Reduction, Unity and the Nature of Science: Kant's Legacy?

MARGARET MORRISON

1. Introduction: Kant, unification and modern physics

One of the hallmarks of Kantian philosophy, especially in connection with its characterization of scientific knowledge, is the importance of unity, a theme that is also the driving force behind a good deal of contemporary high energy physics. There are a variety of ways that unity figures in modern science—there is unity of method where the same kinds of mathematical techniques are used in different sciences, like physics and biology; the search for unified theories like the unification of electromagnetism and optics by Maxwell; and, more recently, the project of grand unification or the quest for a theory of everything which involves a reduction of the four fundamental forces (gravity, electromagnetism, weak and strong) under the umbrella of a single theory. In this latter case it is thought that when energies are high enough, the forces (interactions), while very different in strength, range and the types of particles on which they act, become one and the same force. The fact that these interactions are known to have many underlying mathematical features in common suggests that they can all be described by a unified field theory. Such a theory describes elementary particles in terms of force fields which further unifies all the interactions by treating particles and interactions in a technically and conceptually similar way. It is this theoretical framework that allows for the prediction that measurements made at a certain energy level will supposedly indicate that there is only one type of force. In other words, not only is there an ontological reduction of the forces themselves but the mathematical framework used to describe the fields associated with these forces facilitates their description in a unified theory. Specific types of symmetries serve an important function in establishing these kinds of unity, not only in the construction of quantum field theories but also in the classification of particles; classifications that can lead to new predictions and new ways of understanding properties like quantum numbers. Hence, in order to address issues about

doi:10.1017/S1358246108000039 © The Royal Institute of Philosophy and the contributors 2008

unification and reduction in contemporary physics we must also address the way that symmetries facilitate these processes.

But what does this have to do with Kant, aside from the fact that he too was interested in unity? Several commentators have stressed the different ways that unity functions in Kant's philosophy of science and how it emanates from both reason and the understanding. Indeed there are several different notions or levels of unity at work in the Kantian system encompassing ontology, epistemology and methodology. Moreover, the unity of knowledge and experience acquired through intuition and understanding also requires a transcendental unity of consciousness, which in turn involves an act of synthesis. While the relation between synthesis and unity in the Kantian architectonic is important in its own right, its explication is less crucial for my project in this paper, which is to articulate what, if any, connection exists between the notion of unity embedded in modern physics and Kant's account of unity in his philosophy of science.

At a very basic level Kant's account of synthesis and unity can be summarized as follows: the imagination (via the synthesis of apprehension) functions in a spontaneous way to produce a synthesis of the manifold of intuition and the a priori representations of space and time. The understanding and its categories secures the unity of the appearances represented in intuition under rules (A79/B105). In other words, the concepts of the understanding give unity to the synthesis of a manifold. Despite the crucial role played by the understanding in the unifying process, it is reason that is predominant in achieving the type of unity we associate with scientific knowledge, knowledge that consists in a *system* connected according to necessary laws (A645/B673). In fact, Kant describes systematic unity as 'what first raises ordinary knowledge to the rank of science' or that which 'makes a system out of a mere aggregate' (A832/B860).²

There is however an apparent tension in Kant's presentation of the role of reason as a unifying faculty and in the description of what that unification consists in. Very often Kant seems to suggest that the requirement to seek unity is simply a subjective or logical principle rather than the embodiment of an objective fact about nature; especially since the notion of an 'all encompassing unity' is something that for us is a regulative idea (A647-8/B675-6). Other times Kant

¹ Kitcher (1983), (1986); Buchdahl (1992); Guyer (1990); Morrison (1989).

² I shall henceforth use Norman Kemp Smith's (1929) translation of the *Critique of Pure Reason* (New York: St. Martin's).

stresses the objective aspects of unity but when doing so seems to associate this objectivity with epistemic goals that involve what he calls the 'coherent employment of the understanding' (*ibid*.):

Its function is to assist the understanding by means of ideas, in those cases in which the understanding cannot by itself establish rules, and at the same time to give to the numerous and diverse rules of the understanding unity of system under a single principle, and thus to secure coherence in every possible way (A648/B676).

Given that scientific knowledge involves the process of logical systematization (A832/B860) reason functions in a methodological way to urge us along in the process of constructing a unified system, something that we, nevertheless, typically find in nature when we go in search of it (A653-4/B681-2).³

Although this kind of unity can be seen as Kant's way of separating science from other 'non-scientific' knowledge, it seems to bear little resemblance to the goals of reductive unity constitutive of high energy physics. For that we need to go beyond the systematization of knowledge to embrace a full-blown reductionist *ontology*. In other words, the aim is a unification of *forces* under the framework of a single theory. Although Kant speaks of reducing different kinds of earths and the desire to find a common principle for the earths and salts, in these contexts he typically speaks about the *presupposition* that the unity of reason accords with nature. Even though this relationship between reason and nature is a presupposition, it is nevertheless a demand of reason rather than a convenient heuristic device (*ibid*.). But, the implication is that the source of the unity is first and foremost methodological rather than being grounded in the objects themselves.

However, remarks in *Metaphysical Foundations of Natural Science* (henceforth abbreviated as MAN, followed by the section number), *Opus postumum* (OP)⁴ and other places in the *Critique of Pure*

³ I do not intend this claim as a resolution of the tension; in fact, as we shall see below the relation between the subjective and objective features of unity is a fundamental feature of Kant's transcendental program. The importance of this 'tension' and the role it plays will be discussed in section two.

⁴ Henceforth, I shall be using James Ellington's (1985) translation of the *Metaphysische Anfangsgründe der Naturwissenschaft* (Indianapolis: Hackett); and Förster and Rosen's (1993) translation of the *Opus postumum* (Cambridge: Cambridge University Press).

Reason (KrV) suggest that Kant also embraced the importance of ontological reduction in the sciences. For example in MAN (534) Kant says that 'all natural philosophy consists in the reduction of given forces apparently diverse to a smaller number of forces and powers sufficient for the explication of the actions of the former. But this reduction continues only to fundamental forces, beyond which our reason cannot go.' This is now beginning to sound very similar to the program of reduction and unification that pervades contemporary particle physics. So, the question becomes how to square this approach to scientific ontology with the seeming anti-reductionist epistemology in the KrV. This is important for understanding whether Kant's account of unification bears any relation to the role of unification in contemporary physics. In other words, is there a Kantian legacy we can legitimately trace?

