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High-performance tracking of high-speed supercavitating vehicles with uncertain parameters using novel parameter-optimal iterative learning control

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SUMMARY
This paper proposes a new technique based on a Parameter-Optimal Iterative Learning Control (POILC) to track a command pitch rate of a high-speed supercavitating vehicle (HSSV). The pitch rate of a supercavitating vehicle has non-minimum phase behavior. Thus, tracking is fundamentally limited to poor performance. To solve this problem, a feed-forward control can be used while using the cavitator as a control input in the feed-forward path to modify the slow response caused by non-minimum phase behavior. The main idea of this paper is to apply the cavitator input with high precision as a feed-forward control to improve tracking performance. The exact value of the feed-forward control is achieved using POILC. However, in the presence of uncertainty, zero convergence of POILC algorithms is threatened. It will be shown that applying adaptive weight in the performance index, the convergence is guaranteed in the presence of uncertainty and also when the system is sign-indefinite. The proposed technique includes an optimal planning to make the error norm monotonically convergent to zero. The convergence and perfect tracking will be guaranteed through a Lyapunov candidate. Performance and significance of the proposed supercavitating vehicle control will be verified by simulation.

KEYWORDS: Supercavitating robot; Iterative learning control (ILC); Monotonic convergence; Parametric optimization; Uncertainty; Robustness.

1. Introduction

In recent years, achieving higher speeds in underwater vehicles has become an important issue. Typical designed underwater vehicles are limited to a speed of about 40 m/s.[1] This restriction is due to the drag forces of skin friction in a fluid. Interaction between skin friction and surrounding water always leads to a drag opposing the direction of movement. For several years, researches have conducted to increase the speed of underwater vehicles. However, in order to reach higher speeds, researches have usually focused on increasing propulsion and improving body form. Although these methods improve the speed, less significant improvements have been made in skin friction drag. Later, scientists succeeded in accessing speeds of over 80 m/s by using supercavitating technique, which revolutionized the underwater vehicle industry.[2] The idea is based on creating a gas that surrounds the moving vehicle and therefore makes the least contact between the body and the surrounding water. This indeed leads to considerable reduction in drag force, and subsequently higher speeds. This new approach of supercavitating can be applied for making extremely fast and even supersonic underwater vehicles as seen in Fig. 1.

Although supercavitating is an effective phenomenon to achieve higher speed, it introduces important challenges in modeling and control. The main part of the challenges is governed by cavity, control, navigation and stability of the vehicle that must be dealt with a narrow region located

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Fig. 1. A supercavitating robot.

in the front and behind the vehicle. Some of the important challenges of supercavitating vehicles can be summarized as follows[2,3]:

- Producing and preserving cavity.
- Mass balance of vehicle.
- Control and navigation.
- High-performance tracking.
- Stability.
- Non-minimum phase dynamics.

One of the deals of a supercavitating vehicle for high-performance tracking is non-minimum phase behavior of such systems in pitch channel for depth control.[4] Control surfaces are needed for lift forces. However, there is non-minimum phase behavior by using these fin inputs. The non-minimum phase behavior reduces the tracking performance of the system with respect to control surfaces. It is shown that a feed-forward control of a cavitator can be used to compensate the slow behavior of the system.[5] Determination of a feed-forward control is a great deal for researchers, especially when high accuracy is desired, and it is the main issue. The exact value of a feed-forward control can be achieved using an iterative learning control (ILC). This is an effective method for determining a feed-forward control that can achieve high-performance tracking in the presence of uncertainty at the parameters of the system and also non-minimum phase behavior of a vehicle with respect to conventional controllers (such as predictive control, robust, PID etc.). The main characteristic of an ILC is using recorded data from the previous operations. These data improve control performance from trial to trial such that tracking error is significantly reduced during consecutive tasks. Some important applications of an ILC include robotic (mobile,[6,7] welding,[8] underwater robots[9] etc.), rotating systems,[10] batch chemical processes[11] *et al.*

The idea of an ILC algorithm for academic community was first introduced by Arimoto *et al.* in 1984.[12] Convergence properties (especially convergence to a zero tracking error) of the proposed Arimoto-type ILC algorithms have been completely analyzed. However, it is not always clear how to determine the learning gain of the algorithms to achieve monotonic or fast convergence.[13] In addition, it has been shown that if the learning gain is not chosen optimally, then the algorithms may preserve poor transient performance.[14] An effective improvement approach of these controllers is its combination with optimization problems to iteratively correct the learning gain. This so-called Parameter-Optimal Iterative Learning Control (POILC) algorithm is based on applying an appropriate optimization problem at any iteration, solving the problem and rerunning the algorithm with optimal input. This controller has attractive characteristics (such as simple structure, which is effective to implement in practice) as mentioned in different papers.[14] This algorithm was first proposed by Owens and Feng.[13] At the end of any iteration, the technique produces an optimal learning gain to provide a monotonic convergence. The main shortcoming of the POILC algorithm is high dependence of zero convergence (perfect tracking) upon positivity (or negativity) of a plant matrix description. In other words, if system fails to comply sign-definite (positivity or negativity) condition, convergence to a zero tracking error (as the main purpose of any ILC algorithm) cannot be achieved.[15] Ownes and Feng[13] showed that the algorithm is not convergent to a zero tracking error when the plant is sign-indefinite. The original system can be sign-definite by either using slow sampling rate or substituting all signals by exponentially weighted signals.[13] In ref. [15], a new high-order POILC algorithm was proposed to improve the work of Owens and Feng.[13] The zero convergence was ensured by adding an appropriate set of basic functions into the updating rule. In order to guarantee the zero convergence, a large number of parameters are needed. Another approach can be using inverse of the plant to make system sign-definite.[16] However, the inverse algorithm can be degraded at a high frequency of signals.[17] An approximate polynomial representation of the inverse plant can reduce the effect of high frequency.[17] The behavior of a POLIC algorithm in the vicinity of limit sets can have undesirable convergence properties. To eliminate such limiting behaviors, Owens *et al.*[18] proposed a switching-based approach. In essence the model was assumed to be a perfect one. It was also claimed that further work is needed to consider robustness with respect to uncertainty in model parameters[18] and requires other algorithms for improvement. The efficiency of POILC algorithms is still an arising issue in the presence of model uncertainty and disturbances.[17] These conditions are hard to meet as several systems fail to provide the zero convergence. The Markov parameters are affected by state space matrix parameters. The matrix size is relatively large. Besides, a small variation (e.g. the sampling rate) changes the matrix completely different from the previous one. Therefore, altering some conditions will be a serious problem even for a small change in matrix that violates the assumptions of perfect convergence. These facts necessitate an alternative approach with the aim of finding solutions to control supercavitating vehicle via modifying the conventional POILC. This is achieved through the modification of performance index to obtain zero convergence. Authors try to develop and extend the POILC algorithm to consider such plants that led to the current proposal, i.e. using adaptive weight (ρ_j) in POILC. This convergence will be monotonic such that at any iteration the error is gradually reduced. The proposed modification for a POILC is analytically proved using the Lyapunov theory for the first time. Meanwhile, this approach is shown coping with the lumped uncertainty. The quality of the approach will be shown when pitch rate of the vehicle is properly controlled. The performance will also be improved in the presence of bounded model uncertainty.

The rest of the manuscript is organized as follows.

