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Generation of plasma wave and third harmonic generation at ultra relativistic laser power

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Abstract

This paper investigates the generation of plasma wave and third harmonic generation in hot collision less plasma by a Gaussian ultra intense laser beam, when relativistic and ponderomotive nonlinearities are operative. First, we derive the dynamical equation for the pump laser beam when these two nonlinearities are operative. The solution of pump laser beam has been obtained within the paraxial ray approximation. Filamentary structures of the laser beam are observed. On account of $\vec{V} \times \vec{B}$ force, the generation of plasma wave at second harmonic frequency has been studied in these filamentary structures. Interaction of the plasma wave with the incident laser beam generates the third harmonic. For the typical laser plasma parameters: Nd: YAG laser beam ($\lambda = 1064$ nm), $r_o = 15$ μm, laser power flux equals 6×10^{17} W/cm^2, electron density equals $n_o = 1.9 \times 10^{19}$ per cm^3, the third harmonic yield comes out to be equals to 2×10^{-6}.

Keywords: Third harmonic generation; Intense laser beam; Short pulse lasers

1. INTRODUCTION

Recent advances in ultra intense short laser pulse technology, such as chirped pulse amplification used to generate femtosecond duration pulse, have led to rapid developments in the field of laser induced fusion and particle acceleration research (Jonathan, 1994; Modena *et al.*, 1995; Kuroda *et al.*, 2006; Ozaki *et al.*, 2006). The nonlinear interaction of intense laser short pulse lasers with plasmas has been a subject of experimental and theoretical study due to its relevance to laser driven fusion. These ultra intense short laser pulses interact with plasmas leading to various nonlinear phenomena (Kruer, 1988; Umstadter, 2003), such as self focusing, filamentation, stimulated Raman scattering, stimulated Brillouin scattering, and harmonics generation (Esarey *et al.*, 1993; Foldes *et al.*, 2003; Chirila *et al.*, 2004).

Harmonic generations in ultra intense laser plasma interaction has been studied extensively both experimentally and theoretically in the past. Esarey *et al.* (1993) among others proposed a nonlinear cold fluid model, valid for ultra high intensities and used to analyze relativistic harmonic generation. A linearly polarized ultra intense laser field induces transverse plasma currents, which are highly relativistic and nonlinear, resulting in the generation of coherent harmonic radiation in the forward direction. Recently, Banerjee *et al.* (2002) studied experimentally the harmonic generation in relativistic laser plasma interaction. They showed that relativistic Thomson scattering produced a significant amount of harmonic generation. Wilks *et al.* (1993) presented a method for generating odd harmonics from an intense laser, incident upon a sharp vacuum-overdense plasma interface. With the intensity greater than 10^{18} W/cm^2, these pulses have a pressure greater than 10^3 Mbars, creating large density oscillations and relativistic electron velocities at the surface. This results in efficient odd harmonic generation. Liu *et al.* (1993) reported the result of a harmonic generation experiment in hydrogen using 1 ps, 1 μs laser pulses with a range of intensities extending from below to far above the laser ionization saturation threshold, and determine an upper limit on the conversion efficiency of third harmonic generation in a preformed plasma.

Most of the above mentioned work on harmonic generation at ultra relativistic powers of the laser beam has not included the relativistic and ponderomotive effects simultaneously. By including both the effects, specifically, the ponderomotive effect, the plasma density in the channel is expected to change and should affect the harmonic generation. Moreover, these nonlinear effects will break the laser pulse into small filamentary structures where the laser field

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is very intense. Therefore, both the ponderomotive and filamentation effects should affect the harmonic generation.

In this paper, we have studied the propagation of an ultra intense Gaussian laser beam through plasma. The ultra high intensity of laser beams by relativistic and ponderomotive nonlinearities create plasma channel, where the laser beam creates very intense hot filaments. These intense filaments trap plasma wave at second harmonic frequency because at very high $\vec{V} \times \vec{B}$ force in the channel (Sodha *et al.*, 1976). The amplitude of these plasma waves depends upon the laser beam intensity and background density. These waves have the same phase velocity as the laser beam, and hence their Landau damping is negligible. Therefore, these plasma waves interacting with the incident laser beam generate a third harmonic. In Section 2, we derived the expression for the effective dielectric constant of the plasma in the presence of an ultra intense laser beam, where relativistic and ponderomotive nonlinearities are operative. In Section 3, we studied the solution for laser beam propagation and obtain the numerical results showing the laser intensity evolution in axial and transverse directions, and the corresponding filamentation of the laser beam. In Section 4, we derived the expression for coupling between ultra intense laser beam and electron plasma wave. We also derived the expression for the power of electron plasma wave and studied the behavior with the laser beam and plasma parameters where relativistic and ponderomotive nonlinearities are operative. In section 5, we derived the expression for the power of the third harmonic generation where relativistic and ponderomotive nonlinearities are operative. For typical laser beam parameters; Nd: YAG laser beam ($\lambda = 1064$ nm), $r_o = 15$ μm, $\omega_{po} = 0.03\omega_o$, $V_{th} = 0.1$c and the third harmonic yield comes out to be equal to 2×10^{-6}.

