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CONSUMPTION, LEISURE, AND MONEY

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This paper takes a parametric approach to demand analysis and tests the weak separability assumptions that are often implicitly made in representative agent models of modern macroeconomics. The approach allows estimation and testing in a systems-of-equations context, using the minflex Laurent flexible functional form for the underlying utility function and relaxing the assumption of fixed consumer preferences by assuming Markov regime switching. We generate inference consistent with both theoretical and econometric regularity. We strongly reject weak separability of consumption and leisure from real money balances as well as weak separability of consumption from leisure and real money balances, meaning that the inclusion of a money in economic models would be of quantitative importance. We also investigate the substitutability/complementarity relationship among different categories of personal consumption expenditure (nondurables, durables, and services), leisure, and money. We find that the goods are net Morishima substitutes, but because of positive income effects they are gross complements. The implications for monetary policy are also briefly discussed.

Keywords: Weak Separability, Minflex Laurent Functional Form, Markov Regime Switching.

1. INTRODUCTION

We test the weak separability assumptions that are often implicitly made in many areas of macroeconomic research. Consider, for example, the representative agent's utility function

$$U = f\left(\boldsymbol{c}, \ell, m\right), \tag{1}$$

where $\boldsymbol{c}$ is a vector of the services of consumption goods, ℓ is leisure time, and m is real money balances. It is typically assumed that the utility function (1) is weakly separable in consumption, $\boldsymbol{c}$, from leisure, ℓ, and money, m, implying the utility tree

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$$U = f\left(u\left(\boldsymbol{c}\right), \ell, m\right), \tag{2}$$

and the existence of a consumption aggregate, $u(\boldsymbol{c})$.

Similarly, assuming that the utility function (1) is weakly separable in consumption and leisure from money, implies the utility tree

$$U = f\left(u\left(\boldsymbol{c}, \ell\right), m\right), \tag{3}$$

and suggests that the demand for consumption goods, $\boldsymbol{c}$, and leisure, ℓ, is independent of money, m, as in most of the equilibrium business cycle literature.

Utility tree structures like those in (2) and (3) are frequently treated as maintained hypotheses in both theoretical and applied demand analyses. According to the original definition of weak separability by Leontief (1947) and Sono (1961), they imply that the marginal rate of substitution between any two goods in the weakly separable group is independent from the quantities consumed of any good outside this group. As Cherchye et al. (2015, p. 129–130) put it, "weak separability has several convenient implications. First of all, it allows for representing consumption in terms of two-stage budgeting. This means that, in order to determine the demanded quantities of the goods in the separable group, it suffices to know the prices of the goods in this group and the total within-group expenditure. Further, weak separability is a crucial condition for the construction of group price and quantity indices. Such aggregates can be useful, for example, to compute group cost of living indices to be used in the welfare analysis. Finally, from an empirical point of view, weak separability significantly reduces the number of parameters of the demand system to be estimated in practical applications."

Background. In the literature, there are two approaches to test for weak separability. One approach is "nonparametric," in the sense that it requires no specification of the form of the utility function. This approach, originally developed by Varian (1982), uses the techniques of revealed preference analysis and has been used extensively over the years to test for consistency with preference maximization and for the existence of consistent aggregates. For example, Hjertstrand et al. (2016), building on earlier work by Swofford and Whitney (1987, 1988, 1994), investigate the validity of two important assumptions underlying representative agent models of modern macroeconomics—utility maximization and weak separability—by checking the revealed preference conditions for these assumptions. See also Fleissig and Rossana (2003), Fleissig and Whitney (2003), Jones and De Peretti (2005), and Barnett and de Peretti (2009) for some other applications.

There are advantages and disadvantages of the revealed preference approach to demand analysis. In particular, this approach can be applied to data sets with a small number of observations (thus avoiding degrees of freedom problems) and requires no specification of the form of the utility function (thus being insensitive to model misspecification). However, the revealed preference approach is not without problems. As Fleissig et al. (2000, p. 329) put it, “the main disadvantage is that the tests are not stochastic. Violations are all or nothing; either there is a utility function that rationalizes the data or there is not.” It is also to be noted that the majority of the early studies in the revealed preference approach assume that observed quantities and prices are measured accurately and do not allow for measurement error. In recent years, however, accounting for measurement error in revealed preference tests has been an active research area. See, for example, Fleissig and Whitney (2005, 2008), Elger and Jones (2008), Jones and Edgerton (2009), Cherchye et al. (2015), and Aguiar and Kashaev (2018). It is important to note, however, that although it is possible to account for measurement error in the revealed preference framework, these tests are designed only for measurement error and not for any general notion of stochastic noise. This is different from the parametric approach to demand analysis where the additive error could, in principle, be interpreted to account for more general noise than just measurement error.

Contribution. In this paper, we take the parametric approach to demand analysis to test the weak separability assumptions that are often implicitly made in representative agent models of modern macroeconomics. It allows estimation and testing in a systems context assuming a flexible functional form for the utility function, based on the dual approach to demand system generation developed by Diewert (1974). However, the usefulness of flexible functional forms depends on whether they satisfy the theoretical regularity conditions of positivity, monotonicity, and curvature, and in the older demand systems literature there has been a tendency to ignore regularity. In fact, as Barnett (2002, p. 199) put it in his *Journal of Econometrics* Fellow’s opinion article, without satisfaction of all three theoretical regularity conditions “ ... the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid.” Motivated by these considerations, we pay explicit attention to theoretical regularity, treating the curvature property as a maintained hypothesis.

We use the minflex Laurent (ML) demand system, introduced by Barnett (1983) and Barnett and Lee (1985), which is based on the Laurent series expansion. Moreover, instead of assuming that consumer preferences are fixed as in neoclassical demand theory, we assume Markov regime switching, allowing for complicated nonlinear dynamics and sudden changes in the parameters of the demand functions and the underlying utility function. The Markov switching approach, associated with Hamilton (1989), has been widely followed in the analysis of economic and financial time series—see, for example, Sims and Zha (2006). We model the flexible ML demand system as a function of an unobserved regime-shift variable, governed by a first-order, two-state Markov process.

We use quarterly data for the USA, over the period from 1967:Q1 to 2016:Q4, and in addition to testing the weak separability assumptions underlying a large class of representative agent models of modern macroeconomics, including the real business cycle model, we investigate the substitutability/complementarity relationship among different categories of personal consumption expenditure (nondurables, durables, and services), leisure, and money.

Layout. The rest of the paper is organized as follows. Section 2 briefly sketches related neoclassical demand theory and applied demand analysis. Section 3 presents the ML model, derives the associated system of budget share equations, and discusses the method of imposing global curvature with the objective of achieving theoretical regularity. Section 4 discusses the Markov regime-switching approach and presents the stochastic Markov switching ML demand system. Section 5 discusses the data, Section 6 presents the weak separability results, and section 7 presents the full set of elasticities, regarding the degree of substitution among the different goods. Section 8 discusses the implications of our research for macroeconomics and monetary economics research, and the final section briefly concludes the paper.

2. NEOCLASSICAL DEMAND THEORY

Let us consider an economy with identical households with the following utility function:

$$U = f\,(n, s, d, \ell, m)\,, \tag{4}$$

where n is the real consumption of nondurable goods, s is the real expenditure on services, d is the real service flow from the stock of durable goods, ℓ is leisure time, and m is the real value of money held by the consumer.

The representative household's optimization problem can then be written as

$$\max_{\boldsymbol{x}} u(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} \leq y$$

where $\boldsymbol{x} = (n, s, d, \ell, m)$ and $\boldsymbol{p} = (p_n, p_s, p_d, p_\ell, p_m)$ is the corresponding vector of prices. The solution of the first-order conditions is the Marshallian demand functions

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{p}, y), \tag{5}$$

and the indirect utility function $h(\boldsymbol{p}, y)$. The demand system (5) can also be represented in budget share form $\boldsymbol{w}$, where $\boldsymbol{w} = (w_1, w_2, ..., w_k)$, and $w_j = p_j x_j(\boldsymbol{p}, y)/y$ is the expenditure share of good j. Since the Marshallian demand functions are homogenous of degree zero in $\boldsymbol{p}$ and y, we could write the demand system in budget share form as

$$w_j = w_j(\boldsymbol{v}), \tag{6}$$

where $\boldsymbol{v} = (v_1, v_2, ..., v_k)$ with v_j denoting the income normalized price, p_j/y.

An effective method to derive the demand system in budget share form is to apply Diewert's (1974) modified version of Roy's identity to the indirect utility function, $h(\boldsymbol{p}, y)$—see Barnett and Serletis (2008) for more details.

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3. THE MINFLEX LAURENT MODEL

We use the flexible functional forms method to the approximation of aggregator functions. The advantage of this method is that the corresponding demand system will adequately approximate systems resulting from a broad class of aggregator functions. In particular, we use the ML demand system, documented in detail in Barnett (1983) and Barnett and Lee (1985), to approximate the indirect utility function

$$h(\boldsymbol{v}) = c + 2\boldsymbol{\delta}'\sqrt{\boldsymbol{v}} + \sum_{i=1}^{k} d_{ii}\nu_i + \sum_{i=1}^{k}\sum_{j=1,\, j\neq i}^{k} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} - \sum_{i=1}^{k}\sum_{j=1,\, j\neq i}^{k} h_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}}, \tag{7}$$

where k denotes the number of goods (in our case, $k = 5$), ν_i denotes the income normalized price (p_i/y), c is a constant, and $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_k)'$, and d_{ij} and h_{ij} are all parameters. The ML model is based on a Laurent series expansion, which is a generalization of the Taylor series expansion. It is also known as the ML generalized Leontief model, as it represents a generalization of the generalized Leontief model, proposed by Diewert (1974), and which is based on a Taylor series expansion. See Barnett (1985) for more details.

