Quantum Decoherence and the Approach to Equilibrium*

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We discuss a recent proposal by Albert (1994a; 1994b; 2000, ch. 7) to recover thermodynamics on a purely dynamical basis, using the quantum theory of the collapse of the wave function by Ghirardi, Rimini, and Weber (1986). We propose an alternative way to explain thermodynamics within no-collapse interpretations of quantum mechanics. Our approach relies on the standard quantum mechanical models of environmental decoherence of open systems (e.g., Joos and Zeh 1985; Zurek and Paz 1994). This paper presents the two approaches and discusses their advantages. The problems faced by both approaches will be discussed in a sequel (Hemmo and Shenker 2003).

1. Introduction. Our experience tells us that macroscopic thermodynamic systems invariably evolve towards high entropy states in an irreversible way. In some programs in the foundations of statistical mechanics (both classical and quantum) a central problem is to explain this experience by appealing to the underlying dynamics *alone.* The aim here is twofold. First, to explain the macroscopic thermodynamic phenomena[1] on the basis of the dynamical equations of motion that operate at the microscopic level, possibly using some probabilistic hypotheses. Then, to justify those probabilistic hypotheses by the same underlying dynamics. Hitherto neither of

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1. By expressions such as 'thermodynamic phenomena', 'thermodynamic (law-like) regularities', and similar ones we mean the results of measurements as predicted by the zeroth and second laws of thermodynamics, stating the spontaneous and irreversible approach to equilibrium and entropy increase.

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the two aims has been satisfactorily accomplished (see overviews of this problem by Sklar (1993) and Guttmann (1999)). Albert (1994a; 1994b; 2000, ch. 7) has proposed to solve these problems by appealing to the theory of the collapse of the quantum state by Ghirardi, Rimini, and Weber (GRW) (1986).[2]

In this paper we propose an alternative way of explaining the laws of thermodynamics, in particular the approach to equilibrium and the increase of entropy, using the quantum mechanical dynamics in *no-collapse* theories. Our proposal is supported by results in decoherence theory which strongly suggest that interactions with the environment are crucial for the emergence of quasi-classical and thermodynamic behavior. We use the standard models of so-called environmental decoherence of open systems (Zurek 1982, 1993; Caldeira and Leggett 1983; Joos and Zeh 1985; Giulini et al. 1996 and references therein) and recent results about the evolution of the von Neumann entropy of open (decohering) systems by Zurek, Habib and Paz (1993), Zurek and Paz (1994, 1995); see also (Paz and Zurek 1999, ch. 6, 55–65; Monteoliva and Paz 2000). This paper, however, while focusing on quantum mechanics without collapse, does not defend any specific no-collapse interpretation of quantum mechanics (e.g., pilot-wave, many-worlds or modal theories).[3] The role of decoherence in the recovery of classical mechanics and of thermodynamics in particular has been investigated by many authors (e.g., Zeh 1992, ch. 4, sec. 4.2.2; Wallace 2001 and references therein). The present paper follows a more condensed argument given by Hemmo and Shenker (2001).

The paper is structured as follows. In Section 2 we present the problem of justifying the thermodynamic regularities in classical statistical mechanics. In Section 3 we turn to the quantum mechanical context and we discuss Albert's approach to the problem using the GRW theory of the collapse of the quantum state. In Section 4 we give a brief description of the standard model of environmental decoherence, and we describe recent results by Zurek, Habib, and Paz concerning the connection between decoherence of open systems and the evolution of the von Neumann entropy. In Section 5 we present our approach to the problem in which we make use of both environmental decoherence and the induced dynamics of open systems in no-collapse interpretations of quantum mechanics. In Section 6 we consider the role of probabilities and stochasticity in Albert's GRW

2. In this paper we only consider the extent to which the GRW theory can be successful in the recovery of thermodynamics. This theory faces some serious problems, some of which are mentioned in Section 3 below.

3. See DeWitt and Graham 1973 on many worlds; Healey and Hellman 1998 and Dieks and Vermaas 1998 on modal theories; Cushing, Goldstein, and Fine 1996 and Bub 1997 on the pilot-wave theory.

approach and in our no-collapse approach. In a sequel to this paper (Hemmo and Shenker 2003) we address further problems faced by the two approaches.

2. The Problem. Empirical evidence suggests that irreversibility and the approach to equilibrium are universal (for systems that are isolated and contained in a finite volume).[4] Systems evolve to equilibrium invariably and irrespective of their initial conditions. These ideas form the heart of thermodynamics. Can the universal approach to equilibrium be explained on the basis of the underlying dynamics?

To illustrate the problem consider the following example. A gas cloud is confined to the left-hand side of a container by a partition, which is removed at time t_0. The principles of thermodynamics dictate that the gas will evolve to equilibrium, that is, expand and fill out the container, and remain in the expanded state indefinitely. The universal phenomenon of the approach to equilibrium is classically understood as the macroscopic appearance of occurrences at the microscopic level. This general idea is applicable only in the right circumstances, and these ought to be taken into account.

First, in systems characterized by a small number of degrees of freedom, fluctuations (which disagree with the predictions of thermodynamics) are dominant. The law-likeness of thermodynamics, in particular the approach to equilibrium, emerges only when we move to macroscopic systems with many degrees of freedom.

Second, where quantum mechanical phenomena like superpositions dominate, thermodynamic magnitudes and their evolution are not always well-defined. In fact, we take it that without solving the measurement problem quantum mechanics has no empirical content at all, thermodynamic or otherwise. For this reason, the explanation of thermodynamic behavior within a quantum mechanical setting crucially depends on the way the measurement problem is solved in quantum mechanics. Hence, if one wishes to explain the thermodynamic regularities on the basis of quantum mechanics, one has to consider an interpretation of quantum mechanics in which the measurement problem is solved. These are the circumstances on which both approaches discussed in this paper focus.

One problem in explaining macroscopic occurrences on the basis of the classical microscopic dynamics is that the latter allows for micro-evolutions

4. In this paper we use terms like 'the second law', 'law of entropy increase', 'principle of approach to equilibrium', etc. interchangeably. The exact meaning of the second law of thermodynamics is, however, not clear. See the detailed analysis by Uffink (2001) on this particular topic. It is even an open question whether the second law entails or assumes a time-asymmetric spontaneous evolution to equilibrium (see Brown and Uffink 2001).

that (would) appear at the macroscopic level as anti-thermodynamic (had they occurred), such as the gas remaining in the left hand side of the container, say ten minutes from now. We call micro-evolutions, and the microstates along them, *thermodynamic normal* if the regularities they exhibit correspond to the laws of thermodynamics (in particular the second law) for a suitable time interval *T*. (A time interval is suitable if it is long enough in thermodynamic time scales but short enough so that Poincarè recurrence is unlikely to occur). Micro-evolutions (and the microstates along them) which don't satisfy this condition will be called thermodynamic *abnormal*.[5] The existence of thermodynamic abnormal states, as predicted by the underlying classical dynamics, contradicts the letter of the second law of thermodynamics. This situation has been known for a long time: J. C. Maxwell had proposed his famous Demon to illustrate it (Earman and Norton 1998).

In classical statistical mechanics there are two main grand schools to solving this problem: following Boltzmann and following Gibbs. In the Boltzmannian school the properties of a system (including its entropy) are taken to be properties of the system's microstate through its relation to the system's macrostate. Among the initial microstates there are abnormal ones, namely states that lead to anti thermodynamic evolutions. The problem now is to explain why, despite the possibility of such anti-thermodynamic evolutions, the actual world obeys the laws of thermodynamics (see overviews of this particular problem in the classical context by Sklar (1993), Guttmann (1999), and Albert (2000)). One solution is to argue that, as a matter of fact, the actual initial state and evolution are normal. This is a matter of fact, not of law, and therefore it does not need to be explained beyond its mere stipulation (e.g., Sklar 1993, 210). In order to derive the laws of thermodynamics we must add this fact to the underlying dynamics (call this the *matter-of-fact* approach).

