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Solutions for Space Operations with a Star Finder

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This paper shows a possible new use of a common or computerised star finder for a rapid and easy solution to some space problems such as:
– launch windows;
– satellite fly-over;
– the rise and setting of a satellite;
– Earth or Moon satellite surveys;
– the eclipse of a satellite.

KEY WORDS

1. Space Operations. 2. Graphic Solution.

1. LAUNCH WINDOWS.

1.1. *Introduction.* Knowledge of the attitude of a spacecraft is essential to control the direction of thrusts for orbital manoeuvres and the orientation of onboard sensors. So, at least during key activities such as launch, deployment, and planned satellite operations the desired attitude must be acquired accurately and controlled by onboard measurements (Pocha, 1987). The precision of astronomical attitude checks depend on the positions of the reference stars with respect to the spacecraft and its spin axis. These relative positions obviously vary with time, so that launch window times must be defined to ensure that the most favourable conditions will be met in all critical phases of the flight. A method by C. Chen and J. Wertz makes use of the celestial sphere for astronomical attitude checks (Wertz, 1995).

1.2. *Chen–Wertz method.* Let us consider a spin-stabilized spacecraft, a celestial sphere centred in it, and let A be the spherical point representing the spin axis orientation. Consider the spherical triangle of Figure 1.

S is the Sun, E the centre of the Earth, β and η are the "*Sun and the Earth angles*", which may be automatically measured by onboard sensors, as well as the "*rotation angle*" Φ. At least two measurements are necessary to fix A relative to S and E. Common measurement couples are: β and η, β and Φ, η and Φ. Each measurement leads to a line of position (*LOP*) for A: a circumference for a β or η measurement; a line of equal difference of azimuth for a Φ measurement. The uncertainty of fixing A by two *LOPs* depends on their density δ and their intersection or "*correlation*" angle θ (see Appendix 1). The uncertainty of A is high when δ is low and/or θ is near 0° or 180°. A possible quick summary of some of the method results may be as follows.

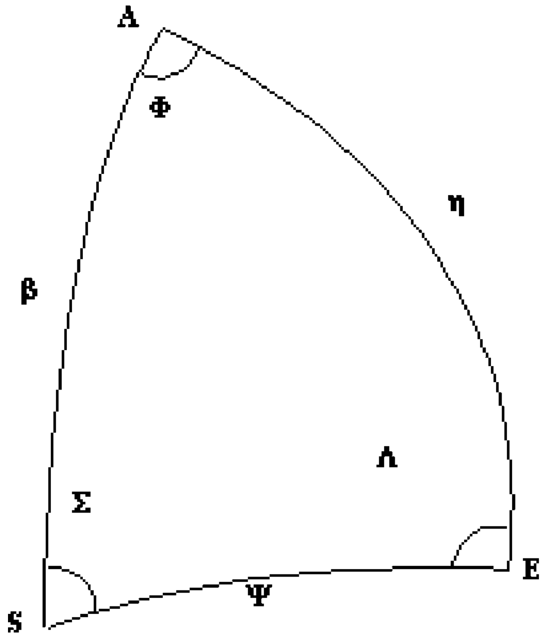

Figure 1. Spherical triangle defined by spin axis, Earth and Sun.

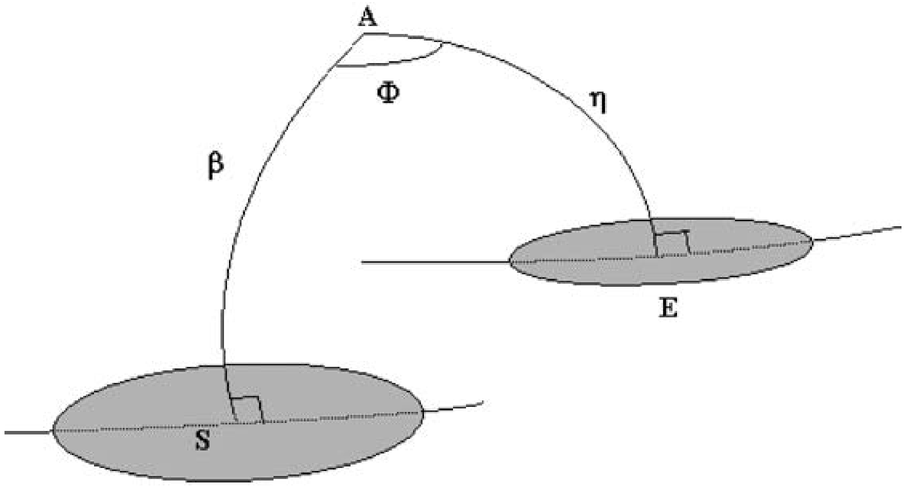

Figure 2. Poor density regions.