The application of Roy's identity to (7) yields the share equations of the ML demand system (for $i = 1, \ldots, k$)

$$w_i = \frac{\delta_i \nu_i^{\frac{1}{2}} + d_{ii}\nu_i + \sum_{j=1, j\neq i}^{k} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} + \sum_{j=1, j\neq i}^{k} h_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}}}{\boldsymbol{\delta}'\sqrt{\boldsymbol{v}} + \sum_{i=1}^{k} d_{ii}\nu_i + \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} - \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} h_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}}}. \tag{8}$$

Note that the share equations (8) are homogeneous of degree of zero in the parameters. Therefore, following Barnett and Lee (1985), we impose the normalization

$$\sum_{i=1}^{k} d_{ii} + 2\sum_{i=1}^{k} \delta_i + \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} d_{ij}^2 - \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} h_{ij}^2 = 1, \tag{9}$$

and the restrictions

$$d_{ij} = d_{ji}, \quad h_{ij} = h_{ji}, \quad d_{ij}h_{ij} = 0, \quad i \neq j. \tag{10}$$

Finally, the curvature condition could be globally imposed by replacing all non-squared parameters by squared parameters—see Barnett (1983).

In order to estimate demand systems such as (8), a stochastic version is specified, assuming that the observed share in the ith equation deviates from the true share by an additive disturbance term ϵ_i. The stochastic specification can be written in matrix notation as

$$\boldsymbol{w}_t = \boldsymbol{w}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t, \tag{11}$$

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where $\boldsymbol{\varepsilon}_t = (\epsilon_{1t}, ..., \epsilon_{kt})'$ is a vector of classical disturbance terms and $\boldsymbol{\theta}$ is the parameter vector to be estimated. Note that the shares satisfy the adding-up property (i.e., they sum to one) and thus the errors also satisfy adding-up (they sum to zero). It is also typically assumed that the resulting disturbance vector $\boldsymbol{\varepsilon}$ is a classical disturbance term, $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \Omega)$, where $\mathbf{0}$ is a null matrix and Ω is the $k \times k$ symmetric positive definite error covariance matrix.

4. THE MARKOV SWITCHING MINFLEX LAURENT

Do consumer preferences change with changes in the overall economic environment? Guiso et al. (2018) find that qualitative and quantitative measures of risk aversion increase substantially after financial crises, and that fear is a potential mechanism that influences financial decisions, whether by increasing the curvature of the utility function or the salience of bad outcomes. Certainly, there are other possible factors such as commodity price shocks, large-scale events (such as wars and financial crises), changes in government policy (such as the introduction of inflation targeting), and technological and institutional changes that might induce significant shifts in consumer tastes and preferences over goods. In this regard, Gordon and St-Amour (2000) show that the regime-switching preferences of households could successfully resolve the equity premium, risk-free rate, and predictability puzzles. They also find that the regime-switching preferences of households are able to explain the sharp swings in assets prices that are characteristic of financial market data.

In this paper, in order to relax the assumption of constant parameters in the aggregator function (7) and the resulting demand system (11), we follow Xu and Serletis (2019) and take the Markov switching approach, associated with Hamilton (1989). We model (8) as a function of an unobserved regime-shift variable, z_t, governed by a first-order Markov process. In this regard, Xu and Serletis (2019) assume that z_t follows a first-order, homogeneous, m-state Markov chain governed by the transition matrix

$$\Pi = \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}$$

where $p_{ji} = P[z_t = j \,|z_{t-1} = i]$, $i, j = 1, ..., m$, and $p_{ji} = 1 - \sum_{k \neq j} p_{ki}$ is the probability of regime j in period t given that the system was in regime i in period $t-1$. In this paper, we assume two regimes, as suggested by Hamilton (1988, 1989), but augment the setup of the transition matrix by allowing time variation in the transition probabilities, as follows:

$$\Pi_t = \begin{pmatrix} p_{11,t} & p_{12,t} \\ p_{21,t} & p_{22,t} \end{pmatrix}, \tag{12}$$

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where $p_{11,t} + p_{21,t} = 1$ and $p_{12,t} + p_{22,t} = 1$, and

$$p_{11,t} = \varphi(\alpha_1 + \beta_1 g_{t-1}) \tag{13}$$

$$p_{22,t} = \varphi(\alpha_2 + \beta_2 g_{t-1}), \tag{14}$$

where α_1, α_2, β_1, and β_2 are all parameters, $\varphi(\cdot)$ is the cumulative density function of a standard normal distribution, and g_{t-1} is the lagged real GDP (z_t) growth rate. Time variation in the transition probabilities allows for better modeling compared with fixed transition probabilities—see, for example, Filardo (1994). Also, we use the real GDP growth rate to model the transition probabilities, because we believe that real GDP growth is the best indicator for the overall macroeconomic environment. This setup is also consistent with the definition of the business cycle, consisting of an expansion and a contraction. Moreover, having two regimes renders the estimation tractable, as the addition of a third regime significantly increases the number of parameters.

Thus, the stochastic Markov regime-switching ML demand system (8) is written as

$$w_{it} = \frac{\delta_{i,z_t} v_{it}^{\frac{1}{2}} + d_{ii,z_t} v_{it} + \sum_{j=1,j\neq i}^{k} d_{ij,z_t}^2 v_{it}^{\frac{1}{2}} v_{jt}^{\frac{1}{2}} + \sum_{j=1,j\neq i}^{k} h_{ij,z_t}^2 v_{it}^{-\frac{1}{2}} v_{jt}^{-\frac{1}{2}}}{\boldsymbol{\delta}_{z_t}' \sqrt{\boldsymbol{v}_t} + \sum_{i=1}^{k} d_{ii,z_t} v_{it} + \sum_{i=1}^{k} \sum_{j=1,j\neq i}^{k} d_{ij,z_t}^2 v_{it}^{\frac{1}{2}} v_{jt}^{\frac{1}{2}} - \sum_{i=1}^{k} \sum_{j=1,j\neq i}^{k} h_{ij,z_t}^2 v_{it}^{-\frac{1}{2}} v_{jt}^{-\frac{1}{2}}} + \epsilon_{it,z_t}, \tag{15}$$

for $i = 1, \ldots, k$. The normalization (9) is written as

$$\sum_{i=1}^{k} d_{ii,z_t} + 2\sum_{i=1}^{k} \delta_{i,z_t} + \sum_{i=1}^{k} \sum_{j=1,j\neq i}^{k} d_{ij,z_t}^2 - \sum_{i=1}^{k} \sum_{j=1,j\neq i}^{k} h_{ij,z_t}^2 = 1, \tag{16}$$

and the restrictions in (10) as

$$d_{ij,z_t} = d_{ji,z_t}, \quad h_{ij,z_t} = h_{ji,z_t}, \quad d_{ij,z_t} h_{ij,z_t} = 0, \quad i \neq j. \tag{17}$$

It should be noted that the imposed restrictions are necessary in terms of achieving the parametric minimality property in Barnett (1985) and do not affect the properties of the ML model. However, one might be concerned with how the Markov regime switching affects the flexibility of the model. We believe that Markov regime switching does not alter the properties of a demand system; it just allows the parameters of the demand system to change across the time dimension.

5. DATA

We use quarterly data for the USA, over the period from 1967:Q1 to 2016:Q4, and follow important contributions in related research areas in order to construct the series that we use in our parametric demand analysis. In particular, we use data on expenditures and prices on nondurable goods, n, durable goods, d, and services, s, from the US Bureau of Economic Analysis. We follow Ni (1995) and Ogaki and

Reinhart (1998) and calculate the service flow from the stock of durable goods as

$$\text{flow} = \zeta(k_{t-1} + d_t)$$

where ζ is the quarterly depreciation rate, k_{t-1} is the stock of durable goods in period $t-1$, and d_t is the expenditure on durable goods. We construct the stock of durable goods based on a benchmark stock for 1966:Q4 from Musgrave (1979), following Fleissig and Rossana (2003). Regarding the user cost of durable goods, we also follow Ni (1995) and Ogaki and Reinhart (1998) and use

$$p_{flow,t} = p_{d,t} - \frac{1-\zeta}{1+R_t} E_t p_{d,t+1},$$

where $p_{d,t}$ is the price of durable goods, R_t is the nominal interest rate, and $E_t p_{d,t+1}$ is the expected price of durable goods. As in Ni (1995), we use a depreciation rate ζ of 6%. The nominal interest rate is the 3-month treasury bill rate from the Federal Reserve Bank of St. Louis. The expected price of durable goods is calculated based on static expectations.

We also use average hourly earnings and average weekly hours of production and non-supervisory employees from the Federal Reserve Bank of St. Louis, and calculate leisure time as in Hjertstrand et al. (2016), who calculate leisure time as 98 h minus average hours worked per week. We use the Divisia M4 monetary aggregate and its user cost from the Center for Financial Stability—see Barnett et al. (2013) for more details regarding the Divisia monetary aggregates. In this regard, we would like to note that the superior performance of the (market-based) Divisia monetary aggregates has been demonstrated by a large number of studies, including Barnett and Chauvet (2011), Hendrickson (2014), Serletis and Gogas (2014), and Belongia and Ireland (2014, 2015), among others. More recently, Jadidzadeh and Serletis (2019) and Dery and Serletis (2019) provide evidence that supports and reinforces Barnett's (2016) assertion that we should use, as a measure of money, the broadest Divisia M4 monetary aggregate, and this is what we do in this paper. The real GDP data series that we use to model the time-varying probabilities is from the Federal Reserve Bank of St. Louis. Finally, we convert all the expenditure series into real per capita terms using the corresponding price series and population.

In Figures 1–5, we provide graphical representations of the normalized price (or user cost), normalized quantity, and share for each of the five goods that enter as arguments in the representative agent's utility function (4)—nondurable goods, n, services, s, durable goods, d, leisure, ℓ, and real money balances, m, respectively. The normalization is based on the median quarter without altering the shares as in Barnett (1983). Our sample starts in 1967:1, when the Center for Financial Stability Divisia monetary aggregates became available, and includes the increased volatility in the money supply in the aftermath of the global financial crisis.