A second approach (within the Boltzmannian school) seeks to give some explanation for this fact by stipulating a probability distribution over the possible initial states of the universe. This distribution entails that most initial states evolve (with certainty) in accordance with the thermodynamic laws, and the minority of initial states evolve (again, with certainty) anti-thermodynamically. This approach goes on to assume that the actual initial state of the universe belongs to the majority. It is *typical*. (Call this the *typicality* approach.) Like the matter-of-fact approach, the typicality approach also needs to postulate something beyond the underlying dynamics in order to recover or explain thermodynamics. The matter-of-fact approach postulates the actual initial state, and the typicality approach pos-

5. These definitions are in agreement with Albert (1994a, 1994b, 2000).

tulates the initial probability distribution and the typicality of the actual initial state.[6]

Albert (2000, ch. 4), recently proposed a third solution to this problem, again in a Boltzmannian framework. Albert hypothesizes that *all* the physically possible initial states of the universe have been thermodynamic normal. This leads to a rejection of the standard understanding of a uniform distribution over the microstates corresponding to the present macrostate. In all three approaches the additional non-dynamical postulates are extremely hard to justify.

The Gibbsian school, by contrast, maintains that the properties of individual systems correspond to ensemble averages, and so the effect of the relatively small number of thermodynamic abnormal trajectories is negligible. The average behaves like the majority, namely, in accordance with the thermodynamic laws. Since the average is said to correspond to measurable quantities pertaining to individual systems, this approach needs to prove the uniqueness of the probability measure which is the basis for calculating the averages. A satisfactory proof based on the underlying dynamics has not yet been found (Sklar 1993; Guttmann 1999). It is known that one can obtain this result whenever the system is ergodic, but ergodicity may be neither sufficient nor necessary to explain thermodynamics. Not sufficient, since it does not provide predictions for finite time intervals. Not necessary, since some interesting and relevant systems may not be ergodic, by KAM's theorem (Walker and Ford 1969; for problems in the ergodic approach see Earman and Redei 1996). And so also the Gibbsian school relies on non-dynamical postulates.[7]

Until recently, no approach to the foundations of statistical mechanics has been able to overcome these difficulties and to rely only on the dynamics in recovering thermodynamics. Albert's (1994a; 1994b; 2000, ch. 7) recent proposal (on which we shall focus here) attempts to provide a way to do precisely this, by taking seriously the fact that the underlying dynamics is quantum mechanical, rather than classical. Albert's proposal belongs to the Boltzmannian school in that it takes entropy to be a property of the microstates of individual systems, rather than a property of

6. A typicality approach is advocated by the Bohmian school in the interpretation of quantum mechanics in the context of justifying the quantum mechanical probability distribution $||\Psi(x, t)\rangle|^2$ over the positions in Bohm's theory, e.g., Goldstein's proposal (in Bricmont et al. 2001).

7. In this school there are additional problems. One is explaining why phase averages yield predictions regarding individual systems. Another is that, since entropy is a property of the probability distribution, it does not change at all if that distribution does not change in time (which is the kind of distribution Gibbs was looking for). To solve this problem, Gibbs devised the idea of coarse graining, which is problematic (Ridderbos 2002).

ensembles or probability distributions over microstates as in the Gibbsian approach. Therefore he focuses on the dynamical evolution of individual systems, rather than on the evolution of ensembles or probability distributions. It seems to us possible to apply Albert's ideas in a Gibbsian framework as well; we do not undertake this here.

3. GRW Jumps and Thermodynamics. Albert (2000, ch. 7) proposes to explain the thermodynamic regularities by relying solely on the stochastic dynamics of the quantum state as prescribed by the quantum theory of the collapse of the wave function proposed by Ghirardi, Rimini and Weber (1986). On his approach it is an intrinsic feature of the GRW dynamics of the quantum state that *every single one* of the possible initial microstates of a thermodynamic system has a high probability to evolve to states which are compatible with the predictions of thermodynamics, and therefore there is no need to add any of the non-dynamic postulates used in the classical case, as described above. As we shall see Albert's approach relies heavily on the fact that the GRW dynamics is genuinely stochastic.

We now turn to a detailed discussion of Albert's approach. We start in Section 3.1 by briefly presenting the GRW theory and how it solves the measurement problem in quantum mechanics. Then, in Section 3.2 we describe Albert's approach as to how to recover the thermodynamic regularities using the GRW collapses. Finally, in Section 3.3 we consider in more detail some features of Albert's approach.

3.1. The GRW Theory. Albert's approach makes an explicit linkage between the GRW solution to the measurement problem in the quantum theory of measurement and the implications of the GRW theory concerning the time evolution of thermodynamic systems. The measurement problem arises in quantum mechanics as a straightforward consequence of applying the Schrödinger linear and deterministc dynamics to the measurement interaction. This dynamics results for a generic measurement interaction in a superposition of the form

$$|\Psi\rangle = \sum_i \mu_i |\psi_i\rangle \otimes |\varphi_i\rangle, \tag{1}$$

where the kets $|\psi_i\rangle$ represent some suitably defined pointer states of the measuring apparatus (typically, the $|\psi_i\rangle$ are eigenstates of the pointer position), and the $|\varphi_i\rangle$ are some states of the system. The problem is that in states of the form (1) the measurement has no definite outcome (except in the special case where all but one of the μ_i are zero), since the final (reduced) state of the apparatus cannot in general be described in terms of an ensemble of systems in a classical mixture (in which the $|\mu_i|^2$ represent the probabilities for each $|\psi_i\rangle$ to actually be the case). The GRW theory (formulated for non-relativistic quantum mechanics) solves this problem by modifying the Schrödinger linear dynamics. In particular, the Schrödinger equation of motion is changed by adding to it a non-linear and stochastic factor (so-called a *jump* factor). On occasion, this jump generates the collapse of the wave function in a way that depends on the mass density of the system (roughly, the frequencies of the collapses are proportional to the number of particles or to the mass density of the system, depending on the model). For our purposes it is enough to present Bell's (1987) version of the elementary and non-relativistic theory. This goes roughly as follows.[8]

Consider the quantum mechanical wave function of a composite N particles system:

$$\psi(t, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \langle \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N | \psi(t) \rangle. \tag{2}$$

The time evolution of the wave function usually (at almost all times) satisfies the deterministic Schrödinger equation. But sometimes *at random* the wave function collapses (these collapses are known as the GRW *jumps*) onto a wave function ψ_ℓ localized in position which has the (normalized) form

$$\psi_\ell = \frac{j(\mathbf{x} - \mathbf{r}_n)\, \psi(t, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)}{R_n(\mathbf{x})}, \tag{3}$$

where $\mathbf{r}_n$ in the jump factor $j(\mathbf{x} - \mathbf{r}_n)$ is randomly chosen from the arguments $\mathbf{r}_1, \ldots, \mathbf{r}_n$ of the wave function immediately before the jump. R_n in (3) is a renormalization factor:

$$|R_n(\mathbf{x})|^2 = \int \mathrm{d}^3\mathbf{r}_1 \ldots \mathrm{d}^3\mathbf{r}_N \, |j\psi|^2, \tag{4}$$

and the jump factor j is also normalized:

$$\int \mathrm{d}^3\mathbf{x} \, |j(\mathbf{x})|^2 = 1. \tag{5}$$

For j GRW suggest the Gaussian:

$$j(\mathbf{x}) = K \exp(-\mathbf{x}^2 / 2\Delta^2), \tag{6}$$

8. For a more detailed discussion of the GRW theory, see GRW 1986; Bell 1987; Ghirardi 2000, and references therein.

where the width Δ of the Gaussian is a new constant of nature: $\Delta \approx 10^{-5}$ cm. Its size is chosen so that the spontaneous collapses will not result in an observable violation of energy conservation.

