwise, technical changes and capital deepening can impact labor shares (as well as growth) in both the long and short run. To illustrate, if the substitution elasticity exceeds unity, an increase in labor-saving technologies will "favor" labor (i.e., raise labor's income share for given factor proportions).[8]

In (at least) the short run, capital and labor-saving technologies, though, can coexist and their relative strength (coupled with substitution possibilities) determines the evolution of factor incomes.[9] Under a balanced growth path (BGP), per-capita growth equals the rate of labor-saving technical development. In the steady state this is necessarily constant and thus a production technology even with a non-unitary elasticity implies stable factor shares and independence between growth and factor shares.

Moreover, in a standard production function, profit maximizing framework, factor prices (returns), and factor volumes are endogenous. In his framework, though, Piketty (2014) puts given movements in factor returns at the heart of the distributional argument. If the real rate of return on capital exceeds that of growth, and factors are characterized by an above unitary elasticity, then the weaker the growth the higher capital's share of income.[10]

If factor shares are indeed not stable, then the empirical relevance of the BGP becomes an open question. Technological biases might instead be a persistent (rather than transitory) determinant of factor share changes. This could be related to globalization—for example, where there is an expansion of the pool of available labor and thus diminished needs to bias technological progress toward the (abundant) labor factor and more toward capital (which itself may be reversed as the constraint of aging becomes binding). Or it could be related to continual and persistent changes in the sectoral make-up of economies, that is, moving from Agriculture to Manufacturing to Services and so on with each stage being characterized by different aggregate elasticities of factor substitution, factor intensities, and/or required factor-saving technologies [Buera and Kaboski (2012)].[11] It could also be related to the situation of different factor types (skilled, unskilled labor, different capital categories). Then, even though there may be a stable steady-state labor share level, *dispersion* of labor income may exist depending on how technologies effect—or are directed toward—the different labor types and in turn how those complement capital inputs.

In the above models, note, technology tends to be modeled exogenously. As we move to endogenous growth models, though, the link between distribution and growth becomes even more nuanced. A general feature of such models is the presence of non-decreasing returns, non-rival technologies, and imperfect competition. In such cases agents' saving rates, human capital accumulation, as well as public policy measures may affect the long-run growth rate. Such public policy and institutional structures can work for good or ill: at one end, for example, subsidies to R&D and educational opportunities [on the assumption that the decentralized economy produces a socially suboptimal allocation, Growiec et al. (2018)], and at the other, accommodating rent seeking by various groups to gain special redistributive, tax or market-power protections.

An important offshoot in such discussions is the extent to which human capital accumulation and occupational choices are distorted by imperfect capital markets. Indeed such factors constitute a good candidate to understand the positive low frequency co-movement between the labor share and growth (that we find). Akin to the inequality and growth literature [Atkinson (2015)], a decline in the labor share would lead to a reduction in human capital accumulation for households that are credit constrained. The strength of this explanation is that human capital accumulation is decentralized at the households' level, and therefore depends on labor income in economies where capital is unevenly distributed.

A final point is causality. Does growth generate or cause inequality, does inequality cause low growth, or are they bi-causal? Kuznets (1955) proposed that inequality tends to increase in the beginning of the development process and fall thereafter (reflecting industrialization and the initial distribution of factor endowments). Thus growth patterns cause inequality patterns. Empirical support and academic acceptance for the theory has weakened considerably in recent years, for example, Deininger and Squire (1998). In a not dissimilar vein, if we assume that sustained growth requires risky innovative behaviors, then we might expect that growth begets inequality. Although if the innovators supplant incumbents undermining existing market power, then this may go the other way. On the other side, inequality may affect growth (positively or negatively) if it generates political pressures for income redistribution. Likewise, if human capital is sub-optimal due to financial constraints, then the causality would go from inequality to growth.

We shall revisit these themes in our conclusions section. The bottom line is that there is no unifying theory linking growth and inequality. And this is why the accumulation of careful evidence is especially important to gather.

3. LONG-TERM DEVELOPMENTS IN THE LABOR SHARE

This section describes the trends in the labor share of income. In particular, we show that the labor share can be characterized by long-term cycles over which short-term fluctuations occur. Comparing the relative importance of these different cycles, the long-term cycles appear to account for more than 50% of the variance in the data in the three countries considered.

3.1. Definition

Labor income is based on National Accounts and sums different components of the compensation of employees. Historical national account data are taken from Piketty and Zucman (2014) (hereafter PZ). The labor share ls is defined as (suppressing time subscripts for convenience):

$$ls = \frac{ce_c \cdot \left(1 + \frac{Y_{hh}}{Y_c}\right) + ce_g}{Y - tx} \tag{1}$$

Total labor income is thus the sum of the compensation of employees paid by corporations ce_c and by the government ce_g. Total labor income is also augmented by the imputation of the labor income of the self-employed defined as the labor share in the corporate sector ce_c/Y_c multiplied by the net domestic product of the non-corporate business sector Y_{hh}. Y_c is the net domestic product of the corporate sector. This imputation assumes that the distribution of income between labor and capital in the non-corporate business sector is identical to the distribution of income between labor and capital in the corporate sector. The denominator is a measure of national income: net domestic product Y minus production taxes, tx. Rognlie (2015) argues that the factor shares must be measured net of depreciation as the recent increase in the rate of depreciation tends to limit the downward (upward) trend in the labor (capital) share. Subtracting production taxes from the denominator corresponds to a measure of the labor share at factor costs. It follows that in the PZ database the labor share and the capital share sum to one and can be used interchangeably.

This definition of the labor share applies for France (1896–2010) and for the UK (1856–2010). However, the primary income distribution accounts start only in 1929 for the USA in the PZ database.[12] Accordingly, we backdate the data prior to 1929 using the database by Groth and Madsen (2016), which provides compensation of employees and value-added data starting in 1898 based on historical source of Liesner (1989). The compensation of employees is given for the corporate non-agricultural private sector. Compared to equation (1), this definition excludes some institutional sectors. However, it is similar to the definition of the labor share in the corporate sector $ce_c/\,(Y - tx)$, which serves as a basis for the imputation of the self-employment labor income and has the advantage of starting in 1898.

3.2. Wavelet Analysis

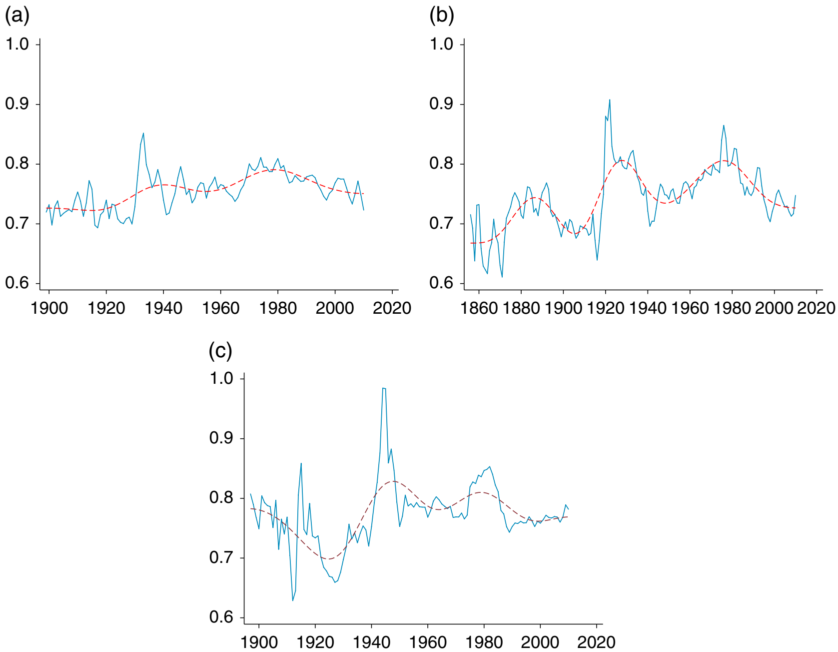
In order to uncover the spectral properties of the labor share and growth, different methodologies are available. One is the well-known Hodrick–Prescott (HP) filter. However, one limitation of this filter is the choice of the smoothing parameter for data other than quarterly data. The HP filter is a poor approximation of the ideal filter with annual data [Baxter and King (1999)].[13] Additionally, it may produce spurious regressions [Harvey and Jaeger (1993)] and produce periodicity and comovement that are not present in the input series [Cogley and Nason (1995)]. An alternative, the Fourier analysis, is able to identify the main cyclical comovements in the data. However, it fails to capture the transient relations in the data and when changes in the cyclical comovements occur. The time information of the data is fully lost [Gençay et al. (2002)].

Overcoming these limitations, wavelet analysis maps all the information of time series into specific frequencies and time. In our empirical work we consider both continuous and discrete wavelet analyses. We use the continuous wavelet analysis to study the variance and co-variance decomposition across time and frequency, as well as to study the phase difference between the labor share and growth. While it may be argued that one should use either the continuous or the discrete methods, the discrete wavelet analysis is used to perform robustness check in the form of regressions controlling for third factors. Applications of wavelet analysis in economics include Ramsey et al. (2010), Gallegati et al. (2011), Aguiar-Conraria et al. (2012), Rua (2012), and Gallegati and Ramsey (2013). In the following sections, different tools are being used such as the wavelet power spectrum (a variance decomposition over time and frequencies) or the wavelet coherency (the co-variance between two variables across time and frequency). These tools are presented in detail in the appendix (see Appendix A).

3.3. Long-Term Movements in Labor Share

Figure 1 displays the labor share data for the countries considered, and Table B.1 in Appendix B lists the summary statistics. This figure also presents the long-term trend in the labor share through the smoothed series using the maximal overlap discrete wavelet transform.[14]

The labor share series are clearly made of short-term fluctuations along long-term cycles. Let us focus on the *long-term* component. In the USA, there are



Notes: The charts in these figures display the labor share and their long-term component in the three countries considered.

FIGURE 1. The labor share of income. (a) USA; (b) UK; and (c) France.

two peaks in 1940 and 1980 and two troughs in 1917 and 1955. The labor share displays essentially three long-term cycles in the UK and two and a half cycles in France and in the USA. In the UK, peaks can be dated to the years 1888, 1927, and 1975 and troughs in years 1906 and 1949.[15] In France, the peaks occurred in 1948 and 1980, and the troughs in 1925, 1964, and 2000.