2. EFFECTIVE DIELECTRIC CONSTANT OF THE PLASMA

We consider the propagation of an ultra intense laser beam along the z direction. The initial intensity distribution of the beam is given by

$$E_o \cdot E_o^*|_{z=o} = E_{oo}^2 \exp\left(-\frac{r^2}{r_o^2}\right). \quad (1)$$

Where r is the radial coordinate of the cylindrical coordinate system and r_o is the initial beam width. The dielectric constant of the plasma is given by

$$\varepsilon_o = 1 - \frac{\omega_{po}^2}{\omega_o^2}, \quad (2)$$

where ω_{po} is the plasma frequency given by $\omega_{po}^2 = 4\pi n_o e^2/m_o$ (with e being the charge of an electron, m_o is the rest mass, and n_o is the density of plasma electrons in the absence of laser beam) and relativistic factor is given by

$$\gamma = \left[1 + \frac{e^2}{\omega_o^2 c^2 m_o^2} E_o \cdot E_o^*\right]^{1/2}$$

The above expression is valid when there is no change in the plasma density. The relativistic ponderomotive force is given by Esarey *et al.* (1988), Borisov *et al.* (1992), and Brandi *et al.* (1993*a*, 1993*b*)

$$F_p = -m_o c^2 \nabla(\gamma - 1).$$

Using the electron continuity equation and current density equation for the second order correction in the electron density equation (with the help of ponderomotive force) and total density is given by Esarey *et al*. (1988), Borisov *et al*. (1992), and Brandi *et al*. (1993*a*, 1993*b*)

$$n = n_o + n_2 = n_o + \frac{c^2 n_o}{\omega_{po}^2}\left(\nabla^2\gamma - \frac{(\nabla\gamma)^2}{\gamma}\right). \quad (3)$$

Now the effective dielectric constant of the plasma at pump frequency ω_o is given by

$$\varepsilon = \varepsilon_o + \phi(E_o E_o^*)$$

where

$$\phi(E_o \cdot E_o^*) = \frac{\omega_{po}^2}{\omega_o^2}\left(1 - \frac{n}{n_o\gamma}\right).$$

Expending dielectric constant around $r = 0$ by Taylor expansion, one can write

$$\varepsilon = \varepsilon_f + \gamma_1 r^2$$

where

$$\varepsilon_f = \varepsilon_o + \frac{\omega_{po}^2}{\omega_o^2}\left[1 + \left(-1 + \frac{a}{\gamma r_o^2 f_o^4 k_p^2}\right)\left(1 + \frac{a}{f_o^2}\right)^{-1/2}\right]$$

$$\gamma_1 = -\frac{\omega_{po}^2}{\omega_o^2}\left[\frac{a}{2\gamma^3 r_o^2 f_o^4} - \frac{3a}{\gamma^2 k_p^2 r_o^4 f_o^6} - \frac{3a^2}{\gamma^4 k_p^2 r_o^4 f_o^8}\right]. \quad (4)$$

Here, $a = \alpha_o A_{oo}^2$ is the square of the dimensionless vector potential $(E = -\partial A/\partial(ct))$, $\alpha_o = e^2/m_o^2 c^4$, f_o is the beam width parameter at z as given by Eq. (9) in Section 3 and $k_p^2 = \omega_{po}^2/c^2$.

3. LASER BEAM PROPAGATION

The wave equation governing the vector potential of the laser beam in the plasma can be written as

$$\frac{\partial^2 A}{\partial z^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial r^2} + \frac{\omega_o^2}{c^2}\varepsilon A = 0 \tag{5}$$

Following Akhamanov *et al.* (1968), the solution to the above equation is

$$A = A'(r,z)\exp[-iS_o(r,z)]. \tag{6}$$

Using Eq. (6) into Eq. (5) and separating the real and imaginary parts we get:

$$2\varepsilon_f \frac{\partial S_o}{\partial z} + \frac{c}{\omega_o}\left(\frac{\partial S_o}{\partial r}\right)^2 = \frac{\omega_o}{c}\gamma_1 r^2 + \frac{c}{\omega_o A'}\left(\frac{\partial^2 A'}{\partial r^2} + \frac{1}{r}\frac{\partial A'}{\partial r}\right), \tag{7a}$$

$$\frac{\omega_o}{c}\varepsilon_f \frac{\partial A'^2}{\partial z} + \frac{\partial S_o}{\partial r}\frac{\partial A'^2}{\partial r} + A'^2\left(\frac{\partial^2 S_o}{\partial r^2} + \frac{1}{r}\frac{\partial S_o}{\partial r}\right) = 0. \tag{7b}$$

The solution to the above-coupled equations can be written as

$$S_o = \frac{r^2}{2f_o}\frac{df_o}{dz} + \phi_o(z), \qquad k_o = \frac{\omega_o}{c}\varepsilon_o^{1/2}, \varepsilon_o = 1 - \frac{\omega_{po}^2}{\omega_o^2},$$

and the intensity of the laser beam

$$A'^2 = \frac{A_o^2}{f_o^2}\exp\left(-\frac{r^2}{r_o^2 f_o^2}\right). \tag{8}$$

Using Eq. (8) into Eq. (7a) and normalization distance $\xi = zc/\omega_o r_o^2$, then we get an expression for f_o

$$\frac{d^2 f_o}{d\xi^2} = \frac{1}{\varepsilon_f f_o^3} - \frac{f_o}{\varepsilon_f}\left[\left(\frac{\omega_{po}^2 r_o^2}{c^2}\right)\frac{a}{2\gamma^3 f_o^4} + \frac{3a}{\gamma^2 f_o^6} - \frac{3a^2}{\gamma^4 f_o^8}\right]. \tag{9}$$