Probabilities enter the theory twice. First, the probability that the collapsed wave function ψ_ℓ is centered around the point $\mathbf{x}$ is given by

$$d^3\mathbf{x}\left|R_n(\mathbf{x})\right|^2. \tag{7}$$

This probability distribution, as can be seen, is proportional to the standard quantum mechanical probability given by the Born rule for a position measurement on a system with a wave function $R_n(x)$ just prior to the jump. Second, the probability in for a GRW jump to take place in a unit of time is

$$\frac{N}{\tau}, \tag{8}$$

where N is the number of arguments in the wave function (in Bell's model it may be interpreted as the number of particles), and τ is another new constant of nature ($\tau \approx 10^{15}$ sec.). Note that the expression (8) does not depend on the quantum wave function, but only on N. This is essentially the whole theory.

As it stands, it seems that this theory cannot be generalized to relativistic and field theories, since the GRW jumps are applied to particles' positions and not field variables, and the collapse rates are determined by particle numbers.[9] As part of an attempt to solve this problem more general models are considered in which the collapse rates are defined such that they are increased exponentially in correspondence with the mass density of the system (Ghirardi 2000).

In both models it is a straightforward consequence of the standard quantum mechanical treatment of composite systems (in particular, of non-factorizable quantum states) that a single GRW jump of any one of the subsystems in the composite is enough to bring about a collapse of the global wave function. For microscopic systems, collapses have extremely low probability to occur, so that the quantum mechanical Schrödinger equation turns out to be almost literally true at all times just as no-collapse quantum mechanics predicts (and experiment confirms). However, for massive macroscopic systems (or for systems with 10^{23} particles) collapses are highly probable at all times. In measurement situations the GRW theory implies that superpositions of macroscopic pointer states of the

9. See Pearle, Ghirardi, and Grassi 1990; Ghirardi 2000, on the problem of generalizing the GRW theory to the relativistic domain.

form (1) collapse with extremely high probability onto the localized states $|\psi_i\rangle$ on time scales that are much faster than measurement times. In particular, the probability that the wave function of the composite of system plus apparatus will stay in the superposition (1) for more than a fraction of a second (i. e., by the time the measurement is complete) vanishes exponentially. Moreover, whenever the wave function in (1) has a spatial spread which is larger than Δ, any GRW jump will result in a localization of the wave function. That is, the jump will reduce the wave function onto one of the terms $|\psi_i\rangle \otimes |\varphi_i\rangle$, in which the pointer is in the localized state $|\psi_i\rangle$, where the probability for the *i*-th term (see equation (7)) is given as usual by the squared amplitude $|\mu_i\rangle^2$. This means that in a sequence of quantum mechanical measurements the GRW jumps result in definite outcomes with frequencies that are (approximately) equal to the Born-rule probabilities $|\mu_i\rangle^2$. The measurement problem is solved as long as measurements involve a macroscopic recording of the result in position (e. g., a moving pointer of a measuring device, particles hitting on macroscopically separated regions of a computer screen, etc.).

The following properties of the GRW theory will be important later. First, the dynamics is fundamentally chancy. The time of the collapse and the center of the Gaussian into which the wave function collapses are determined in a purely chancy way. Second, the time evolution resulting from the GRW dynamics is non-invariant under time reversal. Past states cannot be retrodicted by the GRW theory, not even approximately or in a probabilistic way. Third the GRW jumps, by construction, occur only in position, and in this sense the quantum mechanical 'position basis' is given a physically preferred status.

3.2. Albert's Approach to Thermodynamics. The key idea in Albert's approach is this. In the GRW theory the jumps mean that the system actually undergoes stochastic transitions from one state to another. In the context of the recovery of thermodynamics it is useful to think about the GRW jumps as if they induce *stochastic perturbations* of the Schrödinger trajectory of the system. This means that the GRW trajectory can be seen as a patchwork of segments of different Schrödinger trajectories each of which corresponds to a different initial state of the system. The system jumps from one Schrödinger trajectory to another, such that the net result is an effective stochastic trajectory. In other words, the system performs a random walk in the space of all possible Schrödinger trajectories where the probabilities are given by (7).

The connection to thermodynamics goes as follows (see Figure 1). We want to determine whether the time evolution of a given system is thermodynamic normal or abnormal. Consider the spreading out of a gas in

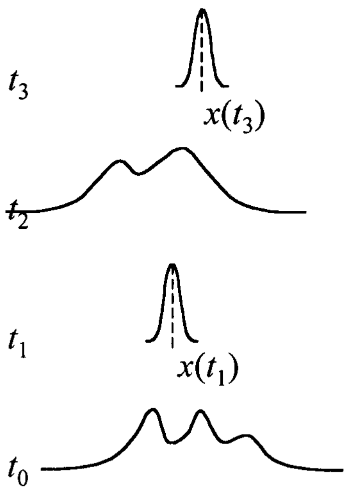

Figure 1. GRW dynamics

the box. When the partition is removed at t_0 the composite wave function of the gas is

$$|\Psi(0)\rangle = \sum_i \lambda_i(0)|\Phi_i\rangle, \tag{9}$$

where the $|\Phi_i\rangle$ are some wave functions in position representation and the $\lambda_i(0)$ are the corresponding quantum mechanical amplitudes. We assume that the wave function of the gas evolves in time in accordance with the GRW dynamics, and that the gas is macroscopic enough, so that there is high probability for a GRW jump to occur during any dynamical time interval Δt which is short in a thermodynamic scale. When a jump occurs the wave function of the gas collapses into a state that is localized around a certain position $x = x_1, x_2, \ldots, x_N$, that is, around some spatial distribution of the gas molecules. For example, at t_1 the wave function collapses into some state $|\psi_1\rangle$ which corresponds to a Gaussian centered around $x(t_1)$. The collapsed state then evolves in accordance with the Schrödinger equation. The high mass density of typical macroscopic systems implies an overwhelmingly high probability for a collapse by t_3 (where the time interval $(t_3 - t_1)$ is short in thermodynamic scales) onto a Gaussian centered around a position $x(t_3)$, where $x(t_3) \neq x(t_1)$.

Since the GRW jumps presumably solve the measurement problem, a sequence of such jumps results in a trajectory in the system's state space which can be described in terms of thermodynamic magnitudes. It is therefore possible, in this case, to determine whether or not the evolution of the system obeys the laws of thermodynamics (e. g., of entropy increase). Suppose now that we write down the GRW equation for a given thermodynamic system, and solve it for all possible initial states. Consider any time interval (t_1, t_3) for which, according to the GRW prescription, a collapse of the quantum state of the system occurs with high probability. For every possible initial state at t_1 there are in general many (possibly infinitely many) possible final states at t_3. For each such evolution, it is then possible to determine whether the evolution is thermodynamic normal or abnormal.