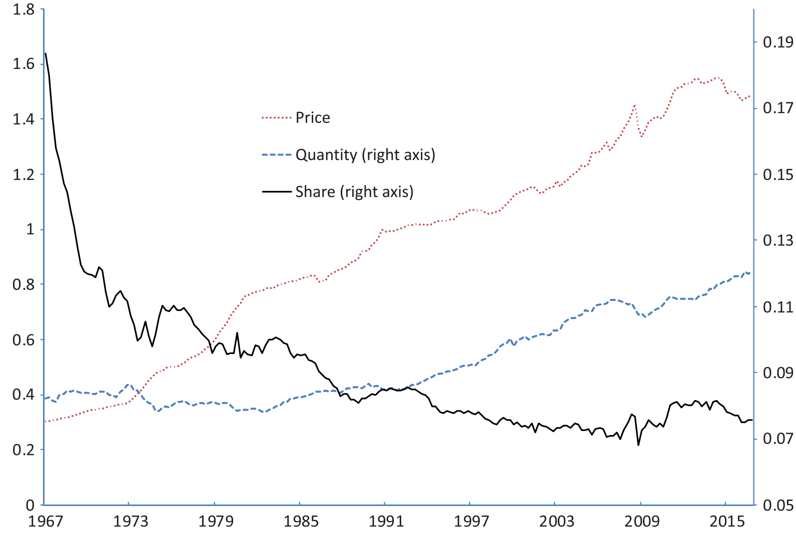

FIGURE 1. Nondurable goods.

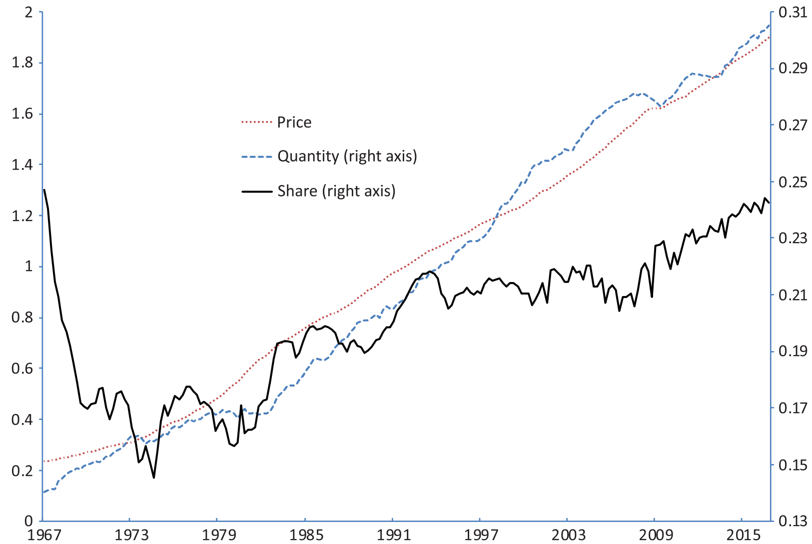

FIGURE 2. Services.

6. ECONOMETRIC ISSUES

As shown by Xu and Serletis (2019), Markov regime-switching demand systems can be estimated using the maximum likelihood method by deleting an arbitrary equation. Moreover, the parameter estimates are invariant with respect to the equation (good) deleted—see Theorems 1 and 2 in Xu and Serletis (2019). Thus,

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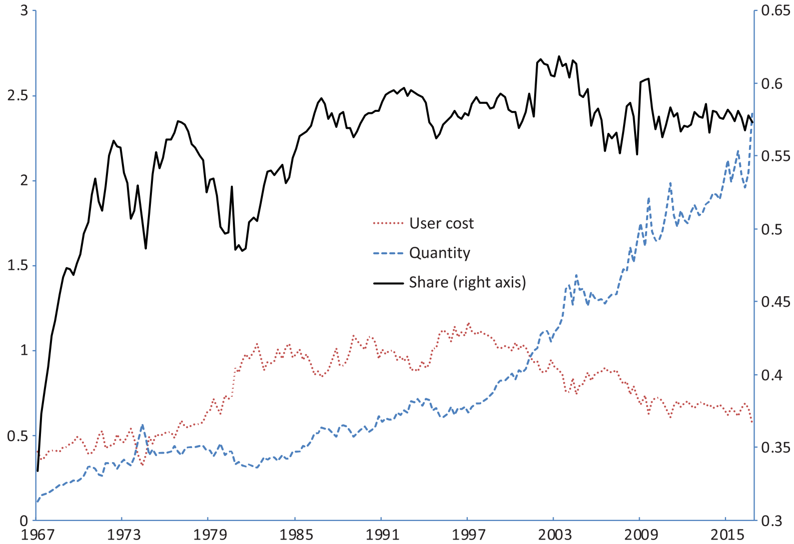

FIGURE 3. Durable goods.

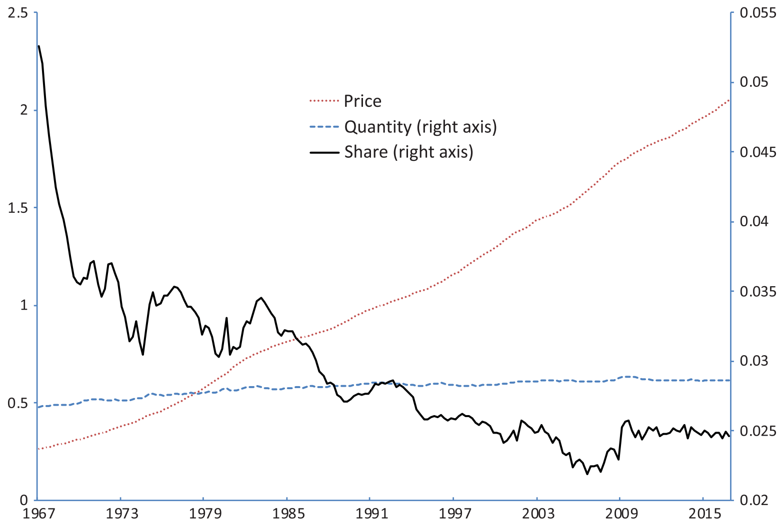

FIGURE 4. Leisure.

we estimate the five-good Markov regime-switching ML demand system, consisting of equations (12), (13), (14), (15), (16), and (17), by deleting an arbitrary equation.

We pay explicit attention to the theoretical regularity conditions of positivity, monotonicity, and curvature. As noted by Barnett (2002), without satisfaction of theoretical regularity, "the second-order conditions for optimizing behavior fail,

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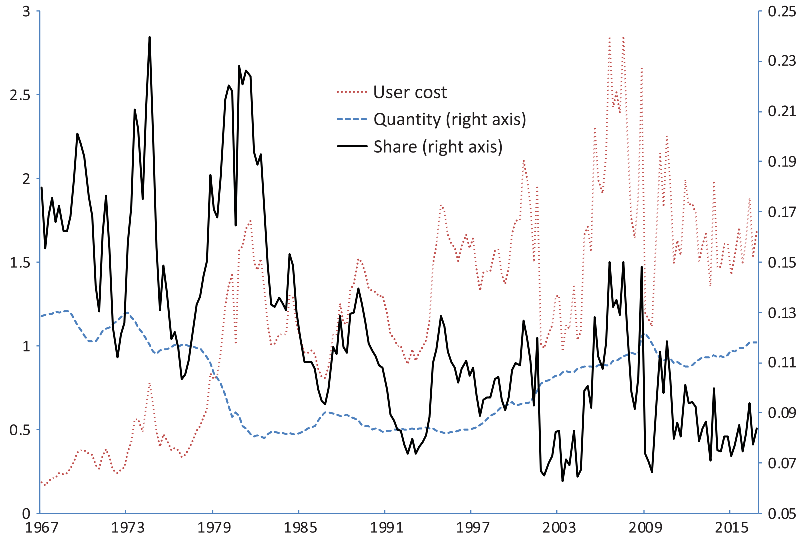

FIGURE 5. Divisia M4.

and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid." The theoretical regularity conditions can be checked as follows (see Barnett and Serletis (2008) for more details):

- Positivity is checked by direct computation of the estimated indirect utility function $\widehat{h}(\boldsymbol{v})$; it is satisfied, if $\widehat{h}(\boldsymbol{v}) > 0$, for all t.
- Monotonicity is checked by direct computation of the values of the first gradient vector of the estimated indirect utility function; it is satisfied, if $\nabla\widehat{h}(\boldsymbol{v}) < 0$.
- Curvature requires that the Slutsky matrix be negative semidefinite and can be checked by performing a Cholesky factorization of that matrix; it is satisfied, if the Cholesky values are nonpositive. See Lau (1978, Theorem 3.2).

With Markov regime switching, we check the theoretical regularity conditions across each of the two regimes. To do so, we follow Kim (1994) and calculate the smoothed probabilities

$$p(z_t = i|I), \quad i = 1, 2$$

where I is the information of the full sample. We then use these smoothed probabilities to assign each data point to the corresponding regime, as follows:

$$\text{Data point at time } t \begin{cases} \text{belongs to regime } i, & \text{if } p(z_t = i|I) \geq 0.95 \\ \text{does not belong to regime } i, & \text{otherwise.} \end{cases}$$

That is, the separation of the data points into the two regimes is done with a 95% confidence level. As we find some violations of curvature when the system is

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estimated without global imposition of curvature, we impose curvature by replacing all of the non-squared parameters by squared parameters, as suggested by Barnett (1983), thus treating the curvature property as a maintained hypothesis.

