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Globally exponential continuous controller/observer for position tracking in robot manipulators with hysteretic joint friction

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SUMMARY
In this work, we present a continuous observer and continuous controller for a multiple degree of freedom robot manipulator with hysteretic joint friction. The fictitious hysteresis state is of course unknown to the controller and must be estimated. The joint velocities are assumed measured here. For this considered plant, we propose and present a continuous observer/controller that estimates or observes the hysteresis state and drives the position tracking error to zero. We prove that the combined tracking error and observer error converges to zero globally exponentially.

KEYWORDS: Hysteresis observer; LuGre hysteretic friction; Coulomb friction; Globally exponential tracking; Robot; Actuators; Drives; Nonlinear controls; Global exponential; Globally asymptotic tracking.

1. Introduction
This work is concerned with the problem of position tracking for a multiple degree of freedom (DOF) robot with hysteretic friction in its joints. It is the same plant as considered in ref. [10]. Such plants contain a fictitious hysteresis state which must be observed to achieve position tracking. For such a plant, we present an observer and a controller, both of which are continuous. We prove globally exponential convergence of the combined tracking/observation error for the proposed continuous observer/controller.

Canudas de Wit *et al.*[1] considered friction models which include hysteresis states for single DOF plants. With velocity measurement assumed available, they provide an observer and controller for globally driving a tracking error to zero. Panteley *et al.*[10] extended the work of Canudas de Wit *et al.*[1] to adaptive global tracking of multiple DOF robot manipulators with hysteretic joint friction and velocity measurement. It is worth mentioning, in the adaptive controls context, that the model in Canudas de Wit *et al.*[1] and Panteley *et al.*[10] is not fully linear in the parameters (see Eqs. (1)–(3) of the present paper, specifically), thus making the adaptive control problem more challenging. In the adaptive controls context, Lin *et al.*[9] use neural networks in an adaptive approach for estimating and compensating hysteresis in piezo-electric actuators, with one objective being to reduce the requirement of a detailed model that is known a priori.

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The present paper also has the same hysteresis model as Canudas de Wit *et al.*[1] However, as in Panteley *et al.*,[10] the present paper is concerned with the full nonlinear model of a multiple DOF robotic manipulator while Canudas de Wit *et al.*[1] were considering a single DOF plant. It is hard to make a one to one comparison of the present paper with Panteley *et al.*[10] partly because the latter is adaptive and the former not. However, one contribution of the present paper as compared to Panteley *et al.*[10] can be viewed as being the fact that the controller and observer are both continuous in the present paper whereas the control torque in Panteley *et al.*[10] is a discontinuous function of time (see Eqs. (3.11), (3.12), (3.4), and (3.6) in Panteley *et al.*[10]). Both are globally exponential in their nonadaptive versions. It is worth mentioning that their (and our) plant (and the plant in Chen and Lin[2] and Canudas de Wit *et al.*[1]) surprisingly does not satisfy the complete uniform observability condition in Teel and Praly;[13] this is due to the fact that the coefficient of the hysteresis state, in the equation of motion (acceleration equation), can be zero at a specific value of the joint velocity.

Friction has long been known to be one of the primary disturbances in robotic positioning systems. Peng and Chen[11] consider the problem of biaxial contouring control in the presence of friction disturbances. Later, these same authors, in Chen and Lin,[2] consider the same hysteretic friction model as in Panteley *et al.*,[10] Canudas de Wit *et al.*,[1] and the present paper. The robotic plant is a 2 DOF one with two orthogonal prismatic joints. Velocity is not measured. Observers are designed for estimating both the hysteresis state and the velocity. The authors prove the closed loop system's signals are ultimately bounded, and good experimental results are obtained.

There have been other papers concerned with hysteresis observers for actuators such as piezo-electric ones obeying hysteresis models such as those in Dominguez *et al.*,[4] Heine,[7] Lin and Yang,[8] Driessen and Duggirala,[5] and Driessen and Kondreddi.[6] All of these were concerned with a single DOF plant unlike the present paper and unlike Panteley *et al.*[10]

In the present paper, we give a precise statement of the problem in Section 2. In Section 3, we define the observer and controller. In Section 4, we give the theorems and proofs of the globally exponential convergence under the proposed continuous observer/controller system. In Section 5 we provide numerical results and in Section 6 conclusions.