Albert (2000, 148–162, and especially, 155–156) now observes the following. First, the GRW jumps can be understood as inducing stochastic perturbations of the quantum state of the gas. We thus have an *internal perturbation mechanism,* as opposed to the external mechanism used in classical open system approaches. As we explained above, the wave function, say of our gas in (9), follows a genuinely stochastic trajectory in the system's state space. Second, and moreover, any GRW collapse induces a set of probability distributions, that is, transition probabilities—given the wave function just prior to the collapse—over the possible wave functions of the system immediately after the collapse. So in order for a GRW collapse to put (with high probability) our gas's wave function on a segment of a Schrödinger trajectory which is thermodynamic normal, what is needed is that the thermodynamic normal states (throughout the set of microstates to which the system can collapse) overwhelmingly outnumber[10] the thermodynamic abnormal ones. Moreover, we need this condition to hold in every microscopic region of the state space. If this turns out to be correct it would mean that after a GRW jump the wave function of the system will be (with high probability) thermodynamic normal, regardless of the history of the system and, in particular, of the state of the system immediately before the collapse. And so, each and every state has an overwhelmingly high probability to evolve to a thermodynamic normal state following a GRW jump. This implies that the property of being thermodynamic normal is stable over time, whereas that of being thermodynamic abnormal is highly unstable. In effect, what is needed is that the GRW probabilities for the collapse transitions reproduce the probabilities of the (ab)normal trajectories calculated from the standard statistical-mechanical measure for any given macrostate of the system. Albert puts forward the hypothesis that as a matter of fact the GRW dynamics provides precisely this. Call this Albert's *dynamical hypothesis.*

Note that Albert's hypothesis need not invoke postulates regarding ini-

10. Albert (1994a, 1994b, 2000, ch. 7) provides only qualitative plausibility arguments for his approach, and so the quantitative terms we use are vague.

tial states and probability distributions thereof. Rather, since the GRW dynamics is genuinely stochastic, whether or not this hypothesis is true depends on the set of transition probabilities generated by the GRW collapses. In this sense, Albert's approach aims at deriving the thermodynamic regularities from the underlying GRW dynamics only, without recourse to initial states or probability distributions thereof.

3.3. Some Advantages of Albert's Approach. Let's spell out in more detail the main advantages of Albert's approach.

One Solution for Two Problems. In Albert's approach, the GRW solution of the measurement problem is also the solution for the problem in the foundations of statistical mechanics, namely, a dynamical justification for the use of probability. Moreover, the systems with the same properties required for the GRW solution of the measurement problem (large or massive systems) are the systems in which statistical mechanics can best recover thermodynamics. For typical thermodynamic systems the GRW jumps are highly probable to occur at all times, either because such systems are typically massive enough, or because they interact with some other massive systems, such as the interactions of the gas's molecules with the box's walls. In such cases the GRW theory gives high probability for a collapse of the quantum wave function, and this means that the thermodynamic system has high probability to be at all times in a localized state. The measurement problem is solved and the thermodynamic magnitudes are well defined. When we add Albert's dynamical hypothesis (regarding the solutions of the GRW equations) we get the probability distributions that are needed for statistical mechanics to work. Recall that the dynamical hypothesis still lacks proof. But should it be proved, Albert's approach will present a unified way in which the GRW razor, so to speak, cuts twice: in the theory of quantum measurement and in the foundations of statistical mechanics. Moreover, it will have two clear advantages, both of which are related to the notion of probability as chance, as follows.

Single Origin of Chance in Physics. First, on the GRW dynamics the collapse of a superposition such as (1) onto one of the terms $|\psi_i\rangle \otimes |\varphi_i\rangle$ is a purely chancy event. The jumps invariably induce transitions from pure states to pure states, where no crucial role is played by ignorance probabilities in mixtures. For example, in (1) the quantum mechanical probabilities $|\mu_i|^2$ describe irreducible and genuine chances for a transition from the superposition (1) to the corresponding localized state $|\psi_i\rangle \otimes |\varphi_i\rangle$. This means that Albert's approach has a clear advantage of parsimony.[11] The epistemic probabilities in classical statistical mechanics are completely re-

11. Of course, there are other respects in which Ockham's razor cuts against the GRW theory, e.g., since the theory postulates *two new* constants of nature.

duced to the GRW quantum mechanical chances. Thus, the classical epistemic probabilities have no essential role in physics on this view.[12]

No Recourse to Probability Distributions over Initial Conditions. The second advantage is this. The GRW collapse dynamics and the probabilities for such collapses around any given spatial point at a given time depend on the wave function of the (total) system only at that time and don't depend on initial conditions (or the initial wave function). In the context of thermodynamics this means that Albert's approach need not rely on statistical postulates regarding initial (micro) conditions or probability distributions over them. Given the hypothesis about the preponderance of the thermodynamic normal evolutions, the GRW jumps have high probability to result at all times in thermodynamic normal trajectories for a typical (macroscopic) thermodynamic system irrespective of (micro) initial conditions.

4. Quantum Decoherence. We now consider an alternative approach to Albert's: namely, the recovery of the thermodynamic regularities on the basis of quantum mechanics without collapse. We shall rely on results in decoherence theory of open (quantum) systems. In this respect our approach belongs to the interventionist (or open systems) tradition in the foundations of classical statistical mechanics. Let us start by briefly describing the standard models of decoherence through the interaction with the environment in no-collapse quantum mechanics (Zurek 1982, 1993; Caldeira and Leggett 1983; Joos and Zeh 1985; Giulini et al. 1996). This is followed by a description of results by Zurek and Paz (1994) concerning the role of environmental decoherence in accounting for the increase of the von Neumann entropy of quantum chaotic systems.

In the standard quantum mechanical models of decoherence the total initial state of a macroscopic system plus environment is usually assumed to be a product state

$$\left|\psi(x_1 \ldots x_N, t)\right\rangle \otimes \left|E\right\rangle, \tag{10}$$

where $|\psi(x_1 \ldots x_N, t)\rangle$ is the quantum state of the system and $|E\rangle$ is some state of the environment. This means, in particular, that the states of the system and of the environment are separable (i.e., not quantum mechanically entangled; here to simplify the models we assume that they are also pure states, but this is not necessary). One of the key features in these models is that the interaction between the system and the environment is assumed to be governed by a Hamiltonian that commutes (approximately) with some observable of the system. That is,

12. This means that problems in the interpretation of probabilities in the classical approaches, such as the connection between probability and relative frequency distributions over infinite ensembles, and the connection between such ensembles and the evolution of single systems in finite times simply don't arise.

$$[H_{int}, \Pi] \approx 0. \tag{11}$$

where H_{int} is the interaction Hamiltonian, and the system observable Π (called the *pointer variable*) is usually taken to be position. In this sense, the standard models of decoherence usually assume that (approximate) position is a preferred basis in the Hilbert space of the system.[13] In general, the time evolved (Schrödinger) state can be written in the form

$$|\Psi(t)\rangle = \sum_i \mu_i(t)|\psi_i\rangle \otimes |E_i(t)\rangle, \tag{12}$$

where the kets $|\psi_i\rangle$ are assumed to be the eigenstates of Π, and the $|E_i(t)\rangle$ are the relative states of the environment. The set of states $\{|\psi_i\rangle\}$ is called the *pointer basis.* The result of the coupling is that the scalar products between the environment states $|E_i(t)\rangle$ in (12) relative to different pointer states $|\psi_i\rangle$ decay exponentially satisfying

$$\left\langle E_i(t+\Delta t)\middle|E_j(t+\Delta t)\right\rangle \approx \delta_{ij} \tag{13}$$

after extremely short times Δt (called *decoherence times*) which are typically around 10^{-23}, sec.[14] The decay of the scalar products in (13) is known as *environmental decoherence.*

Joos and Zeh (1985) derive a master equation for the reduced state of the system assuming recoil-free scattering (e.g., large mass ratio of the decohered system over the scattered particles) and isotropy in the distribution of the incoming particles (photons and molecules). Under these assumptions, the solutions of the equation exhibit exponential decay of the off-diagonal elements. The localization rate is proportional to $e^{-\Lambda(x-y)^2 t}$, depending in general on various factors, such as the strength of the coupling, temperature, and mass ratios. For our purposes the following results are crucial.