The key to answering this questions lies in explicating how the search for unity proceeds at the level of empirical science both for Kant and in modern physics, and the relation of those activities to the Kantian transcendental principles/conditions associated with reason and the understanding. That is, how did Kant see the relation between empirical science and reason and what, if any, bearing does that relation have on contemporary practice. Part of what is at stake here is the form that both physical explanation, and theories more generally, ought to take. For example, contemporary debates in philosophy of science frequently focus on whether we should be satisfied with a multiplicity of levels as exemplified by effective field theories, making the goal of a theory of everything simply a metaphysical hope; or is there reason to think that this latter kind of unity is attainable in practice.⁵ In other words, what is the relation here between physics and metaphysics? These kinds of questions focus on what the limits of unity are for empirical science and what kind of evidence we have for that unity, issues Kant was especially concerned about. In other words, there are three different questions here that we need to address: (1) What is the relation between the empirical and transcendental in Kant's theory of science?; (2) What is the relation between

These are not just questions that preoccupy philosophers of science, they are very much a part of the scientific discussions that address the nature of fundamental physics. Many contemporary physicists are concerned with issues surrounding reduction and emergence and whether the search for a theory of everything is simply a metaphysical hope. See, for example, recent articles by Laughlin and Pines (2000) as well as Weinberg (1993) and Anderson (1972).

physics and metaphysics in contemporary science?; (3) Does the answer to (1) have any impact on (2)?

The relevance of Kant's answers to these questions for modern physics is far from straightforward. While his characterization of unity seems to bear a close relationship to many of the issues surrounding contemporary unification, a closer analysis reveals that that these connections may, in fact, be rather superficial. This is traceable, ultimately, to the relation between the transcendental and empirical, in particular the links between unification and reduction expressed in KrV and MAN. So, while the Kantian legacy in contemporary physics might appear to be rather strong, this is due to the fact that some of his ideas are adopted piecemeal into modern contexts with little or no attention paid to the underlying philosophical framework that legitimates them. But does this really matter? At the object level where science is practiced, perhaps it is enough that the use of symmetry principles, for example, can be interpreted along the lines of logical maxims or transcendental ideas of reason. We simply do not need the entire Kantian architectonic in order to locate his influence in contemporary science. While this might be a tempting line of argument I am doubtful about such a conclusion and in what follows I want to outline the reasons why.

In order to flesh out the story, I begin with a brief discussion of the non-reductive character of Kant's epistemology and go on to discuss how this feature relates to his ideas about unity and reduction in science. From there, I examine the extent to which these views find a place in contemporary physics and whether the reduction and unification present there can be seen as reflecting Kantian principles. I conclude by arguing that because contemporary science has more or less ignored Kant's Copernican revolution, many features the unification project it is engaged in embody the kind of transcendental realism Kant was at pains to avoid.

2. Kant's anti-reductionist epistemology

Rationalists like Descartes and Leibniz, as well as empiricists like Hume were all concerned with establishing the proper foundations for human knowledge. Part of that project involved reducing certain features of knowledge to its elementary constituents. Descartes, for example, attempted to derive physical laws from metaphysical principles and claimed in the *Principles of Philosophy* (section 203) that all of physics follows from the self-evident (clear and distinct) proposition that matter is extension. The latter followed

from the *cogito*, together with the existence of God, and was guaranteed by reason. In fact, in the preface to the *Principles* Descartes remarks that one can derive knowledge of all things in the world from the basic principles of philosophy. A similar type of project, although much different in detail, was envisioned by the monadic metaphysics of Leibniz, who also espoused the reduction of mathematics to logic. While empiricists do not attempt to establish deductive relations founded on the certainty of reason, they nevertheless embrace a reduction of all knowledge to impressions/ideas. Even the Newtonian program of 'deduction from phenomena' embodies reductionist goals as is evident from *Optics* (Query XXXI) where Newton claims that '....to derive two or three general principles of motion from phenomena andto tell us how the properties and activities of all empirical things follow from these manifest principles would be a very great step in philosophy ...'.

What differentiates Kant from this methodological picture is that his goal is not to seek the ultimate justification for the existence of the objects of physics or human knowledge more generally, or to ask whether knowledge of objects is possible, but to ask what makes possible the experience that we do have of objects in the physical world. Transcendental idealism is not concerned with locating the source of knowledge either outside ourselves or in our reason, but rather with denying the dichotomous nature of the reductionist project and in doing so establishing the dual source of knowledge in both experience and the understanding. As Kant argues in the Refutation of idealism, the existence of objects in space outside us is simply not in question. But, because these objects are conditioned, they do not play the same foundational role as the empiricist's impressions. In that sense then Kant is not wedded to the idea that knowledge is based on reduction to a fundamental level involving either reason or the ultimate constituents of experience (whatever they may be).

One might want to claim that the importance of the synthetic a priori as the ground of necessity and universality hints at a kind of reductionism to the extent that these principles function as the foundations that make knowledge possible. However, it is important to note that while synthetic a priori principles certainly structure our experience there is no accompanying deduction of any empirical laws from them; so in that sense their role is very different from the metaphysical truths of Descartes and Leibniz or the foundational project of the empiricists. There is no derivation of scientific laws from transcendental principles. Empirical laws are simply special determinations of the pure laws of the understanding. The latter make the

former possible and it is through them that appearances take on an orderly character (A127-8; B165).

But what about the Newtonian version of reductionism? Newton's goal was to derive explanations of physical phenomena from physical principles, a practice that still very much defines the methodology of contemporary physics. So, the question is to what extent Kant adopted this kind of reductionism within empirical science itself. In order to answer that, we need to look more closely at Kant's views on unity in the context of empirical investigation. This is important for the issue I raised in the introduction, namely whether we can articulate some kind of Kantian legacy in the way science is actually practiced, even if we are unable to extend that legacy to its philosophical foundations. Because of the close relation between unity and reduction in contemporary physics the question is whether Kant's views on unity embody a similar relation. So, there are two issues/questions here: (1) To what extent do Kant's views on unity and reduction go together?; (2) Do Kant's views on the empirical nature of science in any way resemble contemporary practice?