We also pay explicit attention to econometric regularity, as serially correlated residuals are commonly reported in demand systems estimation. In doing so, we test for residual serial correlation in the Markov regime switching ML demand system by calculating the one-step standardized residuals and applying the Q-test. We find significant residual serial correlation and follow Berndt and Savin (1975) to assume an AR(1) process

$$\boldsymbol{\varepsilon}_{t,z_t} = \rho_{z_t}\boldsymbol{\varepsilon}_{t-1,z_{t-1}} + \boldsymbol{\xi}_{t,z_t},$$

where ρ_{z_t} is the serial correlation coefficient, which is regime-dependent, and

$$\boldsymbol{\xi}_{t,z_t} \sim N\left(\mathbf{0}, \boldsymbol{\Theta}_{z_t}\right).$$

However, with Markov regime switching, $\boldsymbol{\varepsilon}_{t,z_t}$ depends on z_t and also indirectly on $\{z_{t-1}, z_{t-2}, \ldots\}$. That is, the error term at time t depends on the entire sequence of regimes up to time t. One has to construct the likelihood function by integrating over all possible paths and as it turns out the estimation is not tractable; this is a problem that typically shows up in the estimation of regime-switching GARCH models. To address this issue, we follow Klaassen (2002) and integrate out the regime z_{t-1} at time $t-1$ using the information of z_t. Therefore, our specification is

$$\boldsymbol{\varepsilon}_{t,z_t} = \rho_{z_t}E_{t-1}\left(\boldsymbol{\varepsilon}_{t-1,z_{t-1}}\middle| z_t\right) + \boldsymbol{\xi}_{t,z_t}, \tag{18}$$

suggesting that the current regime could provide enough information about the previous regime—see also Klaassen (2002). This specification is meaningful since regimes are normally persistent. After integrating out z_{t-1}, the error term $\boldsymbol{\varepsilon}_{t,z_t}$ at time t depends only on the current regime z_t, and the estimation becomes tractable. In particular, the constructed error term at time $t-1$ is given by

$$E_{t-1}\left(\boldsymbol{\varepsilon}_{t-1,z_{t-1}}\middle| z_t\right) = \sum_{i=1}^{2} p(z_{t-1}=i|z_t, I_{t-1})\boldsymbol{\epsilon}_{t-1,\, z_{t-1}=i}.$$

Thus, the two-regime Markov switching demand system that incorporates (18) is given by

$$\boldsymbol{w}_t = \boldsymbol{w}\left(\boldsymbol{\vartheta}\,|\,\boldsymbol{v}_t, z_t\right) + \rho_{z_t}\left(\boldsymbol{w}_{t-1} - \sum_{i=1}^{2} p(z_{t-1}=i|z_t, I_{t-1})\boldsymbol{w}\left(\boldsymbol{\vartheta}\,|\,\boldsymbol{v}_{t-1}, z_{t-1}=i\right)\right) + \boldsymbol{\xi}_{t,z_t},$$

and

$$\boldsymbol{\xi}_{t,z_t} \sim N\left(\mathbf{0}, \boldsymbol{\Theta}_{z_t}\right).$$

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Note that the two-regime Markov switching ML demand system with AR(1) errors still satisfies adding-up and symmetry. Thus, maximum likelihood estimation is possible while maintaining invariance with respect to the equation (good) deleted in the estimation of the model.

7. EMPIRICAL EVIDENCE

We report the estimation results and some hypotheses tests in Table 1. For comparison purposes, we report the results for the single-regime ML model in column 1 and those for the two-regime ML model in columns 2 and 3. In both cases, the model is estimated with the curvature conditions imposed and AR(1) errors as discussed earlier.

According to Table 1, there are significant differences between the single- and two-regime ML models. We find that the single-regime model with curvature and serial correlation correction still produces serially correlated standardized residuals, as can be seen from the $Q(10)$ test statistic p-values for each of the four standardized residuals; $Q(10)$ is the Ljung and Box (1978) Q-statistic for testing serial correlation, (asymptotically) distributed as a $\chi^2(36)$ on the null of no autocorrelation. The two-regime ML model performs better in terms of the Q-statistic; we do notice that the Q-statistic associated with one of the one-step standardized residuals is significant, but the corresponding correlogram (not shown here) indicates that the degree of correlation is low and does not have a pattern.

The superiority of the two-regime Markov switching ML model with curvature imposed and AR(1) errors is also supported by the approximated Bayes factor, when we compare it with its restricted version (the single-regime ML model). In particular, the Bayes factor (BF) is

$$\text{BF} = \frac{\Pr(\text{Data}\,|\text{Unrestricted model}\,)}{\Pr(\text{Data}\,|\text{Restricted model}\,)}.$$

According to Kass and Raftery (1995), the logarithm of the Bayes factor could be approximated using the Bayesian information criterion (BIC), as follows:

$$2\ln \text{BF} = -\,(\text{BIC of unrestricted model} - \text{BIC of restricted model})$$

and if $2\,\ln\text{BF} > 10$ then the superiority of the unrestricted model is decisive. In our case, $2\,\ln\text{BF} = 137.164$, which is strong evidence that the two-regime Markov switching ML model is superior to its restricted version.

To better understand the two regimes, in Figure 6, we plot the smoothed probability of regime 1, $p(z_t = 1|I)$, as well as the variance of the real GDP growth rate, the latter calculated using a moving window of four quarterly observations. We interpret the variance of the real GDP growth rate as an indicator of the level of uncertainty in the economy. As can be seen, regime 1 is generally consistent with contractions in the business cycle and increased economic uncertainty. It includes the 1967–1975 period, which covers the two National Bureau of

TABLE 1. Parameter estimates for the ML model with curvature imposed and AR(1) errors

Goods:

1 = Nondurables; 2 = Services; 3 = Durables; 4 = Leisure; 5 = Money

Parameter	Single regime	Two-regime Markov switching: Regime 1	Two-regime Markov switching: Regime 2
δ_1	0.039 (0.000)	0.027 (0.000)	0.001 (0.542)
δ_2	0.208 (0.000)	0.166 (0.000)	0.159 (0.000)
δ_3	0.069 (0.000)	0.077 (0.000)	0.095 (0.000)
δ_4	0.000 (0.999)	0.000 (0.999)	0.000 (0.999)
δ_5	0.196 (0.000)	0.242 (0.000)	0.209 (0.000)
d_{11}	0.045 (0.000)	0.048 (0.000)	0.057 (0.000)
d_{12}	−0.000 (0.000)	0.000 (0.995)	−0.000 (0.000)
d_{13}	0.000 (0.008)	−0.000 (0.190)	−0.000 (0.000)
d_{14}	0.037 (0.000)	0.035 (0.000)	0.008 (0.633)
d_{15}	0.000 (0.964)	0.021 (0.356)	−0.126 (0.000)
d_{22}	0.000 (0.999)	0.009 (0.089)	0.000 (0.995)
d_{23}	0.000 (0.000)	−0.000 (0.000)	0.000 (0.000)
d_{24}	−0.047 (0.010)	−0.065 (0.000)	0.000 (0.999)
d_{25}	−0.000 (0.996)	0.000 (0.992)	0.174 (0.000)
d_{33}	0.000 (0.999)	0.000 (0.999)	0.000 (0.999)
d_{34}	−0.000 (0.000)	0.000 (0.000)	−0.000 (0.000)
d_{35}	−0.000 (0.000)	−0.000 (0.991)	−0.000 (0.890)
d_{44}	0.018 (0.000)	0.015 (0.000)	0.019 (0.000)
d_{45}	0.050 (0.000)	0.054 (0.000)	0.078 (0.000)
d_{55}	0.001 (0.416)	0.007 (0.000)	0.012 (0.001)
h_{12}	0.130 (0.000)	0.122 (0.000)	0.145 (0.000)
h_{13}	0.037 (0.004)	0.046 (0.000)	0.054 (0.000)
h_{14}	−0.000 (0.993)	−0.000 (0.535)	0.000 (0.633)
h_{15}	−0.000 (0.963)	0.000 (0.266)	−0.000 (0.699)
h_{23}	0.073 (0.000)	0.101 (0.000)	0.092 (0.000)
h_{24}	−0.000 (0.000)	−0.000 (0.902)	−0.000 (0.999)
h_{25}	0.000 (0.995)	−0.000 (0.992)	−0.000 (0.991)
h_{34}	0.024 (0.000)	0.036 (0.000)	0.032 (0.000)
h_{35}	0.170 (0.000)	0.176 (0.000)	0.167 (0.000)
h_{45}	0.000 (0.000)	−0.000 (0.980)	−0.000 (0.000)
ρ	0.998 (0.000)	0.998 (0.000)	0.998 (0.000)
		$j=1$	$j=2$
α_j		1.707 (0.000)	1.548 (0.000)
β_j		6.905 (0.775)	73.016 (0.000)
$Q(10)$ 1 :	0.000	0.000	
2 :	0.001	0.490	
3 :	0.000	0.118	
4 :	0.000	0.086	
Approximated Bayes factor		137.164	

Note: Numbers in parantheses are p-values.

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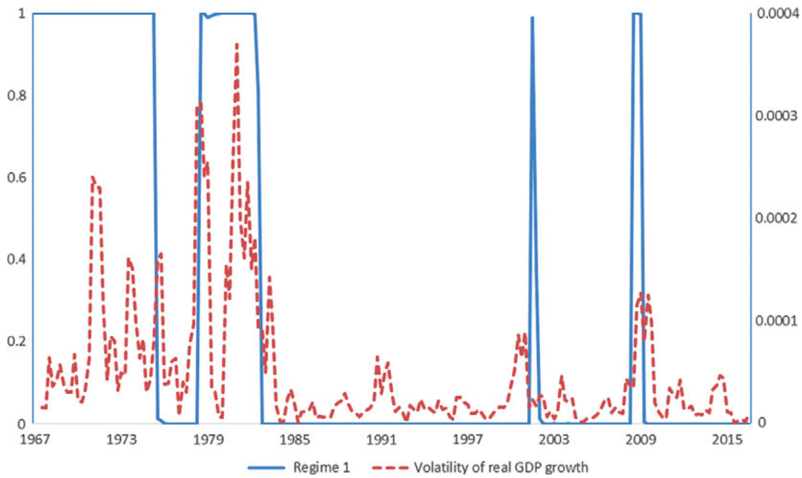

FIGURE 6. Smoothed probabilities of regime 1 and the volatility of real GDP growth.