In Section 2, the dynamic of a supercavitating vehicle is investigated. Also, the tracking problem and convergence of POLIC are studied in this section. In Section 3, a new performance index in terms of the error is developed to generate a desired control signal. Convergence of the closed-loop error at iterations is investigated in Section 4. To evaluate the performance of the proposed method, numerical simulations are presented in Section 5 for a supercavitating vehicle. Finally, conclusions close the work in Section 6.

2. Supercavitating System and Problem Description

In this section, supercavitating vehicle model will be stated. The goal is to maintain a perfect tracking of pitch rate in a prescribed path by deflection of cavitator. Due to the challenges in producing and preserving cavities, mass balance of the vehicle and control of this system are also important issues.[19] One of the most recent researches in this field is conducted by Mokhtarzadeh *et al.*[20]

2.1. Supercavitating vehicle dynamics

The proposed method is based on pitch rate controller (rotation around y-axis), which uses the following two-state longitudinal model based on angle of attack (α) and pitch rate (q) given in ref. [21]:

$$\begin{cases} \dot{\alpha} = \frac{1}{u\,m}(F_{c_z} + F_{T_z} + F_{g_z}) + q, \\ \dot{q} = \frac{1}{I_{yy}}(M_{c_y} + M_{T_y}), \end{cases} \tag{1}$$

where F_{c_z}, F_{T_z} and F_{g_z} are applied forces to body due to cavitator, thrust and gravitation in the z-direction, respectively, and M_{c_y} and M_{T_y} are cavitator and thrust moment acting rotation about the y-axis, respectively. Furthermore, I_{yy}, u and m are moment of inertia in the y-direction, axial velocity

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in the x-direction and vehicle mass, respectively. Indeed, the idea was the linearization of equations of motion and then using the linear controllers.

Generally, applied forces to this robot are much more complicated than airplane. However, since robot quickly reaches to a steady speed (u_0), equations of motion of this robot are essentially similar to an airplane.[1] These equations are linearized about the axial velocity (as u) whereas other small variations are neglected. Therefore, control is basically designed at this velocity.[1] Linearization about u_0 makes it possible to use linear controllers effectively.[2] Finally, the following relation is obtained between the linearized input and output about u_0 using (1),

$$q = G_e \delta_e + G_T u_T + G_c \delta_c, \tag{2}$$

where δ_e, u_T and δ_c are Elevator deflection angle, thrust and cavitator deflection angle, respectively. Also,G_e, G_T and G_c denote the transfer function of pitch rate with respect to Elevator deflection angle, thrust and cavitator deflection angle, respectively. Here the goal is to determine δ_c in order to minimize output tracking error.

Controller should be designed such that during each test, cavitator profile changes properly. Performance of pitch rate should be improved in order to track the desired path (other rotations are considered to be zero). Dynamical equations are divided into two mutually independent longitudinal and lateral equations. These equations are separately defined in the body coordinate system. In this paper, robot is considered to be rigid and symmetrical. As a result, robot has no initial undesired deflection. Usually, mechanical parts are considered such that rotation in lateral equations becomes negligible. Otherwise, a separate control for lateral system has to be designed. In the present study, the goal is to control pitch rate in a predefined path in order to satisfy the performance requirements.

The path and initial conditions are assumed constant in each trial while sampling is performed in time domain during the test procedure. Due to practical restrictions, such as elapsed time (speed), limited amount of fuel and desired paths, it is necessary to make high-performance tracking. To reach these goals and solve the perfect tracking problem, new POILC technique is proposed.

2.2. *Problem statement*

Consider the following linear discrete-time iterative system:

$$\begin{cases} x_j(t+1) = A\,x_j(t) + B\,u_j(t) \\ y_j(t) = C\,x_j(t) + D\,u_j(t) \end{cases}, \quad \begin{matrix} t = 0, 1, \ldots, M, \; j = 0, 1, \ldots \\ x_j(0) = x_0 \end{matrix} \tag{3}$$

where j is iteration number and t is time in each iteration. $x \in \Re^n$ is the state vector, and $u \in \Re$ and $y \in \Re$ are system input and output, respectively. Moreover, x_0 is the initial condition vector of the system, and M is the number of sampling in time domain. Matrices A, B, C and D are real with appropriate dimensions. Without loss of generality, lifted plant can be used for system description (see ref. [15]). In general, the ILC problem is to design control law for system (3) such that by increasing the number of iterations, the following error between $y_j(t)$ and $y_d(t)$ converges to zero:

$$\lim_{j\to\infty} (y_d(t) - y_j(t)) = 0, \quad \text{for} \quad t = l, \; l+1, \ldots, M. \tag{4}$$

Vectors y_d and $y_j$are defined as follows:

$$\begin{aligned} Y_d &= [y_d(l), \; y_d(l+1), \ldots, \; y_d(M)]^T, \\ Y_j &= \left[y_j(l), \; y_j(l+1), \ldots, \; y_j(M)\right]^T. \end{aligned} \tag{5}$$

where T is transpose. The number l indicates the relative degree as in the following:

$$l = \begin{cases} 0 & D \neq 0 \\ \min\limits_s \left\{ s \in \aleph^+ : C A^{s-1} B \neq 0 \right\} & D = 0 \end{cases}. \tag{6}$$

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It should be mentioned that the pitch rate is considered as system output and the cavitator deflection angle is assumed as system input that is determined using the proposed POILC. Also, the state space model in (3) is for transfer function G_c. The main goal of iterative learning control is to satisfy the following relation:

$$\lim_{j\to\infty} \left\| E_j \right\| = 0, \tag{7}$$

where $\| \, . \, \|$ denotes the 2-norm (the Euclidean) and $E_j = [e_j(l)e_j(l+1)...e_j(M)]^T$ stands for the error vector $(e_j(i) = y_d(i) - y_j(i), \;\; i = \{l, \, l+1, \, \ldots \, M\})$. Thus, the high-performance tracking is achieved. The following describes non-convergence problem of Eq. (7).

2.3. ILC problem formulation

It is the aim to provide a proper control law to modify previously achieved convergence. This control has to be generated by the error. Consider the following control law proposed by Owens and Feng[13]:

$$u_{j+1} = u_j + \beta_{j+1} \, e_j, \tag{8}$$

where β_{j+1} (learning gain) is an optimal parameter and must be tuned in an optimal approach. Error will be defined as a discrepancy of the desired trajectory from the current path in *j*th iteration as in the following form:

$$e_j(t) = y_d(t) - y_j(t). \tag{9}$$

As soon as e_j is formed in the *j*th iteration, learning coefficients can be determined for $j+1$ iteration such that the following performance index is minimized:

$$J_{\text{Old}}(\beta_{j+1}) = \left\| E_{j+1} \right\|^2 + w\beta_{j+1}^2. \tag{10}$$

The closed-loop error must be achieved at any iteration to determine the learning gain. First, by applying system (3) it can be written that

$$x_j(t) = A^t x_0 + \sum_{k=0}^{t-1} A^{t-1-k} B u_j(k). \tag{11}$$

By using (11) and also system output in state space (3), the following equation is concluded:

$$Y_j = GU_j + d, \tag{12}$$

where:

$$U_j = \left[u_j(0), u_j(1), \ldots u_j(N-1)\right]^T, Y_j = \left[y_j(l), y_j(l+1), \ldots y_j(M)\right]^T. \tag{13}$$

with the following Markov parameters:

$$G = \begin{bmatrix} g_l & 0 & 0 & \cdots & 0 \\ g_{l+1} & g_l & 0 & \cdots & 0 \\ g_3 & g_2 & g_l & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ g_M & g_{M-1} & g_{M-2} & \cdots & g_l \end{bmatrix}. \tag{14}$$

Matrix d in (12) indicates the role of initial condition, kept constant during each iteration. g_i, $i = l, \;\; l+1, \ldots, \;\; M$ by $(N = M - l + 1)$, and $g_l \neq 0$ are the Markov parameters of system (3). Similarly, for a model with a relative degree of $l \neq 0$, the Markov coefficients will be $g_i = CA^{i-1}B$.