Let us now look at each cycle in more detail. In the three countries, the post-World War II cycle is hump shaped. The cycle takes place over the period 1949–2010 in the UK, 1964–2000 in France, and 1955–2010 in the USA. The labor share peaks in 1975 in the UK at 80%, in 1980 in France at 81%, and in the 1980 at 79% in the USA. The trough is reached in 2000 in France at 76%. In 2010, the labor shares reach a low point at 0.72% in the UK and at 0.75% in the USA. The amplitude of the cycles is quite large: respectively, 4, 8, and 4 percentage points of GDP in France, the UK, and the USA, respectively. In the UK and in the USA, the labor share in 2010 is back to its value of 1949 and 1955, respectively, while in France the labor share undershoots in the 1980s and 1990s reaching a level in 2000 below the 1964 level.

Over the first half of the 21st century, the dynamic of the labor share varies across countries, although a strong resemblance exists between France and the USA. The labor share in the UK goes through a full cycle between 1906 and 1949 with a peak in 1927 at 80%. In 1949 the labor share is 5 percentage points higher than in 1906. In France the labor share completes a full cycle between 1925 and 1964. The labor share first increases from 0.70 in 1925 to 0.82 in 1948, before declining to 0.78 in 1964. Similarly in the USA, the labor share completes a cycle starting at 0.72 in 1917, reaching a peak at 0.77 in 1940 before to decrease back to 0.75.

In the first half of the 21st century, the evolution of the labor share through the smoothed component is slightly different from the visual inspection of the raw data made in Figure 1 between the UK and the other two countries. The main reason is that while both the UK and France have experienced a large increase in the labor share during World War I, the permanent impact that war arguably had on the labor share in the UK appears as a peak in the smoothed component while it does not in France.

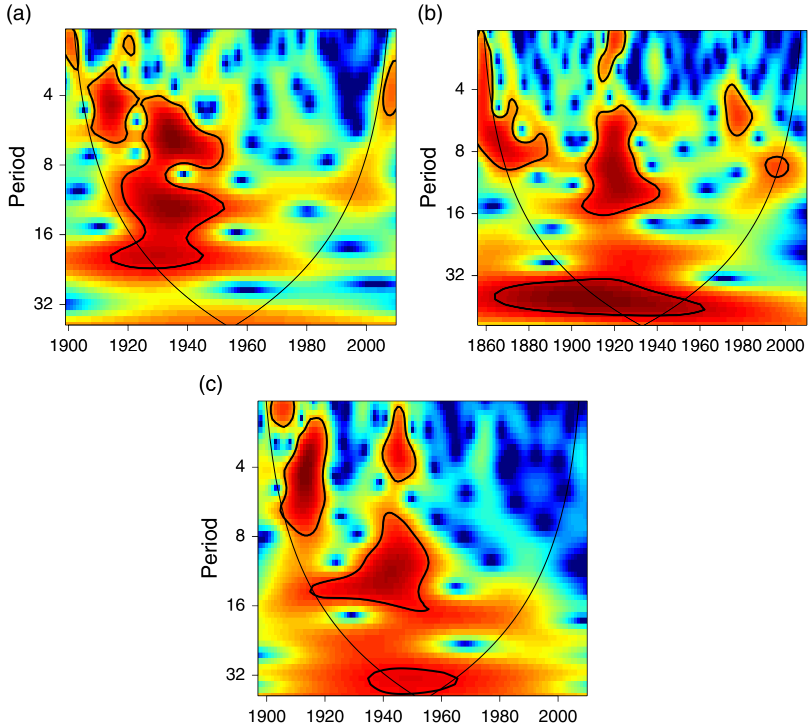
In the UK, a third cycle takes place between 1856 and 1906. The labor share first increases by 8 percentage points up to 1888 before declining by 6 percentage points until 1906. In the UK the two cycles in the second half of the 19th century and in the first half of the 20th century have taken place along an increasing trend.

Piketty (2014) describes the evolution of the capital share in France and the UK as a U-shape function of time over the period going from the beginning of the 19th century and the beginning of the 21st century.[16] The existence of secular (200 years) inverted U-shape cycle in the labor share cannot be detected by the wavelet analysis given that our sample does not cover the early 19th century. However, a similar pattern can be observed from descriptive statistics. The average labor share is 0.73 in France between 1896 and 1939, 0.78 over the period 1950–1974 (excluding World War II), and then 0.76 between the period 1993–2010 (excluding the 1970s and 1980s). Similarly, in the UK, the labor share is on average, respectively, at 0.73, 0.77, and 0.73 over the same subperiod (the first period starting in 1856 rather than 1896). In the USA, the labor share is 0.73, 0.77, and 0.76 over the same subperiods.

3.4. Variance Decomposition across Frequencies: Power Spectrum

Figure 1 shows that the labor share is made of different types of cycles, reflecting both long- and short-term fluctuations. It is important, though, to identify the most relevant frequency for the analysis of the labor share. In Figure 2, the power spectrum analysis corresponds to the local variance of a time series across time and across frequencies. The wavelet power spectrum is an energy density in the time–frequency plane. The horizontal axis denotes the time and the vertical axis the frequency. The concentration of the information is represented by color intensity: warmer colors standing for higher power. Regions surrounded by a bold line are regions significant at 10% against the null that the data-generating process is stationary.[17] The cone of influence identifies the regions affected by the edge effects.[18]

The power spectrum analysis for the labor share has similarities in the three countries considered (see Appendix A for a presentation of the power spectrum). The information is concentrated (represented by red colors) before World War II



Notes: The three figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet power spectrum is an energy density (or variance distribution) in the time–frequency plane. The WPS identifies the time and frequency at which information is concentrated. The warmer colors stand for high power.

FIGURE 2. Labor share—variance decomposition across time and frequency: power spectrum. (a) USA; (b) UK; and (c) France.

pointing to the fact that economic fluctuations have dampened since the second half of the 20th century. This may be related to the more widespread and efficient use of stabilization and social welfare policies after World War II. However, there are regions with high power in the 1970s, which capture the large increase in the labor share that took place as a result of low unemployment and wage indexation mechanisms and stronger labor bargaining power. The power spectrum for the labor share tends to indicate that there is little information at the highest frequencies as represented by the blue color. By contrast, information is concentrated at frequencies lower than business cycle frequencies, which (somewhat unfortunately perhaps) has received most of the attention so far in the literature. This is represented by the red color that covers the entire time span for frequencies larger than 16 years.

The power spectrum thus indicates that medium-run fluctuations account for a majority of the volatility in the labor share in the USA. The energy decomposition indicates that the low frequencies account for the majority of the variance in the labor share: 64%, 45%, and 47% for the UK, France, and the USA, respectively. This is indicative of the existence of a medium-to-long-term component in the evolution of the labor share. These long swings in the labor share bring a new perspective on the allegedly constancy of the labor share. Additionally, it raises the question of their impact on growth. This is the question to which we now address ourselves.

4. LABOR SHARE AND GROWTH

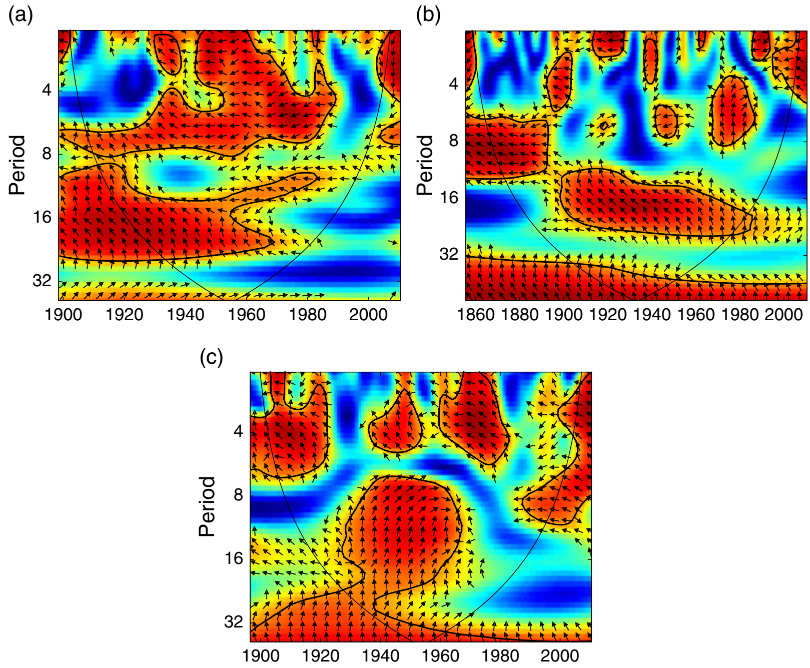
This section studies the relationship between labor share and growth across different time scales. The main result is that the impact of labor share on growth changes sign with the frequency considered. An increase in the labor share reduces growth in the short term but enhances growth in the long term. This result is robust to the inclusion of control variables in a regression-based analysis.

There are two additional results. Looking at the co-variance across frequencies, we show that correlations are stronger at lower frequencies than at higher frequencies. This overwhelmingly highlights that the relevant frequency to study the comovement between the labor share and growth is the long term rather than the business cycle. In addition, looking at relative phase, the labor share tends to lead growth in the long run. This indicates that the positive coefficient estimated for the low frequency is unlikely to be driven by simultaneous causality between the two variables. To perform this analysis, real per-capita GDP series are taken from the Maddison project.[19]

4.1. Co-variance across Frequencies and Phase Difference Analysis

The wavelet coherency as defined in appendix equation (A4) is the cross-wavelet power normalized by the power spectrum of both series (see Appendix A for a presentation of the cross-wavelet power). The cross-wavelet power analyzes the time–frequency dependencies between two time series, capturing the co-variance between these variables in the time–frequency domain. In Figure 3, the solid line corresponds to regions significant at 5%. The color scale is similar to the power spectrum presented earlier. The red color indicates strong correlation between labor share and growth for a given time and given frequency. The coherency analysis between growth and the labor share shows that there are strong correlations at certain points in time at the highest frequencies. However, the region of significant coherency expands with the scale to cover the entire period for the frequency > 32 years.

The importance of the co-variance in the long term is particularly striking in the UK for which we have time series starting in 1856. In France, the co-variance is associated with red colors for the period 1896–2010 for the frequency 32 years and beyond. In the USA, the information is more evenly distributed across scales and the long-term coherency is only significant at 10%. The main conclusion from



Notes: The three figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet coherency captures the co-variance between two variables in the time–frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The warmer colors stand for high power, or high coherency. An arrow pointing right (left) means that both series are in (anti) phase. An arrow pointing up (down) means that labor share is leading (lagging) growth by 90°.