The intensity of the laser beams with relativistic and ponderomotive nonlinearities are expressed by Eqs. (8) and (9). We studied the variation of laser beam intensity with distance along the laser beam propagation direction and radial direction. Figure 1a demonstrates the generated filaments of laser beam when an ultra intense laser beam propagates through the plasmas in the presence of relativistic and ponderomotive nonlinearities. Figure 1b demonstrates the generated filaments of laser beam when an ultra intense laser beam propagates through the plasmas in the presence of relativistic nonlinearity. The following set of parameters has been used in the numerical calculations; laser beam Nd: YAG ($\lambda = 1064$ nm), $r_o = 15$ μm, $\omega_{po} = 0.03\omega_o$, $V_{th} = 0.1$c (speed of light).

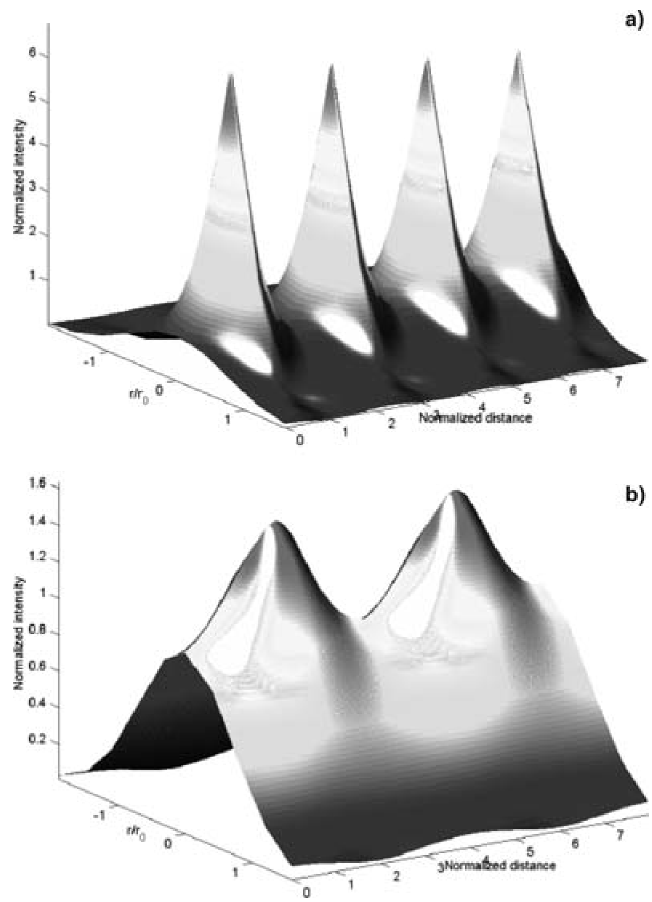

Fig. 1. (**a**) Variation in laser beam intensity with normalized distance (ξ) and radial distance (r) for $a = 0.8$, when relativistic and ponderomotive nonlinearities are operative. (**b**) Variation in laser beam intensity with normalized distance (ξ) and radial distance (r) for $a = 0.8$, when only relativistic nonlinearity is operative.

4. GENERATION OF PLASMA WAVE

To analyze the generation and growth of electron plasma waves, we start with the following set of equations. (1) The continuity equation

$$\frac{\partial N}{\partial t} + \nabla\cdot(NV) = 0. \tag{10}$$

(2) The momentum equation

$$m\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right] = -eE - \frac{e}{c}V\times B - 2\Gamma emV - \frac{3K_O T_e}{N}\nabla N. \tag{11}$$

(3) The Poisson's equation

$$\nabla . E = -4\pi eN\cdot \tag{12}$$

Where N is the total electron density, E is the sum of electric vectors of the laser beam and the self-consistent field, V is the sum of drift velocities of the electron in the laser field and self-consistent field. Using Eqs. (10), (11), and (12), we get the following equations for N:

$$\frac{\partial^2 N}{\partial t^2} - v_{th}^2 \nabla^2 N + 2\Gamma_e \frac{\partial N}{\partial t} + \frac{\omega_{po}^2}{\gamma}\left(\frac{n}{n_o}\right) N - \frac{e}{m}(E \cdot \nabla N) = \nabla \cdot \left[\frac{N}{2}\nabla(V \cdot V^*) - V\frac{\partial N}{\partial t}\right] \qquad (13)$$

where $v_{th}^2 = 3k_o T_e/m$ is the electron thermal velocity. Eq. (13) is the general equation governing the time independent and time dependent component of the election density inside the plasma. Time dependent component of electron density contains harmonics of the pump frequency, harmonics of the frequency of the plasma wave and their combinations. Here we are interested in the component of the electron density (N_2), which is at second harmonic frequency, and it is given by

$$-(2\omega_o)^2 N_2 - v_{th}^2 \nabla^2 N_2 + 4i\Gamma_e \omega_o N_2 + \frac{\omega_{po}^2}{\gamma}\left(\frac{n}{n_o}\right) \times N_2 - \frac{e}{m}(E_2 \cdot \nabla n_o) = \nabla \cdot \left[\frac{N_o}{4}\nabla v_o^2 + \frac{N_2}{4}\{\nabla(v_o \cdot v_o^* + v_2 \cdot v_2^*)\} + \frac{N_2}{8}\nabla v_2^2\right] \qquad (14)$$