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Using (12) results in

$$Y_{j+1} - Y_j = GU_{j+1} + d - GU_j - d. \tag{15}$$

In other words:

$$Y_{j+1} = Y_j + GV_j, \tag{16}$$

where:

$$V_j = U_{j+1} - U_j. \tag{17}$$

It can be seen from (16) that

$$Y_d - Y_{j+1} = Y_d - Y_j - GV_j. \tag{18}$$

Actually, Eq. (18) constructs the error vector as in the following form:

$$E_{j+1} = E_j - GV_j. \tag{19}$$

According to the definitions of vectors E_j and Y_j, (17) can be rewritten as:

$$V_j = \beta_{j+1} E_j. \tag{20}$$

Substitution of V_j from (21) into (19) yields the following closed-loop error:

$$E_{j+1} = G_c(j+1)E_j, \tag{21}$$

where:

$$G_c(j+1) = I - \beta_{j+1}G\ . \tag{22}$$

Equation (22) is the matrix of closed-loop system at iterations, and I is the identity matrix with appropriate dimensions.

The learning gain is updated such that the performance index is minimized at any iteration. The learning gain for minimizing the performance index (10) is as follows:

$$\beta_{j+1} = \frac{E_j^T G E_j}{w + E_j^T G^T G E_j}\ . \tag{23}$$

By substituting the learning gain into performance index and applying induction, the following relation is finally achieved:

$$\lim_{j\to\infty} \frac{\left(E_j^T G E_j\right)^2}{w + E_j^T G^T G E_j} = 0\ . \tag{24}$$

However, the important goal of any ILC algorithm is high-performance tracking, i.e. $\lim_{j\to\infty} \|E_j\| = 0$. The denominator of (24) is strictly positive, and as a result the following limit is held,

$$\lim_{j\to\infty}\ E_j^T G E_j = 0. \tag{25}$$

From (25) cannot be concluded that $\lim_{j\to\infty} \|E_j\| = 0$, where $\|E_j\|$ denotes the 2-norm (the Euclidean norm) of the error vector. The error norm converges to zero when $G + G^T$ is sign-definite ($G + G^T > 0$ or $G + G^T < 0$).[13] The proposed idea is that if $\lim_{j\to\infty}\ \rho_{j+1}^2 \|E_j\|^2 = 0$ is satisfied instead of Eq. (25) such that $\lim_{j\to\infty}\ \rho_{j+1}^2 \neq 0$, then it can be concluded that $\lim_{j\to\infty} \|E_j\| = 0$.

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The goal is to define an adaptive weight ρ_{j+1} in the performance index to provide zero convergence. The proposed index is defined to guarantee monotonic convergence to a zero tracking error.

3. Proposed POILC Design

Index J_{Old} fails to guarantee perfect convergence of the error norm to zero for sign-indefinite systems.[13] Therefore, another technique must be introduced to create high-performance tracking. The idea is to define an alternative performance index. This is based on using an adaptive weight (ρ_j) in the performance index to guarantee the convergence for sign-indefinite systems. The performance index is proposed as follows:

$$J_{\text{New}}(\beta_{j+1}) = \rho_{j+1}^2 \|E_j\|^2 + w\beta_{j+1}^2, \tag{26}$$

where w is the scalar weight to limit the amplitude of the control effort. It should be noted that required control signal would be generated from this criteria when the error is optimized. Furthermore, the criteria provide a monotonic convergent and an update law for the learning gain.

The main goal of the proposed POILC is to provide convergence to a zero tracking error for systems in which conventional algorithms were not able to guarantee the convergence. It is also the aim to show shortcoming of the conventional POILC[13] in the supercavitating problem due to irregularity in the Markov coefficient matrix of $G + G^T$. Here $\|E_j\|^2 = E_j^T E_j$ and w is a positive parameter. Coefficient ρ_{j+1} is updated from the following equation:

$$\rho_{j+1}^2 = 1 + S^2\beta_{j+1}^2 + F_j\beta_{j+1}, \tag{27}$$

where coefficients S and F_j are calculated from the following relations:

$$\begin{aligned} S^2 &= \lambda_{\max}(G^T G), \\ F_{j+1} &= \text{sign}(-\Phi_j)\lambda_{\max}\{-(G + G^T)\ \text{sign}(-\Phi_j)\} \neq 0. \end{aligned} \tag{28}$$

$\lambda_{\max}$ (.) is the largest Eigenvalue and sign (.) denotes the sign function. $\beta_{j+1}$is a stationary point of function (26), which is obtained by

$$\frac{\partial J_{\text{New}}(\beta_{j+1})}{\partial \beta_{j+1}} = 2\{w + \Psi_j\}\beta_{j+1} + \Phi_j = 0 \Rightarrow \beta_{j+1}^* = -\frac{1}{2}(w + \Psi_j)^{-1}\Phi_j, \tag{29}$$

where denominator terms, i.e. Ψ_j and Φ_j are as follows:

$$\begin{aligned} \Psi_j &= \|E_j\|^2 S^2, \\ \Phi_j &= \|E_j\|^2 F_j. \end{aligned} \tag{30}$$

Furthermore, S denotes maximum of singular values G, whilst F_j, Ψ_j and Φ_j are some auxiliary variables to shorten the length of equations.

This proposed index solves the problem of convergence to a zero tracking error using adaptive weight. In the following, it will be shown that the performance index finally converges to zero to ensure perfect tracking. Using the above preliminaries, the convergence of the proposed POILC will be investigated in the next section.

4. Convergence of the Proposed POILC

As expressed in Section 1, perfect tracking is a challenging issue in POILC. Therefore, the convergence requirement(s) should be satisfied for such systems. In this section, it will be shown that by using criterion (26), monotonic convergence to a zero tracking error can be achieved. Primarily consider the following Lemma.

Lemma. *The following inequality for an upper bound of the error holds in all iterations.*

$$\|E_{j+1}\| \le \rho_{j+1}\|E_j\|. \tag{31}$$

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Proof. Refer to Eq. (21), the following relation holds for norm relations:

$$\|E_{j+1}\| = \|G_c(j+1)\| \, \|E_j\|, \quad \forall j \geq 0. \tag{32}$$

It is known that:

$$\|G_c(j)\|^2 = \lambda_{\max}(G_c^T(j)\, G_c(j)). \tag{33}$$

Accordingly, inequality in (22) provides:

$$G_c^T(j+1)\, G_c(j+1) = (I - \beta_{j+1} G)^T (I - \beta_{j+1} G) = I + \beta_{j+1}^2 (G^T G) - \beta_{j+1}(G^T + G). \tag{34}$$

Regarding the relations of maximum and minimum amount of Eigenvalues in ref. [22], substitution of Eq. (34) into (33) yields:

$$\lambda_{\max}\left(I + \beta_{j+1}^2 (G^T G) - \beta_{j+1}(G^T + G)\right) \leq 1 + \beta_{j+1}^2 S^2 + \beta_{j+1} F_j = \rho_{j+1}^2 \,. \tag{35}$$

In the meantime, by using (32) and (22) in (35), the following inequality is yielded:

$$\|E_{j+1}\| = \|G_c(j+1)\| \, \|E_j\| \leq \rho_{j+1} \|E_j\|. \tag{36}$$

Note that the equality still holds. This means that error does not increase. However, in the next section, it will be proved that equality holds when the error norm becomes zero (i.e. only in infinity).