As I noted above, there are various kinds of unity described by Kant, one of which is the province of the understanding and another, the domain of reason. The former involves a synthetic unity, the ground of which is contained 'a priori in the original sources of knowledge in our mind' (A125). But these 'subjective conditions' must also be 'objectively valid, being the possibility of knowing any object at all' (ibid.). The order and regularity we experience in nature consists in a connection of appearances and this connection is the unity produced by the understanding in the activity of thinking. As Kant notes in the transcendental deduction, we introduce order and regularity and hence unity to experience. Although this unity makes the practice of science possible there is another kind of unity that constitutes, if you will, the *doing* or *practice* of science itself; that is, the organization and systematization of empirical knowledge into a coherent system of laws. The latter is the task of reason but in directing us to search for the 'absolute totality of the conditions of all appearances' (A416/B384), we must recognize that these conditions are not given as objects of experience and hence the unity that we seekthe 'whole of all appearances' (A328)—is an unattainable ideal. Hence, the requirement that we 'find for the conditioned knowledge of the understanding the unconditioned whereby its unity is brought to completion' (B364), is a process that is itself uncompletable.

The search for something that is not a possible object of experience involves what Kant calls the hypothetical employment of reason; it is not constitutive and hence the unity it prescribes is grounded in what Kant terms a 'logical' principle concerned with knowledge in its unified form (A648/B676). Although the term 'principle' denotes a universal proposition obtained solely from concepts, the principles of reason depend on thought alone (A302) and are associated with problematic concepts for which there is no corresponding object. This is in contrast to the principles of the understanding which are grounded in the synthetic a priori features of knowledge. As a logical principle, the demand to seek unity is intended to secure the coherent employment of the understanding. Hence, this 'logical' employment of reason concerns itself with the attempt to reduce the knowledge obtained through the understanding to 'the smallest number of principles (universal conditions) and thereby achieve the highest possible unity' (A305), as in the construction of a logical system.

However, nothing follows from the logical demands of reason concerning a unified nature per se. Yet Kant claims that the principle requiring us to seek unity is a necessary principle, not because we achieve unity through the observation of objects in nature, but because it is only through the coherent employment of the understanding that an empirical criterion of truth is possible. In other words, empirical truth is not possible without the systematic employment of the understanding, something that is in turn made possible by the demand of reason. As Kant indicates in the discussion at B84-5/A60, the only access we have to empirical truth is via a negative condition, namely the agreement of knowledge with the general and formal laws of the understanding. But the understanding cannot function unless it can unify its various laws into a coherent system. Hence, the degree to which we can achieve this is what makes possible the ability to judge the truth or falsity of the knowledge acquired through the understanding. In other words, coherence allows for the possibility of a judgment about truth-it is not equated with truth; it provides an epistemic condition, not a semantic mark, or an ontological correspondence. Although reason's demand for systematic unity is a regulative constraint designed to secure a measure of coherence, this coherence is necessary to systematize knowledge of objects acquired through the understanding, a crucial component for characterizing science:

...to say that the constitution of the objects or the nature of the understanding which knows them as such, is in itself determined

to systematic unity, and that we can in a certain measure postulate this unity a priori, without reference to any such special interest of reason, and that we are therefore in a position to maintain that knowledge of the understanding in all its possible modes (including empirical knowledge) has the unity required by reason, and stands under common principles from which all its various modes can, in spite of their diversity, be deduced—that would be to assert a transcendental principle of reason, and would make the systematic unity necessary, not only subjectively and logically, as method, but objectively also (A648/B676).

What the above quote suggests is that the objectivity that we ascribe to the idea of unity is one that is mirrored in our knowledge of the empirical world. Although Kant does discuss various examples of the kind of reduction and unification we experience when doing empirical science, this endeavour while made possible by reason, is not, he claims, the justification of the principle. In the discussion of the chemist engaged in reducing different kinds of earths, the hypothesis of a common principle for earths and salts, if successful, imparts probability to the explanation that they are indeed unified. But this Kant refers to as a 'selfish purpose' that must be dinstinguished from the *idea* of reason which does not here 'beg but command' (A653/B681).

What this means is that these two activities are clearly distinct. More specifically, the empirical hypothesis of unity and the search for unity among scientific entities is not to be identified with the idea of reason, which is a transcendental principle. The latter, however, makes possible the former; and in that sense they are inextricably linked. Consequently, the objectivity Kant refers to in the quote above has to do with the fact that the understanding could not engage in the search for unity, were it not for the demands of reason. The relation between the subjective, methodological aims and 'objective' features are explicit in Kant's claim that if the logical principle of genera is to be applied to nature (objects that are given to us), then it presupposes a transcendental principle. And 'in accordance with this latter principle, homogeneity is necessarily presupposed in the manifold of possible experience (although we are not in a position to determine in an a priori fashion its degree); for in the absence of homogeneity, no empirical concepts, and therefore no experience, would be possible' (A654/B682). This suggests that although the kind of unity we seek in physics and other sciences is an empirical unity grounded in a logical principle or maxim, it is nevertheless a unity whose justification lies ultimately

in a transcendental principle of reason.⁶ And, it is a unity we seek not as a result of inductive success but because it is necessary for experience and 'constitutes' the activity of acquiring knowledge and doing science. In other words, it is objective because it is linked via the understanding to objects of experience.

That this kind of unity involves a type of reduction is further evident from Kant's remarks at (A663/B691) about the discovery of the elliptical nature of planetary orbits and ultimately a unity in the cause of all the laws of planetary motion, namely, gravitation. In extending these notions and by making use of the specific logical principles of affinity (continuity), unity (homogeneity) and manifoldness (specificity or variety), we further explain things that experience can never confirm, such as the paths of comets and the uniting of distant parts of the universe, a universe that is held together by one and the same moving force. These principles recommend: 1) in the case of homogeneity, to seek unity in variety; 2) in the case of manifoldness, to seek variety under unity; and 3) with respect to affinity or continuity, to seek similarities between things that recognize both unity and variety, as in the classification of entities into natural kinds. The principle of continuity is especially important because it arises from a union of the other two. It is only in ascending to the higher genera and descending to the lower species that we obtain the idea of 'systematic connection in its completeness' (A658/ B686). In other words, we are able to see how differences in phenomena are nevertheless related to each other in so far as they 'one and all spring from one highest genus' (*ibid*.). For example, in noticing that the comets deviate from true circular orbits we draw inferences about their hyperbolic paths. The principle of continuity permits the kind of speculation that is necessary for the formulation of inductive hypotheses that take us beyond the immediate consequences of empirical experience, but not beyond possible experience.