FIGURE 3. Labor share and growth—covariance decomposition across time and frequency: Wavelet coherency. (a) USA; (b) UK; and (c) France.

the wavelet coherency is that the relevant frequency of analysis is the medium term and long term rather than at the business cycle.

The arrows in Figure 3 represent the relative phase between the two series as defined in equation (A6). The direction in the arrow can be interpreted as follows. An arrow pointing right indicates that the two variables are in comovement. The arrow pointing left indicates that the two variables are in anti-phase. The arrow pointing up indicates that the labor share is leading growth by 90°; while one pointing down means that the labor share is lagging growth by 90°.

In the countries considered, in areas characterized by high common power (the red color in the figure), the arrows are pointing up at low frequency (32 years and beyond) indicating that the labor share is leading growth by 90°. There are small differences across countries. In France, this result also holds at intermediate frequencies (8–32 years) for a period centered over the middle of the 21st century. Similarly in the USA, the labor share is also leading growth at frequencies 8–32 over the period 1900–1960.

At higher frequencies, the arrows tend to point left in areas of high common power indicating an anti-phase movement between the labor share and growth. In France and in the USA, the arrows are pointing left at high frequencies comprised between 2 and 8 years. In the UK, the labor share and growth are in anti-phase over the period 1850–1900 for frequencies comprised between 6 and 16 years as well as over the period 1940–1960 for frequencies comprised between 6 and 10 years. Interestingly in the UK, there seems to be a phase shift over the first part of the 20th century for frequencies 16–32 years as the labor share is first in anti-phase and then starts leading growth.

We can draw two main results from the coherency analysis. First, to repeat, the relevant frequency to analyze the relationship between the labor share and growth is clearly the medium run and the long run. This stands in contrast with the usual business cycle frequency perspective on this topic. Second, the choice of the frequency to analyze the comovement in the labor share and growth is not neutral as the labor share and growth are in anti-phase in the short term while the labor share leads growth in the long term. In other words, the labor share has a negative impact on growth in the short term and a positive impact on growth in the long term.

The difficulty when studying the impact of labor share on growth is that the sign of the effect might be driven by simultaneous causality. However, the relative phase sheds a new light on the issue of endogeneity between the labor share and growth. The labor share leading growth at low frequencies is an indication that the labor share is not endogenous to growth. The peaks and troughs of the labor share are a good predictor of the peaks and troughs of growth.[20] However, the relative phase should be interpreted carefully as a lead by 90° could also mean a lag by 270°. Reproducing the same exercise for the capital share shows opposite results in terms of phase shift: the capital share is lagging growth when the labor share is leading growth and inversely. The associated figures are not reproduced here but are available on request.

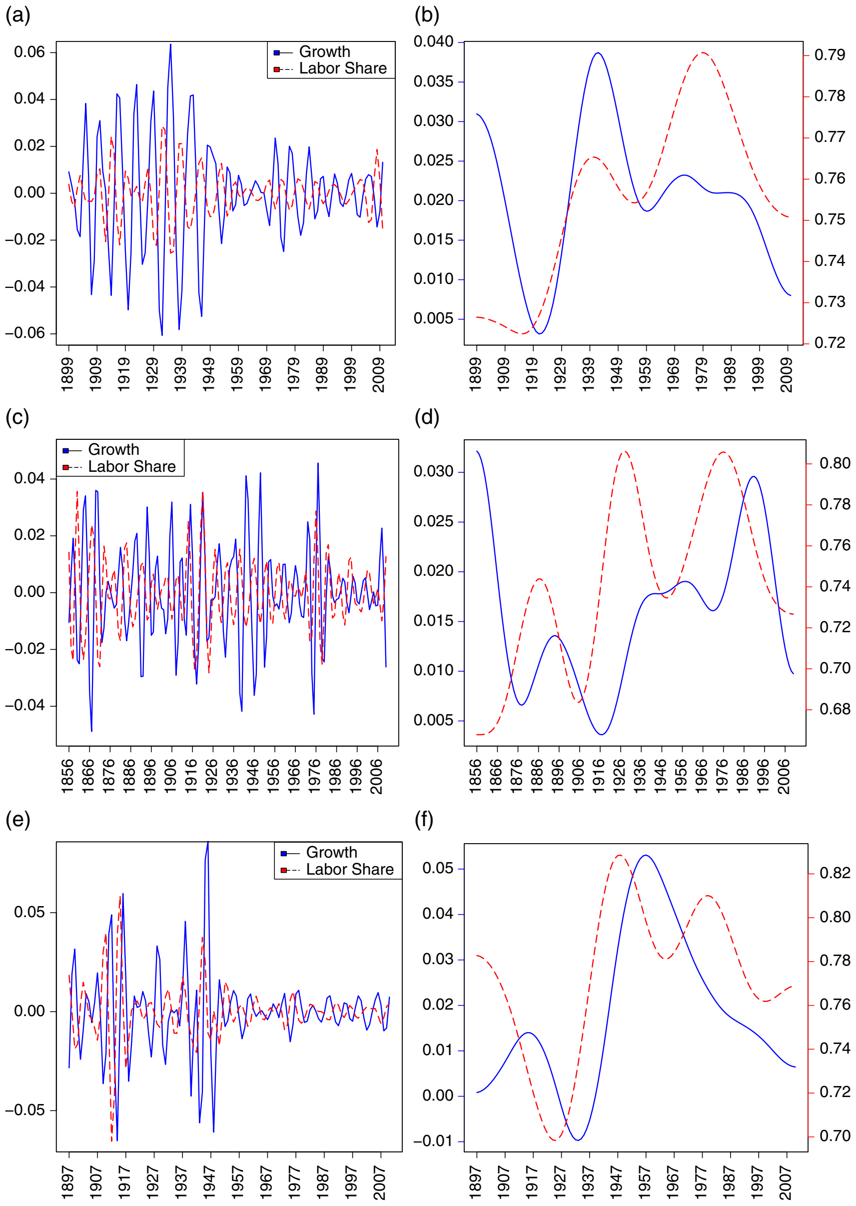
4.2. Regressions across Time Scale

The previous subsection highlighted the changing relationship between functional income distribution and growth. In this section, we perform a regression analysis to test whether this result is robust to the inclusion of a set of control variables typically used in the endogenous growth literature. For this purpose, we make use of the discrete wavelet transform as presented in Appendix A.2.[21]

Figure 4 displays the outcome of the discrete wavelet transform for growth (the blue line) and the labor share (the red dashed line) over the following frequency ranges (recalling our earlier definitions): [22]

Short Run	D_1	$2-4$ years	D_2	$4-8$ years
Medium Run	D_3	$8-16$ years	D_4	$16-32$ years
Long Run	S_4	> 32 years		

In each country, visual inspection tends to show that at highest frequencies D_2, growth and the labor share are in anti-phase capturing the negative correlation of the labor share at the business cycle frequency. However, when the scaling



Note: This figure displays the discrete wavelet filter for labor share and economic growth for the following frequencies: $D_2 : 4 - 8$ years and S_4 beyond 32 years. The frequencies $D_1 : 2 - 4$ years, $D_3 : 8 - 16$ years, and $D_4 : 16 - 32$ years are available on request.

FIGURE 4. Phase relationships short term and long term. (a) US: $D_2 : 4 - 8$ years; (b) US: $S_4 > 32$ years; (c) UK: $D_2 : 4 - 8$ years; (d); UK: $S_4 > 32$ years; (e) France: $D_2 : 4 - 8$ years; and (f) France: $S_4 > 32$ years.

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level increases the two series seem to become gradually in-phase with the labor share leading growth. In the long term, the labor share peaks and troughs seem to anticipate the growth peaks and troughs. This visual impression is in line with the previous phase-shift analysis. In France and in the UK, two exceptions arise at the beginning of the period (the period 1897–1910 in France and 1856–1890 in the UK) and during the 1970s characterized by high wage growth and low unemployment.

Correlations tend to confirm the visual inspection described above, which points to a change in the relationship between growth and the labor share across frequencies.[23] The sign associated with the labor share and growth is negative at the frequencies D_1–D_4, then positive in the long run S_4. Small differences exist across countries. The sign of the correlation is positive but not significant at the frequencies D_3 and D_4 in France. In the UK, the sign is positive in S_4 but significant only if the 19th century data are excluded.

We complement the correlation by performing regressions as described in equation (2) for each time scale $j = [D_1, D_2, D_3, D_4, S_4]$. The dependent variable is the growth rate of real GDP per capita Δy and the independent variable is the labor share of income. The vector of control variables x includes real GDP per capita at $t-1$, lagged investment to GDP ratio, as well as the lagged inflation rate. The real GDP per capita captures the proposition that the lower the initial GDP per capita the larger the growth potential. The investment to GDP ratio measures the contribution of the accumulation of capital to growth. The inflation rate is a proxy for economic volatility, which is generally seen as detrimental to economic growth. These control variables are standard in the inequality and growth literature as in Barro (2000) or Voitchovsky (2005). This equation is derived from a simple equation $\Delta y_t = F\,(y, y^*)$, where the growth rate in the economy depends on the level of per-capita output y and the long-run level of per-capita output y^*. In line with the neoclassical model with diminishing returns, growth is inversely related to the level of economic development. The long-run level of output y^* depends on government policies and institutions. The smoothed component of each of these control variables is used in the regression:

$$\Delta y_{j,t} = \alpha_j + \beta_{ls_j} ls_{j,t-i} + \beta_{x_j} x_{j,t-1} + \beta_I D_{WWI} + \beta_{II} D_{WWII} + \epsilon_t. \qquad \textbf{(2)}$$

The regression also adds two war dummy variables, D_{WWI}, D_{WWII}. Equation (2) is also estimated for two different lag structures $i = 0$ and $i = 1$ for the labor share variable (see Table 2). Equation (2) is also likely to have auto-correlated errors especially at low frequency since the wavelet analysis uses a combination of sinusoid functions. This may generate a non-consistent estimate of the variance of the OLS estimates. Accordingly, equation (2) is estimated using an OLS regression with heteroscedastic and auto-correlation consistent estimator (HAC).[24]

We introduce the control variables sequentially. First, the regression is estimated with no controls ($\hat{\beta}_{x_j} = \hat{\beta}_I = \hat{\beta}_{II} = 0$). Then, we introduce dummies for the two World Wars. Finally, the equation is estimated with all control variables and for two lag structures ($i = 0$ and $i = 1$). Results are displayed in Tables 1 and 2. In