Here v_o and v_2 are drift velocities of electron in the pump field and self consistent field (at frequency $2\omega_o$), respectively, ω_o is the pump wave frequency. The wave equation governing the total electric field inside the plasma is given by

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J}{\partial t}, \qquad (15)$$

where J is the total current density. In writing Eq. (15), we neglected the term $\nabla\,(A.\nabla \log \varepsilon)$ which is justified as long as $(\omega_{po}^2/\omega_o^2\ \varepsilon)\log\ \varepsilon \ll 1$, where ϵ is the effective dielectric constant of the plasma. Equations for the coherent vector potential inside the plasma and vector potential of the third harmonic generation is given by using the vector potential

$$\nabla^2 A_o + \frac{\omega_o^2}{c^2}\left(1 - \left(\frac{n}{n_o}\right)\frac{\omega_{po}^2}{\omega_o^2}\right) A_o = 0 \qquad (16a)$$

and

$$\nabla^2 A_3 + \frac{\omega_s^2}{c^2}\left(1 - \left(\frac{n}{n_o}\right)\frac{\omega_{po}^2}{\omega_s^2}\right) A_3 = \frac{\omega_{po}^2}{2c^2}\frac{N_2}{n_o}A'_o, \qquad (16b)$$

where N_2 is governed by Eq. (14) and $\omega_s = \omega_o + 2\omega_o$ is the frequency of third harmonic generation. Using the zero order solution of Eq. (14), N_2 is given by Soda *et al.* (1978)

$$-4\omega_o^2 N_2 - v_{th}^2 \nabla^2 N_2 + 4i\Gamma_e \omega_o N_2 + \frac{\omega_{po}^2}{\gamma}\left(\frac{n}{n_o}\right) \times N_2 + \frac{e^2 n}{4m_o^2\gamma^2 c^2}\left(\frac{c^2}{v_{th}^2}\right)\nabla^2 A_o'^2 = 0. \qquad (17)$$

The last term in Eq. (17) suggest that one component of N_2 must propagate as exp $(-2ik_o z)$ and the second component as exp $(-ikz)$, where $k = (4\omega_o^2 - \omega_p^2)/3v_{th}^2$ is the propagation vector of the plasma wave supported by the hot plasma. Hence N_2 may be written as

$$N_2 = N_{20}(r,z)\exp(-ikz) + N_{12}(r,z)\exp(-2ik_o z). \qquad (18)$$

Using N_2 in Eq. (17), and equating the coefficient of $\exp(-ikz)$ and $\exp(-2ik_o z)$, we get,

$$-4\omega_o^2 N_{20} - v_{th}^2\left[\frac{\partial^2 N_{20}}{\partial r^2} + \frac{1}{r}\frac{\partial N_{20}}{\partial r} - 2ik\frac{\partial N_{20}}{\partial z} - k^2 N_{20}\right] + 4i\Gamma_e \omega_o N_{20} + \frac{\omega_{po}^2}{\gamma}\left(\frac{n}{n_o}\right) N_{20} = 0 \qquad (19)$$

and

$$-4\omega_o^2 N_{12} - v_{th}^2\left[\frac{\partial^2 N_{12}}{\partial r^2} + \frac{1}{r}\frac{\partial N_{12}}{\partial r} - 4ik_o\frac{\partial N_{12}}{\partial z} - 4k_o^2 N_{12}\right] + 4i\Gamma_e \omega_o N_{12} + \frac{\omega_{po}^2}{\gamma}\left(\frac{n_e}{n_o}\right) N_{12} + \frac{e^2 n}{4m_o^2\gamma^2 c^2}\left(\frac{c^2}{v_{th}^2}\right)\nabla^2 A_o'^2 = 0 \qquad (20)$$

To solve these equations, we used the Eikonal approximation. Hence,

$$N_{20} = N'_{20} e^{-iks}$$

and

$$N_{12} = N'_{12} e^{-2ik_o s_o}.$$

Using these expressions of N_{20} and N_{12} in Eqs. (19) and (20), respectively, we obtain the following equations.

$$\left(\frac{\partial s}{\partial r}\right)^2 + 2\frac{\partial s}{\partial z} = \frac{1}{k^2 N'_{20}}\left[\frac{\partial^2 N'_{20}}{\partial r^2} + \frac{1}{r}\frac{\partial N'_{20}}{\partial r}\right] + \frac{4\omega_o^2}{k^2 v_{th}^2}\left[1 - \frac{\omega_{po}^2}{4\omega_o^2}\left(\frac{n}{\gamma n_o}\right)\right] \qquad (21a)$$

$$\frac{\partial N_{20}'^2}{\partial z} + \frac{\partial s}{\partial r}\frac{\partial N_{20}'^2}{\partial r} + N_{20}'^2\left[\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right] + \frac{4\Gamma_e \omega_o N_{20}'^2}{k v_{th}^2} = 0 \qquad (21b)$$