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2. Problem Statement

We have a robotic plant with hysteretic friction (same plant as Panteley *et al.*[10]):

$$M(q)\ddot{q} + f(q,\dot{q}) = u - \sigma_0 z - \sigma_1 \dot{z} - \sigma_2 \dot{q}, \quad (1)$$

$$\dot{z} = -\sigma_0 \gamma^{-1}(\dot{q}) diag\,(|\dot{q}|)\, z + \dot{q}, \quad (2)$$

where $q \in R^n$ is the vector of measured joint angles of the robot, $\dot{q} \in R^n$ is the vector of measured joint velocities, $u \in R^n$ is the control torque, and where $M(q) = M(q)^T > 0$ is a smooth function of q; $f(q,\dot{q})$ is a smooth function of q and $\dot{q}$. The state $z \in R^n$ is the fictitious hysteresis state that is *unknown* to the controller, and $\sigma_0 > 0$, $\sigma_1 > 0$ and $\sigma_2 > 0$ are diagonal constant matrices with $\sigma_0 > \sigma_1$. The factor $diag\,(|\dot{q}|)$ is the diagonal matrix whose ith diagonal entry is $|\dot{q}_i|$, and $\gamma^{-1}(\dot{q})$ is the matrix inverse of the diagonal matrix, $\gamma(\dot{q})$, whose ith diagonal entry is as follows:

$$\gamma_{ii} = \alpha_{0i} + \alpha_{1i} \exp(-(\dot{q}_i/\alpha_{2i})^2), \quad (i = 1, \ldots, n), \quad (3)$$

where $\exp(\cdot) = e^{(\cdot)}$ is the natural exponential and where the α_{0i}, α_{1i}, and α_{2i} are positive constants.

We have a bounded desired trajectory

$$\left\{q_d^T, \dot{q}_d^T, \ddot{q}_d^T\right\}^T \in L_\infty, \quad (4)$$

The objective is to design a continuous controller and a continuous observer, with state estimate $\hat{z}$ of z such that the below error vector

$$\Omega \equiv ((q - q_d)^T, (\dot{q} - \dot{q}_d)^T, (\hat{z} - z)^T)^T \quad (5)$$

converges to zero globally exponentially.

Remark 2.1: It is well known that the eigenvalues of $M(q)$ are bounded (and bounded below away from zero). We will denote by the constant $\lambda_{\min}$ the smallest eigenvalue of $M(q)$ over all q and by the constant $\lambda_{\max}$ the largest eigenvalue of $M(q)$ over all q, i.e.,

$$\lambda_{\min} \|w\|^2 \le w^T M(q) w \le \lambda_{\max} \|w\|^2, \quad \forall q, \quad \forall w. \quad (6)$$

Remark 2.2: Actual values of the parameters σ_0, σ_1, σ_2, and the α_{ji} would be obtained from a system identification. (*see*, e.g., page 327 of Chen and Lin[2]).

3. Proposed Observer/Controller

The proposed observer/controller is given as follows. Define the position tracking error e as

$$e \equiv q - q_d \quad (7)$$

and the generalized tracking error s as

$$s \equiv \dot{e} + \alpha e, \quad (8)$$

where $\alpha > 0$ is a scalar constant. Then the observer/controller is

$$\dot{\hat{z}} = -\sigma_0 \gamma^{-1}(\dot{q}) diag\,(|\dot{q}|)\, \hat{z} + \dot{q} - K_2 M s, \quad (9)$$

$$u = -K_1 M s - \dot{M} s + f(q,\dot{q}) + \sigma_0 \hat{z} + \sigma_1 \dot{\hat{z}} + \sigma_2 \dot{q} + M\ddot{q}_d - \alpha M \dot{e}, \quad (10)$$

where $K_1 > 0$ and $K_2 > 0$ are constant diagonal gain matrices where K_1 satisfies

$$\sigma_0 - \sigma_1 K_1 > 0. \quad (11)$$

(This restriction in Eq. (11) is not a significant constraint since the actual values of σ_0 are quite large compared to those of σ_1.)

Remark 3.1: The observer in Eq. (9) and control in Eq. (10) are arrived at by enforcing negativity of the Lyapunov function derivative in the following section.

4. Analysis and Convergence Proof

In this section we prove that the proposed observer/controller of Section 3 meets the convergence and boundedness goals defined in Section 2.

Theorem 4.1: The observer and controller defined in Section 3 meets the goals given in the problem statement of Section 2; namely $((q - q_d)^T, (\dot{q} - \dot{q}_d)^T, (\hat{z} - z)^T)^T \to 0$ globally exponentially, and all closed loop signals remain bounded.