Kant goes on to claim that this logical principle of *continuum specierum* (*formarum logicarum*) must presuppose a transcendental law (*lex continui in natura*) because otherwise the understanding might follow a path that is contrary to that prescribed by nature. Initially, this seems to suggest that nature itself (i.e. objects of experience) exhibits this type of continuity and as a result determines the way we ought to proceed in our investigations. But this interpretation

⁶ Because scientific knowledge constitutes a logical system the practice of constructing such a system involves logical principles. But, as Kant insists, these logical principles only have methodological force because they are grounded in transcendental principles.

is quickly dispelled by Kant's claim that the law must not rest on empirical grounds because, if it did it, would 'come later than the systems, whereas in actual fact it has itself given rise to all that is systematic in our knowledge of nature' (A660/B688). In other words, if the law were empirical, we would derive claims about systematization from our observations, making the process purely contingent. The formulation of the laws is not due to any hidden experimental design or by putting them forward in a hypothetical manner. However, when their content is confirmed empirically, this yields evidence for the hypothesis that the presupposed unity is well grounded and that the laws of parsimony of causes, manifoldness of effects, and affinity of the parts of nature, are in accordance with both reason and nature itself.⁷

But how should we understand this 'accordance'? Is it only sometimes that these laws lead us to the right conclusions? Apparently not, because even though these principles contain 'mere ideas' for the guidance of the empirical employment of reason Kant claims that they nevertheless posses, as synthetic a priori propositions, objective but indeterminate validity and, as such, serve as rules for possible experience (A663/B691). This means that they function as rules for possible experience insofar as they apply directly to the understanding, and hence indirectly to objects. Without them we would not have, for example, a concept of genus, or any other universal concepts that belong to the understanding. Indeed the understanding itself (and hence experience) would be non-existent (A654/B682). This is because the application of the logical principle of genera to nature presupposes a transcendental principle whereby homogeneity is necessarily presupposed in the manifold of possible experience (even though we cannot determine, in an a priori fashion, the degree to which this takes place). In the absence of this, no empirical concepts would be possible. So, although the transcendental principles can be employed as heuristic devices in the 'elaboration of experience' (A 663/B691), they also 'carry their recommendation directly in themselves and not merely as methodological devices' (A661/B689). While

⁷ Similarly, in the case of specification, empirical inquiry soon comes to a stop in the distinction of the manifold, if it is not guided by the antecedent transcendental law of specification, which not only leads us to always seek further differentiation but suspects these differences even where the senses are unable to disclose them (A657/B685). Such discoveries are possible, Kant claims, only under the guidance of an antecedent rule of reason. We assume the presence of differences before we prescribe the understanding the task of searching for them.

we each may attend to different logical maxims/principles in different contexts, taken as a set they are necessary for the empirical employment of the understanding and experience in general. However, the method of looking for order in nature in accordance with transcendental principles, and the maxims that require us to regard such order as grounded in nature, leave undetermined where and to what extent that order will be found (A668/B696).⁸

What we have seen here is Kant's attempt to resolve the tension between the subjective logical employment of maxims and the objectivity that their relation to transcendental principles allegedly guarantees. The epistemology of transcendental idealism requires that we locate the source of unity in both reason and nature in the same way that knowledge in general has a dual source in experience and the understanding. Hence to ask whether nature constitutes a unified whole, independent of the requirement that we must seek unity in nature, is simply the wrong kind of question to ask. As we have seen above, we must presuppose a unified world if we are to have a coherent employment of the understanding and hence experience at all. But, the necessity of the presupposition should not be equated with a necessity in nature, which is why the maxims we employ at the level of empirical investigation have the status of logical principles, whose ultimate justification comes via their transcendental counterparts.

At the beginning I mentioned similarities between Kant's use of transcendental and logical principles in his argument for unity, the role of reduction in his account of empirical science, and the way that these ideas might be instantiated in modern physics via symmetry principles. And, as we saw above, Kant's logical and transcendental principles function as heuristic devices that guide our search for knowledge, knowledge that accords with both reason and nature itself. However, as heuristics they are more than just useful tools, not only because experience is limited in its ability to point us in new directions, but because, according to Kant, the discovery that absorbent earths are different kinds was possible only under the guidance of an antecedent rule of reason. In these kinds of situations reason is 'proceeding on the assumption that nature is so richly diversified that we may presume the presence of such differences and therefore prescribe to the understanding the task of searching for them' (A657/B686). In other words, reason points us in certain directions even when our senses are unable to do so. This way of

⁸ The important point is that the methodology is grounded, ultimately, in transcendental principles.

approaching empirical/theoretical investigation bears certain similarities to the way symmetries are used in particle physics, especially in cases where they function in the construction of theories that would not have been possible on phenomenological grounds alone (e.g. the electroweak theory). In order to explore some of these parallels, let me now turn to a brief discussion of symmetries to see how closely the comparisons can be made. ¹⁰

3. Symmetries as transcendental principles?

Symmetries function in the extension of our theoretical knowledge via the prediction and classification of elementary particles, as well as in the development of theories themselves, a situation analogous to Kant's principle of continuity. The notion of symmetry that is especially relevant here is the group theoretic one which defines symmetry as invariance under a specified group of transformations and applies not only to spatial figures but also to more abstract objects like mathematical (dynamical) equations. This feature makes the group theoretical apparatus especially useful in constructing physical theories. Essentially one can proceed in two ways, either by examining the symmetry properties of equations that one is interested in; or, starting with symmetries we assume have physical significance, and using those to search for dynamical equations that have certain properties. For example, the classification of hadrons through the representations of the SU(3) group suggested that these particles had certain similarities which led, eventually, to the quark hypothesis. Similarly, the Glashow-Weinberg-Salam (GWS) electroweak theory developed out of a synthesis of the SU(2) and U(1) symmetry groups that was spontaneously broken by the Higgs mechanism. The standard model adds to this combination the SU(3) group governing the strong interactions, which is now associated with the color quantum number of quarks. In both cases, and

⁹ My remark about the impossibility of theory construction on the basis of phenomenology is simply meant to indicate that there were no reasons to assume that electromagnetism bore any relation to the weak force. In the former case the particle carrying the force is the massless photon, while in the latter much heavier massive bosons were required. No indication that two such theories could be unified emerged from the physical phenomenology.