The above-mentioned Lemma confirms that the tracking error converges to zero monotonically by applying the controller in (8) together with the optimal learning gain in (29). The following theorem proves the perfect convergence. □

Theorem 1. *Consider system (3). The updating control law (8) with optimal learning gain (29) guarantees monotonic convergence of the closed-loop error norm at iterations. In other words, the following two relations are held:*

$$\|E_{j+1}\| < \|E_j\|, \tag{37}$$

$$\lim_{j \to \infty} \|E_j\| = 0. \tag{38}$$

Indeed, inequality (37) shows a monotonic convergence, while Eq. (38) is an alternative expression of Eq. (4) to confirm convergence to zero.

Proof. In order to prove the convergence of the system, let the Lyapunov candidate function be:

$$\Gamma_j = \left\|E_j\right\|^2. \tag{39}$$

It is a positive function. Therefore, it has the necessary condition of the Lyapunov function. Using Lemma in the next iteration, the Lyapunov candidate function is stated as:

$$\Gamma_{j+1} = \left\|E_{j+1}\right\|^2 \leq \rho_{j+1}^2 \left\|E_j\right\|^2. \tag{40}$$

If a Lyapunov candidate is strictly decreasing, then it will be a Lyapunov function. It is therefore necessary to prove the decreasing feature. Replacement of the optimal learning gain of (29) in (35)

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obtains:

$$\begin{aligned}
&\left\|E_j\right\|^2\left(1+\beta_{j+1}^2 S^2+\beta_{j+1} F_j\right) \\
&=\left\|E_j\right\|^2\left(1+\frac{1}{4} \frac{F_j^2\left\|E_j\right\|^2\left(w+\left\|E_j\right\|^2 S^2-w\right)}{\left(w+\left\|E_j\right\|^2 S^2\right)^2}-\frac{1}{2} \frac{F_j^2\left\|E_j\right\|^2}{w+\left\|E_j\right\|^2 S^2}\right) \\
&=\left\|E_j\right\|^2\left(1-\frac{1}{4} \frac{F_j^2\left\|E_j\right\|^4}{w+\left\|E_j\right\|^2 S^2}-\frac{1}{4} \frac{w F_j^2\left\|E_j\right\|^2}{\left(w+\left\|E_j\right\|^2 S^2\right)^2}\right)<\left\|E_j\right\|^2 .
\end{aligned} \tag{41}$$

Equation (41) shows that the error norm is strictly decreasing. In other words, the following relation always holds:

$$\Gamma_{j+1}<\Gamma_j \Rightarrow \Gamma_{j+1}-\Gamma_j<0 . \tag{42}$$

On the other hand, a Lyapunov candidate possesses all the conditions of a Lyapunov function. Consequently, the error dynamic is convergent, i.e. error in each iteration is less than the previous one until it converges to zero.

To prove Eq. (38), necessary condition to maintain a perfect tracking will be assessed. Consider again the index in (26) when (28) is substituted in (27):

$$J_{\text{New}}(\beta_{j+1}^*)=\|E_j\|^2 \rho_{j+1}^2+w \beta_{j+1}^2=\|E_j\|^2-\frac{1}{4} \frac{F_j^2\|E_j\|^4}{w+\|E_j\|^2 S^2}. \tag{43}$$

Through mathematical induction, the following inequality is concluded:

$$\begin{aligned}
\|E_0\|^2 &> \|E_0\|^2-\frac{1}{4} \frac{F_0^2\|E_0\|^4}{w+\|E_0\|^2 S^2} \\
&> \|E_0\|^2-\frac{1}{4} \frac{F_0^2\|E_0\|^4}{w+\|E_0\|^2 S^2}-\frac{1}{4} \frac{F_1^2\|E_1\|^4}{w+\|E_1\|^2 S^2} \\
&> \|E_0\|^2-\frac{1}{4} \frac{F_0^2\|E_0\|^4}{w+\|E_0\|^2 S^2}-\frac{1}{4} \frac{F_1^2\|E_1\|^4}{w+\|E_1\|^2 S^2}-\frac{1}{4} \frac{F_2^2\|E_2\|^4}{w+\|E_2\|^2 S^2} \\
&> \ldots \\
&> \|E_0\|^2-\frac{1}{4} \sum_{k=0}^{j} \frac{F_k^2\|E_k\|^4}{w+\|E_k\|^2 S^2} \Rightarrow
\end{aligned}$$

$$0<\lim_{j \rightarrow \infty} J_{\text{New}}(\beta_{j+1}^*) \leq \|E_0\|^2-\lim_{j \rightarrow \infty} \sum_{k=0}^{j} \frac{1}{4} \frac{F_k^2\|E_k\|^4}{w+\|E_k\|^2 S^2}<\|E_0\|^2<\infty. \tag{44}$$

Given that the error norm is finite initially (it is not infinite), by converging the iterations to infinity in this infinite series, the term in the series of (44) must converges to zero (otherwise, the value of series will be infinite). Thus, the following limit is always held:

$$\lim_{j \rightarrow \infty} \frac{F_j^2\|E_j\|^4}{w+\|E_j\|^2 S^2}=0 . \tag{45}$$

Denominator in Eq. (45) is always seen positive and nonzero. Since $F_j \neq 0$, this equation holds as long as $\lim_{j \rightarrow \infty}\|E_j\|=0$ and this completes the proof. □

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The above theorem states that the error converges monotonically to zero. The following results are also valid. Although the result 1 and 2 are valid in ref. [13], the same results are achieved here when the weight is adaptively updated, which can be seen in (26)–(30).

Result 1. The following expression always holds for an optimal learning gain (29):

$$\lim_{j\to\infty} \beta_j = 0. \tag{46}$$

Proof. Denominator in Eq. (29), i.e. $(w + \Psi)$ term, is strictly positive whereas its numerator is an error norm function. Since the error norm converges to zero, it can be concluded that the optimal learning gain also converges to zero. □

Result 2. The error norm converges monotonically to zero. The following inequality always holds for index (26) and the error norm using optimal parameter (29):

$$\|E_{j+1}\|^2 < J_{\text{New}}(\beta^*_{j+1}) < \|E_j\|^2 \ . \tag{47}$$

Proof. According to (26) and (31), the first part of inequality in (47) holds ($\|E_{j+1}\|^2 < J_{\text{New}}(\beta^*_{j+1})$). The performance index is greater than error at any iteration. Since the performance index is the summation of a positive term with error, now it must be proved that this value of the error is less than the error in the previous section. Thus, the error will be convergent. Since the second side of index $\|E_j\|^2\rho^2_{j+1} + w\beta^2_{j+1}$ is positive, as a result it is always greater than $\|E_{j+1}\|^2$. On the other hand, Eq. (43), $J(\beta^*_{j+1}) = \|E_j\|^2 - \frac{1}{4}\ F_j^2\|E_j\|^4(w + \|E_j\|^2S^2)^{-1}$, shows that the second part of (47) can be clearly concluded ($J_{\text{New}}(\beta^*_{j+1}) < \|E_j\|^2$). □