Table 1. Regression across frequencies

	Dependent variable $\Delta y_{j,t}$				
Frequency (years)	D_1 2 − 4	D_2 4 − 8	D_3 8 − 16	D_4 16 − 32	S_4 > 32
			USA 1899–2010		
ls_t	-1.938^{***}	-1.547^{***}	-0.911^{***}	-0.576^{***}	0.153^{***}
	(0.730)	(0.215)	(0.263)	(0.206)	(0.077)
War dummies	No	No	No	No	No
R^2	0.17	0.42	0.30	0.16	0.13
ls_t	-1.929^{***}	-1.550^{***}	-0.873^{***}	-0.446^{***}	0.102^{*}
	(0.657)	(0.175)	(0.270)	(0.210)	(0.062)
War dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.18	0.43	0.31	0.34	0.39
			UK 1857–2010		
ls_t	-1.258^{***}	-0.424^{*}	-0.327^{***}	-0.154^{**}	0.008
	(0.445)	(0.255)	(0.106)	(0.071)	(0.034)
War dummies	No	No	No	No	No
R^2	0.23	0.08	0.22	0.09	0.04
ls_t	-1.264^{***}	-0.427^{*}	-0.377^{***}	-0.180^{**}	0.006
	(0.454)	(0.253)	(0.109)	(0.085)	(0.034)
War dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.23	0.08	0.25	0.11	0.10
			UK 1898–2010		
ls_t	-1.927^{***}	−0.062	-0.180^{**}	-0.257^{***}	0.056^{*}
	(0.422)	(0.295)	(0.071)	(0.059)	(0.030)
War dummies	No	No	No	No	No
R^2	0.34	0.04	0.10	0.21	0.08
ls_t	-1.950^{***}	−0.061	-0.199^{***}	-0.323^{***}	0.049^{**}
	(0.437)	(0.239)	(0.071)	(0.058)	(0.025)
War dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.34	0.04	0.13	0.27	0.21
			France 1898–2010		
ls_t	-1.445^{***}	-0.439^{***}	0.154	0.020	0.298^{***}
	(0.293)	(0.131)	(0.159)	(0.117)	(0.063)
War dummies	No	No	No	No	No
R^2	0.16	0.08	0.017	0.01	0.36
ls_t	-1.461^{***}	-0.436^{***}	0.229	0.159	0.354^{***}
	(0.278)	(0.129)	(0.248)	(0.147)	(0.068)
War dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.17	0.09	0.20	0.13	0.45

Notes: This table presents the regressions results across each time scale and for each country with and without dummies for the two World Wars. The equation estimated is (2). The estimation method is HAC-OLS. The weights follow Newey–West. The constant is suppressed to save space. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

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TABLE 2. Regression using the smooth component S4

Dependent variable $\Delta y_{s4,t}$				
cst	ls_t	ls_{t-1}	Controls^a	R^2
		USA: 1899–2010		
−0.243***	0.339***		Yes	0.69
−0.272***		0.380***	Yes	0.72
		UK: 1898–2010		
−0.020*	0.025		Yes	0.40
−0.030**		0.034*	Yes	0.41
		UK: 1857–2010		
−0.012	−0.035		Yes	0.49
−0.018		−0.025	Yes	0.50
		France: 1898–2010		
−0.083***	0.072***		Yes	0.87
−0.098***		0.093***	Yes	0.88
		Panel: 1899–2010		
No	0.101***		Yes	0.72
No		0.109***	Yes	0.72

Notes: This table presents the regressions using the smoothed component $S4$ for each country. The equation estimated is (2) with Δy the growth rate of real GDP per capita, ls the labor share. [a]The control variables are real GDP per capita, investment share in GDP, CPI inflation (all lagged one period/year), and dummies for the two World Wars. t and $t-1$ indicate the lag structure. The estimation method is HAC-OLS. The weights follows Newey–West. For the panel estimation, country and year fixed effects have been included. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

the three countries, the regressions confirm the results of the continuous wavelet analysis, namely that the relationship between growth and the labor share changes across frequencies from negative at high frequencies to positive at low ones. In France, the sign associated with the labor share is negative at the frequencies D_1 and D_2, not significant at frequencies D_3 and D_4 and positive and significant at frequency S_4. The coefficient is increasing with the scale considered from -1.4 to 0.3. In the UK, the sign associated with the labor share is negative from D_1 to D_4 and not significant for S_4 over the period 1857–2010. Similarly to France, the coefficient increases with the time scale considered from -1.2, -0.4, -0.3, and -0.15. Excluding the 19th century data for the UK produces a positive and significant coefficient for the long-term series $\hat{\beta}_{ls_{s4}} = 0.056$. In the USA, the labor share has a negative impact on growth at frequencies D_1–D_4, the sign turning positive at frequency S_4. Here as well the sign increases with the scale considered from -1.9 to 0.15.

Table 1 also displays the estimation of equation (2) augmented with war dummies. Controlling for the two World Wars is necessary as economic structures have been deeply affected by both events. In particular, our earlier Figure 1 shows that the labor share tends to become closer to unity during war periods. The main result is that adding dummies does not modify the results described above. If there

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is an impact, it concerns the smoothed components S_4. The long-term coefficients are slightly increased in France and slightly reduced in the UK and the USA. The standard errors are affected only marginally.

In this first set of regressions, the independent variable enters the regression contemporaneously. As indicated by the relative phase, the labor share is a leading indicator of growth in the areas of common power. This may be interpreted as pointing that endogeneity is less of an issue when estimating equation (2) at t, especially at low frequency.

Regarding the smoothed component, the R^2 ranges from 0.10 to 0.45 depending on the country considered. Regressions using wavelets differ from regressions with time series. The objective is not to fit the raw data as well as possible, but to show whether the sign of the relation changes across time scales. It follows that the lagged dependent variable is not added as a regressor. The details and smooth components are sinusoid functions that display strong auto-correlation especially at low frequencies. A lagged dependent variable as a regressor would appear strongly significant and may overshadow the relation existing with other explanatory variables. In the absence of a lagged dependent variable the R^2 is mechanically lower.

In Table 2, we estimate equation (2) with the complete set of controls focusing on the long term corresponding to the 32 years and beyond frequency (labeled S_4). Equation (2) is also estimated for two different lags for the independent variable ($i = 0$ and $i = 1$). A first result is that the inclusion of control variables does not affect the result discussed previously. The coefficient associated with the labor share is positive and significant in the long term. Compared to Table 1, introducing control variables reduces the coefficient associated with the labor share in France and increases it in the USA. In the UK, the coefficient is still not significant when the regression is performed over 1857–2010 but turns positive and significant when excluding the 19th century data (1898–2010) when the labor share enters with a lag. In France and in the USA, estimating equation (2) with a lag does not alter the results. The coefficient increases from 0.07 to 0.09 in France and from 0.34 to 0.38 in the USA. The last two lines of Table 2 summarize the result of the estimation for a panel of the three countries for 1899–2010. The panel estimation includes country fixed effects and year fixed effects. The sign associated with the labor share is positive and significant regardless of the lag structure ($i = 0$ and $i = 1$) and after controlling for third factors.

In Appendix C, we check whether these results are robust to the use of alternative filters. We reproduce the regression exercise described in Tables 1 and 2 filtering the series with the Christiano–Fitzgerald (CF) bandpass filter [Christiano and Fitzgerald (2003)]. Our main result (namely, that the sign of the coefficient associated with the labor share changes from negative at high frequency to positive at low frequency) remains under the CF filter. The appendix discusses in detail the choice of this alternative filter as well as the similarities and differences in the filtered series (see Figure C.1 and Tables C.1 and C.2 in the appendix).

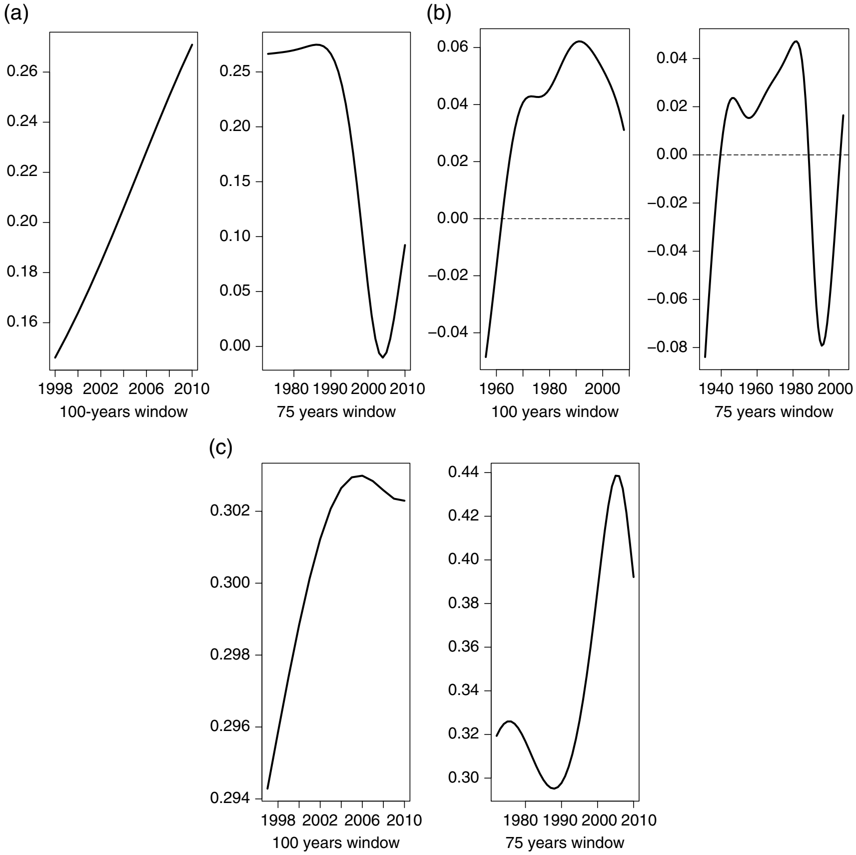
We now perform further robustness check in particular regarding changes in the definition of the labor share. We explore two main aspects: (i) sub-components of the wage bill and (ii) the importance of capital depreciation in the measurement of the labor share. Given that we have two different data source for the UK and France on the one hand and for the USA on the other hand, this section can only be performed for the former two countries.

As defined in equation (1), the labor share is made of different elements of the wage bill in the corporate sector and in public administration and an imputed value of the wage bill in unincorporated enterprises owned by households' member. In order to gain insights on the contribution of these different elements on the sign of the coefficient between labor share and growth we proceed in two steps.