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and

$$-4\omega_o^2 N'_{12} + 4k_o^2 v_{th}^2 + (\omega_{po}^2 n/\gamma n_o)N'_{12} - \frac{e^2 k_o^2 A_{oo}^2 n}{\gamma^2 m_o^2 c^2 f_o^2}\left(\frac{c^2}{v_{th}^2}\right) \times \exp\left(-\frac{r^2}{r_o^2 f_o^2}\right) \cong v_{th}^2\left[\frac{\partial^2 N'_{12}}{\partial r^2} - 4k_o^2 N'_{12}\left(\frac{\partial s_o}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial N'_{12}}{\partial r} - 8k_o^2 N'_{12}\frac{\partial s_o}{\partial r}\right]. \quad (22)$$

To obtain a solution for N'_{12}, it can be shown that the right-hand side of Eq. (22) can be put equal to zero in the zero order approximation, and N'_{12} is given by

$$N'_{12} \approx -\left(\frac{n}{n_o}\right)\left(\frac{c^2}{v_{th}^2}\right) \times \frac{e^2 k_o^2 n_o A_{oo}^2 \exp(-r^2/r_o^2 f_o^2)}{\gamma^2 m_o^2 c^2 f_o^2\left[4\omega_o^2 - 4k_o^2 v_{th}^2 - \omega_{po}^2 n/\gamma n_o\right]}. \quad (23)$$

Following Akhmanov *et al.* (1968), the solution for Eqs. (21a) and (22b) in the paraxial ray approximations are given by

$$s = \frac{r^2}{2f}\frac{df}{dz} + \phi(z)$$

$$N'^2_{20} = \frac{B^2}{f^2}\exp(-r^2/a_o^2 f^2)\exp(-4\Gamma_e \omega_o z/k v_{th}^2), \quad (24)$$

where f is the dimensionless beam width parameter of the beam of the electron plasma wave at second harmonic frequency governed by

$$\frac{d^2 f}{d\xi^2} = \frac{r_o^4}{a_o^4 f^3} - \frac{f}{4}\left(\frac{c^2}{v_{th}^2}\right)\left[\left(\frac{\omega_{po}^2 r_o^2}{c^2}\right)\frac{a}{2\gamma^3 f_o^4} + \frac{3a}{\gamma^2 f_o^6} - \frac{3a^2}{\gamma^4 f_o^8}\right] \quad (25)$$

B and a_o are constants which will be determined by using boundary conditions, we assume that the amplitude of the plasma wave is zero at $z = 0$, so

$$B = -\left(\frac{n}{n_o}\right)\left(\frac{c^2}{v_{th}^2}\right)\frac{k_o^2 e^2 A_{oo}^2 n_o}{\gamma^2 m_o^2 c^2[4\omega_o^2 - 4k_o^2 v_{th}^2 - \omega_{po}^2 n/\gamma n_o]}$$

and

$$r_o = \sqrt{2}a_o.$$

Therefore, N_2 is given by

$$N_2 = N_{20}e^{-ikz} + N_{12}e^{-2ik_o z}. \quad (26)$$

Before proceeding further, we analyze the dispersion relation of the plasma wave arising on account of thermal effects in the plasma, it is given by

$$4\omega_o^2 \cong \omega_{po}^2 + k^2 v_{th}^2.$$

The phase velocity is $\cong v_{th}/\left(\sqrt{1 - \omega_{po}^2/4\omega_o^2},\right)$ and for a propagating laser beam ($\omega_{po} < \omega_o$) it varies from one to two times the thermal velocity of particles. For a propagating mode of the pump wave, $k\ \lambda_d \gg 1$, where λ_d is the Debye length. This shows that it is a very short wavelength wave and it will be heavily Landau damped. Therefore, in the expression of N_2 using Eq. (18), only the last term contributes, and we get

$$N_2 \approx \left[-\left(\frac{n}{n_o}\right)\left(\frac{c^2}{v_{th}^2}\right)\frac{k_o^2 e^2 n_o A_{oo}^2 \exp\left(-r^2/r_o^2 f_o^2\right)}{\gamma^2 m_o^2 c^2[4\omega_o^2 - 4k_o^2 v_{th}^2 - \omega_{po}^2 n/\gamma n_o]}\right] \times \frac{e^{-2ik_o(z+s_o)}}{f_o^2} \quad (27)$$

By using the Poisson equation, we get the scalar potential ϕ_2

$$\phi_2\phi_2^* = \left(\frac{4\pi e}{2k_o}\right)^2 \times \left[\left(\frac{n}{n_o}\right)\left(\frac{c^2}{v_{th}^2}\right)\frac{k_o e^2 n_o A_{oo}^2 \exp(-r^2/r_o^2 f_o^2)\sin 2k_o(z+s_o)}{2\gamma^2 m_o^2 c^2[4\omega_o^2 - 4k_o^2 v_{th}^2 - \omega_{po}^2 n/\gamma n_o]}\right]^2 \frac{1}{f_o^4}$$

The power of plasma wave at frequency $2\omega_o$, incident across the transverse cross section at z can be obtained by integrating $\phi_2\phi_2^*$ over r from 0 to ∞, i.e.,

$$P_2 = 4k_o^2\int_0^\infty 2\pi r\ \phi_2\phi_2^* dr$$

$$P_2 = \frac{\pi P_o}{3}\left[\left(\frac{n}{n_o}\right)\left(\frac{c^2}{v_{th}^2}\right)\frac{\omega_{po}^2 V_{oo}\sin 2k_o(z+s_o)}{c\gamma^2 f_o[4\omega_o^2 - 4k_o^2 v_{th}^2 - \omega_{po}^2 n/\gamma n_o]}\right]^2. \quad (28)$$