Note. F_j and S^2 in (28) indicate an upper bound of error. In order to converge the error norm monotonically to zero other amounts may be assigned. It can be therefore concluded that F_j and S are treated as design parameters (even the same for Φ_j and Ψ_j) by which the error norm converges to zero (parameter S^2 may vary at each iteration). An appropriate tuning of these parameters, which determines a lower value for upper bound, ensures a fast convergence. In this regard, the following relation can also be used instead of (30):

$$\Psi_j = E_j^T G^T G\, E_j, \quad \Phi_j = F_j\|E_j\|^2. \tag{48}$$

4.1. Convergence analysis in presence of uncertainty in the model

Due to model simplification, uncertainty is usually involved in its structure. This uncertainty can be due to assumptions made during modeling process, system aging, change in working conditions, linearization etc. Consider system (3) in presence of an uncertainty, which can be rewritten in the following form:

$$\{x_j(t+1) = (A+\Delta A)x_j(t) + (B+\Delta B)u_j(t),\ y_j(t) = (C+\Delta C)x_j(t) + (D+\Delta D)u_j(t), \tag{49}$$

where matrices ΔA, ΔB, $\Delta C$and ΔD denote uncertain system dynamics. In this case, according to (3), a standard linear form of closed-loop error at iterations with uncertainty in the system can be represented in the following form[23]:

$$E_{j+1} = G'_c(j+1)\,E_j\ . \tag{50}$$

for

$$G'_c(j+1)\ = 1 - \beta_{j+1}G'\ , \tag{51}$$

where G' stands for the same as G except that A, B, C and D are replaced with $A+\Delta A$, $B+\Delta B$, $C+\Delta C$ and $D+\Delta D$, respectively. Existence of such uncertainty can be shown as $G' = G + \Delta G$.

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Therefore, (21) is rewritten in the following form as a lumped uncertainty:

$$G'_c(j+1) = 1 - \beta_{j+1}(G + \Delta G), \tag{52}$$

where:

$$\Delta G = \begin{bmatrix} \delta_l g_l & 0 & 0 & \cdots & 0 \\ \delta_{l+1} g_{l+1} & \delta_l g_l & 0 & \cdots & 0 \\ \delta_3 g_3 & \delta_2 g_2 & \delta_l g_l & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \delta_M g_M & \delta_{M-1} g_{M-1} & \delta_{M-2} g_{M-2} & \cdots & \delta_l g_l \end{bmatrix} = P \times G, \tag{53}$$

where matrix P is a lower triangular and is as follows:

$$P = \begin{bmatrix} \delta'_l & 0 & 0 & \cdots & 0 \\ \delta'_{l+1} & \delta'_l & 0 & \cdots & 0 \\ \delta'_{l+2} & \delta'_{l+1} & \delta'_l & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \delta'_M & \delta'_{M-1} & \delta'_{M-2} & \cdots & \delta'_l \end{bmatrix}. \tag{54}$$

It is also supposed that matrix P is an upper bounded by

$$\|P\|^2 \le \mu\ . \tag{55}$$

The goal is to present convergence conditions in presence of uncertainty.

Theorem 2. *System in (50) is monotonically convergent to zero if matrix P (in (54) as a bound of uncertainty) is upper-bounded by:*

$$\mu \le \frac{1}{4}\ . \tag{56}$$

Proof. Parameter β_{j+1} is again assessed in the same way as in (29). Taking the norm of (51) gives:

$$\left\|G'_c(j+1)\right\| = \left\|I - \beta_{j+1}(G + \Delta G)\right\| \le \left\|I - \beta_{j+1}G\right\| + \left\|\beta_{j+1}\Delta G\right\|. \tag{57}$$

Substituting (34) into (57) gives:

$$\left\|I - \beta_{j+1}G\right\| + \left\|\beta_{j+1}\Delta G\right\| \le \rho_j^2 \|G\|^2 + \left\|\beta_{j+1}G\right\|^2 \mu = 1 + \beta_{j+1}F_j + \beta_{j+1}^2 S^2 + \mu\,\beta_{j+1}^2 S^2. \tag{58}$$

Similarly for the same Lyapunov candidate as in (39), Eq. (32), i.e. $\|G_c(j+1)\|\ \|E_j\|$ gives:

$$\begin{aligned} \Gamma_{j+1} &= \left\|E_{j+1}\right\|^2 \le \left\|G'_c(j)\right\|^2 \left\|E_j\right\|^2 \\ &\le \left(1 + \beta_{j+1}F_j + \beta_{j+1}^2 S^2 + \mu\,\beta_{j+1}^2 S^2\right)\left\|E_j\right\|^2 \\ &\le \left(1 - \frac{1}{4}\frac{\left\|E_j\right\|^2 .F_j^2}{w + \left\|E_j\right\|^2 .S^2} + \mu\frac{\left\|E_j\right\|^4 .F_j^2}{\left(w + \left\|E_j\right\|^2 .S^2\right)^2} S^2\right)\left\|E_j\right\|^2 \\ &\le \left\|E_j\right\|^2 - \mu\frac{w\left\|E_j\right\|^4 .F_j^2}{\left(w + \left\|E_j\right\|^2 .S^2\right)^2} - \frac{\left\|E_j\right\|^4 .F_j^2 (0.25 - \mu)}{w + \left\|E_j\right\|^2 .S^2}. \end{aligned} \tag{59}$$

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According to the last part, if condition (58) holds, then closed-loop error is converging. This means the error norm is monotonically decreasing. However, to prove zero convergence, resorting to mathematical induction, (58) gives:

$$0 < \lim_{j\to\infty} \left\| E_{j+1} \right\| \leq \|E_0\| - \lim_{j\to\infty} \sum_{k=0}^{j} \mu \frac{w \, \|E_k\|^4 \,.F_k^2}{\left(w + \|E_k\|^2 \,.S^2\right)^2} < \|E_0\| < \infty \,. \tag{60}$$

Similar to (44), a series with infinite terms can be finite as long as the argument of the summation converges to zero. Since μ and F_k are nonzero, it is necessary that the error norm converges to zero, $\lim_{j\to\infty} \|E_j\| = 0$. This confirms that (38) is satisfied and error norm monotonically converge to zero. □

The above theorem shows that if uncertainty is upper-bounded by (55), i.e. $\mu \leq 0.25$, then the system is convergent to zero. The unity means the norm of the plant. Thus, $^1/_4$ means the lumped uncertainty is as big as up to 25% of the plant norm (which seems big!). The proposed controller copes with such a big uncertainty. On the other hand, this relation denotes that by adding μ, convergence to a zero tracking error will be more slow since a positive term is added to the decreasing error norm. It causes convergence to zero with smaller acceleration. However, this bound is conservative.

4.2. Convergence analysis with measurement noise

In this section, a measurement noise Q_j is added to the output of system (3) by the following form:

$$y_j(t) = G\, u_j(t) + Q_j(t), \tag{61}$$

where $Q_j(t)$ is a realization of stochastic process.[24] This makes Eq. (21) to become:

$$E_{j+1} = G'_c\,(j+1)\; E_j + D_j, \tag{62}$$

where $D_{j+1} = Q_j - Q_{j+1}$. In order to assess capability of the proposed algorithm against noise, three different kinds of noise are considered.