First, we reproduce the different regressions using the corporate labor share $ls_{c_1} = ce_c/Y_c$. The corporate labor share is not directly comparable with the definition used throughout the paper. However, it is an interesting benchmark as it captures functional income distribution in the corporate sector. We then reproduce the regressions taking each element of the wage bill in isolation and keeping the denominator as defined in equation (1). The comparison with the aggregate labor share definition is more straightforward. However, the interpretation is more difficult as these sub-components do not capture functional income distribution. In fact, the labor share in the administrative sector is 1 by definition. The wage bill in unincorporated enterprises owned by households' member is a simple imputation based on the corporate labor share. The main result from this robustness check is that the wage bill in the corporate sector as well as the wage bill in the public sector positively impact growth at low frequency. By contrast, the wage bill associated with self-employment appears with a negative sign at low frequency (see Tables D.1, D.2, and D.3 in the appendix). At higher frequencies, the negative sign seems to be driven by the corporate wage bill and self-employment wage bill.

A second aspect of our robustness check is whether labor share is a measured net of capital depreciation. In equation (1), total income in the denominator is a measured net of capital depreciation. This point has been made by Rognlie (2015), who shows that the increase in capital depreciation in the recent decades has an important impact on functional income distribution. Since we use gross domestic product (rather than net domestic product) on the left-hand side of equation (2), we perform robustness check using gross labor share (see Tables D.5 and D.6 in the appendix). The coefficients across frequencies are unaffected by the change in the definition for France. For the UK, the coefficient associated with the gross labor share at low frequency is now negative but not significant for the time period 1898–2010 (when including control variables).

Finally, we perform a robustness check regarding the time period considered (using the labor share as defined in equation (1)). The advantage of using historical data is that it allows to perform single country estimation, while existing studies using data typically from the 1970s rely often on pooled estimations. The shortcoming of using historical data is that the sign of the relation may change over time as the countries considered have experienced profound changes over the century. The changing relationship over time may also explain some of the results described in the previous section as for instance the changing level of significance for the frequency S_4 in the UK depending on the time coverage considered.



Notes: These figures display the 100 years and 75 years rolling window regression for France, the UK, and the USA using the smoothed component *S*4. The estimation method is OLS-HAC.The dashed line is the zero line. The horizontal axis displays the median year of the period over which the regression is performed.

FIGURE 5. Rolling regression 100 years window—S4. (a) USA; (b) UK; and (c) France.

To better capture the (possible) instability in the relation between the labor share and growth, we perform the estimation described in equation (2) using 100 years and 75 years rolling window. The results are described in Figure 5. In France, the 75 years rolling window estimation shows a positive coefficient between 0.3 and 0.44. The coefficient is quite stable around 0.3 for the 100 years rolling window correlation. For both windows the coefficient is significantly different from zero. In the USA, the coefficient fluctuates between 0.05 and 0.25 for the 75 years window and between 0.1 and 0.25 for the 100 years window. Similarly to France, the coefficients are significant. Last, in the UK the coefficient associated with the labor share is negative and significant when regressions are performed using data for the 19th and early 20th centuries. The coefficient then turns positive and significant. This may explain that the coefficient is not significant for S_4 in the regression using the entire sample presented in Table 1. A last interesting point is that the sign associated with the labor share is positive and increasing after World War II in all three countries. This section illustrates that a second advantage of using historical data is that the impact of income distribution on growth can be seen to have changed over time.

5. CONCLUSION

The issue of how national income is distributed among factors is a long-standing and controversial topic. Ricardo (1821) proclaimed it the "principal problem of Political Economy." Our contribution is to use newly available historical time series on shares long enough to capture long-term cycles in the labor share and run single country estimation. Moreover, we adopt a relatively under-utilized technique, wavelet analysis, to robustly uncover its different frequencies and its real comovement characteristics. Our approach stands in contrast with many existing papers, which rely on time series with a short time dimension and pooled data.

Our results can be summarized as follow. First, the labor share exhibits long swings. These long swings account for the major part of the variance in the data. Long-term movements in the labor share in the UK, in France, and in the USA bring a fresh perspective to the debate regarding the stability of factor shares. Moreover, economic models which analyze factor share movements with business cycle mechanism are apparently capturing relatively little of its underlying variation.

Second, the impact of labor share on growth changes sign across frequencies from negative in the short term to positive in the long term. This pattern is confirmed by both the relative phase and the regression analysis. In addition, the positive sign at low frequencies is further validated by rolling window regressions showing that the coefficient increases over time. The positive sign estimated in the long term is especially important as the coherency analysis indicates that the long term is the relevant time scale to conduct this study. Finally, in terms of "causality" our study would suggest that inequality (as captured by labor shares) leads growth; this has been a point of considerable controversy in the literature.

There is, as earlier noted, no unifying theory of how (and if) inequality and growth are related. Our contribution adds to the literature by bringing further evidence based on very long sample data and wavelet methods. Moreover, given the limitations of our data, it is not feasible for us to discriminate between different explanations.

Notwithstanding, if were to draw some general conclusions, our perspective tends to one of scepticism of the concept of balanced growth. In an economy where technological biases are fluctuating and where different economic sectors are developing, growing, and contracting at different rates, that economy is characterized by balanced growth and stable functional income shares and does not seem to be borne out by the data. Although, of course, if we look at snap shots of the data (say over 30, 50 years) we may find conforming (if potentially illusory) signs of balanced growth. At the same time, given that the labor share is necessarily bounded, growth cannot deliver endless gains to its income share. A more granular analysis of the link seems a promising future research direction, as does an examination of possible threshold effects between growth and inequality.

NOTES

1. See the various discussions in Kurz (2010), Kramer (2011).

2. We define total labor income as the sum of the compensation of employees paid by corporations and by the government, with a correction for self-employed income. See Section 3.1.

3. In defining the long run we loosely combine two approaches. First, there is the idea of asymptotic growth theory which supports no specific numerical value (for how long is the long run) but instead refers to the operation of the economy on its balanced growth path. Second, there are time series definitions. For this purpose we report frequency ranges of 2–8 years, the medium run which are assumed to be a frequency range of 8–32 years and the long run which is beyond 32 years .

4. Elsby et al. (2013) look at US data starting in 1948. Karabarbounis and Neiman (2014) look at a panel of countries since 1975. IMF (2017) reviews the recent literature on the apparent decline in the labor income share in recent decades in the international context.

5. Regressions using raw data would not reveal the changing relationship between labor share and growth across frequencies. Alternative methodologies, such as HP filters, have been criticized for over differencing, which may cause spurious autocorrelation especially when using annual data. Other frequency methods such as Fourier transforms lose the time dimension of the data, while wavelets combine both the frequency and the time dimensions.

6. In other words, increasing inequality depresses growth in the long term.

7. Factor income shares will always be constant since any change in factor proportions or biases in technology (i.e., whether technology change impacts labor more than capital) will be exactly offset by a change in factor prices. Although strictly speaking under Cobb Douglas, technology cannot be identified. Moreover the notion that Cobb Douglas can be characterized in the long run given Pareto-distributed innovations Jones (2005) also fails to address of the visible variation in factor shares beyond business cycles.

8. Whereas if the elasticity exceeds unity increased capital deepening decreases the labor share. These relationships reverse if the elasticity lies below one.

9. If capital-saving technical change is permanent, then the output per-capita growth exceeds permanently the rate of labor saving technical change and no convergence to equilibrium exist. de La Grandville and Klump (2000) consider the case where economic growth becomes positively related to the size of the substitution elasticity.

10. A more extensive review of Piketty's arguments can be found in the Wealth and Inequality symposium in the *Journal of Economic Perspectives*, 2015.

11. Conversely, growth models stress that inequality has a negative impact on growth through the interplay between credit constraint and human capital accumulation in the long term. However, this strand of the literature focuses on personal rather than functional income distribution [Halter et al. (2014)].

12. These definitions refer to Tables US.11 and US.10.

13. The usual smoothing parameters for annual data (100 or 400) involve leakage at low frequencies as well as significant compression and exacerbation (frequency response lower than 1 or higher than 1).

14. The long term corresponds to frequencies beyond 32 years.

15. In the UK and the USA, the question arises whether the low point observed in 2010 at the end of our sample corresponds to a trough.

16. This U-shaped cycle is based on time series with a 10-year frequency.

17. An alternative level of significance of 5% for instance has a small impact on the size of the regions.

18. The edge effect designates the challenge of estimating the wavelet transform at the edge of the signal.

19. See http://www.ggdc.net/maddison/maddison-project/home.htm. As a measure of output we use real Gross Domestic Product.

20. Although Granger (1969) in his original article uses spectral analysis concept to describe causality, Granger tests have been mainly applied using time series techniques that render them unsuitable to the present exercise.

21. An alternative methodology to study the impact of the labor share on growth would have been to perform a growth accounting exercise and to disentangle the channels going through labor productivity and real wage. However, in the absence of employment data, this exercise cannot be conducted.

22. The other frequencies are not displayed due to space limitations.

23. The correlations are available on request.

24. The HAC estimator adjusts the covariance matrix by applying weight to account for autocorrelation. The HAC estimator in this paper uses pre-whitening of the error term and the weights are chosen following Newey and West (1987).

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APPENDICES

A: WAVELET ANALYSIS

In this section, we present the main concepts behind the continuous wavelet analysis and the discrete wavelet analysis. We also present the tools used in the main text to analyze the variance (wavelet power spectrum) and co-variance (wavelet coherency) across time and frequency.

A.1. Continuous Wavelet Analysis

In the continuous wavelet framework, a time series $x(t)$ is projected onto the time–frequency space using the concept of *mother wavelet function*. A mother wavelet is a function of time that spans on the real space, $\Psi(t) \in L^2(\mathbb{R})$ and satisfies the following admissibility condition $0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|}{|\omega|} d\omega < \infty$. C_ψ is the admissibility condition and $\Psi(\omega)$ denotes the Fourier transform, a function of angular frequency ω. Assuming that $\Psi(t)$ is a function with sufficient decay, the previous admissibility condition can be restated as follows $\Psi(0) = \int_{-\infty}^{\infty} \Psi(t)\, dt = 0$. The assumed decaying property of the mother wavelet function enables the localization in both time and frequency. From a mother wavelet function $\Psi(t)$, one can get a set $\psi_{\tau,s}(t)$ of wavelet daughters by scaling and translating $\Psi(t)$:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}} \Psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R},\ s \neq 0, \tag{A1}$$

where s is a parameter that controls the width of the wavelet and τ is a parameter that controls the location of the wavelet in the time domain. Taking scale values s (smaller) larger than 1, means that the mother wavelet function is (compressed) dilated to capture (high) low-frequency features of the data. The continuous wavelet transform (CWT) of a time series $x(t)$ is a projection of $x(t)$ onto a specific mother wavelet function $\Psi(t)$:

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t-\tau}{s}\right) dt, \tag{A2}$$

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where * denotes the complex conjugate. Under the admissibility condition, the CWT does not alter the energy (variance) of $x(t)$ (Henceforth, $x(t)$ can be recovered from its CWT). In our work we use one particular analytic mother wavelet function, the Morlet wavelet function $\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}$. The motivation being that the joint time–frequency concentration of the Morlet wavelet function is optimal (see Gallegati and Ramsey (2013)).