1.2.1. *Density condition.* For β/η measurements there is no density condition because for circular *LOPs* δ is constant and equals 1. For measurements including Φ see Figure 2. *E* must not be within the shaded area centred on *S*, and *S* must not be in the shaded area centred on *E*. How large these areas are depends on the values of β and η. Related formulas are in Appendix 1: by giving δ the minimum acceptable value (i.e. 0·5), from formula (1) in Appendix 1, the Φ values corresponding to a series of Σ values from 0° to 360° can be computed. Then, for each (Σ, Φ) the corresponding series of (Σ, Ψ) can be computed from the triangle in Figure 1. These are the polar

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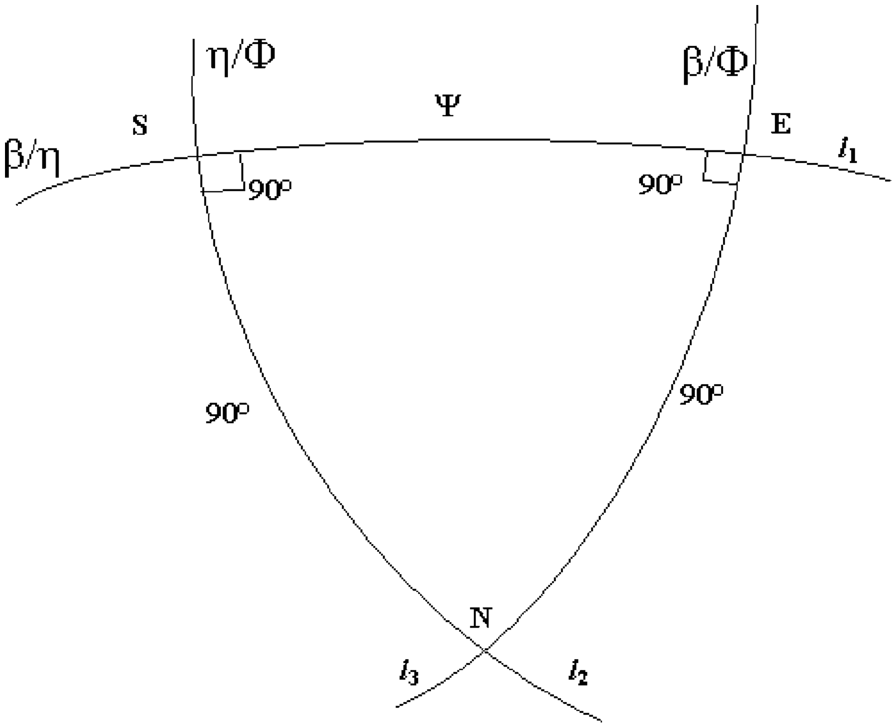

Figure 3. Null point and critical lines.

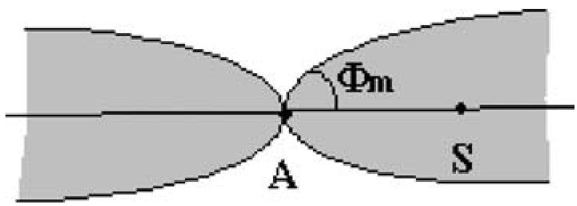

Figure 4. High correlation regions for β/η measurements.

co-ordinates of points on the edge around S in Figure 2. Similarly the edge around E can be found from formula 2 in Appendix 1.

1.2.2. *Correlation condition*. Figure 3 represents points and lines on the celestial sphere under consideration.

N is the "*Null point*" given by $N = S \times E$. In N the density of *LOPs* of an equal rotation angle Φ is minimum. In order to avoid a critical value for the correlation angle θ, A must be far from l_1 or from l_2 or from l_3 according to the pair of measurements chosen: β/η, η/Φ or β/Φ as shown in Figure 3. As only the positions of S and E relative to A can be monitored, let us translate these conditions as follows:

(a) β/η Measurements: A far from $l_1 \Rightarrow E$ far from the line joining A and $S \Rightarrow E$ outside areas like those shaded in Figure 4, whose axis and edges are great circles on the celestial sphere. The angle Φ equals the correlation angle between circular *LOPs* involved in these measurements; its minimum acceptable value Φ_m (Figure 4) may be posed as equal to thirty degrees.