Situation 1: In case Q_j is the same at any iteration (for example, noise is a sinusoidal function in time axis), $D_j = 0$ and a monotonic convergence to a zero tracking error is achieved. This means that the measurement noise at the system output does not affect zero convergence. This can be originated due to environment effects, i.e. $D_j = 0$. This means that there is no such undesired effect on the convergence, and a zero monotonic convergence is attained.

Situation 2: A deterministic and predictable noise occurs differently in any iteration. This situation is similar to the case where an uncertainty exists in the model. Therefore, the negligibility of this kind of noise will be shown in appropriate section in presence of such an uncertainty.

Situation 3: Finally, a random and stochastic noise will be filtered by such a relevant active processing.

5. Simulation

Consider the dynamics of a supercavitating vehicle in state space format. The system parameters are $I_{yy} = 5.1847$ kg m^2, m $= 22$ kg and $u_0 = 77$ m/s. The model is digitized using the sample time of 0.001 sec. The relative degree of the system is 1 ($l = 1$).[4] The goal is that the system output tracks the following command for pitch rate. Therefore, a slow pitch rate control is of interest. From practical points of view, a typical desired pitch rate is as follows:

$$q_{\text{com}} = \begin{cases} 0.2\left(1 + \sin\left(\dfrac{2\pi}{T}t - \dfrac{\pi}{2}\right)\right), & 0 \leq t \leq T \\ 0, & t > T \end{cases}. \tag{63}$$

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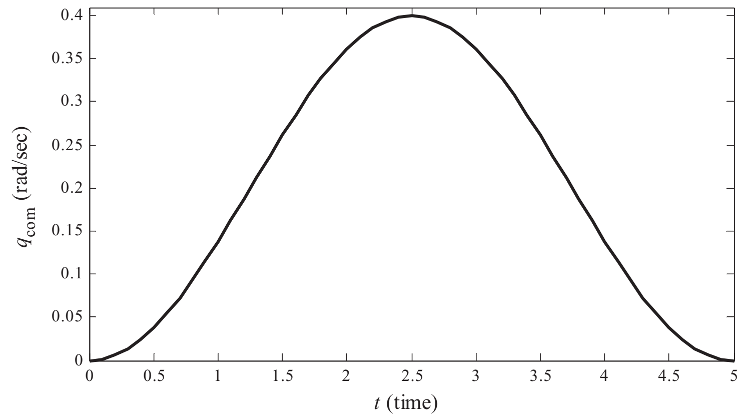

Fig. 2. A desired pitch rate command.

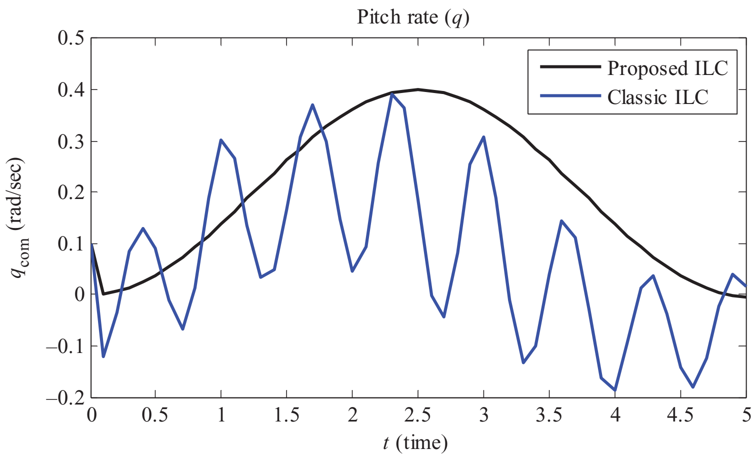

Fig. 3. The system pitch rate using two conventional and the adaptive weight POILC.

Figure 2 shows the desired output of the system in 5 sec ($T = 5$). For longer times, according to Eq. (63), desired output is kept constant at zero (the unit for the pitch rate in all the figures is rad/sec).

The Markov parameters of the system are obtained using Eq. (14). Eigenvalue of $G + G^T$ consists of both positive and negative ones. This declares that $G + G^T$ is sign-indefinite which fails to satisfy the definite requirements in ref. [13]. Thus, the closed-loop error fails to converge as seen in Fig. 3 if the conventional POILC[13] is used.

It can be seen from Fig. 3 that the system output could track the desired output (command pitch rate) after 300 iterations. In contrast, the proposed ILC technique offers a perfect tracking. In this section, three different cases are investigated while simulations are performed in the MATLAB^{TM} environment.

Case 1. A perfect model is of use: The model is considered perfect (with no uncertainty and noise). Figure 4 represents a simulation result of the error norm and the desired pitch rates in 300 iterations. Term $F_j \neq 0$ denotes that the maximum Eigenvalue of $G + G^T$ is nonzero. Consequently, necessary condition to use the proposed technique to provide a perfect convergence is satisfied. Initial conditions are $x_0 = [0,\ 0.1]^T$ and $w = 1$ whereas index (26) is also considered as in the following:

$$J(\beta_{j+1}) = \|E_j\|^2 \rho_{j+1}^2 + \beta_{j+1}^2. \tag{64}$$

As this figure shows, pitch rate controller has provided a suitable command signal. It is seen that command tracking was satisfactory and a perfect tracking is achieved.

Figure 5 also shows a 3D view per time and iteration. When the number of trials is increased, tracking is improved. Similar conclusions are achieved when other design parameters are tuned.

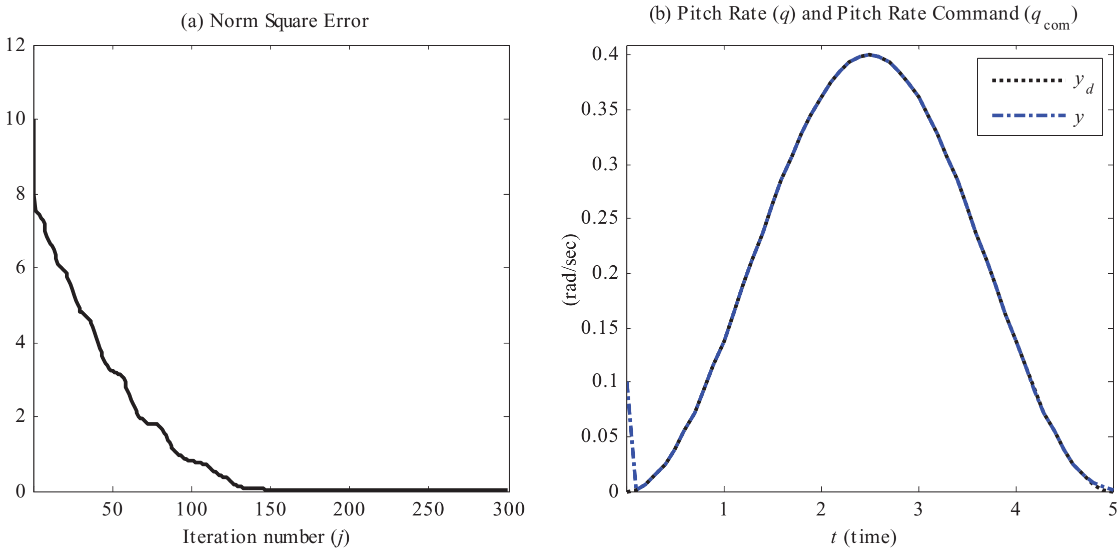

Fig. 4. (a) Error norm and (b) pitch rate and command pitch rate in a perfect model.