Economic Research recessions, 1969–1970 and 1973–1975, the latter caused by the quadrupling of oil prices as a result of the restriction of oil production by the Organization of the Petroleum Exporting Countries during the Arab–Israeli war of 1973. It includes part of the 1978–1980 recession, which was almost an exact replay of the 1973–1975 recession, and the October 1979 to October 1982 period when the Federal Reserve de-emphasized the federal funds rate as an operating instrument and conducted a policy of monetary targeting, using non-borrowed reserves (the monetary base minus discount loans) as the primary operating instrument and monetary aggregates as intermediate targets. It includes the short 2001 recession, caused by a combination of aggregate demand shocks, including the tech bubble burst in March 2000, the September 11, 2001 terrorist attacks, and the 2001 Enron bankruptcy and other corporate accounting scandals. Regime 1 also covers the 2007–2008 Great Recession and global financial crisis, originated in the unregulated shadow banking system. We refer to regime 2 as the low uncertainty regime.

We evaluate the persistence of each regime by the estimates of $\hat{\alpha}_j$ and $\hat{\beta}_j$, $j = 1, 2$. We find that the high uncertainty (recession) regime is more persistent compared to the low uncertainty one, since $\hat{\alpha}_1 > \hat{\alpha}_2$. The $\hat{\beta}_j$ parameters indicate how lagged real GDP growth affects the persistence of the current regime. Given that $\hat{\beta}_1$ and $\hat{\beta}_2$ are both positive ($\hat{\beta}_1 = 6.905$ with a p-value of 0.775 and $\hat{\beta}_2 = 73.016$ with a p-value of 0.000), we find that higher lagged real GDP growth makes the current regime more persistent, irrespective of the current regime. This effect, however, is rather asymmetric, because $\hat{\beta}_2$ is much larger than $\hat{\beta}_1$ and is statistically significant, suggesting that continuous real GDP growth is important for maintaining the low uncertainty regime.

7.1. Weak Separability Tests

We now turn to test the main empirical implication of the classical theory of consumer demand that the representative agent's utility function is weakly separable in consumption goods and leisure as follows:

$$U = f\left[g\left(n, s, d, \ell\right), m\right]. \tag{19}$$

This assumption restricts the effects of money on real economic activity and allows one to focus on the details of the demand for the services of consumption goods, $\boldsymbol{c} = (n, s, d)$, and leisure, ℓ, ignoring the services of money, m, as in the following problem:

$$\max_{\boldsymbol{c},\ell} \quad u\left(\boldsymbol{c}, \ell\right) \quad \text{subject to} \quad \boldsymbol{p}_c'\boldsymbol{c} + p_\ell \ell = y$$

where $\boldsymbol{p}_c'$ is a vector of the prices of the consumption goods, p_ℓ is the price of leisure time (assumed to be the wage rate), and y is the expenditure on the services of consumption goods and leisure. This assumption also implies the existence of a monetary aggregate—see Hjertstrand et al. (2016) who test for the existence of a monetary aggregate using the nonparametric approach.

According to Blackorby et al. (1991), the necessary and sufficient conditions for the utility tree structure (19) are

$$\frac{\partial \tilde{x}_i\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)\right)}{\partial p_m} = \mu_m \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}\frac{\partial x_m(\boldsymbol{p}, y)}{\partial y}, \quad i \in \left(n, s, d, \ell\right),$$

where $\tilde{x}_i\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)\right)$ is the Hicksian (compensated) demand function and μ_m is a proportionality factor. Following Moschini et al. (1994), we impose three independent weak separability restrictions (evaluated at the mean of the data), which in the context of the Markov regime-switching framework need to be imposed on each regime ($i = 1, 2$). The restrictions are

$$\frac{\partial \tilde{x}_n\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m}{\partial \tilde{x}_s\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m} = \frac{\partial x_n(\boldsymbol{p}, y\,|z_t = i)/\partial y}{\partial x_s(\boldsymbol{p}, y\,|z_t = i)/\partial y}$$

$$\frac{\partial \tilde{x}_n\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m}{\partial \tilde{x}_d\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m} = \frac{\partial x_n(\boldsymbol{p}, y\,|z_t = i)/\partial y}{\partial x_d(\boldsymbol{p}, y\,|z_t = i)/\partial y}$$

$$\frac{\partial \tilde{x}_n\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m}{\partial \tilde{x}_\ell\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)|z_t = i\right)/\partial p_m} = \frac{\partial x_n(\boldsymbol{p}, y\,|z_t = i)/\partial y}{\partial x_\ell(\boldsymbol{p}, y\,|z_t = i)/\partial y}.$$

Note that the test is local, since the restrictions are imposed at the mean of the data, and not at every data point. Therefore, our conclusions are made based on the performance at the mean.

We test the null hypothesis of the restricted model (19) against the unrestricted model (4), using a likelihood ratio (LR) test in which the test statistic is calculated as $LR = -2(\log L_r - \log L_u)$, where L_r is the log-likelihood value of the restricted model and L_r that of the unrestricted model. In our case, LR has a $\chi^2(6)$ distribution, and $LR = 437.737$ with a p-value of 0.000, thus rejecting the null hypothesis that nondurables, services, durables, and leisure are weakly separable from money. Moreover, since the tests of restrictions in large demand systems are biased toward rejection—see, for example, Laitinen (1978) and Meisner (1979)—we also perform the size-corrected likelihood ratio test. Following Moschini et al. (1994), the size-corrected likelihood ratio statistic, LR_c, is

$$LR_c = LR\left(\frac{nT - 0.5(N_r + N_u) - 0.5n(n+1)}{nT}\right),$$

where n is the number of equations in the demand system, T is the number of observations, and N_r and N_u denote the number of parameters in the restricted and unrestricted model, respectively. LR_c is asymptotically distributed as a $\chi^2(j)$, where j is the number of restrictions. In our case, with $j = 6$, $LR_c = 337.652$ with a p-value of 0.000, thus strongly rejecting the weak separability assumption. This suggests that the inclusion of a monetary aggregate in economic models would be of quantitative importance.

Next, we test if the consumption goods are weakly separable from leisure and real money balances, and the consistency of the implicit two-stage optimization decision. That is, we test the null hypothesis

$$U = f\left[g\left(n, s, d\right), \ell, m\right],$$

against the alternative hypothesis of (4). Weak separability of $\boldsymbol{c} = (n, s, d)$ from leisure, ℓ, and money, m, implies the existence of a consumption services utility function (consumption aggregate), $u\left(\boldsymbol{c}\right)$, and the utility tree given in (2), allowing one to focus on the details of the demand for the services of consumption goods ignoring leisure and money, as in the following problem:

$$\max_{\boldsymbol{c}} \quad u\left(\boldsymbol{c}\right) \quad \text{subject to} \quad \boldsymbol{p}_c'\boldsymbol{c} = y,$$

where now y is the expenditure on the services of consumption goods. The corresponding necessary and sufficient conditions for the existence of a consumption aggregate are

$$\frac{\partial \tilde{x}_i\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)\right)}{\partial p_\ell} = \mu_\ell \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}\frac{\partial x_\ell(\boldsymbol{p}, y)}{\partial y}, \quad i \in (n, s, d)$$

$$\frac{\partial \tilde{x}_i\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right)\right)}{\partial p_m} = \mu_m \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}\frac{\partial x_m(\boldsymbol{p}, y)}{\partial y}, \quad i \in (n, s, d),$$

and involve the imposition of the following four restrictions in each of the two regimes (for a total of eight restrictions):

$$\frac{\partial \tilde{x}_n\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_\ell}{\partial \tilde{x}_d\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_\ell} = \frac{\partial x_n(\boldsymbol{p}, y \,| z_t = i)/\partial y}{\partial x_d(\boldsymbol{p}, y \,| z_t = i)/\partial y}$$

$$\frac{\partial \tilde{x}_s\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_\ell}{\partial \tilde{x}_d\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_\ell} = \frac{\partial x_s(\boldsymbol{p}, y \,| z_t = i)/\partial y}{\partial x_d(\boldsymbol{p}, y \,| z_t = i)/\partial y}$$

TABLE 2. Weak separability tests based on other monetary aggregates

Monetary aggregate	Null hypothesis: $\boldsymbol{c}$ and ℓ are weakly separable from m	Null hypothesis: $\boldsymbol{c}$ is weakly separable from ℓ and m
Divisia M1		
Likelihood ratio statistic	419.539 (0.000)	697.948 (0.000)
Size-corrected likelihood ratio statistic	323.615 (0.000)	538.367 (0.000)
Divisia M2		
Likelihood ratio statistic	387.451 (0.000)	414.910 (0.000)
Size-corrected likelihood ratio statistic	298.862 (0.000)	320.043 (0.000)
Divisia M3		
Likelihood ratio statistic	460.517 (0.000)	451.924 (0.000)
Size-corrected likelihood ratio statistic	355.228 (0.000)	348.595 (0.000)

Note: Numbers in parentheses are p-values.

$$\frac{\partial \tilde{x}_n\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_m}{\partial \tilde{x}_d\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_m} = \frac{\partial x_n(\boldsymbol{p}, y \,| z_t = i)/\partial y}{\partial x_d(\boldsymbol{p}, y \,| z_t = i)/\partial y}$$

$$\frac{\partial \tilde{x}_s\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_m}{\partial \tilde{x}_d\left(\boldsymbol{p}, h\left(\boldsymbol{p}, y\right) | z_t = i\right) / \partial p_m} = \frac{\partial x_s(\boldsymbol{p}, y \,| z_t = i)/\partial y}{\partial x_d(\boldsymbol{p}, y \,| z_t = i)/\partial y}.$$

The corresponding LR test statistic is 437.526 with a p-value of 0.000, and the size-corrected LR test statistic is 337.489 with a p-value of 0.000. Thus, we reject the hypothesis that nondurables, services, and durables are weakly separable from leisure and money. This means that the inclusion of money in dynamic optimizing general equilibrium models would be of quantitative importance. We discuss the implications of our findings for monetary policy and business cycle analysis in more detail in Section 8.