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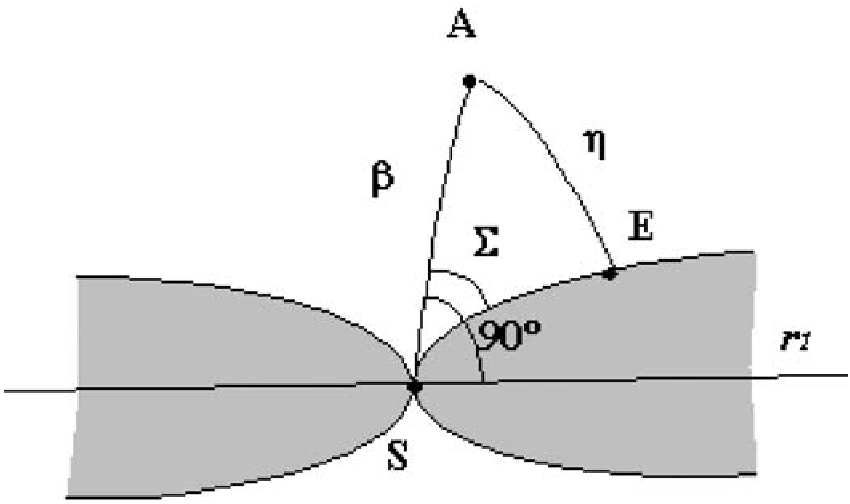

Figure 5. High correlation regions for η/Φ measurements.

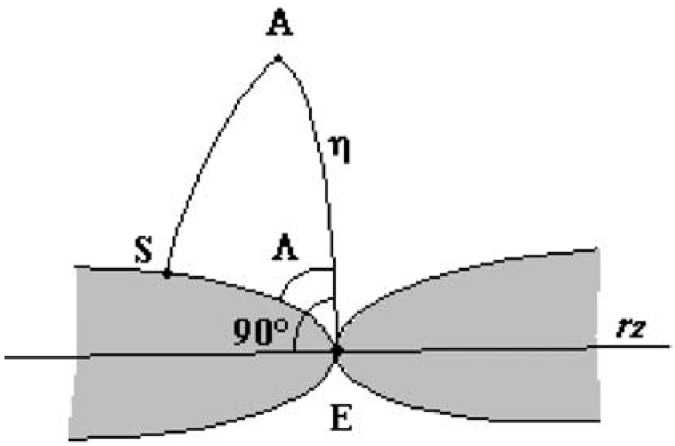

Figure 6. High correlation regions for β/Φ measurements.

(b) η/Φ Measurements: A far from $l_2 \Rightarrow E$ far from the line r_1 perpendicular in S to $\overset{\frown}{AS}$; in fact, if A is on l_2, E will be on $r_1 \Rightarrow E$ outside areas like those shaded in Figure 5. The angle Σ in Figure 5 can be computed using formula 5 in Appendix 1 by posing the minimum acceptable value, say 30° for θ: the related CEP value doubles. (From formula 6 in Appendix 1.)

(c) β/Φ Measurements: A far from $l_3 \Rightarrow S$ far from the line r_2 perpendicular in E to $\overset{\frown}{AE} \Rightarrow S$ outside areas like those shaded in Figure 6. Angle Λ in Figure 6 can be computed using formula 4 in Appendix 1, again giving θ the minimum acceptable value but, for this case, further considerations are possible. The value for η that appears in formula 4 is variable during satellite motion around the Earth, so the strip width in Figure 6 is valid for a single instant in which the attitude control should be made (i.e. in a satellite position where a key operation is needed).

A representation of the critical areas around point E, as those shaded in Figure 6, is not practical because E is moving on its orbit while S and A are fixed. It is better to trace uncertainty areas around A and/or S points and observe which part of E's orbit is inside that, if any. We can refer to an uncertainty sector limited by lines of equal Λ, that pass through S and A. To get the limiting Λ values, refer to formula 4 in Appendix 1 with $\theta = \pm 30°$. The limiting lines can then be traced by points, calculating their polar co-ordinates relative to S, i.e. (Σ, Ψ) with the said limiting values of Λ and η as a variable parameter.