Here $p_o = (\omega_o^2/8\pi c)\pi r_o^2 A_{oo}^2$ and $V_{oo} = (eA_{oo}/cm_o)$. Eq. (28) represents the power of electron plasma wave at twice the pump wave frequency. Figure 2a demonstrates the generated electron plasma wave intensity where an ultra intense laser beam propagates through the plasmas in the presence of relativistic and ponderomotive nonlinearities. Figure 2b demonstrates the same where an ultra intense laser beam propagates through the plasmas in the presence of only relativistic nonlinearity. Figure 4 demonstrates the variation of the power of electron plasma wave at twice pump wave frequency where relativistic and ponderomotive nonlinearities are operative.

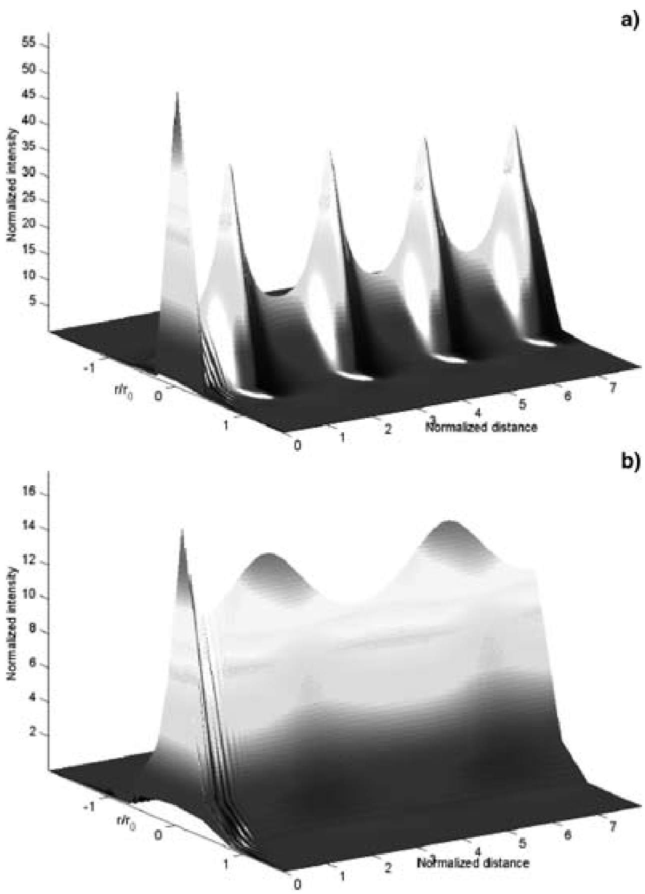

Fig. 2. (**a**) Variation in electron plasma wave intensity at second harmonic frequency with normalized distance (ξ) and radial distance (r) for $a = 0.8$, when relativistic and ponderomotive nonlinearities are operative. (**b**) Variation in electron plasma wave intensity at second harmonic frequency with normalized distance (ξ) and radial distance (r) for a = 0.8, when only relativistic nonlinearity is operative.

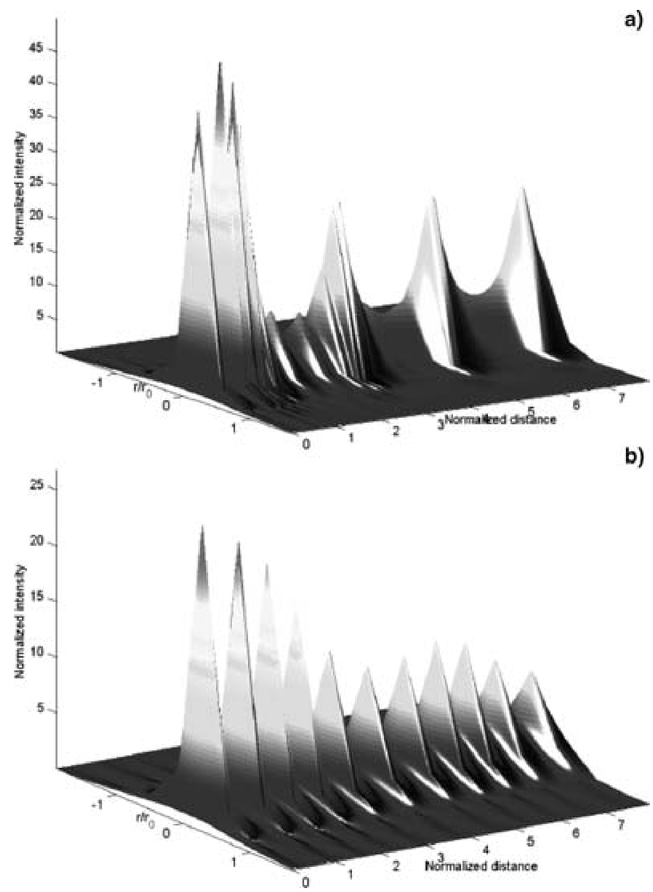

Fig. 3. (**a**) Variation in intensity of second harmonic with normalized distance (ξ) and radial distance (r) for $a = 0.8$, when relativistic and ponderomotive nonlinearities are operative. (**b**) Variation in intensity of second harmonic with normalized distance (ξ) and radial distance (r) for $a = 0.8$, when only relativistic nonlinearity is operative.