To investigate the robustness of our results to the use of alternative Divisia monetary aggregates, in Table 2, we present results for the Divisia M1, Divisia M2, and Divisia M3 aggregates. As can be seen, the test statistics reject the corresponding null hypotheses, suggesting that our evidence is robust to different Divisia definitions of the money supply. It should be noted, however, that our results are based on quarterly data, and that it is possible that the negative separability results might be due to sluggish adjustments by the economic agent to fluctuations in the determinants of consumption, leisure, and money demand.

7.2. Elasticities of Substitution

Next, we turn to an examination of the substitutability/complementarity relationship among nondurables, services, durables, leisure, and money across the low and high uncertainty regimes using elasticities of substitution. There are

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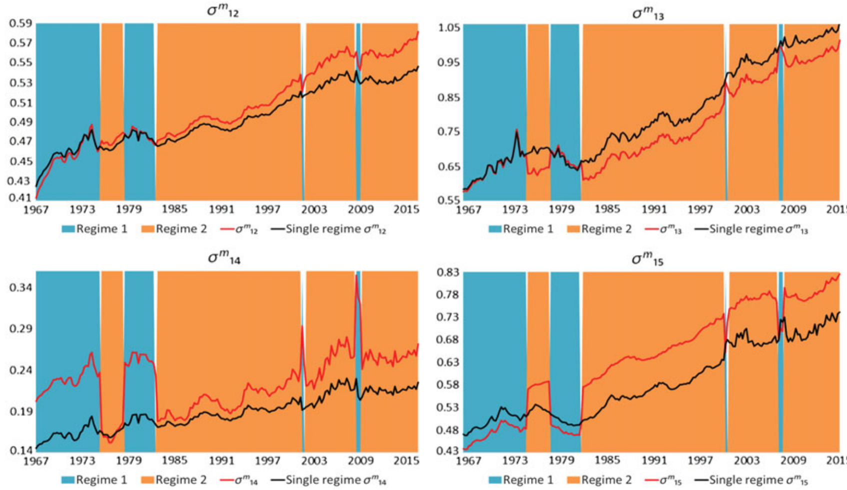

FIGURE 7. Morishima elasticities of substitution for nondurables.

four commonly used elasticities of substitution to assess the relationship between goods. These are the (uncompensated) Marshallian cross-price elasticity, $\eta_{ij} = \partial \log x_i/\partial \log p_j$, where x_i is the Marshallian demand function, the (compensated) Hicksian cross-price elasticity, $\eta^h_{ij} = \partial \log x^h_i/\partial \log p_j$, where x^h_i is the Hicksian demand function, the (compensated) Allen-Uzawa elasticity of substitution, $\sigma^a_{ij} = \sigma^a_{ij} = \eta^h_{ij}/s_j$, and the (compensated) Morishima elasticity of substitution, $\sigma^m_{ij} = \eta^h_{ij} - \eta^h_{jj}$. However, as noted by Blackorby and Russell (1989), with more than two goods the Allen-Uzawa elasticity of substitution may be uninformative, and the Morishima elasticity of substitution, σ^m_{ij}, is the correct measure of substitution.

The Morishima elasticity of substitution looks at the impact on the optimal ratio x_i/x_j when the price of good j, p_j, changes holding the price of good i, p_i, fixed. Goods will be Morishima substitutes ($\sigma^m_{ij} > 0$) if an increase in p_j causes q_i/q_j to increase and Morishima complements ($\sigma^m_{ij} < 0$) if an increase in p_j causes q_i/q_j to decrease. We plot the complete set of the Morishima elasticities of substitution in Figures 7–11, and for comparison purposes, we report elasticities for both the single-regime and the two-regime ML models. However, since the two-regime model is superior compared to its restricted (single-regime) version, in what follows, we only discuss the elasticities based on the two-regime model (the red lines in the figures). As can be seen, the Morishima elasticities of substitution are always positive for all pairs of goods (suggesting substitutability) and exhibit large swings across the two regimes, generally being higher in the high uncertainty (recession) regime.

We are interested not only in the net elasticities of substitution but also in the gross elasticities of substitution, since when the price of one good changes (holding all other things constant), the consumer will end up on a different indifference

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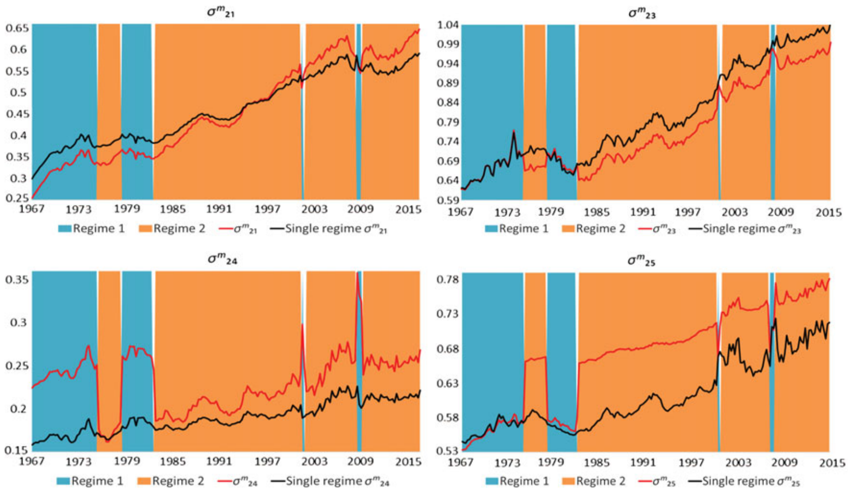

FIGURE 8. Morishima elasticities of substitution for services.

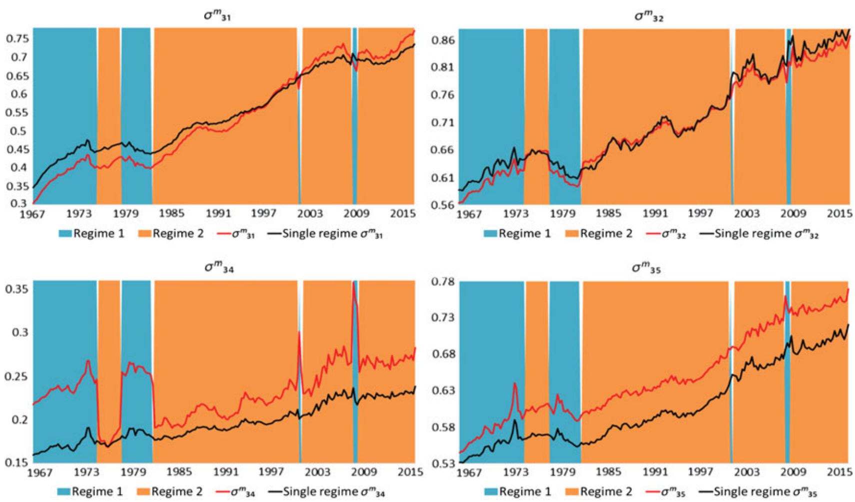

FIGURE 9. Morishima elasticities of substitution for durables.

curve, because of the income effect. Therefore, we plot the complete set of the Marshallian cross-price elasticities in Figures 12–16, in the same fashion as for the Morishima elasticities of substitution in Figures 7–11. We also plot the income elasticities in Figure 17. Clearly, the Marshallian cross-price elasticities vary significantly across the two regimes and indicate that all pairs of goods are typically gross complements (the more consumers buy of one good, the more they will buy of the other). The reason is that there is a moderate and positive income elasticity of demand for each good; as can be seen in Figure 17, nondurables, services,

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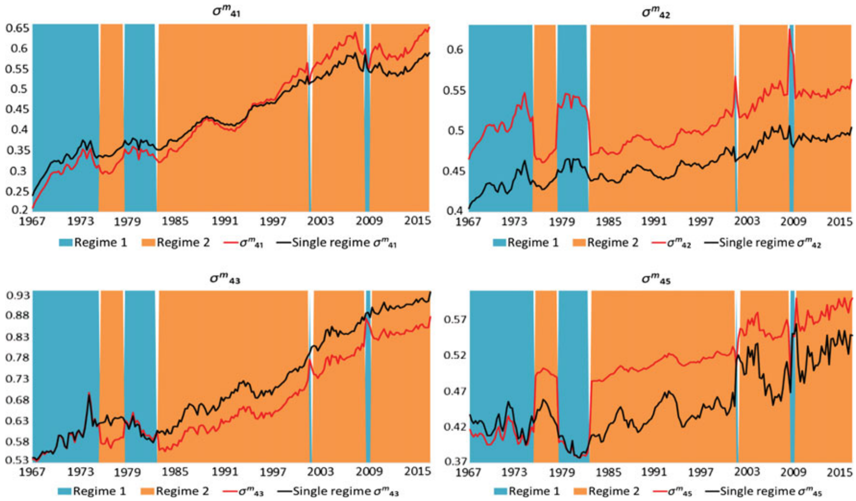

FIGURE 10. Morishima elasticities of substitution for leisure.