The attraction of the continuous wavelet analysis is that it offers tools to study the properties of a series across time and frequency. The local variance of a time series $x(t)$ across time and frequencies is the wavelet power spectrum:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2. \tag{A3}$$

The wavelet power spectrum was used in Section 2. Additionally, the co-variance between two series $x(t)$ and $y(t)$ across time and frequencies is the wavelet coherency:

$$R_{x,y}(\tau, s) = \frac{\left|S\left(W_{x,y}(\tau, s)\right)\right|}{\left[S\left(|W_x(\tau, s)|^2\right) S\left(\left|W_y(\tau, s)\right|^2\right)\right]^{1/2}}. \tag{A4}$$

$R_{x,y}$ lies between 0 (no correlation) and 1 (highly correlated). S is a smoothing operator in both time and frequencies. $\left|W_{x,y}(\tau, s)\right|$ is the cross-wavelet power, which is defined as the absolute value of the cross-wavelet transform $W_{x,y}(\tau, s)$:

$$W_{x,y}(\tau, s) = W_x(\tau, s)\, W_y^*(\tau, s), \tag{A5}$$

where * denotes the complex conjugate. The continuous wavelet analysis also gives us information about the leads and lags between two time series. The phase difference $\phi_{x,y}(\tau, s)$ of two time series shows how the causal relationship between these two time series evolves across time and frequencies:

$$\phi_{x,y}(\tau, s) = \arctan\left(\frac{\mathcal{I}\left(W_{x,y}(\tau, s)\right)}{\mathcal{R}\left(W_{x,y}(\tau, s)\right)}\right), \tag{A6}$$

where the phase angle is computed from the smoothed cross-wavelet transform instead of the cross-wavelet transform. $\mathcal{R}(X)$ and $\mathcal{I}(X)$ denote, respectively, the real and the imaginary part of X. The phase shift is represented by arrows in the coherency figure as illustrated in the next section. The direction of the phase arrow indicates the following relationship between two variables: left anti-phase, right in-phase, down $x(t)$ leading $y(t)$ by 90°, up $y(t)$ leading $x(t)$ by 90°.

A.2. Discrete Wavelet Analysis

The discrete wavelet analysis is a discretization of the continuous wavelet analysis. The multiresolution wavelet analysis (MRA) decomposes a time series x_t into several components with different cycle periodicity (Main references include Gençay et al. (2002), and Percival and Walden (2006)):

$$x_t = s_{J,t} + \sum_{j=1}^{J} d_{j,t}, \ t = 0, \ldots, N-1, \tag{A7}$$

where J denotes the number of scales. The wavelet detail $d_{j,t}$ represents the change in the time series x_t on a scale of length $\lambda_j = 2^{j-1}$ and captures the oscillations of x_t within a

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window of $\left[2^j, 2^{j+1}\right]$ periods. The wavelet smooth $s_{J,t}$ is the long-term changes in x_t and captures the oscillations of x_t over more than 2^{j+1} periods (years in our case).

The MRA is performed using the maximal overlap discrete wavelet transform (labeled MODWT). The MODWT of a time series x_t is represented by the following matrix equation:

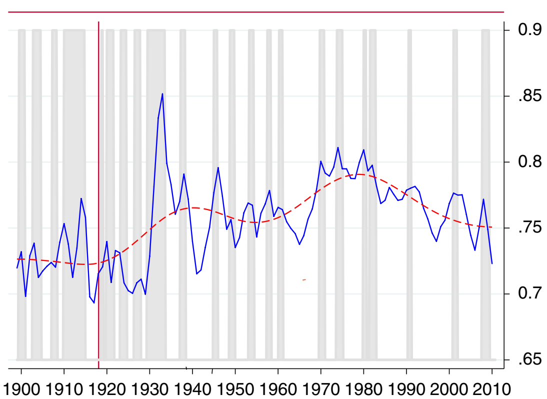
$$\boldsymbol{w} = \mathcal{W}\boldsymbol{x}, \tag{A8}$$

where $\boldsymbol{x} = (x_0, x_1, \cdots, x_t, \cdots, x_{N-1})'$ is a $(N \times 1)$ vector of the observations of x_t, $\boldsymbol{w} = \left(\boldsymbol{w}_1, \cdots, \boldsymbol{w}_j, \cdot, \boldsymbol{w}_J, \boldsymbol{v}_J\right)'$ is the $((J+1)\,N \times 1)$ vector of MODWT coefficients, and $\mathcal{W}$ is a $((J+1)\,N \times N)$ matrix defining the MODWT.

The main practical issue is the choice of an appropriate wavelet filter. We choose to use the least asymmetric wavelet filter of width 8, *LA* (8). This width is large enough to obtain a filter close to the ideal bandpass filter but small enough to minimize the number of wavelet coefficients affected by the boundaries. In addition, this filter is nearly symmetric.

The choice of the number of scales to consider is also crucial as it defines the frequency of the long-term oscillations of the time series x_t. We choose a conservative rule-of-thumb $J = \log_2\left(\frac{N}{L-1} + 1\right)$, which gives us the following frequencies: $D_1 :$ $2-4$ annual frequency; $D_2 : 4-8$ annual frequency; $D_3 : 8-16$ annual frequency; $D_4 :$ $16-32$ annual frequency; and, $S_4 :> 32$ annual frequency. Finally, we use the reflection boundary conditions, which provide continuity at the boundaries and which does not affect either the sample mean or the sample variance of the original data (In order to reduce the impact of the edge effect, the data are replicated by reflection as follows: $x_0, x_1, \ldots, x_{N-2}, x_{N-1}, x_N, x_{N-1}, x_{N-2}, \ldots, x_1, x_0$. The reflected data (time reversed) are pasted at the end of the original time series).

B: ADDITIONAL RESULTS



Notes: These figures display the labor share and their long-term component in the three countries considered. For the USA, NBER recession periods (in gray shaded areas) are overlaid at the appropriate frequency.

FIGURE A.1. The US labor share of income with NBER recession dates.

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TABLE B.1. Descriptive statistics of labor share series

				Full Sample		Post-War		
Country	Mean	Median	Std. Dev.	Min	Max	Min	Max	Normality
				Levels Series				
USA	0.756	0.757	0.031	0.693 **1917**	0.852 **1933**	0.723 **2010**	0.811 **1974**	[0.234]
UK	0.743	0.747	0.052	0.611 **1871**	0.908 **1922**	0.704 **1922**	0.865 **1976**	[0.399]
France	0.773	0.769	0.054	0.629 **1912**	0.985 **1944**	0.743 **1989**	0.984 **1945**	[0.000]
				Long-Run Component, S4				
USA	0.756	0.758	0.021	0.722 **1915**	0.791 **1979**	0.751 **2010**	0.791 **1979**	[0.000]
UK	0.742	0.740	0.0410	0.668 **1857**	0.806 **1928**	0.727 **2010**	0.806 **1976**	[0.000]
France	0.773	0.779	0.035	0.698 **1925**	0.829 **1948**	0.762 **1999**	0.829 **1948**	[0.000]

Notes: This table presents the descriptive statistics of the level labor share series. The **dates** below the max min columns indicate the year of occurrence. The normality test is the Shapiro-Francia W′ test (probability values reported in squared parentheses).

C: ROBUSTNESS: CHRISTIANO–FITZGERALD BANDPASS FILTER

In this appendix, we check whether the results described above are affected by the choice of an alternative filter. There are two main bandpass filters: the Baxter–King (BK) filter [Baxter and King (1999)] and the Christiano–Fitzgerald (CF) filter [Christiano and Fitzgerald (2003)]. Wavelet filters are both time and frequency localized. Contrastingly, the BK filter and the CF filter have properties formulated in the frequency domain only. Both the BK filter and the CF filter approximate the ideal filter through a two-sided moving average of the time series. They differ with respect to the symmetry of the weights. The BK formulates symmetric weights. This implies that the filter does not produce phase shift in the filtered series. However, this requires to drop observations at the beginning and at the end of the sample. The CF filter uses asymmetric weights, which imply that no observations are lost. Contrasting to wavelets, both filters also make assumptions regarding the data-generating process (*iid* for BK and random walk for CF). We choose the CF filter as the filtered series have the same number of observations as the wavelet filter, which makes the comparison more straightforward. In addition, the assumption that variables follow a