We will show how to use a common star finder in the case of β/η measurements, while for this and the other two cases, where calculations are involved, we have developed and will illustrate a computerised star finder.

1.3. *Launch windows by a common star finder – β/η measurements.* As is well known, a star finder is a representation of the celestial sphere made on a set of disks: a "base" (BD) and a set of transparent disks (TD).

The stars and their scales of right ascension (RA) and polar distances are sketched on the BD. On the TD (one for each latitude), the pattern of the zenith Z, the verticals, the parallels of altitude and the horizon with the altitude and azimuth scales are sketched in the same cartographic representation as the BD; so the said pattern will come out deformed dependant on the latitude. The TD can be superimposed on the BD and rotated around the centre (pole P) till the meridian PZ crosses the point on the RA scale indicating the local sidereal time. So the position of Z between the stars can be visualised for each time. To obtain launch windows we proceed as follows:

a) Mark on the BD the points S, π (spacecraft orbit's pole), A by their RA and polar distances. The position of A is that chosen for the mission. The RA of π is 90° from the RA of the ascending node of the orbit, which is equal to the local sidereal time of the spacecraft's transit through the node. Its polar distance is related to the orbit's inclination. As for the position of S, it obviously relates to the date, and therefore an ephemeris must be used.
b) Mark on the BD the Earth's orbit relative to the spacecraft or at least the significant positions of E, i.e. the end points E_1 and E_2 of the orbital segment in which attitude has to be verified. For this purpose, choose the TD whose Z point can be best superimposed on π marked on the BD, i.e. the co-latitude of Z should be equal to the polar distance of π; the horizon on this TD will then be the Earth's orbit.
c) Mark the sectors of uncertainty shaded in Figure 4. For this purpose, a second TD will be used whose Z point can best be superimposed on A. The boundaries of these sectors are the verticals at each side of the vertical passing through S at a distance of Φ_m from this one.
d) Look at the position of the arc $\overset{\frown}{E_1E_2}$ and the sectors of uncertainty. Suitable launch windows correspond to $\overset{\frown}{E_1E_2}$ outside these sectors and can be assessed by rotating π around the pole because a given π rotation means an equal rotation of the arc $\overset{\frown}{E_1E_2}$ and an equal span of launch time. Obviously, the launch windows relate to the position of S and therefore to the date.

1.4. *Launch windows by a computerized star finder.* Software has been set up for this purpose. Firstly, the options of method and parameters may be inserted using a

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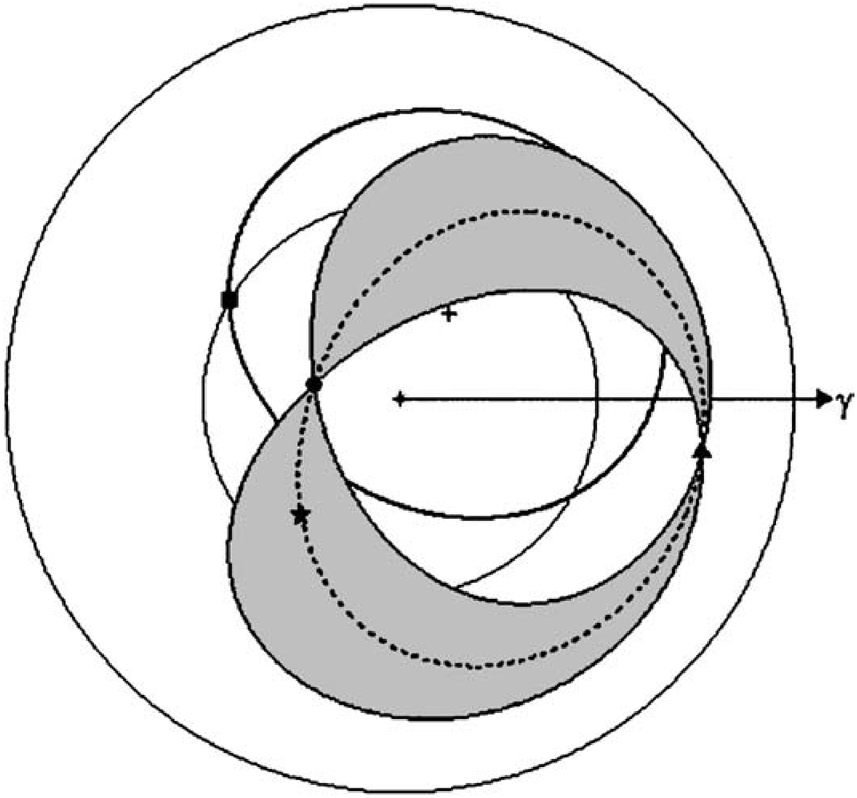