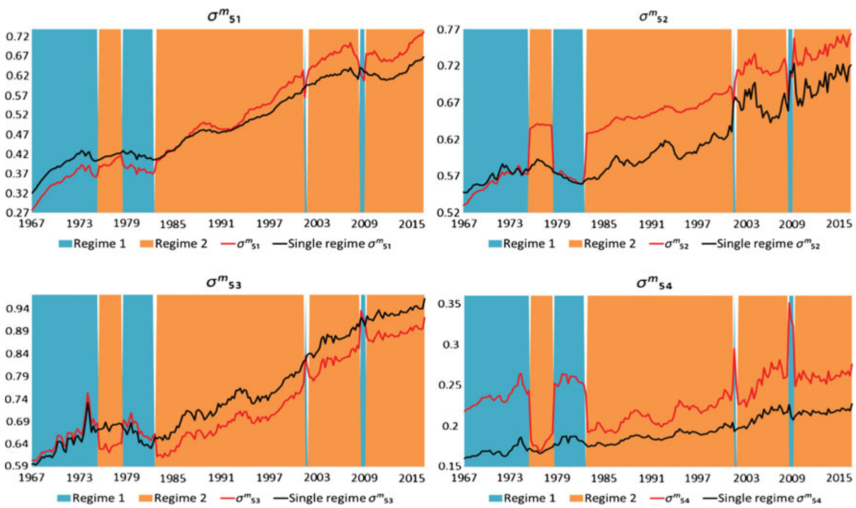

FIGURE 11. Morishima elasticities of substitution for money.

and leisure are near-normal goods in both regimes. On the other hand, durables and money are generally luxuries, consistent with Pakoš (2011) and Hoffman and Rasche (1991), respectively. Thus, given the positive income elasticity of each good, an increase in the price of any good induces a sizable and negative income effect on the consumer, causing a reduction of expenditure on every good.

Moreover, the estimated Marshallian cross-price elasticities η_{15}, η_{25}, η_{35}, and η_{45} have important implications for monetary policy. They all tell how consumers substitute for money when the user cost of money changes. In particular,

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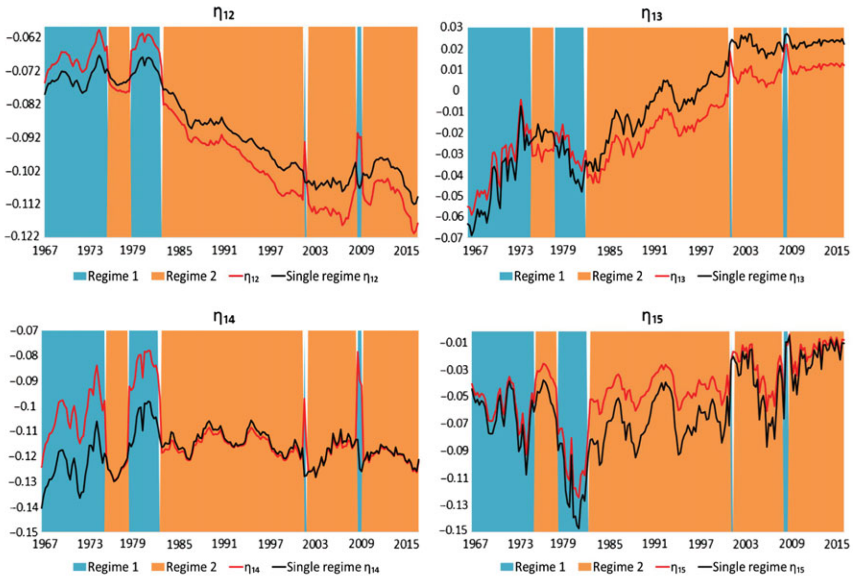

FIGURE 12. Marshallian cross-price elasticities for nondurables.

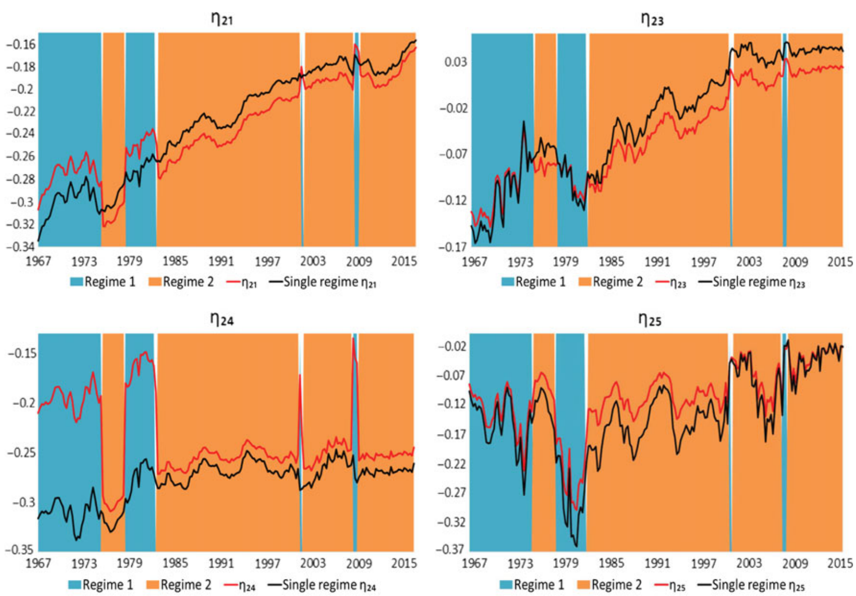

FIGURE 13. Marshallian cross-price elasticities for services.

since changes in interest rates directly affect the user cost of money, these elasticities are useful in terms of understanding and predicting consumer behavior when there is a change in the stance of monetary policy. Given that η_{15}, η_{25},

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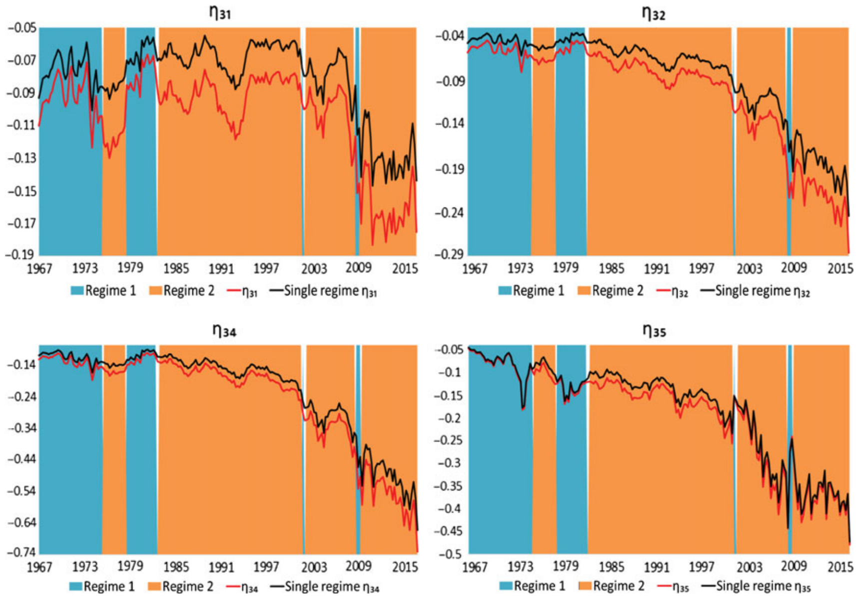

FIGURE 14. Marshallian cross-price elasticities for durables.

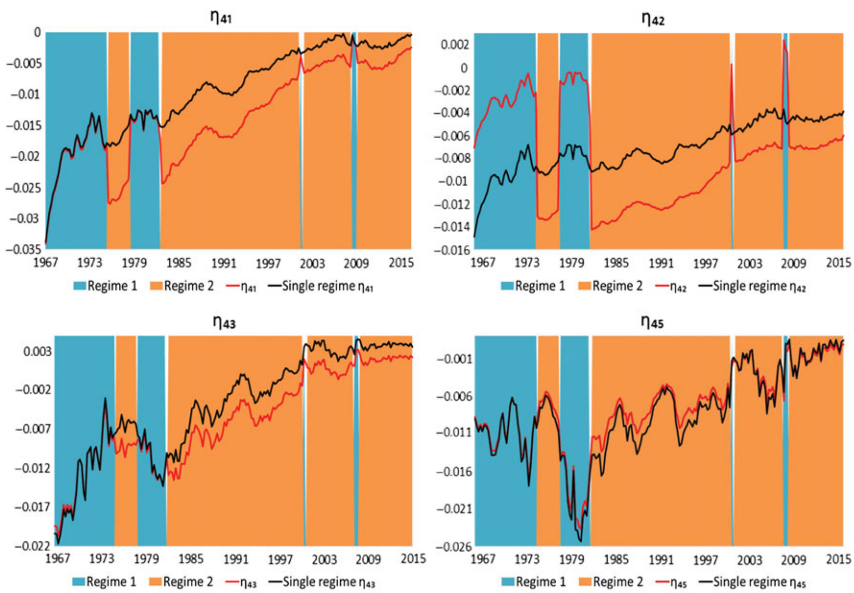

FIGURE 15. Marshallian cross-price elasticities for leisure.

η_{35}, and η_{45} are mostly negative, we can argue that consumers will reduce spending on other goods if the user cost of money increases. On the other hand, a monetary policy that lowers the user cost of money will increase spending on

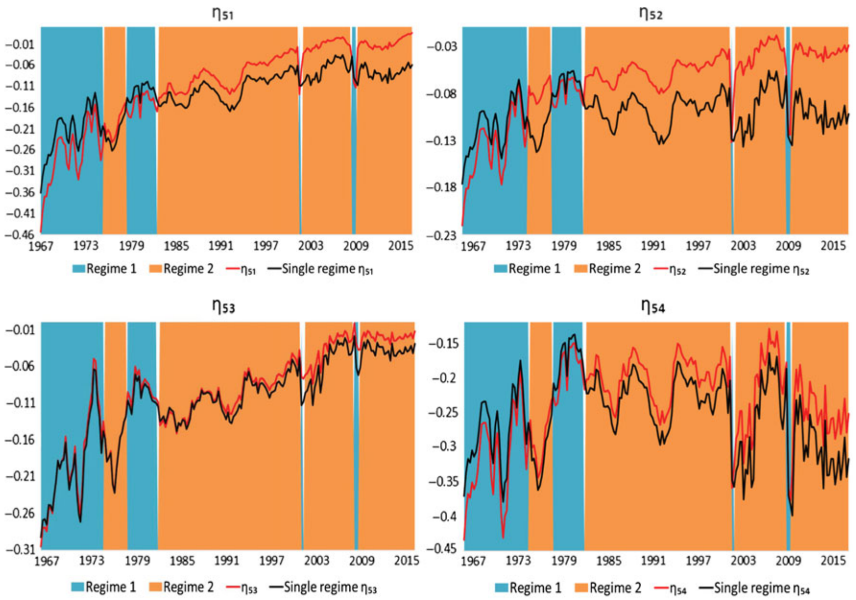

FIGURE 16. Marshallian cross-price elasticities for money.