5. POWER OF THIRD HARMONIC GENERATION

The effective dielectric constant of the plasma at the third harmonic, frequency is given by

$$\varepsilon_3(\omega_s) = \varepsilon_{3f}(\omega_s) + \gamma(\omega_s) r^2. \tag{29}$$

Here

$$\varepsilon_{3f}(\omega_s) = \left(1 - \frac{\omega_{po}^2}{\omega_s^2}\right) + \frac{\omega_{po}^2}{\omega_s^2}\left[1 + \left(-1 + \frac{a}{\gamma f_o^4 r_o^2 k_p^2}\right)\left(1 + \frac{a}{f_o^2}\right)^{-1/2}\right]$$

and

$$\gamma(\omega_s) = -\frac{\omega_{po}^2}{\omega_s^2}\left[\frac{a}{2\gamma^3 r_o^2 f_o^4} - \frac{3a}{\gamma^2 k_p^2 r_o^4 f_o^6} - \frac{3a^2}{\gamma^4 k_p^2 r_o^4 f_o^8}\right].$$

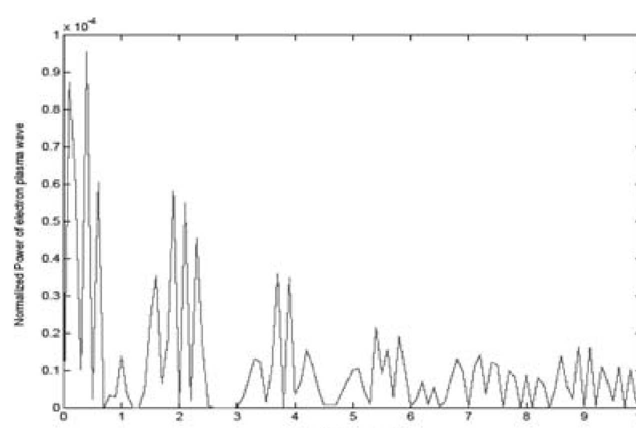

Fig. 4. Variation in normalized power of electron plasma wave at second harmonic frequency with normalized distance (ξ) for $a = 0.8$, when relativistic and ponderomotive nonlinearities are operative.

The solution for Eq. (16b) in terms of vector potential, can be written as

$$A_3 = A_{30} \exp\Big(-i(k_3 z + s_3)\Big) + A_{31} \exp\Big(-3i(k_o z + s_o)\Big) \quad (30)$$

Using this solution in Eq. (16b) and separate real and imaginary parts, we get

$$2k_3 \frac{\partial s_3}{\partial z} + \left(\frac{\partial s_3}{\partial x}\right)^2 + \left(\frac{\partial s_3}{\partial y}\right)^2 = \frac{1}{A_{30}}\left(\frac{\partial^2 A_{30}}{\partial x^2} + \frac{\partial^2 A_{30}}{\partial y^2}\right) + \gamma(\omega_s) r^2 \quad (31a)$$

$$k_3 \frac{\partial E_{30}^2}{\partial z} + A_{30}^2\left(\frac{\partial^2 s_3}{\partial x^2} + \frac{\partial^2 s_3}{\partial y^2}\right) + \frac{\partial s_3}{\partial x}\frac{\partial A_{30}^2}{\partial x} + \frac{\partial s_3}{\partial y}\frac{\partial A_{30}^2}{\partial y} = 0 \quad (31b)$$

and

$$A_{31} \cong \frac{\omega_{po}^2}{2c^2}\frac{\omega_s}{\omega_o}\left(\frac{N_{12}}{n_o}\right)\frac{A_{oo}}{f_o}\frac{\exp(-r^2/r_o^2 f_o^2)}{(k_3^2 - 9k_o^2)} \quad (32)$$

The solution for Eqs. (31a) and (31b) in the paraxial ray approximation are given by

$$s_3 = \frac{r^2}{2}\frac{k_3}{f_3}\frac{df_3}{dz} + \phi_3(z)$$
$$A_{30}^2 = \frac{(B')^2}{f_3^2}\exp\left(-\frac{r^2}{a_o^2 f_3^2}\right) \quad (33)$$

and f_3 is the dimensionless beam width parameter of the third harmonic generation governed by

$$\frac{d^2 f_3}{d\xi^2} = \frac{1}{f_3^3} - \frac{f_3}{9}\left[\left(\frac{r_o^2\omega_{po}^2}{c^2}\right)\frac{a}{2\gamma^3 f_o^4} + \frac{3a}{\gamma^2 f_o^6} - \frac{3a^2}{\gamma^4 f_o^8}\right]. \quad (34)$$

Moreover, the constant B' and a_o are determined by the boundary condition that the third harmonic is zero at $z = 0$.

$$B' = -\frac{\omega_{po}^2}{c^2}\left(\frac{N_{20}}{n_o}\right)_{z=0}\frac{A_{oo}}{(k_3^2 - 9k_o^2)}.$$