Proof of Theorem 4.1: Define the observer error $\tilde{z}$:

$$\tilde{z} \equiv \hat{z} - z. \quad (12)$$

Differentiation of Ms (with s from Eq. (8)) while substituting in the control in Eq. (10) gives

$$\frac{d}{dt}(Ms) = \sigma_0 \tilde{z} + \sigma_1 \dot{\tilde{z}} - K_1 M s. \quad (13)$$

Subtracting the observer Eq. (9) minus the plant Eq. (2) gives

$$\dot{\tilde{z}} = -\sigma_0 \gamma^{-1} diag\,(|\dot{q}|)\, \tilde{z} - K_2 M s. \quad (14)$$

From Eq. (13), we have, for the Laplace variable p and transfer function $G(p)$ below:

$$Ms = G(p)\tilde{z} \equiv (pI + K_1)^{-1}(\sigma_0 + \sigma_1 p)\,\tilde{z} = \sigma_1 \tilde{z} + (pI + K_1)^{-1}(\sigma_0 - \sigma_1 K_1)\,\tilde{z} \quad (15)$$

or

$$Ms = \sigma_1 \tilde{z} + y, \quad \dot{y} + K_1 y = (\sigma_0 - \sigma_1 K_1)\,\tilde{z}. \quad (16)$$

Define the simple Lyapunov function V :

$$V \equiv \tfrac{1}{2} y^T y + \tfrac{1}{2} \tilde{z}^T \left(K_2^{-1}(\sigma_0 - \sigma_1 K_1)\right) \tilde{z} \quad (17)$$

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(recalling (11)). Differentiation of V while inserting (16) for $\dot{y}$ and (14) for $\dot{\tilde{z}}$ gives:

$$\dot{V} = -y^T K_1 y + y^T(\sigma_0 - \sigma_1 K_1)\tilde{z} - \tilde{z}^T\left(K_2^{-1}(\sigma_0 - \sigma_1 K_1)\right)$$
$$\sigma_0\gamma^{-1}\text{diag}(|\dot{q}|)\tilde{z} - \tilde{z}^T K_2^{-1}(\sigma_0 - \sigma_1 K_1)K_2(Ms). \quad (18)$$

The third term is nonpositive and K_2 cancels in the last term, which leaves

$$\dot{V} \leq -y^T K_1 y + y^T(\sigma_0 - \sigma_1 K_1)\tilde{z} - \tilde{z}^T(\sigma_0 - \sigma_1 K_1)(Ms). \quad (19)$$

Substituting Ms in the last term from (16) gives:

$$\dot{V} \leq -y^T K_1 y + y^T(\sigma_0 - \sigma_1 K_1)\tilde{z} - \tilde{z}^T(\sigma_0 - \sigma_1 K_1)$$
$$\times\sigma_1\tilde{z} - \tilde{z}^T(\sigma_0 - \sigma_1 K_1)y. \quad (20)$$

The second and fourth terms cancel, leaving

$$\dot{V} \leq -y^T K_1 y - \tilde{z}^T(\sigma_0 - \sigma_1 K_1)\sigma_1\tilde{z}. \quad (21)$$

It readily follows that $\{y^{\mathrm{T}}, \tilde{z}^T\}^T$ converges to zero globally exponentially. Our change of variables is as follows:

$$\begin{Bmatrix} y \\ \tilde{z} \end{Bmatrix} = \underbrace{\begin{bmatrix} M & -\sigma_1 \\ 0 & I \end{bmatrix}}_{\equiv T}\begin{Bmatrix} s \\ \tilde{z} \end{Bmatrix}. \quad (22)$$

In view of property (6), we have that $\|T^{-1}\|$ and $\|T\|$ are bounded for all $q \in R^n$ or there exist constants B_1 and B_2 such that

$$\{\|T^{-1}\| \leq B_1,\ \|T\| \leq B_2\}, \quad \forall q \in R^n \quad (23)$$

from which we conclude that $\{s^T, \tilde{z}^T\}^T$ also converges to zero globally exponentially. Finally, from the strictly stable error dynamics in Eq. (8), it readily follows that $\{e^T, \dot{e}^T, \tilde{z}^T\}^T$ converges to zero globally exponentially.