First, in the standard models of decoherence, in particular, models in which there is a pointer basis, (13) and (12) imply that the reduced state of the decohering system approaches the diagonal form:

$$\rho_s(t) \approx \sum_i |\psi_i\rangle|\mu_i(t)|^2\langle\psi_i|, \tag{14}$$

13. More generally, this basis is fixed by the dynamically conserved quantities.

14. The relaxation times of the system are typically extremely long, in some models on the order of 10^{40} sec. Also, the decoherence times of the system are much shorter than the dynamical times even for very weakly dissipative systems.

within times comparable to Δt. Second, in these models the diagonal form (14) of the reduced state is stable over time (i.e., the scalar products (13) remain vanishingly small). Third, Zurek, Habib, and Paz (1993) consider the decoherence interaction of a harmonic oscillator with an environment in thermal equilibrium. They show explicitly in the weak coupling limit that the pointer states $|\psi_i\rangle$ correspond to so-called *coherent states,* i.e., narrowly peaked Gaussians in both position and momentum. In their model, coherent states are the most stable states for the system in the sense that they produce the least von Neumann and linear entropy, so that $\rho_s(t)$ becomes maximally mixed when diagonalized by coherent states. In this sense one can say that in the standard models decohering systems follow quasi-classical trajectories. In order to explain this last sentence a few more details are needed.

4.1. Decoherence and the von Neumann Entropy. It turns out that decoherence plays an essential role in accounting for the emergence of classical behavior in quantum mechanics as well as in the recovery of thermodynamic behavior. In the case of chaotic systems Zurek and Paz (1994) have shown how to recover the classical dynamics from the underlying quantum dynamics. In their models they show why the classical evolution cannot be recovered for closed systems, whereas open decohering systems invariably exhibit an evolution which is approximately classical. As we shall see in Section 5 their results may be taken to support our proposal for recovering thermodynamics from quantum mechanics, since it implies that the von Neumann entropy of decohering systems increases in the course of time. Note, however, that it is questionable whether or not the von Neumann entropy is the exact quantum mechanical counterpart of the thermodynamic entropy (Shenker 1999; Henderson 2002).[15]

Before explaining this point in detail, let us describe qualitatively how Zurek and Paz derive their results. In the case of classical chaotic systems there are essentially two constraints on the dynamical evolution: (i) trajectories with initially close segments diverge exponentially, and (ii) the flow of the probability distribution is volume preserving, by Liouville's theorem. These two constraints together have the consequence that the accessible phase space region (containing the states which are the time evolutions of the initial states) assumes a structure which is highly striated on increasingly finer length scales, at exponential rates.

In the case of quantum systems the dynamics of quantum chaotic systems on phase space is usually taken to be described by the so-called Wig-

15. We think that von Neumann's original argument to the effect that the quantum mechanical entropy is equivalent to the thermodynamic entropy is wrong and does not establish its claim. We address this question in a forthcoming paper.

ner function which yields the probability distribution over position and momentum.[16] In general, the Wigner function cannot be straightforwardly interpreted as a probability distribution since it sometimes takes negative values. But for approximate measurements of position and momentum on scales of $\hbar^n$ it does yield (formally) a probability-like distribution (and so a proper interpretation of quantum mechanics may take advantage of this feature). In their detailed quantitative analysis Zurek and Paz (1994) arrive at two extreme results.

(1) *Closed Systems.* If the system is completely closed (e.g., there are no decoherence interactions), the chaotic dynamics results in exponential divergence of neighboring trajectories.[17] But this is compensated by an exponential contraction in the opposite directions, so that the total volume of regions in the phase space over which the Wigner function is nonzero remains constant throughout this evolution, in agreement with Liouville's theorem. However, on finer length scales the contraction generates interference fringes which the Wigner function cannot follow, since if it were to do so, then it could not be positive throughout $\hbar^n$-sized regions of phase space. This means that the dynamics deviates from the classical evolution. In particular, the system doesn't follow (not even approximately) classical trajectories (even in the context of a proper interpretation of quantum mechanics).

16. The equation of motion for the Wigner function is given by the Moyal bracket:

$$\{H,W\}_{mb} = -i\sin\left(i\hbar\{H,W\}_{pb}\right)/\hbar,$$

where $\{H, W\}_{pb}$ is the Poisson bracket describing the classical evolution of the Wigner function W, and H is the Hamiltonian. This yields the evolution equation

$$\dot{W} = \{H,W\}_{pb} + \sum_{n\geq 1}\frac{(-1)^n\hbar^{2n}}{2^{2n}(2n+1)!}\partial_x^{2n+1}V(x)\partial_p^{2n+1}W + \\ + 2\gamma\partial_p(pW) + d\partial_p^2 W,$$

where the first term gives the classical Liouville flow, the second (higher derivatives term) describes the quantum mechanical corrections for the evolution of a closed system, and the last two terms result from the interaction with the environment field. These last terms describe, respectively, the relaxation of the system (where γ is the relaxation rate) and diffusion (where $d = 2\gamma m k_B T$; m is the mass of the system, k_B is the Boltzmann constant, and T is the temperature of the field). The last diffusive term induces the suppression of quantum interference (represented by the off-diagonal elements in the reduced state $\rho_s(t)$ of the system) in the reduced dynamics (Zurek and Paz 1994; Joos 1996, ch. 3, sec. 3.2.3) and references therein.

17. This is because in the case of closed systems the last two terms in the evolution equation of the Wigner function are equal to zero.

(2) *Open Systems.* When the system undergoes a decoherence interaction with the environment there is an initial (relatively short) time interval in which the dynamical evolution is approximately reversible, volume preserving, and follows the classical Liouville flow. But for times larger than the decoherence time of the system the Wigner function delocalizes in position and decoherence ensues. This means that there is an effective collapse of the total state (12) onto the corresponding mixture much before the evolution deviates from the classical evolution.[18] As a result the Wigner function evolves towards a mixture of localized Gaussian states on a time scale that is given approximately by

$$t_c \ln(I/\hbar), \tag{15}$$

where t_c is the divergence rate of the chaotic trajectories of the system (i.e., the time scale at which the classical dynamics develops fine structure below $\hbar^n$-sized regions of phase space), and I is the action. This is in fact the crossover time at which the quantum corrections in the dynamical evolution of the system become effective. For times larger than (15) the Gaussain states in the Wigner function evolve independently of each other following approximately the classical evolution. But since interference terms in the reduced dynamics are washed out (in correspondence with (14)), the total phase space volume over which the Wigner function spreads increases monotonically. This process goes on approximately on time scales at which equilibrium is reached.

Thus, according to Zurek and Paz (1994), closed quantum systems cannot exhibit a chaotic behavior, and in this sense they violate the predictions of classical mechanics. But for open systems, as a result of decoherence classical behavior is recovered (of course, only under a proper interpretation of quantum mechanics; see the next section). In this case (and only in this case) the system may be said (in no-collapse interpretations of quantum mechanics) to follow quasi-classical trajectories (as it were, it doesn't have enough time to deviate from the classical evolution). In particular, if quantum mechanics is correct and we also take into account the effects of decoherence, there is only a small delay in the actual divergence rate of the trajectories of the system relative to the rates given by classical mechanics. Note in this context that the crossover time (15) at which the dynamics of the system becomes effectively classical depends on the classical divergence rate of the trajectories and not on decoherence rates (see more details in Zurek and Paz 1994; Monteoliva and Paz 2000).

The effect of this dynamics on the quantum mechanical (von Neumann) entropy $-k\mathrm{Tr}\,\rho\ln\rho$ is this. In case (1) of isolated systems the von Neumann

18. Note that this means that the diffusion in the evolution of the Wigner function reduces the quantum coherence on *decoherence* time scales.

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entropy remains approximately constant (and identically zero if the system starts out in a pure state). And so in this case classical thermodynamics cannot be recovered.