There have been some suggestions in the literature about interpreting symmetry principles (and their role in physics) as Kantian transcendental principles, see Falkenburg (1988) and Mainzer (1996). While this initially appears as an attractive approach, my claim is that it is ultimately unsuccessful, for the reasons I discuss below.

indeed in quantum field theory in general, theoretical development proceeds in a top-down way using symmetries and the mathematics of group theory rather than physical phenomenology.

The internal symmetries of particles are related to quantized properties like isospin, charm and other quantum numbers. These symmetries are identified with invariances under phase changes of the quantum states, and are especially important because if the classification includes all the quantum numbers for characterizing a particular particle, then it is possible to define the particles in terms of their transformation properties. In other words, a particle can be defined as a unitary transformation of, say, the inhomogeneous Lorentz group. Symmetry classifications are also used in the prediction of new particles, like the omega minus in connection with hadron classification, where the prediction was made on the basis of surplus structure in the mathematics. Symmetry arguments that led to the development of the GWS theory in turn predicted the existence of the W^\pm and Z^0 vector bosons.

We can see then how these symmetries function in a way analogous to Kant's principle of continuity, allowing us to group phenomena together according to certain classificatory schemes which highlight similarities and differences that can in turn be used to form the basis for theoretical hypotheses. Symmetries also lead to restrictions on theory development because quantum field theories (QFTs) are typically characterized by the symmetries of the fields and interactions, i.e. the requirement of invariance with respect to a transformation group results in restrictions to the form of the theory, its equations and the kinds of quantities that appear in it. For example, a constraint on modern QFTs is that they be gauge invariant in order to be renormalizable. Kant's principle of continuity operates in a similar way: it constrains our theorizing in that it assumes a continuity of kinds of motion under a common principle. Although these hypotheses and classifications are subject to empirical confirmation, it is the *postulated* similarity/continuity implicit in the symmetry principles/groups (and for Kant in the principle of continuity) that forms the foundation for the inferences we make.

In addition to sanctioning these kinds of inferences, symmetries also play a unifying role in physics. ¹² Grand Unified Theories (GUT) unify what are considered the three fundamental gauge symmetries:

 $^{^{11}\,}$ For an interesting account of the omega minus case, see Bangu (2008).

The very notion of symmetry itself is related to unity in the sense that the symmetry transformations of a group relate the elements to each other and to the whole. See Morrison (2000).

hypercharge, the weak force, and quantum chromodynamics (QCD). They are based on the idea that at extremely high energies all symmetries have the same gauge coupling strength, which is consistent with the notion that they are really different manifestations of a single overarching gauge symmetry. From a 'physical' point of view, this means that at energies 10¹⁴ GeV the weak, strong and electromagnetic forces are unified into a single field. But what exactly is the relation between these symmetries and the objects they supposedly govern? Are the symmetries simply mathematical heuristic devices? Or are they themselves features of the physical world that in turn explain other features? One might be tempted to say that the enormous success of symmetries in the prediction, explanation and unification of phenomena is a reason to assign them ontological status. In other words, they function so successfully because they constitute the structure of the physical world.¹³

More persuasive, however, are the arguments for the epistemic status of symmetries and it is here where parallels with Kant become especially relevant. In his famous 1967 work entitled Symmetries and Reflections, Eugene Wigner characterizes symmetries as properties of theories or natural laws that describe phenomena, rather than properties of the phenomena themselves. We can claim that they are indirectly related to objects because the requirement that certain laws be invariant under certain symmetries further constrains the kinds of events that are physically possible. In fact, for Wigner the symmetries of space and time are pre-requisites for discovering laws of nature. He claims that if the correlations between events changed from day to day, and were different for different points of space, it would be impossible to discover any laws. Symmetry principles provide the epistemological (and heuristic) guide required to uncover what otherwise might remain unknown to us. It is this meta-theoretical status that makes them especially important in theory construction and in our ability to know the physical world.

These remarks echo those made by Kant regarding the role of transcendental principles (specification) in the discovery of different kinds of absorbent earths, and in the extension of our knowledge using the principle of continuity. Indeed if we adopt Wigner's interpretation of symmetries the parallels are quite remarkable. As with transcendental principles, symmetries relate only indirectly

Two further arguments for the ontological status of symmetries are given on the basis of the geometrical symmetries of spacetime and the relation between symmetries and conservation laws as shown by Noether's theorem. For details of these arguments, see Brading and Brown (2003).

to objects, yet they provide the conditions under which those objects can be known or discovered (as part of a system of knowledge). In speaking of symmetries Wigner claims that there is a structure in the events around us and it is this structure, i.e. the correlations between events, that science wishes to discover. And, in a rather striking resemblance to Kantian epistemology, he remarks that we would not live in the same sense we do, if events around us had no structure, making symmetries appear as conditions for the possibility of experience.

Yet, there is an ambiguity in Wigner's account that also mirrors the tension in Kant's discussion of homogeneity, specificity and continuity as both maxims/principles and as characteristics of objects and properties in nature. Wigner claims that without the symmetries of spacetime we would have no 'stability' that could ground our investigation into the laws of nature. But, this seems to imply that while these symmetries must be reflected in our laws, they must also be features of space-time itself. More precisely, they must be identified with the geometrical structure of the physical world, which in turn must be interpreted ontologically, if we are to guarantee the kind of stability Wigner refers to. In other words, it is the ontological aspect of the symmetries that produces the stability required for laws of nature to exist. So, even though we employ symmetries in a kind of meta-theoretical way, the justification for doing so is ultimately ontological.

As we saw above, Kant's remarks sometimes suggest an ontological reading of the transcendental principles via the logical maxims; however, the epistemological program of KrV clarifies the way in which this needs to be understood. The following quote encapsulates the relationship between the epistemic and ontological aspects of the transcendental principles, a relationship that is only possible from within the framework of transcendental idealism:

Now since every principle which prescribes a priori to the understanding thoroughgoing unity in its employment, also holds, although only indirectly, of the object of experience, the principles of pure reason must also have objective reality in respect of that object, not, however, in order to *determine* anything in it, but only in order to indicate the procedure whereby the empirical and determinate employment of the understanding can be brought into complete harmony with itself (A694/B666).

The point here harkens back to the discussion at the beginning about the non-reductive aspects of Kant's critical program. Because of the interactive relationship between the understanding and the objects of experience, we do not isolate foundational features of knowledge in

either objects themselves or our conceptual framework. While Wigner's account of symmetries bears a *prima facie* similarity to Kant's discussion of transcendental principles, the need to 'specify' a location for symmetries, or 'identify' them as part of the physical world, undermines any attempt to strengthen the analogy. A brief reflection on Kant's epistemology reveals why.