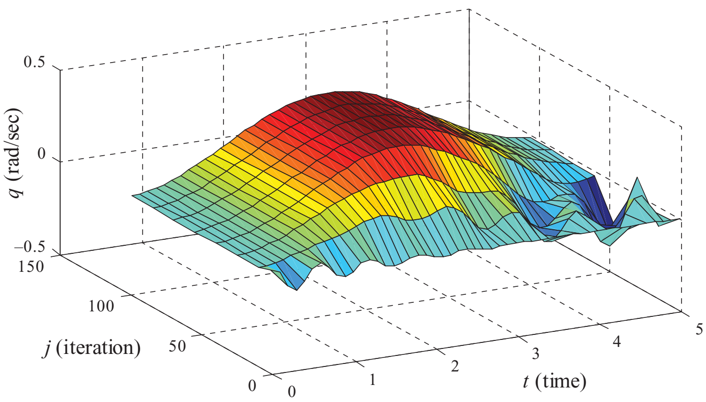

Fig. 5. 3D view of pitch rate per number of iterations and time.

Eventually at the beginning the tracking is not perfect yet. This is because the controller coefficient for an output at zero time consists of D, which shows that the relative degree is 1. This signifies that the control plays no effective role in the output.

Case 2. Perturbed by uncertainty: In this case, the model is perturbed by uncertainty. The perturbation may be involved by a Taylor expansion-based linearization about constant axial velocity or depth change and unexpected forces. A small deviation from the operating point certainly influences system parameters and changes model parameters. In this case consider the system with the following uncertainty:

$$\Delta A = \delta_a A,\ \ \Delta B = \delta_b B,\ \ \Delta C = \delta_c C\ , \tag{65}$$

where $|\delta_a| \le \bar{\delta}_a$, $|\delta_b| \le \bar{\delta}_b$ and $|\delta_c| \le \bar{\delta}_c$,[23] in which the upper bar stands for the maximum amount of $\bar{\delta}_a = 0.21$, $\bar{\delta}_b = 0.13$ and $\bar{\delta}_c = 0.11$. Using Eqs. (51) and (52) helps to find P (uncertainty matrix). A 2-norm immediately gives an upper bound of the Markov metric coefficients:

$$\mu = 0.2397\ \le\ 0.25. \tag{66}$$

Therefore, necessary condition to achieve monotonic convergence to a zero tracking error is met. Relevant simulation result in Fig. 6 confirms the findings.

By comparing the error norm in Figs. 6 and 7, it can be concluded that the system converges more slowly in the presence of uncertainty. It can be seen from (59) that if μ is added, then the error will

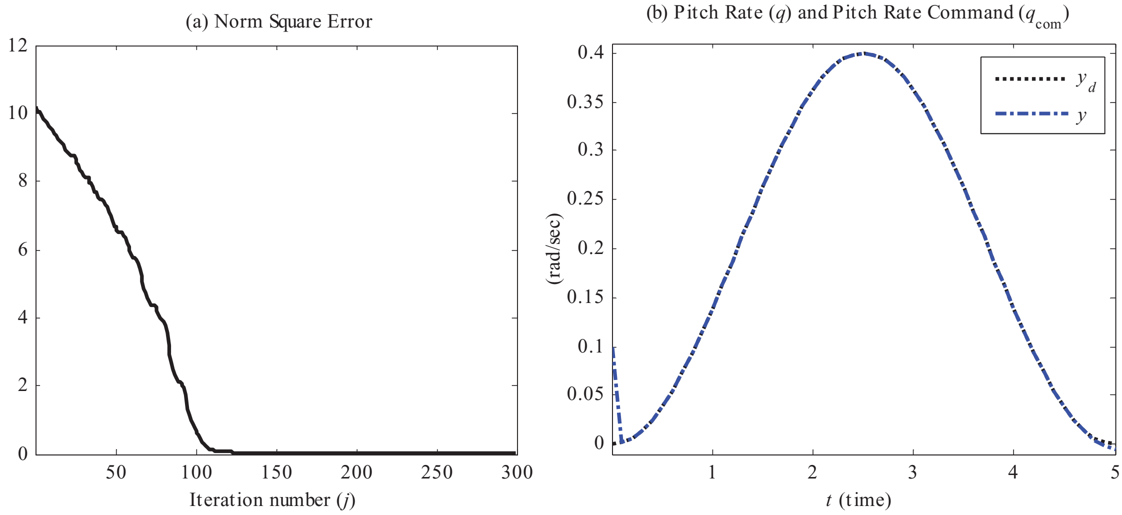

Fig. 6. (a) Error norm and (b) pitch rate and command pitch rate in the system with the uncertainty limit $\mu = 0.2397$.

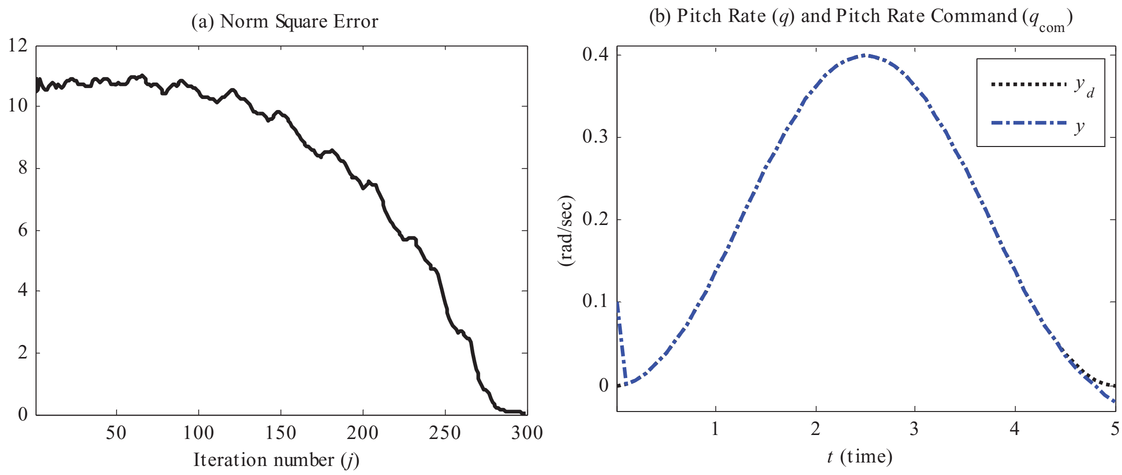

Fig. 7. (a) The error norm and (b) pitch rate and command pitch rate in presence of the uncertainty limit $\mu = 1.0274$.

decrease with a smaller value at next iteration. In other words, the rate of error decreasing is inversely dependent on μ such that if its value exceeds a certain limit, then the system will not be convergent. Figure 6(a) shows the error norm when such a larger uncertainty as in (66) perturbs the system,

$$\bar{\delta}_a = 0.21,\ \bar{\delta}_b = 0.23,\ \bar{\delta}_c = 0.2\ . \tag{67}$$

It is seen that convergence rate is yet lower than the previous case. This is because the upper bound is as $\mu = 1.0274$. Therefore, the error is going to deviate from its nominal value as shown in Fig. 7(b).

Case 3. In presence of measurement noise: The effect of uncertainty was investigated by means of theoretical study and, of course, by simulation in the previous section. In this section, the system is contaminated by some unknown measurement noise. Several resources produce noise on output sensors and gyroscopes. Although the noise characteristics are usually unknown, the Probability Spectral Density (PSD) is known. In this case, a noise with a normal distribution variance of 0.005 and zero mean[25] has perturbed the system output.