The vector potential (A_3) produced due to third harmonic generation; we get from Eq. (30).

$$A_3 = -A_{oo}\left(\frac{\omega_{po}^2}{c^2}\right)\left[\frac{H_1}{f_3}\exp\left(-\frac{r^2}{r_o^2}\left(1 + \frac{1}{f_3^2}\right)\right)\exp\Big(-i(s_3 + k_3 z)\Big) + \frac{H_2}{f_o^3}\times\exp\left(-\frac{2r^2}{r_o^2 f_o^2}\right)\exp\Big(-3i(k_o z + s_o)\Big)\right] \quad (35)$$

Here

$$H_1 = \left(\frac{n}{n_o}\right)_{z=0}\frac{c^2}{v_{th}^2}\frac{e^2 k_o^2 A_{oo}^2}{m_o^2\gamma^2 c^2\left(4\omega_o^2 - 4k_o^2 v_{th}^2 - (\omega_{po}^2/\gamma)\left(n/n_o\right)_{z=0}\right)(k_3^2 - 9k_o^2)}$$

and

$$H_2 = \left(\frac{n}{n_o}\right)\frac{3c^2}{2v_{th}^2}\frac{e^2 k_o^2 A_{oo}^2}{m_o^2\gamma^2 c^2\left(4\omega_o^2 - 4k_o^2 v_{th}^2 - (\omega_{po}^2/\gamma)(n/n_o)\right)(k_3^2 - 9k_o^2)}$$

Therefore the third harmonic power P_3 incident across the transverse cross section at z

$$P_3 = \frac{\omega_o^2}{8\pi c}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} A_3 A_3^* dy$$

$$P_3 = P_o\left(\frac{\omega_{po}^2}{c^2}\right)^2 \times\left[\frac{H_1^2}{2(1+f_3^2)} + \frac{H_2^2}{4f_o^4} + \frac{2H_1 H_2 f_3 \cos[(k_3 - 3k_o)z]}{[f_o^2(1+f_3^2) + 2f_3^2]}\right]. \quad (36)$$

Here we developed the theory for the third harmonic generation and derive the expression for the third harmonic power (Eq. 36) where relativistic and ponderomotive nonlinearities are operative. Figure 5 demonstrates the variation of the power of third harmonic generation where both relativistic and ponderomotive nonlinearities are operative.

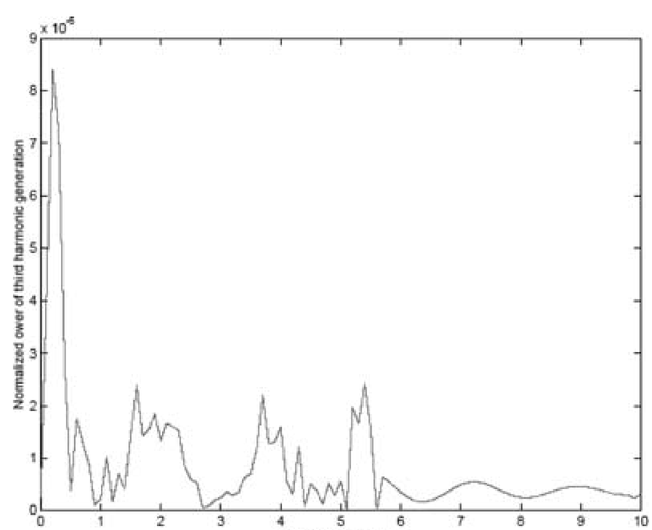

Fig. 5. Variation in normalized power of third harmonic generation with normalized distance (ξ) for $a = 0.8$, when relativistic and ponderomotive nonlinearities are operative.

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6. RESULTS AND DISCUSSION

In the present investigation, we studied the propagation of an ultra intense laser beam through plasmas. This ultra intense laser beam interacts with plasma electrons and these electrons will gain momentum, but the mass of electron will be increased due to relativistic effect, and their dynamics will be influenced. Simultaneously, ponderomotive force push electrons toward the direction perpendicular to the laser beam propagation; this will change the electron density of the plasma channel. The plasma channel alternatively behaves as a converging and diverging lens, and then ultra intense laser beam will be alternatively focused and defocused. These focusing and defocusing effects make very intense hot filaments of the laser beam in a plasma channel. In this investigation, we studied the effect of ponderomotive force on the intensity of filaments. For Nd glass laser, we got maximum normalized intensity of laser beam $A'^2/A_o^2 = 1.6$ where only relativistic nonlinearity is operative, but when relativistic and ponderomotive nonlinearities are operative, we got maximum intensity of laser beam $A'^2/A_o^2 = 6$.

Further we investigated these ultra intense hot filament traps plasma waves at second harmonic frequency because of very high $\vec{V} \times \vec{B}$ force in the channel. The amplitude of these waves is proportional to the square of the amplitude of the electromagnetic wave, there phase velocity is the same as that of the transverse wave and hence their Landau damping is negligible. The maximum power of these electron plasma waves comes out to be $P_2/P_o = 0.8 \times 10^{-4}$. Therefore, these plasma waves lead to third harmonic generations, and the maximum power of third harmonic generation comes out to be $P_3/P_o = 8 \times 10^{-6}$. In fast igniter fusion, the interaction of main laser pulse with compressed core could produce high level of third harmonic generation.

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