The use of symmetries in modern physics raises philosophical questions about their status. Are they ontological features of the world, mathematical objects that function as heuristic devices, epistemic conditions imposed on laws, or perhaps all three? If we opt for the epistemic reading, the most easily defendable of the three, we can simply say that the equations in our theories must obey certain invariances in order for the calculations to give physically meaningful results, i.e. for our theories to be renormalizable. Similarly, Noether's theorem says that for every symmetry of a Lagrangian a corresponding conservation law can be derived. The symmetry here is actually the covariance of the form that the physical law takes with respect to a one-dimensional Lie group of transformations; so, in that sense, it is really a mathematical notion. Although it is associated with an invariance which is the conserved physical, measurable quantity, nothing follows from this about the physical status of the symmetry. In other words, questions still arise as to why symmetries should have epistemic importance over and above their mathematical function in theory construction. My point here is that in order to characterize symmetry principles as transcendental, we must have some principled epistemological reason for doing so. But none emerges outside of the Kantian framework, nor can we embed the practice of modern physics into that framework. We could, of course, simply say that symmetries have the status of logical maxims and function in a heuristic way as constraints on theorizing. But this is not necessarily Kantian in spirit, and as Kant is quick to point out, logical maxims require corresponding transcendental principles for their legitimation.

In her discussion of symmetry principles, Falkenburg suggests that Kant's concept of a systematic unity of Nature provides an interpretation for the unifying function and frequent empirical success of symmetry principles. Elementary particles that are unified through an internal symmetry represent a Kantian system, and the symmetry principles that enable us to discover this structure function as regulative principles. These symmetry principles, like Lorentz invariance, are 'presupposed in the general assumption that there is a systematic unity of Nature, and can be subsumed under the regulative principles which guide the acquisition of knowledge', something Kant derives

from the rational idea of unity.¹⁴ When our employment of symmetries is successful at the phenomenological level, they 'point to a systematic unity of fundamental structures in Nature'. It is the mathematical structure of symmetry groups that makes it possible to search for phenomena that are part of these structures. Mainzer makes a stronger statement for the role of symmetries by claiming that the way they function to determine the characteristics of a physical system (e.g. the form of the interactions) aligns them with what Kant would call 'the conditions for the possibility of an object at all'. 15 In Falkenburg's case the symmetries are part of the presupposition that Nature is unified, but given the necessity of this assumption in Kant's system the symmetries take on a more substantive role than simply regulative heuristics. The problem with this stronger reading and with Mainzer's account is that they elevate symmetry principles to a role that outstrips their function in modern physical theory. In order to associate them with Kant's account of the transcendental necessity and objectivity of ideas of reason, they must have universal applicability. However, as we know, symmetries in physics simply do not enjoy this type of universality.

Before proceeding any further, it is important to note that the Kantian account of a scientific theory involves more than simply the systematization of knowledge. The system must be capable of yielding law-like connections between its parts, connections that require the use of mathematics for their explication. In fact, in MAN (470) Kant explicitly identifies science with the application of mathematics. Given this remark, one option is to embed symmetries into the Kantian program as mathematical objects and account for their relation to physical entities via the strategy for the mathematization of nature described in KrV and MAN. The specific textual details of that strategy are complicated and controversial, and ultimately not necessary for my argument here. Instead let me simply address some of the broader claims made by Kant about mathematization to see whether a role for symmetries might be located within this framework.

3.1 Symmetries as mathematical objects

Some of Kant's remarks about the relationship between mathematics and experience are initially promising; remarks that resemble those made about the principle of continuity: 'Mathematics provides the

Falkenburg (1988), p. 134.
Mainzer (1996), p. 287.

most splendid example of the successful extension of pure reason by itself without the assistance of experience' (A712/B740). This ability to extend knowledge via inferences derived from a priori constructions is what makes mathematics indispensable for physics in the formulation of laws that have both predictive power and certainty. But, because mathematics is concerned not with existence but with the possibility of the relations of things in time and space, we also need metaphysics which deals with what belongs to the existence of things (substance, cause, etc.). As Kant remarks, in 'natural science metaphysics and the art of measurement (the application of mathematics to the measurement of objects in experience) shake hands' (A726/B754).

For our purposes here, the important issue is how this 'shaking hands' actually proceeds. In the case of knowledge extension described above. Kant's reference is to synthetic a priori judgments arrived at through a process of construction in intuition. In his discussion of the axioms of intuition, Kant claims that 'all intuitions have extensive magnitude' is a transcendental principle of the mathematics of appearances and greatly enhances our a priori knowledge. Indeed, it is 'this principle alone that makes mathematics, in its full precision applicable to objects of experience...' (A165/B206). This kind of mathematization of nature has its foundation in the synthesis of space and time, forms of pure intuition, which make possible the apprehension of appearance and every outer experience of objects. Hence, 'whatever pure mathematics establishes in regard to the synthesis of the form of apprehension is also necessarily valid of the objects of apprehension' (A166/B207). In other words, all appearances are given a priori as extensive and intensive magnitudes and consequently subject to mathematization. This, however, is a general claim about the possibility of objects of experience, and their relations to each other in space and time. As Kant points out, "... in mathematical problems existence is not the question, but only the (mathematical) properties of objects in themselves' (A719/ B747). The full integration of metaphysics and mathematics requires that we are able to show how mathematics applies to the kinds of specific objects that physics deals with, i.e. motion, forces etc.; the kinds of objects that are governed by the analogies of experience. This is crucial for natural science since a 'doctrine of nature will only contain as much science proper as there can be mathematics applied in it' (MAN 471). In other words, objects must be mathematizable if they are to be measurable.

In MAN 524-5, Kant stresses that we must start with a metaphysics that explicitly enables the applicability of mathematics to nature,

rather than starting with mathematical principles alone (as in the Galilean tradition) and assuming, without proof, that they apply to physical objects. The project of spelling out the interaction between mathematics and metaphysics begins in KrV and extends through MAN and the OP. Questions about the very ability to apply mathematics to nature and the justification for doing so lay at the heart of Kant's critical program. But, as we shall see below and in the following section, this is a program that embodies marked dissimilarities with the relation between the mathematical and the physical embedded in contemporary physics and also in the physics of Kant's own time. Descartes, for example, professed to have reduced physics to the laws of mathematics (letter to Mersenne, 11 March 1640), and although he may have been less than successful the project of providing a mathematical account of nature was certainly extended by Newton and has become definitive of modern physics as well. Since the time of the Pythagoreans, mathematics and harmony have been the keys to revealing the laws of the universe and contemporary particle physics is a manifestation of this in its use of mathematical symmetry groups.