Figure 7. An example of high correlation regions for β/η method.

suitable graphic interface. The poor precision zones will appear on the screen (properly coloured) together with the Earth's orbit. Some examples are in Figures 7, 8 and 9.

The ascending node of the Earth's orbit is indicated by a small square: displacing this around the equator i.e. the central circumference, means considering launch times delayed by equal amounts. The whole orbit consequently will be displaced and it will be possible to evaluate at any time which part of it is engaged in the low precision areas. Launch windows can thus be defined in this way.

2. SATELLITE FLY-OVER. The launch time of a satellite on a given orbit is required so that it will fly over a specified terrestrial point. On the "*base disk*" of the aforesaid star finder, now representing the terrestrial sphere, the terrestrial point being sought should be traced using latitude and longitude scales. We shall refer directly to a computer program and this base disk consequently appears on the screen. By inserting orbital parameters, the terrestrial trace of the satellite's motion also appears, together with the vernal point, in generic position i.e. for a generic time. The trace can be computed by points. Gruppo COSAT, 1989, shows a method to obtain this using a common star finder. The vernal point and the trace may be rotated together till the latter crosses the specified terrestrial point. This rotation

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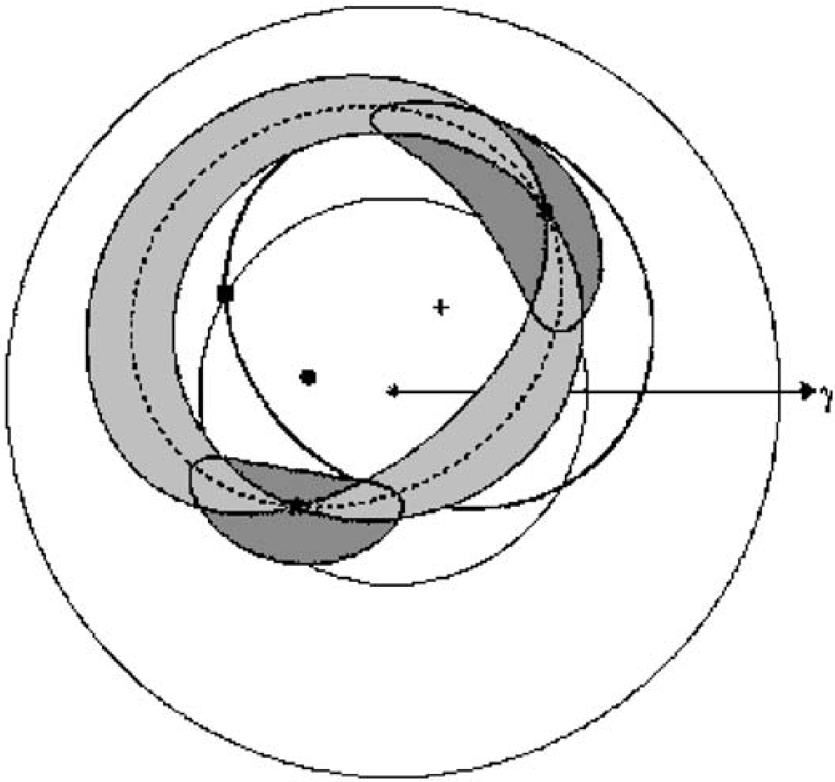

Figure 8. An example of poor density and high correlation regions for η/Φ method.

means equal time-span, and the final time, for the date, will appear on the screen. This is the time of the fly-over and from it the launch time can be easily calculated. Inversely, if the fly-over must happen at a specified time, the longitude of the ascending node must be varied by displacing the vernal point with respect to the trace once the latter crosses the specified terrestrial point: the position of the former must correspond to this time. The said longitude can now be read on the screen as the equatorial distance between the vernal point and the trace's ascending node.