In case (2) Zurek, Habib, and Paz 1993 and Zurek and Paz 1994 show, in the simple model of a decoherence interaction of a harmonic oscillator with an environment in thermal equilibrium, that decoherence yields an increase in the von Neumann entropy as a monotonic function of the *volume* in phase space (see also Joos and Zeh 1985, 235–236.) The rate of increase of the von Neumann entropy is proportional to the degree of mixing of $\rho_s(t)$ (depending logarithmically on the number of eigenstates of $\rho_s(t)$). Assuming, for instance, that the diagonal elements of $\rho_s(t)$ in (14) are approximately equal, the von Neumann entropy is $\ln N$. In the case of chaotic systems the von Neumann entropy increases (before equilibrium is reached) at a rate that is approximately equal to the divergence rate of the chaotic trajectories (Zurek and Paz 1994, 1995; Paz and Zurek 1999, ch. 5; Monteoliva and Paz 2000), approaching (asymptotically) the classical Kolmogorov-Sinai rate for entropy production (Zurek and Paz 1995). This means that the rate at which the von Neumann entropy increases is approximately the classical one. In the case of decohering systems which are not chaotic, the von Neumann entropy also increases but at a much slower rate.[19]

To sum up: In case (1) of isolated systems classical thermodynamics and classical mechanics lead to predictions that disagree with the predictions of standard quantum mechanics. This holds as long as the system is truly isolated from its environment so that there are no decoherence interactions. This is correct of course only if the von Neumann entropy does correspond to the usual notion of entropy as it is defined in thermodynamics and classical statistical mechanics (as we noted above, this is questionable). In case (2) of open decohering systems the classical predictions are recovered. The Wigner function breaks down (due to the effective collapse of the quantum wave function) to Gaussian states each of which follows approximately quasi-classical trajectories (as given by the Poisson bracket). Because of the persistent (effective) reduction of interference terms (in correspondence with the diagonal form of the reduced state $\rho_s(t)$ in (14)) the total volume of phase space over which the Wigner function is nonzero increases monotonically (so that Liouville's theorem no longer describes it). In these cases we obtain a monotonic (and effectively irreversible) increase in the von Neumann entropy at approximately the classical rates.

19. The rate of increase in von Neumann entropy in the case of chaotic systems is independent of the diffusion coefficient appearing in the evolution of the Wigner function. In the nonchaotic case the entropy production depends only on the value of the diffusion coefficient. See further details in Zurek and Paz (1994, 1995); Monteoliva and Paz (2000).

5. Thermodynamics without Collapse. Our project is similar to Albert's, namely, to justify the use of probability as it is used in classical statistical mechanics, but using quantum mechanics as the underlying dynamics. This project is different from the one undertaken by Zurek and Paz (1994). Their project was to show that environmental decoherence brings about an increase in the von Neumann entropy. We (and Albert), on the other hand, argue that decoherence brings about an approach to equilibrium in the classical sense of, for example, an evolution towards the most probable macrostate. The concepts of entropy, equilibrium, etc. that we use are those appearing in classical statistical mechanics, in either its Boltzmannian version or the Gibbsian one. We shall be willing to use the von Neumann entropy only to the extent that it corresponds to those classical notions, and this correspondence is yet to be established (as we have already remarked).

What, then, is the role of the above results concerning the von Neumann entropy in our argument? These results will serve as a significant support for a hypothesis that we shall make, but the hypothesis is reasonable on other grounds as well, and so our proposal does not depend on the above results. But before proceeding to undertake this project, let us make some further remarks on the extent to which the von Neumann entropy corresponds to any classical notion of entropy based on phase probabilities. Some problems arise in this context.

First, decoherence by itself does not solve the measurement problem in quantum mechanics. The interference terms in the superposition (12) are not eliminated, but rather diffused into the degrees of freedom of the environment. It is true that the effects of interference between the different terms in (12) are effectively undetectable for times longer than the decoherence time Δt of the system, and in the pointer basis the reduced state $\rho_s(t)$ has the form of a classical statistical mixture. But $\rho_s(t)$ in (14) is an improper mixture. We still lack an explanation for why our experience singles out only one of the $|\psi_i\rangle$ as actually occurring on each occasion. This means that $\rho_s(t)$ cannot be taken to represent a probability distribution, and the diagonal elements $|\mu_i(t)|^2$ cannot be interpreted as probabilities of the corresponding states $|\psi_i\rangle$.

Second, in the context of thermodynamics, the measurement problem translates into a problem about the meaning of the phase space functions and of the thermodynamic properties of the system. Take first the Wigner function. Due to decoherence it behaves formally like a probability distribution whenever there exists a stable pointer basis (of coherent states). But even in these cases (where the Wigner function takes only positive values, and its evolution is effectively irreversible) the reduced state $\rho_s(t)$ still doesn't correspond to a probability distribution. And thus the reduced dynamics as described by the Wigner function (or by the evolution of the reduced state $\rho_s(t)$) cannot be said to follow quasi-classical trajectories.

Moreover, since $\rho_s(t)$ is not a probability distribution, the von Neumann entropy too cannot be given a Gibbsian interpretation in terms of a probability distribution. Similarly, a Boltzmannian notion of entropy based on dividing the diagonal elements in $\rho_s(t)$ into sets corresponding to macrostates makes no sense. In a Boltzmannian approach entropy is a physical relation between a given microstate of an individual system and a given macrostate. In classical statistical mechanics a macrostate is associated with volume in phase space, and the entropy of the microstate of the system at a given time is the logarithm of the standard measure of the volume which includes this microstate at this time. Thus, it is part and parcel of the Boltzmannian notion of entropy that the system actually be in a given microstate (and a fortiori in a given macrostate). This means that the von Neumann entropy (and the Wigner function) cannot be properly understood in a Gibbsian approach, nor in a Boltzmannian approach. In quantum mechanics the application of such approaches requires a solution to the measurement problem.

Some of the above problems can be solved by appealing to no-collapse interpretations of quantum mechanics (modal, many-worlds, and pilot-wave theories). In such interpretations there are extra dynamical laws (over and above the Schrödinger equation) according to which $\rho_s(t)$ in (14) represents a genuine probability distribution over the $|\psi_i\rangle$. The state of the system at each time is associated with one of the states $|\psi_i\rangle$ (call them *effective states*) corresponding to the diagonal elements of $\rho_s(t)$. And there are transition probabilities between any two such effective states at different times.[20] In such interpretations decoherence is usually used in order to explain (on the basis of the dynamics of the quantum state) why the different terms in the time-evolved superposition (12) effectively cease to interfere.[21] And then the diagonal form of $\rho_s(t)$ (as in (14)) together with the interpretation of $\rho_s(t)$ as describing probabilities is taken to explain in such interpretations the so-called *effective collapse* of the state. As is well known, any one of the above interpretations of quantum mechanics faces its own problems (e. g., in the context of measurement theory and relativistic generalizations), and so whether or not it may be taken as a foundation of classical statistical mechanics will depend on how these problems

20. The interpretation of the reduced state as describing the probability distribution over the $|\psi_i\rangle$ at a single time is the same in the pilot-wave, modal, and many-worlds theories. However, these theories differ in their account of the multitime (joint and transition) probabilities.

21. Note that the pointer basis which is taken here as preferred in decomposing $\rho_s(t)$ is fixed by the condition in (11).

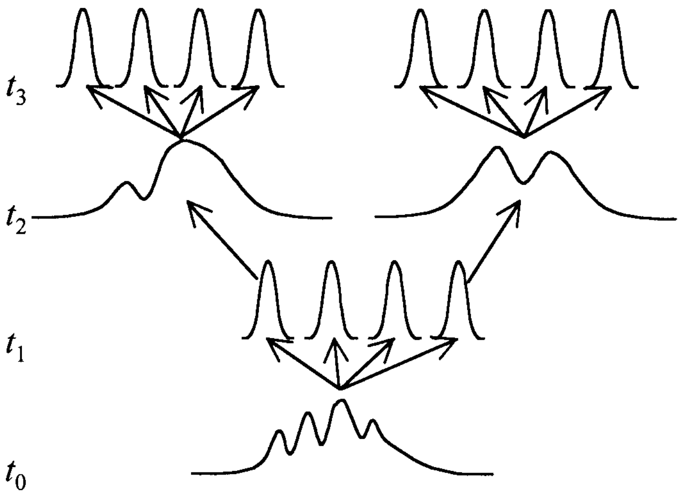

Figure 2. Environmental decoherence

will be solved. In this paper we do not address these issues and do not advocate choosing one of the above interpretations.