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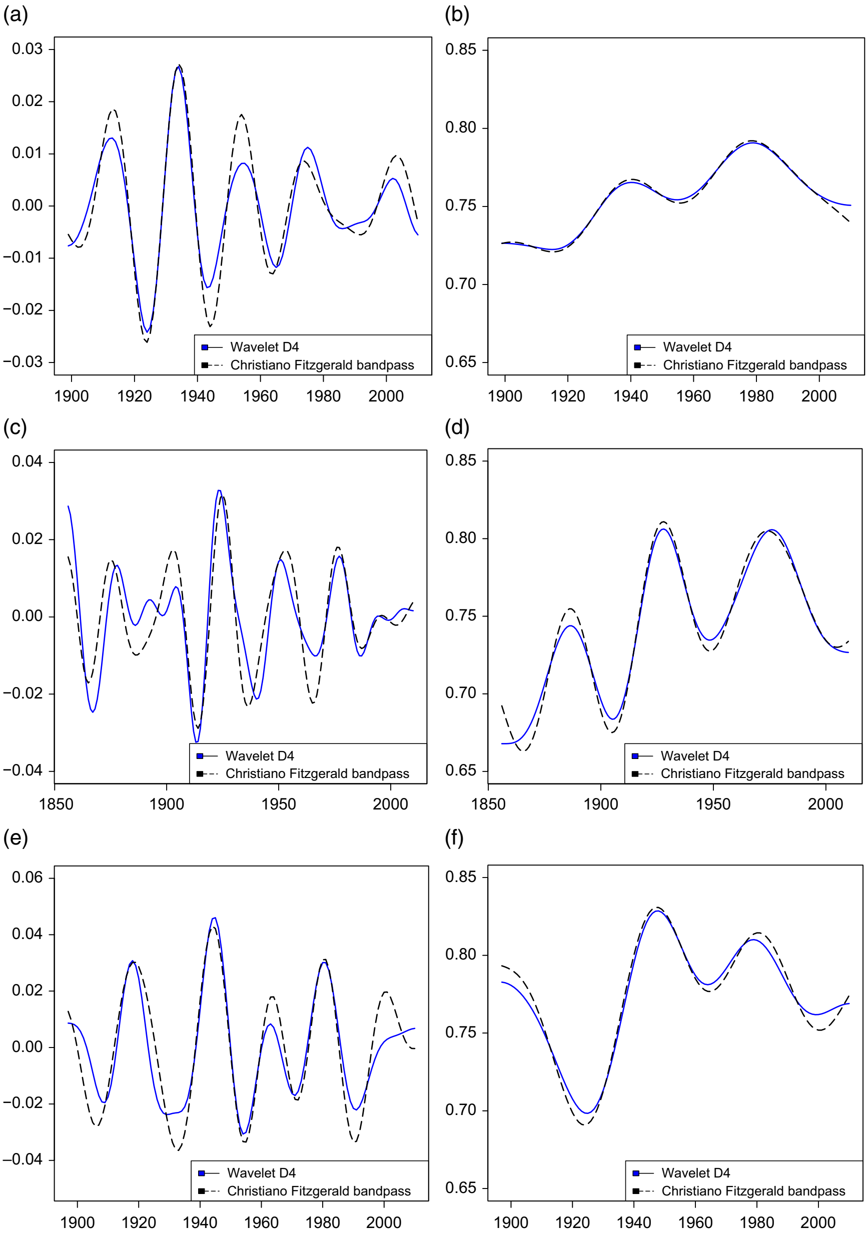

Note: This figure displays the wavelet and CF bandpass filter for the labor share for two frequencies D_4 16–32 years and S_4 Beyond 32 years.

FIGURE C.1. Labor share—CF bandpass filter. (a) USA: D_4 16–32 years; (b) USA: $S_4 >$ 32 years; (c) UK: D_4 16–32 years; (d) UK: $S_4 >$ 32 years; (e) France: D_4 16–32 years; and (f) France: $S_4 >$ 32 years.

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TABLE C.1. Regression across frequencies—CF bandpass filter

	Dependent variable $\Delta y_{j,t}$				
Frequency (years)	D_1 2 − 4	D_2 4 − 8	D_3 8 − 16	D_4 16 − 32	S_4 > 32
			USA 1899–2010		
ls_t	−2.208***	−1.533***	−0.998***	−0.550***	0.139***
	(0.400)	(0.164)	(0.148)	(0.126)	(0.050)
D_{WWs}	No	No	No	No	No
R^2	0.22	0.44	0.29	0.15	0.06
ls_t	−2.189***	−1.540***	−0.972***	−0.435***	0.050
	(0.405)	(0.164)	(0.153)	(0.123)	(0.048)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
R^2	0.22	0.45	0.30	0.30	0.27
			UK 1857–2010		
ls_t	−1.18***	−0.296**	−0.374***	−0.249**	0.017
	(0.197)	(0.114)	(0.047)	(0.038)	(0.017)
D_{WWs}	No	No	No	No	No
R^2	0.19	0.04	0.29	0.22	0.05
ls_t	−1.189***	−0.297**	−0.410***	−0.272***	0.016
	(0.199)	(0.115)	(0.051)	(0.039)	(0.018)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
R^2	0.19	0.04	0.31	0.25	0.06
			UK 1898–2010		
ls_t	−1.836***	0.050	−0.236**	−0.283***	0.066***
	(0.279)	(0.150)	(0.042)	(0.043)	(0.025)
D_{WWs}	No	No	No	No	No
R^2	0.27	0.04	0.22	0.28	0.06
ls_t	−1.867***	0.049	−0.247***	−0.314***	0.061**
	(0.284)	(0.151)	(0.047)	(0.045)	(0.024)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
R^2	0.28	0.04	0.24	0.31	0.12
			France 1898–2010		
ls_t	−1.384***	−0.446***	0.178	0.081	0.227***
	(0.301)	(0.136)	(0.112)	(0.065)	(0.043)
D_{WWs}	No	No	No	No	No
R^2	0.16	0.09	0.02	0.01	0.19
ls_t	−1.398***	−0.448***	0.224**	0.179**	0.284***
	(0.307)	(−0.448)	(0.224)	(0.179)	(0.045)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
R^2	0.16	0.09	0.18	0.09	0.26

Notes: This table presents the regressions results across each time scale and for each country with and without dummies for the two World Wars (D_{WWI} and D_{WWII}). The equation estimated is equation 2. The estimation method is HAC-OLS. The weights follow Newey–West. The constant is suppressed to save space. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

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TABLE C.2. Regression using the smooth component S4—CF bandpass filter

Dependent variable $\Delta y_{s4,t}$				
cst	ls_t	ls_{t-1}	Controls	R^2
		USA: 1899–2010		
−0.249***	0.348***		Yes	0.55
−0.289***		0.402***	Yes	0.58
		UK: 1898–2010		
−0.02	0.038		Yes	0.19
−0.03**		0.049*	Yes	0.20
		UK: 1857–2010		
0.014	−0.043		Yes	0.10
0.005		0.008	Yes	0.10
		France: 1898–2010		
−0.118***	0.13***		Yes	0.67
−0.131***		0.148***	Yes	0.69
		Panel: 1899–2010		
No	0.109***		Yes	0.56
No		0.121***	Yes	0.57

Notes: This table presents the regressions using the smoothed component $S4$ for each country. The equation estimated is (2) with Δy the growth rate of real GDP per capita, ls the labor share. The control variables are real GDP per capita from previous period, lagged investment share in GDP, lagged CPI inflation, and D_{WWI} and D_{WWII} the dummies for World War I and II. In France, lagged CPI inflation is not included as a control variable. See the text for a discussion. t and $t-1$ indicate the lag structure. The estimation method is HAC-OLS. The weights follow Newey–West. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

random walk imply that the CF filter better captures the low frequency rather than the BK filter. The power spectrum of a random walk is defined as a constant over frequency squared [Everts (2006)].

The parameters of the CF filter are chosen to correspond to the frequencies of the wavelet filter. Figure C.1 displays the labor share series corresponding to the "16–32 years" frequency as well as to the "32 years and beyond" frequency under both the wavelet filter and the CF filter. A striking feature is the similarity between the two series. The main difference is that the CF filtered series display larger amplitudes than the wavelet series for all frequencies. This may be due to the fact that wavelets are localized in the time dimension while CF filters are global. This implies that one extreme value, such as the extreme values for the labor share during the two World Wars would be transmitted to all the filtered series in the case of the CF filter but not in the case of the wavelet filters. There are two smaller differences between wavelets and CF filters. There are small phase shifts between the two filtered series, especially for France and for the United Kingdom. In addition, the boundaries of the series differ slightly depending on the filter used. The same issues arise when looking at GDP growth (not represented here due to space limitation).

The main result highlighted in Section 4, that is, the sign of the coefficient associated with the labor share changes from negative at high frequency to positive at low frequency, remains under the alternative filter. In that respect, Table C.1 is almost identical to Table 1. There are two small differences. The coefficient associated with the "beyond 32 years" frequency turns insignificant in the USA. On the contrary, the sign of the coefficients associated with the frequency "8–16 years" and frequency "16–32 years" is now significant (and positive) in France. In addition, the sign associated with the labor share in the long run is also robust to the inclusion of control variables under the CF bandpass filter as shown in Table C.2. Note that the sign associated with the labor share is positive and significant in France when GDP per-capita level and/or investment rate are included as control variables. The coefficient associated with the labor share is 0.293^{***} when controlling for GDP per-capita level and 0.130^{***} adding investment rate as well. These coefficients become 0.306^{***} and 0.148^{***} with lagged labor share. The sign turns insignificant when the filtered series for price inflation is added to the other control variables. The reason for this sudden change and whether it is related to the quality of the filtered series needs to be further researched.

D: ROBUSTNESS: DECOMPOSITION OF LABOR SHARE

In this section of the appendix, we present additional regressions tables for:

1. Sub-components of the labor share.
2. "Gross" labor share (as opposed to net labor share).

The exact definitions behind this robustness exercise are presented in the main text of the paper at the end of Section 4.2 (alternatively these definitions can be found in the note below the regression tables).

TABLE D.1. Labor share sub-components and growth—UK

	UK 1898–2010				
	(1)	(2)	(3)	(4)	(5)
	$D1$	$D2$	$D3$	$D4$	$S4$
C_1	−1.756***	−0.191	−0.093*	0.001	0.002
	(0.387)	(0.210)	(0.051)	(0.039)	(0.028)
D_{WWs}	No	No	No	No	No
C_2	−1.706***	−0.068	−0.077**	0.010	0.061***
	(0.499)	(0.220)	(0.039)	(0.035)	(0.015)
D_{WWs}	No	No	No	No	No
SE_1	−3.732***	−0.594	−0.124	0.078	−0.081***
	(1.109)	(0.738)	(0.192)	(0.086)	(0.014)
D_{WWs}	No	No	No	No	No
G_1	0.769***	0.124	0.043	−0.066***	0.124***
	(0.211)	(0.215)	(0.047)	(0.025)	(0.029)
D_{WWs}	No	No	No	No	No
	Including World Wars fixed effects				
C_1	−1.772***	−0.196	−0.101*	0.010	0.001
	(0.385)	(0.211)	(0.055)	(0.061)	(0.029)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
C_2	−1.705***	−0.072	−0.079*	0.027	0.061***
	(0.504)	(0.222)	(0.042)	(0.060)	(0.014)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
SE_1	−3.743***	−0.632	−0.072	0.158	−0.075***
	(1.103)	(0.751)	(0.246)	(0.122)	(0.014)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
G_1	0.774***	0.133	0.026	−0.126***	0.131***
	(0.206)	(0.218)	(0.055)	(0.031)	(0.031)
D_{WWs}	Yes	Yes	Yes	Yes	Yes

Notes: This table presents the regressions for different subcomponents of the labor share: (i) $ls_{c_1} = ce_c/Y_c$; (ii) $ls_{c_2} = ce_c/\,(Y - tx)$; (iii) $ls_{se_1} = (ce_c Y_{hh}/Y_c)\,/\,(Y - tx)$; and (iv) $ls_{g_1} = ce_g/\,(Y - tx)$. The regression is estimated twice, without and with wars fixed effects (D_{WWI} and D_{WWII}). The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, this table only displays the coefficient associated with the labor share. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