In modern physics, however, the Cartesian goal of reduction to mathematics seems to have found a new voice. In Dreams of a Final Theory Steven Weinberg, for example, remarks that at the fundamental level symmetries are all there is. 16 This is not an attempt to explain the application of mathematics to nature or to justify that relation but rather to simply reduce the physical to the mathematical. The group theoretical description of the purely kinematical properties like spin for quantum relativistic systems associates each relativistic wave function with some unitary representation of the Poincaré group, and as such one can say that an elementary particle is simply a unitary irreducible representation of the group.¹⁷ Given that definition, an elementary particle can be characterised by its mass and spin where spin turns out to be simply a group invariant characterising the unitary representation of the relativity group associated with the wave equation. Consequently, one thinks of spin not as the physical rotation associated with a particle, but rather as a symmetry, a way of mathematically

¹⁶ Weinberg (1993).

An irreducible representation is one that cannot be split up into smaller pieces, each of which would transform under a smaller representation of the same group. All the basic fields of physics transform as irreducible representations of the Lorentz and Poincaré groups. The complete set of finite dimensional representations of the rotation group O(2) or the orthogonal group comes in two classes, the tensors and spinors.

stating that a system can undergo a certain rotation. Even in the Dirac equation, spin appears as a consequence of the transformation law of the solutions under rotation. The tradition continues with modern attempts at unification such as string theory, which serves as a paradigmatic example of mathematics replacing the physical.

This kind of methodology bears certain similarities to what Kant calls the mathematico-mechanical mode of explication. Its advantage is that one can make quick progress when doing physics because one has the certainty of mathematics and can proceed synthetically because there are no constraints due to the concrete nature of physical existents. 18 However, too much freedom is given to the imagination because the method fails to pursue rigorous explanation which includes, for Kant, an investigation into the forces associated with matter. Hence, this kind of approach is tantamount to metaphysical speculation because the process of applying mathematics to the empirical world involves the unjustified assumption that the world has a mathematical character. By contrast, Kant's account of the justification for the mathematization of nature is bound up with our cognitive principles and with establishing a metaphysical foundation which facilitates the application of mathematics, a project that is markedly different from the mathematization characteristic of early modern and contemporary physics.¹⁹ In that sense then, it would be a mistake to associate the unity achieved in contemporary mathematical physics with Kant's ideas regarding the relation between physics and mathematics. In attempting to assimilate Kant's ideas to a modern framework, there is one final possibility that merits consideration and that is whether Kant's account of forces bears any relation to modern accounts of unity via reduction to fundamental forces.

4. Unity and the reduction to fundamental forces

At the beginning I mentioned how GUTs and Theories of Everything involve the reduction of the four fundamental forces and how symmetry functions as the methodology that structures the mathematical foundations of field theories. I have tried to persuade you that despite their heuristic/methodological role we cannot associate symmetries

Kant identifies this mode of explication with the atomistic or corpuscular philosophy (MAN 533).

The specific details of that story have been systematically spelled out by Friedman (1992a), Plaass (1965) and others.

with Kant's view of systematic unity via transcendental principles. However, if we shift our focus from the transcendental to the empirical level, from the meta-methodology of physics to object level practice, perhaps we can identify reduction to fundamental forces as a unification strategy common to both Kant and modern physics.²⁰

Leaving aside the various intricacies associated with reduction in contemporary physics (i.e. the way the symmetry groups determine the form of the theory) the goal is relatively straightforward and can be captured quite accurately in Kant's claim that all natural philosophy consists in the reduction of given forces apparently diverse to a smaller number of forces: a reduction that carries on to the level of fundamental forces (MAN 534). Kant's account of forces is bound up with his theory of matter and the explication of the dynamical approach to understanding material nature. In fact, Kant claims that the concept of matter (and varieties of matter) should be reduced to moving forces because in space no activity or change can occur apart from motion. In that sense, the possibility of matter is proved by reducing it to these forces which are an a posteriori fact of experience.²¹ So far this picture accords quite well with how forces and particles are understood in modern quantum field theory. While each of the fundamental forces has a corresponding quantum or vector boson associated with it, many contemporary physicists view the force fields as primary.²²

However, the perceived agreement is merely superficial because Kant complicates the matter by asking 'Who claims to comprehend the possibility of fundamental forces?' (MAN 524). And, the situation gets worse, because he further claims that these forces (attractive and repulsive forces *in general*) cannot be constructed (525) nor conceived (conceptualized) (534); they can only be assumed. Actual

I realize, of course, that one cannot in principle separate the transcendental and empirical levels, but what I have in mind here is a claim about how Kant saw reduction and forces as essential features of the practice of physics.

Kant claims that the concept of force supplies us with a 'datum' for a 'mechanical construction' (MAN 498), a requirement for proving the 'real possibility' of matter. Here again the details surrounding Kant's theory of matter rise important philosophical points of interpretation, see especially Friedman (2001b) and Carrier (2001).

There is a good deal of philosophical debate about the nature of the field and the role of particles in QFT. While these debates are extremely interesting, the details are not really relevant for the issue I am addressing in the paper.

forces 'can only be given empirically', because the possibility of fundamental forces can never be made comprehensible (A207/B252).

Given the importance of forces in Kant's theory of physics we need to ask what exactly these claims might mean—how should we interpret them and what are the implications for empirical science? A physical concept that is inconceivable (MAN 513) is one that cannot be derived from another more basic concept, and fundamental forces are those that are not further derivable. The challenge then for Kant's dynamical picture is to connect these forces (and hence matter) with mathematics to produce a unified physics that is not grounded in contingency. In other words, the coherency required for systematization cannot come via reduction to fundamental forces alone because, given the remarks above, they lack the status required to underwrite the project. While we experience forces given in nature, fundamental forces in general appear to occupy a different role.