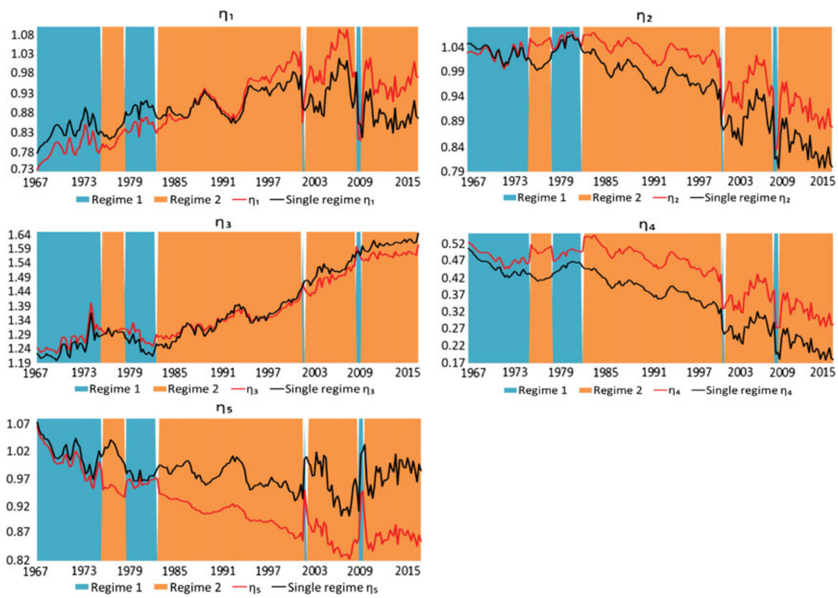

FIGURE 17. Income elasticities.

nondurables and services significantly in the high uncertainty recession regime, suggesting that monetary policy could effectively stimulate economic activity during recessionary times. Correspondingly, η_{51}, η_{52}, η_{53}, and η_{54} play an important role in understanding the demand for money. According to Figure 16, these elasticities are all negative, suggesting that economic agents will reduce the demand for money when the prices of nondurables, services, durables, and leisure increase.

8. IMPLICATIONS FOR MONETARY POLICY

Our results seem to suggest that there is something fundamentally misguided in the current mainstream approach to monetary policy and business cycle analysis. Today's approach to monetary policy is based on the new Keynesian model and completely ignores the role of money. The following simple system (ignoring fiscal policy variables), from McCallum and Nelson (2011), is representative of the basic model

$$y_t = b_0 + E_t y_{t+1} + b_1 \left(i_t - E_t \Delta p_{t+1}\right) + v_t, \quad b_1 < 0 \tag{20}$$

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \kappa \left(y_t - \bar{y}_t\right) + u_t, \quad 0 < \beta < 1, \kappa > 0 \tag{21}$$

$$i_t = \mu_0 + \mu_1 \Delta p_t + \mu_2 \left(y_t - \bar{y}_t\right) + e_t, \quad \mu_1 > 1, \mu_2 \geq 0, \tag{22}$$

where E_t is the expectations operator conditional on information available at time t, y_t is the log of output, p_t is the log of the price level (so that Δp_t represents inflation and $E_t \Delta p_{t+1}$ expected inflation), i_t is the short-term nominal interest rate, and $y_t - \bar{y}_t$ the output gap. Equation (20) is a forward-looking expectational IS function, equation (21) is a Phillips curve relationship, and equation (22) is a monetary policy rule of the Taylor (1993) type, showing how the central bank sets the short-term nominal interest rate, i_t, consistent with the Taylor principle.

One of the features of the new Keynesian modeling approach is that the system of equations (20), (21), and (22) includes no money measure. In this approach, monetary policy is made with regard to the short-term nominal interest rate, i_t. This model, however, seems inappropriate when the utility tree structures in (2) and (3) are rejected by the data. Our results suggest that a sensible way to do monetary policy and business cycle analysis is to include money as an argument in the utility function, as in equation (1), consistent with the dynamic, optimizing general equilibrium models of the Sidrauski (1967) and Brock (1974) type. Another common way to do monetary analysis is to assume that previously accumulated money balances are required for the purchase of consumption and/or investment goods, as in the cash-in-advance models of Stockman (1981), Svensson (1985), Lucas and Stockey (1987), and Cooley and Hansen (1989).

Finally, we would like to note that in this paper, we assume rather than test the existence of a monetary aggregate, m. This assumption is necessary to make the estimation tractable and is partly justified by earlier contributions, such as Swofford and Whitney (1987, 1988, 1994) and Hjertstrand et al. (2016), among others. It is to be noted, however, that Jadidzadeh and Serletis (2019) assume that the economic agent's utility function (1) is weakly separable as follows:

$$U = f\left(\boldsymbol{c}, \ell, m\left(\boldsymbol{m}\right)\right), \tag{23}$$

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where $\boldsymbol{m} = (m_1, m_2, ..., m_{15})$ is a vector of the services of all 15 monetary assets that are included in the broadest US monetary aggregate, and focus on the details of the demand for the services of monetary assets, ignoring the services of consumption goods, $\boldsymbol{c}$, and leisure, ℓ, as in the following problem:

$$\max_{\boldsymbol{m}} \quad m(\boldsymbol{m}) \quad \text{subject to} \quad \boldsymbol{q}'\boldsymbol{m} = y_{m},$$

where $\boldsymbol{q} = (q_1, q_2, ..., q_{15})$ is the corresponding vector of monetary asset user costs and y_m is the expenditure on the services of monetary assets. In the context of the corresponding highly disaggregated monetary asset demand system (encompassing the full range of monetary assets), they address the issue of optimal monetary aggregation. Their statistical tests reject the appropriateness of the aggregation assumptions for all the money measures published by the Federal Reserve as well as for a large number of groupings suggested by earlier studies.

These observations suggest that the main objective of future empirical work in this area should be to test the weak separability assumptions that are implicitly made in representative agent models of modern macroeconomics, estimate the degree of substitutability among consumption goods, leisure, and monetary assets, and address the issue of optimal monetary aggregation in the context of a large integrable demand system, encompassing the full range of consumption goods, $\boldsymbol{c}$, leisure, ℓ, and monetary assets, $\boldsymbol{m}$, as in the following unrestricted structure:

$$U = f(\boldsymbol{c}, \ell, \boldsymbol{m}).$$

The principal reason for this is that nearly all of the published work (including this paper) has generally been carried out in the context of highly aggregated demand systems. Aggregation tends to dramatically reduce the number of goods, so the existing empirical evidence based on small, highly aggregated demand systems is unrepresentative of the substitutability/complementarity and separability relationships among consumption goods, leisure, and liquid assets. As Pudney (1980, p. 875) puts it, “the accurate measurement of cross-price demand responses at a low level of aggregation over goods is of central importance for the majority of potential applications of orthodox microeconomic analysis. The difficulty of making such measurements is widely appreciated, however. Lack of data and lack of sufficient independent variation within the sets of data that are currently available combine with the ‘curse of dimensionality’ that inevitably afflicts very detailed demand studies to make ‘unrestricted’ estimation of these demand responses a practical impossibility.” We leave this as an area for potentially productive future research.

9. CONCLUSION

We take a parametric approach to empirical demand analysis that allows estimation and testing in a systems-of-equations context. We use the ML flexible functional form to approximate the underlying unknown utility function and also relax the assumption of fixed consumer preferences by assuming Markov regime switching. Thus, allowing for complicated nonlinear dynamics in the parameters of the demand functions and the utility function, we generate inference consistent with both theoretical regularity (by treating the curvature property as a maintained hypothesis) and econometric regularity, the latter by correcting for residual serial correlation, following Berndt and Savin (1975) and assuming an AR(1) process.

We test whether consumption goods and leisure are weakly separable from real money balances and the consistency of the implicit two-stage optimization decision. We strongly reject the null hypothesis of weak separability, implying that the demand interactions between goods, leisure, and money are likely to be substantial. We also test for the existence of a consumption aggregate and strongly reject the null hypothesis. The evidence is consistent with earlier work by Abbott and Ashenfelter (1976) and Barnett (1979) and suggests (among other things) that the inclusion of a money in economic models would be of significant quantitative importance. Finally, we investigate the substitutability of goods, leisure, and money and assess the implications for monetary policy.

In this paper, we followed Jorgenson and Lau (1975), Denny and Fuss (1977), and Moschini et al. (1994) and tested for approximate (local) weak separability (i.e., weak separability at some representative point in the data, which is the mean of the data). It is possible that imposing the restrictions at another point in the sample, over a neighborhood of data points in the sample, or at every point in the sample, could actually lead to a different conclusion. In this regard, as Blackorby et al. (1977) show, global imposition of weak separability destroys the local flexibility properties of flexible functional forms such as the translog and the generalized Leontief. In that case, weak separability tests become tests of the joint null hypothesis of weak separability and of an unrealistically restrictive specification for the aggregator function. It should also be noted that Barnett and Choi (1989) conduct a series of Monte Carlo exercises to examine the capability of flexible functional forms to provide correct inferences about approximate (local) separability. They use the translog, the absolute price version of the Rotterdam model, the generalized Leontief, and the third-order translog, and find that none of these models is well suited for testing for approximate weak separability. Whether the ML model used in this paper loses its local flexibility properties when weak separability is globally imposed is another area for future research.

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