3. RISE AND SETTING OF A SATELLITE. Again, as in the preceding paragraph, the star finder "*base disk*" appears on the computer screen with the observer's point and latitude-longitude scales, instead of stars. Again, by inserting orbital parameters, the terrestrial trace of the satellite motion also appears along with the vernal point in the right position relative to the former.

a) Let us rotate the vernal point, and the trace with it, to the position corresponding to the time of the satellite node crossing.
b) By pointing the mouse on a terrestrial point, the line of apparent rising or setting of the satellite for that point appears, as a particular curve in the projection concerned.

Note, the said curve is not the horizon but a parallel of altitude whose altitude equals satellite horizontal parallax minus the depression of the horizon (and astronomical

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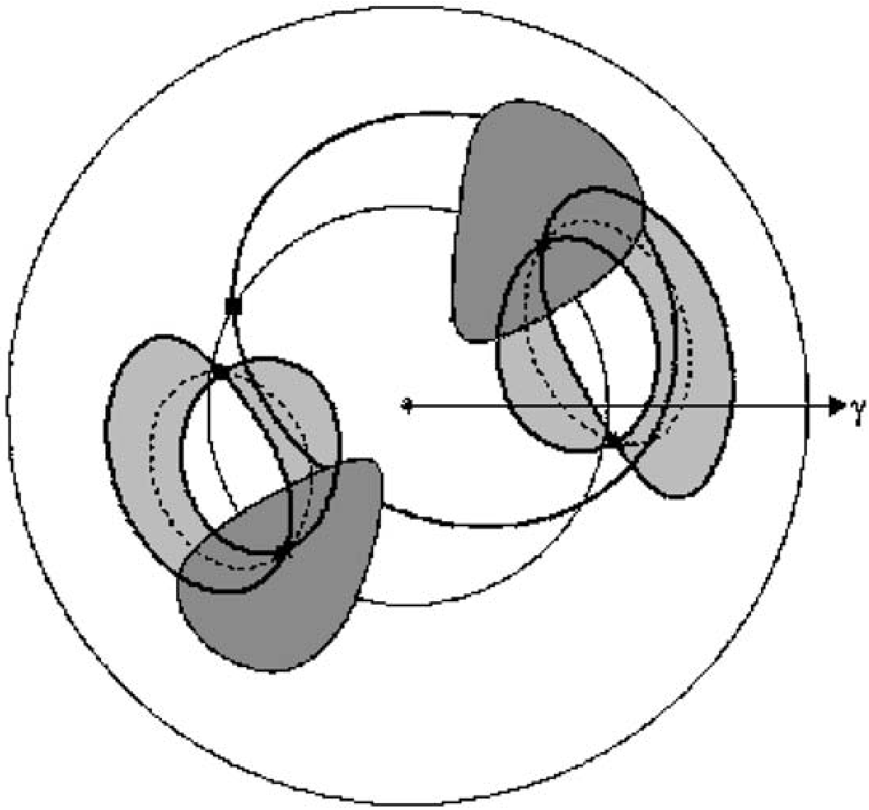

Figure 9. An example of poor density and high correlation regions for β/Φ method.

refraction) calculated on the basis of the satellite distance and the observer's eye level. On this apparent horizon there is an azimuth scale, so the rising and setting azimuths can be viewed. Furthermore rising and setting times will be computed from the latitudes of rising and setting points, by calculating the corresponding orbital arguments of latitude trigonometrically.

4. EARTH OR MOON SURVEY BY SATELLITES. Again, the base disk is the equator projection of the Earth or Moon's surfaces (note that in the considered projection, both the hemispheres are represented). For a given date and hour the umbra terminator and the sub-satellite point are traced. The former can rotate by pressing the arrow keys: each rotation angle is a given time span. For the new times the sub-satellite point is computed again. So the umbra and daylight transits can be determined for one or more satellites. A satellite sub point can also rotate on its parallel by pressing a key: each rotation is an equal variation of the node time transit. This may be useful for orbit design purposes. A separate marginal sketch shows the satellite position on its orbit at each position of the sub point on the "*base disk*". In this way we can, for instance, evaluate if our satellite is flying over an umbra or sunlit land at perimoon.