So, what exactly are fundamental forces and how do they function in the context of physical theory? Do they bear any resemblance to the fundamental forces prominent in contemporary physics? A return to KrV provides a clue. At A648-50/B676-78 Kant discusses what he calls the causality of a substance, a power or force (Kraft). Despite the appearances of many different forces we are required, by a logical maxim, to reduce the diversity of these forces and hence detect a hidden identity. Although logic is not capable of deciding whether a fundamental force exists, the idea of such a force is the 'problem involved in a systematic representation of the multiplicity of forces' (A649/B677). In other words, the notion of a fundamental force acts like a regulative idea of reason that guides our empirical inquiry. This enables us to compare the relatively fundamental forces in order to ascertain the similarities between them with the goal of bringing them nearer to a 'single radical absolutely fundamental force'.

While Kant does not claim that such a force (power) is necessarily realised, he claims we must seek it in the interests of reason, as part of the endeavour to bring systematic unity to our knowledge of experience. However, like the ideas of homogeneity, continuity and specification, when we pass to the transcendental employment of the understanding we find that the notion of a fundamental force is not understood simply as a problem for the hypothetical use of reason, but it is taken to have objective reality. This reality consists not only in virtue of the postulation of systematic unity of various forces of a substance, but also as giving expression to an apodictic principle of reason (A650/B678). But, says Kant, even if we fail in the reduction of diverse forces we still presuppose that such a unity

exists, not only for the case we are considering, but for matter in general. 'In all such cases reason presupposes the systematic unity of the various forces, on the ground that special natural laws fall under more general laws, and that parsimony in principles is not only an economical requirement of reason, but is one of nature's own laws' (A651/B679). In fact, he goes on to say that the employment of a logical principle requires that we also presuppose a transcendental principle, whereby systematic unity is a priori assumed to be necessarily inherent in objects. Otherwise, reason cannot treat the diversity in nature as disguised unity and derive this unity from a fundamental force. If reason is free to admit that all forces are heterogeneous, then the search for such unity would be inconsistent with the constitution of nature. Hence, the law of reason that bids us to seek unity is necessary since without it we would have no reason, no coherent employment of the understanding and no criterion of empirical truth.

The claim, then, is that Kant's notion of a fundamental force has essentially the same status as the transcendental principles discussed above. The process of reducing given powers or forces to a smaller number, in accordance with the logical demands of reason, is part of the empirical process of doing physics and to that extent we need to presuppose that objects of experience conform to the demands placed on the understanding by reason. But, the idea that we could actually complete this reduction and discover a fundamental force is a transcendental principle, and, as such, the product is not to be met with in experience. Moreover, as in the case of the logical maxims, the very notion of a fundamental force or power functioning as a methodological rule only makes sense if it is grounded in a transcendental principle. We must, however, be cautioned against associating this with a *metaphysical* claim about how nature is constituted. The following quotation nicely encapsulates the subtlety of Kant's position:

the investigation of metaphysics behind what lies at the basis of the empirical concept of matter is useful only for the purpose of leading natural philosophy as far as possible in the investigation of the dynamical grounds of explication, because these alone admit hope of determinate laws, and consequently of a true rational coherence of explanations. This is all that metaphysics can ever hope to accomplish...and hence on behalf of the application of mathematics to natural science respecting the properties by which matter fills space....to regard these properties as dynamical and not as unconditioned original

positions....as a merely mathematical treatment would postulate (MAN 534).

To assume we can reach this fundamental level as a product of empirical inquiry and to assume that we can apply, without prior justification and argument, mathematics to empirical objects is to adopt the metaphysics of transcendental realism Kant hoped to banish in the antinomies. So, while the *practice* of reduction and the quest for fundamental forces in modern physics has an exact analogue in Kant's theory of science, the justification of that practice and the product could not be more different. The reduction of forces in modern physics is grounded in the assumption that we can complete the project, a project that for Kant, cannot, even in principle, be completed. That is to say, embedded in the practice of modern physics is a metaphysics that is ultimately incoherent.

5. Conclusions

We have seen then that neither the current approach to unity via symmetries as the methodology of modern physics, nor the ontological project of reduction to fundamental forces can be given a Kantian interpretation or justification. Although some empirical similarities exist between the two frameworks, it would be a mistake to identify these too closely, since any Kantian construal will ultimately locate the justification for the practice of unification and reduction not in the physics itself but in human reason; a view that seems decidedly at odds with the metaphysical realism implicit in contemporary physics. In some places this difference seems less evident, as in Kant's remark about the status of laws within his notion of a system. For example, we presuppose the unity of various powers on the ground that special natural laws fall under more general ones and that 'parsimony in principles is not only an economical requirement of reason but one of nature's own laws' (A650/B678). Because for Kant this has the status of a presupposition, its justification is ultimately linked with what is necessary in order to have experience in the first place and what is necessary for a specific kind of scientific experience. At A657/B685 Kant claims that we have an understanding only on the assumption of varieties in nature and that nature's objects exhibit a certain homogeneity. This is because it is only through the diversity that is contained in a concept that its application can be sanctioned; and that use of concepts is nothing other than the activity of the understanding. In other words, the question of

whether systematic unity is an objective fact about nature is simply the wrong question to ask; not only is it objective, it is necessary.

In the end Kant's necessity of systematicity finds its expression in the faculty of reflective judgment, as outlined in the First Introduction to the Third Critique (the Critique of Judgment) and in that Critique itself. That transition raises many questions about the status and justification of systematicity and whether Kant can successfully uphold the project he outlined in KrV. Answers to those questions require a separate work.²³ In closing I would like to simply draw attention to what I think is perhaps one of Kant's most ingenious remarks, one that embodies what I hope philosophy of physics has learned from Kant even if the practice of physics has not. The remark concerns what he calls the second advantage of metaphysics which is '...knowing what relation the question has to empirical concepts, upon which all our judgments must at all times be based' (Ak. 2:368).²⁴ Kant claims that this advantage is the least known and most important and is attained at a fairly late stage after long experience. Surely, this is one of if not the most important task(s) for philosophy of science, and it is here where Kant's legacy is most prominently felt. In attempting to trace a lineage from Kant's epistemological work and its relation to his theory of physics to the methodology of present-day science, we must be ever mindful of Kant's justification of the practice of mathematical physics. But that justification appears, in most respects, fundamentally at odds with the presuppositions motivating the contemporary search for unity and reduction to fundamental forces. Of course, this is not to say that contemporary physics would not benefit from a Kantian reconstruction, but only to point out that its practices do not reflect those philosophical leanings.

⁴ Kant (1766), English translation (1992), p. 354.

For a detailed discussion of these issues, see Guyer (1990).