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TABLE D.2. Labor share sub-components and growth—UK

	UK 1857–2010				
	(1)	(2)	(3)	(4)	(5)
	$D1$	$D2$	$D3$	$D4$	$S4$
C_1	−1.235***	−0.373**	−0.157**	0.010	−0.039
	(0.372)	(0.189)	(0.063)	(0.036)	(0.025)
D_{WWs}	No	No	No	No	No
C_2	−1.500***	−0.191	−0.100**	0.008	0.006
	(0.437)	(0.233)	(0.041)	(0.034)	(0.015)
D_{WWs}	No	No	No	No	No
SE_1	−2.043**	−1.100***	−0.393*	0.103*	−0.012
	(0.803)	(0.390)	(0.215)	(0.061)	(0.013)
D_{WWs}	No	No	No	No	No
G_1	0.763***	0.122	0.038	−0.065***	0.047**
	(0.209)	(0.214)	(0.048)	(0.025)	(0.022)
D_{WWs}	No	No	No	No	No
	Including World Wars fixed effects				
C_1	−1.240***	−0.381**	−0.189***	0.020	−0.041
	(0.377)	(0.186)	(0.069)	(0.048)	(0.026)
D_{WWs}	Yes	Yes	yes	yes	yes
C_2	−1.499***	−0.200	−0.112**	0.017	0.005
	(0.440)	(0.231)	(0.044)	(0.054)	(0.015)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
SE_1	−2.043**	−1.126***	−0.455*	0.144**	−0.012
	(0.809)	(0.377)	(0.244)	(0.068)	(0.013)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
G_1	0.767***	0.130	0.022	−0.117***	0.050**
	(0.204)	(0.216)	(0.056)	(0.031)	(0.024)
D_{WWs}	Yes	Yes	Yes	Yes	Yes

Notes: This table presents the regressions for different subcomponents of the labor share: (i) $ls_{c_1} = ce_c/Y_c$ (ii) $ls_{c_2} = ce_c/\ (Y - tx)$ (iii) $ls_{se_1} = (ce_c Y_{hh}/Y_c)\ /\ (Y - tx)$ (iv) $ls_{g_1} = ce_g/\ (Y - tx)$. The regression is estimated twice, without and with wars fixed effects (D_{WWI} and D_{WWII}). The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, this table only displays the coefficient associated with the labor share. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

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TABLE D.3. Labor share sub-components and growth—France

	France 1898–2010				
	(1) $D1$	(2) $D2$	(3) $D3$	(4) $D4$	(5) $S4$
C_1	-1.314^{***}	-0.429^{***}	0.140	-0.045	0.126
	(0.253)	(0.112)	(0.125)	(0.118)	(0.087)
D_{WWs}	No	No	No	No	No
C_2	-4.230^{***}	-1.411^{***}	0.226	0.010	0.072^{***}
	(0.894)	(0.508)	(0.189)	(0.254)	(0.026)
D_{WWs}	No	No	No	No	No
SE_1	-0.653	-0.588	2.139^{***}	0.575^{**}	-0.034^{*}
	(0.795)	(0.366)	(0.479)	(0.277)	(0.018)
D_{WWs}	No	No	No	No	No
G_1	-0.648	-0.132	-0.815^{**}	-0.263^{**}	0.123^{***}
	(1.032)	(0.544)	(0.382)	(0.112)	(0.041)
D_{WWs}	No	No	No	No	No
	Including World Wars fixed effects				
C_1	-1.329^{***}	-0.426^{***}	0.200	0.076	0.148
	(0.241)	(0.111)	(0.205)	(0.145)	(0.099)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
C_2	-4.228^{***}	-1.403^{***}	0.264	0.184	0.073^{**}
	(0.870)	(0.504)	(0.317)	(0.312)	(0.029)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
SE_1	-0.633	-0.585	2.219^{***}	0.956^{***}	-0.033
	(0.806)	(0.369)	(0.503)	(0.267)	(0.020)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
G_1	-0.651	-0.026	-0.992^{**}	-0.186	0.123^{***}
	(1.023)	(0.579)	(0.482)	(0.172)	(0.042)
D_{WWs}	Yes	Yes	Yes	Yes	Yes

Notes: This table presents the regressions for different subcomponents of the labor share: (i) $ls_{c_1} = ce_c/Y_c$ (ii) $ls_{c_2} = ce_c/\,(Y - tx)$ (iii) $ls_{se_1} = (ce_c Y_{hh}/Y_c)\,/\,(Y - tx)$ (iv) $ls_{g_1} = ce_g/\,(Y - tx)$. The regression is estimated twice, without and with wars fixed effects (D_{WWI} and D_{WWII}). The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, this table only displays the coefficient associated with the labor share. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

TABLE D.4. Labor share sub-components—regression using the smooth component S4

Dependent variable $\Delta y_{s4,t}$ Labor share sub-component							
$C_{1,t}$	$C_{1,t-1}$	$C_{2,t}$	$C_{2,t-1}$	$SE_{1,t}$	$SE_{1,t-1}$	$G_{1,t}$	$G_{1,t-1}$
UK: 1898–2010							
−0.007	−0.001	0.029*	0.033*	−0.075***	−0.072***	0.110***	0.106***
(0.016)	(0.017)	(0.017)	(0.017)	(0.016)	(0.016)	(0.022)	(0.021)
UK: 1857–2010							
−0.046**	−0.038*	−0.033**	−0.031**	0.024	0.025	0.038	0.044*
(0.020)	(0.020)	(0.015)	(0.014)	(0.017)	(0.017)	(0.024)	(0.023)
France: 1898–2010							
0.054	0.068	0.107*	0.123*	−0.076**	−0.067***	0.392***	0.401***
(0.055)	(0.054)	(0.066)	(0.067)	(0.033)	(0.033)	(0.045)	(0.047)

Notes: This table presents the regressions using the smoothed component $S4$ for each country and for different sub-components of the labor share. The equation estimated is (2) with Δy the growth rate of real GDP per capita and the following sub-component of the labor share: (i) $ls_{c_1} = ce_c/Y_c$ (ii) $ls_{c_2} = ce_c/\,(Y - tx)$ (iii) $ls_{se_1} = (ce_c Y_{hh}/Y_c)\,/\,(Y - tx)$ (iv) $ls_{g_1} = ce_g/\,(Y - tx)$. †The control variables are real GDP per capita, investment share in GDP, CPI inflation (not for France), all lagged one period/year and dummies for the two World Wars. t and $t-1$ indicate the lag structure. The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, only the coefficient for the labor share is reported in the table. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

TABLE D.5. Labor share and growth—"gross" labor share

	UK 1898–2010				
	(1)	(2)	(3)	(4)	(5)
	$D1$	$D2$	$D3$	$D4$	$S4$
Gross labor share	-2.075^{***}	-0.151	-0.230^{**}	-0.350^{***}	-0.074^{**}
	(0.607)	(0.296)	(0.097)	(0.076)	(0.032)
D_{WWs}	No	No	No	No	No
Gross labor share	-2.110^{***}	-0.152	-0.256^{***}	-0.458^{***}	-0.071^{**}
	(0.619)	(0.300)	(0.094)	(0.086)	(0.032)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
	UK 1857–2010				
	(1)	(2)	(3)	(4)	(5)
	$D1$	$D2$	$D3$	$D4$	$S4$
Gross labor share	-1.223^{***}	-0.540^{***}	-0.424^{***}	-0.157^{*}	-0.096^{***}
	(0.464)	(0.193)	(0.120)	(0.085)	(0.029)
D_{WWs}	No	No	No	No	No
Gross labor share	-1.229^{**}	-0.543^{***}	-0.489^{***}	-0.182^{*}	-0.093^{***}
	(0.475)	(0.194)	(0.121)	(0.100)	(0.028)
D_{WWs}	Yes	Yes	Yes	Yes	Yes
	France 1898–2010				
	(1)	(2)	(3)	(4)	(5)
	$D1$	$D2$	$D3$	$D4$	$S4$
Gross labor share	-1.847^{***}	-0.594^{***}	0.271	0.176	0.347^{***}
	(0.332)	(0.161)	(0.265)	(0.144)	(0.053)
D_{WWs}	No	No	No	No	No
Gross labor share	-1.868^{***}	-0.589^{***}	0.364	0.365^{**}	0.396^{***}
	(0.331)	(0.160)	(0.318)	(0.158)	(0.055)
D_{WWs}	Yes	Yes	Yes	Yes	Yes

Notes: This table presents the regressions for an alternative definition of the labor share: $ls = \frac{ce_c \cdot \left(1+\frac{Y_{hh}}{Y_c}\right)+ce_g}{Y-tx}$ where Y_{hh}, Y_c and Y are measured before capital depreciation (Y_{hh} and Y_c are gross value added of their respective sector and Y is GDP) while they were previously measured net of depreciation. This table presents the regressions results across each time scale and for each country with and without dummies for the two World Wars (D_{WWI} and D_{WWII}). The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, this table only displays the coefficient associated with the labor share. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

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TABLE D.6. Regression using the smooth component S4—"gross" labor share

Dependent variable $\Delta y_{s4,t}$				
cst	ls_t	ls_{t-1}	Controls	R^2
		UK: 1898–2010		
0.022	−0.021		Yes	0.48
0.013		−0.008	Yes	0.47
		UK: 1857–2010		
0.068**	−0.082**		Yes	0.30
0.059**		−0.069*	Yes	0.29
		France: 1898–2010		
−0.091***	0.100***		Yes	0.87
−0.101***		0.116***	Yes	0.88

Notes: This table presents the regressions using the smoothed component $S4$. The equation estimated is (2) with Δy the growth rate of real GDP per capita, ls the labor share. The control variables are real GDP per capita from previous period, lagged investment share in GDP, lagged CPI inflation and D_{WWI} and D_{WWII} the dummies for World War I and II. The regression is estimated for an alternative definition of the labor share: $ls = \frac{ce_c \cdot \left(1+\frac{Y_{hh}}{Y_c}\right)+ce_g}{Y-tx}$ where Y_{hh}, Y_c and Y are measured before capital depreciation (Y_{hh} and Y_c are gross value added of their respective sector and Y is GDP) while they were previously measured net of depreciation. See the text for a discussion. t and $t-1$ indicate the lag structure. The estimation method is HAC-OLS. The weights follow Newey–West. Due to space limitation, this table only displays the coefficient associated with the labor share. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.