5. ECLIPSE OF A SATELLITE. The Earth centred celestial sphere is now represented by the "*base disk*" (on the screen), that is the equator plane. By orbital parameters, the satellite orbit on the celestial sphere is calculated and mapped with its ecliptic ascending and descending nodes. For each node, the midpoint of the time span in which the eclipses occur is that of the passage of the Sun through the other node and the initial and final dates can be easily calculated from the radius of the Earth's cone of umbra at the satellite distance. For orbit design purposes, if eclipse is not desired for particular dates, the ecliptic segment of eclipse for such dates can be marked and the orbit rotated till the nodes are out of that segment: the corresponding longitude of the equatorial node can be read on the screen. Of course also other orbital parameters can be varied and the orbital situation tested on the screen.

Appendix 1.

Let's consider the triangle of the Chen–Wertz method (Figure 1). The spherical lines of position (*LOP*) of A for arc measurements β and η are circumferences, centred in S in the case of β and in E in the case of η. The *LOPs* of "*rotation angle*" measurements ϕ are lines of equal "*difference of azimuth*" passing through S and E.

The density δ of the *LOPs* may be seen as the inverse of the well-known *dilution of precision*. For circular *LOPs* δ is constant and equals 1.

The density of the ϕ *LOPs* is:

$$\delta = \left|\frac{\sin\phi}{\sin\beta}\right| \sqrt{\cos^2\beta + \cot\Sigma} \tag{1}$$

in terms of β, Σ and:

$$\delta = \left|\frac{\sin\phi}{\sin\eta}\right| \sqrt{\cos^2\eta + \cot\Lambda} \tag{2}$$

in terms of η, Λ.

The uncertainty of the fix A depends also on the "*correlation angle*" θ, according to which the two *LOPs*, fixing A, intercept each other.

For measurements β/η is:

$$\theta = \phi. \tag{3}$$

For measurements β/ϕ is:

$$\theta = \tan^{-1}\left[\frac{\cot\Lambda}{\cos\eta}\right] = \tan^{-1}\left[\frac{\tan\eta}{\tan\beta\sin\phi} - \cot\phi\right]. \tag{4}$$

For measurements η/ϕ is:

$$\theta = \tan^{-1}\left[\frac{\cot\Sigma}{\cos\beta}\right] = \tan^{-1}\left[\frac{\tan\beta}{\tan\eta\sin\phi} - \cot\phi\right]. \tag{5}$$

The "*Circular Error Probability*" *CEP* of the fix A is:

$$CEP = \frac{1}{\sqrt{2}\sin\theta}\left[\left(\frac{u_1}{\delta_1}\right)^2 + \left(\frac{u_2}{\delta_2}\right)^2 + 2\left(\frac{u_1}{\delta_1}\right)\left(\frac{u_2}{\delta_2}\right)|\cos\theta|\right]^{\frac{1}{2}}, \tag{6}$$

where u_1 and u_2 are the uncertainties of the two measurements.

If u_1 and u_2 are three times the standard deviation of the measurements, the probability of A being in a circumference centred in the fix of the radius equal to the *CEP* is 0·989.

Appendix 2. Operational remarks about launch windows.
β/η Measurements. Uncertainty may be due only to the correlation angle. It may be influenced by the position of the spin axis relative to the satellite orbit: from Figure 4 it may be seen that there are favourable conditions when the Earth E crosses the great circle $\overset{\frown}{AS}$ normally, in particular when the crossing is in A. In this case it is free from uncertainty areas; it happens when the spin axis is in the satellite's orbital plane and by an opportune choice of this plane: the angle Φ must be 90°.

β/Φ Measurements. Both the density and the correlation uncertainty areas become null as they degenerate into the two points S and its antipode S^{-1}, when β approaches 0° or 180°. Their maximum extension is when $\beta=90°$. So we have favourable conditions by pointing the spin axis towards or near the Sun or its antipode. Moreover, Figure 6 shows that we can say that if β on that date is unfavourable, a choice of orbital plane relative to A should be made as in the case β/η; or else the date and/or attitude should be changed.

η/Φ Measurements. The density and correlation conditions are now incompatible. For $\beta=0°$ or 180° there are favourable density conditions; poor density areas degenerate into the two points S and S^{-1} while poor correlation areas appear to be the largest ones. Vice versa for β approaching 90°. Anyway for β near 0° or 180°, if the spin axis is in the satellite orbital plane and if this crosses the Sun, the uncertainty correlation regions are not crossed by E (Figure 5) and the uncertainty density regions are small.

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