5.1. The Proposal. Consider systems which conform to the standard models of decoherence (Caldeira and Leggett 1983; Joos and Zeh 1985; Zurek, Habib, and Paz 1993) in which there is a stable pointer basis, and in which decoherence yields localization of the effective states of the system.[22] In the standard models the total state of system plus environment has the form (12), that is

$$|\Psi(t)\rangle = \sum_i \mu_i(t)|\psi_i\rangle \otimes |E_i(t)\rangle, \tag{16}$$

where the effective states $|\psi_i\rangle$ diagonalizing the reduced state of the system are coherent states. The coherent states in these models are the maximally stable states under the time evolution (including the decoherence interaction).

Figure 2 illustrates the evolution of a state during a time interval longer than the decoherence time of the system. The total state above evolves in accordance with the Schrödinger equation. The different terms in the superposition (16) which are approximately product states of the form

22. More pathological cases in which decoherence does not lead to a pointer basis (of localized states) are discussed in Hemmo and Shenker 2003, sec. 5.

$$\left|\psi_i\right\rangle \otimes \left|E_i(t_1)\right\rangle \tag{17}$$

evolve approximately in accordance with the Schrödinger free evolution, independently of each other, because the $|E_i(t_1)\rangle$ don't reinterfere. In this sense we obtain an effective collapse of the state (16). In general, we can assume that the independent time evolution of each of the branches induces some spread in position (t_2 in Figure 2). When this spread becomes larger than the coherence length, decoherence will operate again (t_3 in Figure 2), and as a result the reduced state $\rho_s(t)$ of the system will become mixed in each of the time evolved terms (17). But decoherence insures that $\rho_s(t)$ will be diagonalized, again, by coherent states. And so in all branches of the total state the new effective states of the system at t_3 are coherent states coupled to (approximately) orthogonal $|E_i(t_3)\rangle$. The states $|\psi_i\rangle$ at t_3 are now centered around some spatial points $x_i(t_3)$ that are, in general, different from $x_i(t_1)$.

We may now assume that a single effective state at t_1 (i. e., one of the $|\psi_i\rangle$ at t_1) does not uniquely determine a single effective state at t_3. That is, we assume that the transitions between the effective states are genuinely stochastic (the two-time correlations are not one-to-one). This depends on the details of the extra dynamics of the no-collapse theory in question: in some modal and many-worlds interpretations the dynamics is indeed genuinely stochastic.[23] On this assumption the result is that the effective state of the thermodynamic system changes in a stochastic way in the course of decoherence. It may be convenient to think about these transitions as if they induce random perturbations on the system's trajectory. The crucial point is that these transitions don't in fact depend on initial conditions (over and above those needed to secure decoherence). In this sense they play exactly the same role played by the GRW jumps in Albert's approach.

Suppose now that we write down the Schrödinger equation for a given thermodynamic system, and solve it for all possible initial states. Take the time interval (t_1, t_3). For every possible initial state at t_1 (which for simplicity we assume is approximately pure) there are many possible evolutions that branch out from it, corresponding to different relative states of the environment (many possible final states at t_3). Compare all the pairs of states, one of which is a possible initial state at t_1 and the other is one branching-out evolution of it at t_3. It is then possible to determine, for

23. For a detailed discussion of the modal interpretation of quantum mechanics, and in particular the extra stochastic dynamics, see Bacciagaluppi 1998; Bacciagaluppi and Dickson 1999. In the pilot-wave theory the velocity equation is deterministic, and so the trajectory of the system is fixed by the initial conditions and the dynamics. In this theory there are no genuinely stochastic transitions between different trajectories along which the system can evolve. In the many worlds theory the question of whether or not there are stochastic transitions when a state of the form (16) branches is under dispute.

each such pair, whether the transition from the state at t_1 to the state at t_3 is thermodynamic or antithermodynamic.

We now argue that the thermodynamic evolutions overwhelmingly outnumber the antithermodynamic ones. The reason is this. The effective evolution is made of segments, each of which is of the t_1-to-t_3 type (Figure 2). Due to the genuinely stochastic nature of the selection of segments which make up the evolution, if most transitions between effective states are thermodynamic normal, then the overall evolution will be thermodynamic, regardless of whether or not any of the segments which make it up happens to be thermodynamic abnormal. Recall that we have defined a trajectory to be thermodynamic normal (abnormal) if the succession of states obeys (violates) the laws of thermodynamics. A prerequisite is, of course, that the thermodynamic magnitudes be well defined, and in the context of quantum mechanics this means, in particular, that the measurement problem be solved.

In order to prove the above *if* clause, we proceed in two stages. First, we put forward a *dynamical hypothesis* that the overwhelming majority of the above t_1-to-t_3 transitions are thermodynamic normal. More precisely, our hypothesis says that the stochastic transitions of the extra variables reproduce during decoherence processes the standard measure as used in classical statistical mechanics. If this is correct it would mean that the decoherence interaction induces perturbations (i. e., stochastic transitions between effective states) that are enough to put the effective wave function of the system with high probability and with high enough rates on thermodynamic normal trajectories. This hypothesis is a counterpart of Albert's dynamical hypothesis (spelled out in Section 3.3), and it needs of course to be proved within a given no-collapse theory.

Second, to support our hypothesis in the framework of quantum mechanics without collapse we turn to the results in decoherence theory by Zurek and Paz described in the previous section. These results demonstrate that the process of decoherence brings about an increase of the von Neumann entropy. This entropy is of a Gibbsian type. But given the proximity between the results of applying the Gibbsian and Boltzmannian approaches in the right circumstances,[24] it seems highly reasonable that entropy in a Boltzmannian approach increases as well due to decoherence. Consequently, it is highly reasonable that in the course of decoherence most of the t_1-to-t_3 type of evolutions are thermodynamic normal for all possible initial states of the system which lead to decoherence.[25] Note that

24. Although they are conceptually very different, and differ in results as well, as already emphasized by Jaynes (1965).

25. We discuss the role of initial conditions in Section 6 and in Hemmo and Shenker 2003.

Zurek and Paz's results support our dynamical hypothesis only insofar as the von Neumann entropy is equivalent to or is a counterpart of the thermodynamic entropy. As we mentioned before, however, this idea is under dispute. According to our proposal it may be possible to recover the thermodynamic regularities without recourse to the von Neumann entropy by relying directly on our dynamical hypothesis.

6. Probabilities and the Role of Stochasticity. As we saw in Albert's approach the stochastic jumps of the GRW theory (given his dynamical hypothesis) have two important consequences: (i) the trajectory of a system will be (with high probability) thermodynamic normal independently of whether or not the initial state of the system was thermodynamic normal or abnormal (thus thermodynamics is obtained as a pure result of the GRW dynamics); and (ii) the probabilities in classical statistical mechanics are entirely reduced to the quantum mechanical probabilities, as the latter are construed by the GRW theory. This second point means that all probabilities in physics may be construed as objective probabilities (pure chances), and in particular it means that ignorance probabilities need play no fundamental role in physics.

By contrast, Albert (2000, 152–153) further argues that the stochastic dynamics in no-collapse interpretations of quantum mechanics (e. g., many-worlds or many minds interpretations, or the stochastic evolution of the extra variables in modal interpretations) will not in general induce the right transitions required for thermodynamic evolutions. This is because in these interpretations the evolution of the quantum state is given by the deterministic Schrödinger equation, and this evolution also determines completely both the evolution of the probabilities and the evolution of the set of all physical properties of the system. One may say that in these interpretations the set of probabilities and the set of properties of the system provide, as it were, an envelope of possibilities which evolves in time in a completely deterministic fashion. Therefore, Albert concludes that in no-collapse quantum mechanics one cannot establish results analogous to points (i) and (ii) above, and so this is a clear advantage of his approach.