A simulation is carried out using noisy data. The error norm can be seen in Fig. 8. It is the aim to investigate the effect of external noise on the supercavitation system when it is under the control of an ILC. According to the closed-loop error equation for a bounded random noise, the convergence to zero

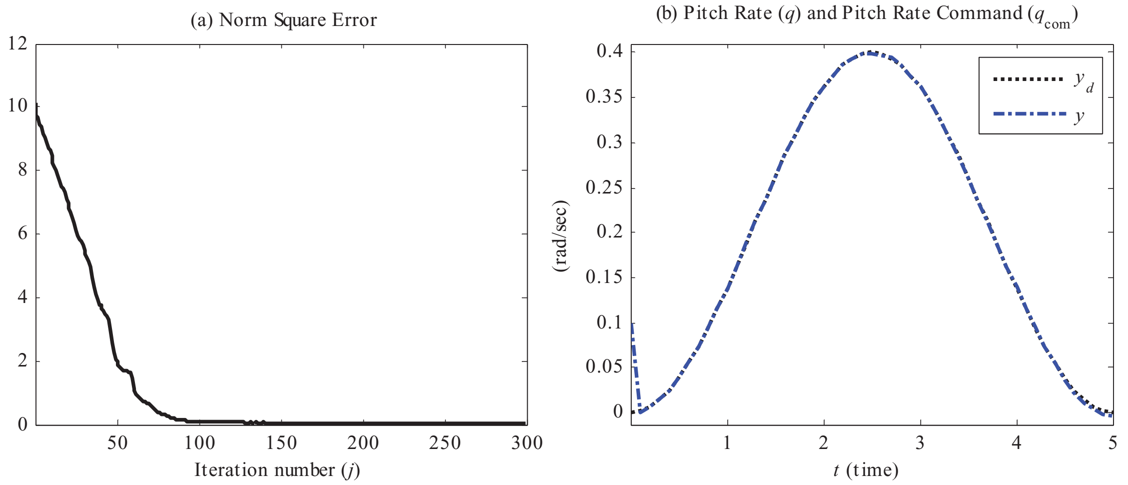

Fig. 8. (a) The error norm and (b) system and the command pitch rate in a noisy environment.

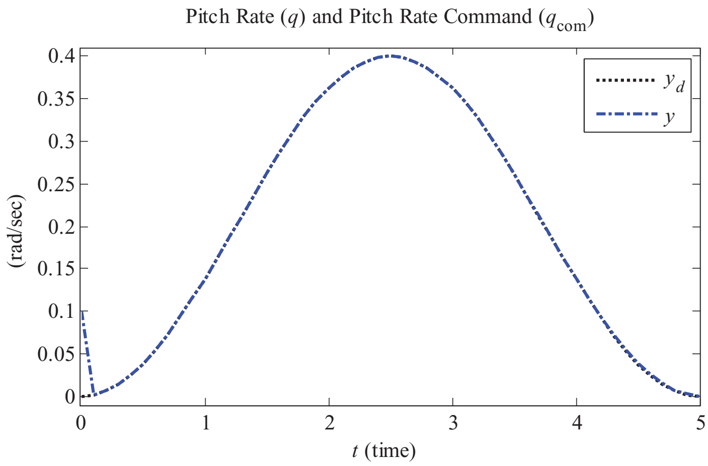

Fig. 9. The system (y) and the command pitch rate (y_d) in presence of the uncertainty limit $\mu = 1.0274$.

has failed. However, the output error is kept bounded even for bounded random noise. Meanwhile, when noise happens with the same dynamics at any iteration, the proposed algorithm is still capable to cope with. Simulation results confirm a perfect convergence with a satisfactory accuracy.

1. *At the beginning of the result*: System with nonzero relative degree is not controllable at initial time. Consider a system in the state space representation format using A, B, C and D. The output in the jth iteration and $t = 0$ can be written as follows:

$$y_j = C\,X_0 + D\,u_j, \tag{68}$$

where X_0 is a vector of initial state. Since at any iteration $D = 0$, the system with relative degree 1 is not initially controllable ($y_j = C\,X_0$). That is why at the beginning the output of the system is deviated from zero.

2. *At the end of the graph*: At $t = 5$, there is uncertainty in the system, Figs. 6–8, which reduces convergence with respect to an ideal case. Wider bound of uncertainty reduces the convergence rate, which can be seen in Figs. 6 and 7. Therefore, more iteration is needed to learn from this. For example, in Fig. 7 for $j = 350$, the system reaches the desired output at $t = 5$.

It is possible to reduce the number of iterations using the weight w. It is because w restricts amplitude of the control effort.[13] Figure 10 shows the effect of different values of w. This means

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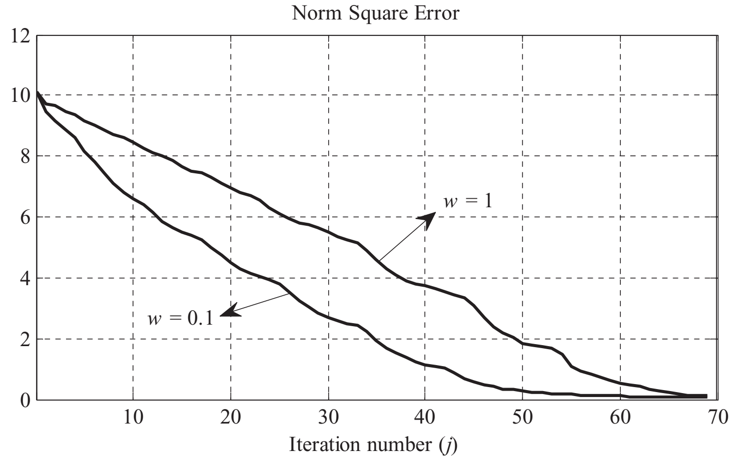

Fig. 10. The effect of weight in the convergence rate.

reduction in w increases the rate of convergence. However, the choice of $w = 0$ gives $J_{\text{New}} = \rho_j^2 \parallel E_j \parallel^2$, which is non-optimal.

6. Conclusions

In the present paper, a new approach is proposed to achieve perfect tracking in the POILC technique. In this approach, a feed-forward control signal is produced by the error of the output from the desired one. The required control signal is then manipulated in a law such that a new developed performance index for POILC is achieved. Necessity of a monotonic convergence to a zero tracking error is proved by some relevant theorems. These theorems guarantee that convergence to a zero tracking error can successfully be achieved by the recent development in the performance index. In accordance with the theorems, when the system is free from uncertainty and noise, a perfect convergence of tracking error to zero is provided. In case the model contains uncertainty, a required bound of uncertainty is assessed to achieve convergence. Simulations on a supercavitating vehicle have shown that larger bounds of uncertainty may endanger the monotonic convergence or even the zero convergence. This signifies that the limit of uncertainty must be precisely determined in advance. In the meantime, when the system is perturbed by a (measurement) noise, the zero convergence is achieved. However, the ultimate error is located in a bounded region dependent on the variance of noise. The proposed technique is performed when a pitch rate control of a supercavitating system is of interest. The simulation results signify the performance of the proposed technique and the developed performance index to provide desired feed-forward control signal. Nonetheless, the quality of the technique in a noisy environment needs a far more knowledge of noise to guarantee a perfect tracking of command pitch rate.

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