However, the foregoing reasoning applies only to isolated quantum systems. That is, as long as the thermodynamic system is isolated, its behavior is fixed entirely by the evolution of its quantum state alone. Moreover, if by the Schrödinger equation the quantum state of an isolated

26. In Bohm's theory the position of both open and closed systems follows always deterministic trajectories. In modal interpretations the evolution of the set of the extra variables associated with the (global) properties of both open and closed systems follows the Schrödinger evolution (i.e., it is deterministic). In the case of closed systems there are typically no stochastic transitions also in the values of those properties. But for open systems the dynamics of the extra variables is generally stochastic (Bacciagaluppi

system evolves along a thermodynamic abnormal trajectory, then the system will violate the predictions of thermodynamics, independently of whether or not other (so-called 'hidden') variables of the system undergo stochastic or deterministic transitions.[26] But, as argued above, in no-collapse interpretations the effects of decoherence in the case of open systems are crucial for the explanation of thermodynamic evolutions. In particular, since decoherence results in stochastic transitions between the system's effective states (at different times), the latter states (as we explained above) evolve along trajectories which will be in general different from the trajectory along which the total quantum state evolves. It is correct that the trajectory along which the total state evolves is fixed deterministically by the initial state of the system. But in the case of interacting systems (i. e., as in decoherence) the trajectories along which the effective states of the system evolve are not fixed deterministically by the initial quantum state of the system (except in deterministic theories like Bohm's), and they may be completely independent of it.

Let us see how these ideas can be understood in, for example, modal interpretations. Recall our schema in Figure 2. In modal interpretations the extra variables evolve between t_1 and t_3 in a genuinely stochastic fashion, such that at t_3 one of the Gaussians is chosen by the stochastic hidden variable. The chosen Gaussian is referred to in modal interpretations as the *actual* (or effective) state of the system. In our schema this means that the effective transitions from one Gaussian state to the next are chancy or stochastic. As explained in the previous section this brings about the thermodynamic behavior. In this way we can see how our approach yields the consequences (i) and (ii) of Albert's approach.

Because of the stochastic nature of the dynamics in modal interpretations, for example, whether or not the actual trajectory of a system is thermodynamic normal (or abnormal) on those interpretations will not depend on the initial quantum state of the system. And so the thermodynamic regularities can be explained along the lines sketched above on a purely dynamical basis, just as in the GRW theory. Furthermore, when the system evolves along decohering trajectories the probabilities in classical statistical mechanics are entirely reduced to the quantum mechanical probabilities.[27]

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and Dickson 1999). In many worlds theories the evolution of closed systems is always deterministic and there is no corresponding splitting of worlds. Splitting (and in some versions stochastic evolution) occurs only due to interactions (typically measurement-like interactions).

27. But note that statistical postulates are needed in order to make sure that the system will evolve along decohering trajectories in the first place. These statistical postulates need not have anything to do with the quantum mechanical probabilities. We discuss this issue as well as other postulates about initial conditions in decoherence theory in Hemmo and Shenker 2003; see also Arntzenius' (1998) analysis of this topic in the context of modal interpretations.

The role of stochasticity in our approach, however, is different in one important respect from its analogue in the Albert-GRW approach. Despite the fact that due to the interaction with the environment the transitions between the effective states of the system are stochastic (Figure 2), the evolution of the total wave function is time reversible, since it is governed by the Schrödinger equation.[28] For example, the total wave function may recohere in the future, and this will (or will not) happen regardless of whether or not the evolution of the extra variables is stochastic. And since the Schrödinger equation is deterministic, the evolution of the total wave function depends entirely on initial conditions. For this reason the stochasticity in our approach (although it may correspond to a genuine chancy evolution of the extra variables) does not entail genuine thermodynamic irreversibility. This is unlike the Albert-GRW approach. In quantum mechanics thermodynamic (ir)reversibility is thus an outcome of the evolution of the total wave function and does not depend on the behavior of extra variables. Here too these variables remain hidden.

Note that in Bohm's theory there can be no stochastic transitions or stochastic perturbations of the trajectory of an open system which don't completely depend on the initial conditions. Of course, our analysis using decoherence is perfectly applicable also in this theory. But since the theory is completely deterministic and time reversible (both the Schrödinger dynamics and Bohm's velocity equation are deterministic and time reversible), initial conditions in the recovery of thermodynamics (such as the initial distribution postulate $\||\Psi\rangle|^2$) play essentially the same role as in the classical approaches in statistical mechanics. We discuss in more detail the role of initial conditions in both Albert's approach and in our approach in a sequel of this paper (Hemmo and Shenker 2003).

7. Summary and Open Questions. Albert proposes to solve some problems in the foundations of statistical mechanics using the GRW approach to quantum mechanics. Had this been the only way to solve these problems, the GRW approach would have gained a significant advantage over alternative interpretations of quantum mechanics. For this reason it is important to learn that no-collapse approaches to quantum mechanics can yield similar results. In particular, the recovery of the thermodynamic regularities in Albert's approach relies on a dynamical hypothesis to the effect that the GRW dynamics produces the standard (classical) probability measure over the microstates. In our no-collapse approach a similar dynamical hypothesis is also required with respect to effective transitions in

28. The Schrödinger equation is time reversible in the usual sense that it is invariant under temporal reflection and complex conjugation.

decoherence processes. Moreover, in both Albert's and our approach these hypotheses are equally justified given the properties of, respectively, the GRW dynamics and decoherence. It is significant to note that in a no-collapse approach, unlike Albert's approach, our hypothesis appears to be supported by the results of Zurek and Paz (1994)—assuming, once again, that the von Neumann entropy corresponds to the thermodynamic entropy. A deeper understanding of these results requires further investigation of precisely what is the notion of entropy in quantum mechanics, and its relation to, say, quantum mechanical mixing and entanglement, and the von Neumann entropy in particular.

As noted above, Albert's proposal has significant points of strength. It proposes a single origin for chance in physics, in both quantum mechanics and statistical mechanics, namely the GRW stochastic equations of motion. And it entails that there is no need to postulate special initial microconditions when accounting for the low entropy in the past and in the future. Yet, it is still necessary in Albert's approach to assume a past macrostate of low entropy, Albert's past hypothesis, in order to get the right (thermodynamic) retrodictions about the past; this is discussed in some detail in Hemmo and Shenker 2003. In this context a deeper understanding of the status and role of Albert's past hypothesis requires further investigation.

Also our no-collapse proposal explains both quantum mechanical probability and statistical mechanical probability in a unified way. But this seems to hold only for dynamical situations in which decoherence endures. It is of prime imprortance in this context to investigate the kind and role of the statistical postulates concerning initial conditions in the standard models of decoherence. As to the past hypothesis, our approach requires that we assume not only a low entropy initial state, but also initial microconditions which gurantee that decoherence but not recoherence takes place in our past and future (see Hemmo and Shenker 2003 for a detailed discussion of the past hypothesis).

It now remains to be seen how the two approaches account for some seemingly problematic cases. For example, in the case of small and light gases the GRW-predicted rates for collapses may be extremely low. In these cases it seems that the thermodynamic behavior of the gases cannot be explained by a GRW-based approach. Another example is the spin echo experiments in which the effective isolation of the system may be problematic for a decoherence-based approach. Some of these problems in the context of the GRW theory are addressed by Albert (2000, ch. 7). In the sequal of this paper (Hemmo and Shenker 2003) we describe these difficulties in detail, and propose ways to solve them in Albert's collapse approach as well as in our no-collapse approach.

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