Remark 4.1: It is known (see, e.g., Panteley *et al.*[10]) that z is bounded in the following way:

$$|z_i(t)| \leq \max\left[|z_i(0)|,\ (\alpha_{0i} + \alpha_{1i})/(\sigma_0)_{ii}\right],$$
$$\forall t \geq 0, \quad (i = 1, \ldots, n) \quad (24)$$

so that $\hat{z}$ is bounded, i.e., $\hat{z} \in L_\infty$.

In view of Eq. (24), we additionally conclude that all closed loop signals remain bounded. This completes the proof of **Theorem 4.1**. **QED**.

5. Numerical Examples

In this section we present numerical examples. We consider a 2 DOF revolute robotic manipulator with hysteretic friction. From Craig[3] (with reference to plant Eq. (1)):

$$M_{11} = m_2 L_2^2 + 2m_2 L_1 L_2 c_2 + (m_1 + m_2)L_1^2,$$
$$M_{12} = M_{21} = m_2 L_2^2 + m_2 L_1 L_2 c_2, \quad M_{22} = m_2 L_2^2, \quad (25)$$

where $c_i \equiv \cos(q_i)$ and where the m_i are the link masses and L_i the link lengths. And,

$$f_1 = -m_2 L_1 L_2 s_2 \dot{q}_2^2 - 2m_2 L_1 L_2 s_2 \dot{q}_1 \dot{q}_2$$
$$+ m_2 L_2 g c_{12} + (m_1 + m_2)L_1 g c_1, \quad (26)$$
$$f_2 = m_2 L_1 L_2 s_2 \dot{q}_1^2 + m_2 L_2 g c_{12}, \quad (27)$$

where $s_i \equiv \sin(q_i)$ and $c_{12} \equiv \cos(q_1 + q_2)$ and where g is the gravitational constant. The parameter values considered are, with all SI units:

$$m_i = 20, \quad L_i = 1, \quad g = 9.81. \quad (28)$$

The following parameter values of the hysteretic friction are similar to those in the references of Section 1 (see, e.g., Chen and Lin[2]):

$$\sigma_0 = 10^5\text{diag}(1, 1), \quad \sigma_1 = \sigma_0^{1/2}, \quad \sigma_2 = \text{diag}(350, 350),$$
$$\{\alpha_{0i} = 7.5,\ \alpha_{1i} = 3.32,\ \alpha_{2i} = 0.001\}, \quad (i = 1, 2). \quad (29)$$

The desired trajectory is

$$q_d = \{1 - \cos(\omega_1 t),\ 1 - \cos(\omega_2 t)\}^T,$$
$$\omega_1 = 2, \quad \omega_2 = 3.5. \quad (30)$$

The initial conditions are (in view of Eq. (24))

$$q(0) = \{0.1,\ 0.2\}^T, \quad \dot{q}(0) = \{0, 0\}^T,$$
$$\left\{\hat{z}_i(0) = -10^{-5}/2,\ z_i(0) = -10^{-5}\right\}, \quad (i = 1, 2). \quad (31)$$

The control gains used were

$$K_1 = \text{diag}(10, 10), \quad K_2 = \text{diag}(0.1, 0.1), \quad \alpha = 1.0. \quad (32)$$

Figure 1 shows the position tracking errors versus time. Figure 2 shows the control torques versus time.

We see that the torque changes quickly as the velocity changes sign (e.g., in joint 1 near about $t = 6.5$ s), as can be revealed by comparing Figs. 2 to 3, where Fig. 3 shows the position and reference position of joint 1 and Fig. 4 that of joint 2. The only way to reduce this $\dot{u}$ is to replace the plant with an "unstiffened" one or reduce the acceleration/speed of the reference trajectory, $q_d(t)$. (Of course, for the standard friction compensator term, of the form $\mu sign(\dot{q})$, $\dot{u}$ is unbounded.)

If the friction compensation term, $\sigma_0\hat{z} + \sigma_1\dot{\hat{z}} + \sigma_2\dot{q}$, of the control u in (10) is removed, the tracking performance deteriorates as shown in Fig. 5.

(If only the $\hat{z}$-containing terms of u in Eq. (10) are removed, at least as significant of deterioration in tracking performance also occurs.)

Remark 5.1: From a practical perspective, one may not want to choose K_2 too large. This is due to the control term (from Eqs. (9) and (10)), $-\sigma_1 K_2 Ms$, and the somewhat large actual values of σ_1. Combined together, the product $\sigma_1 K_2$ could lead to a large control value if K_2 were also somewhat large.

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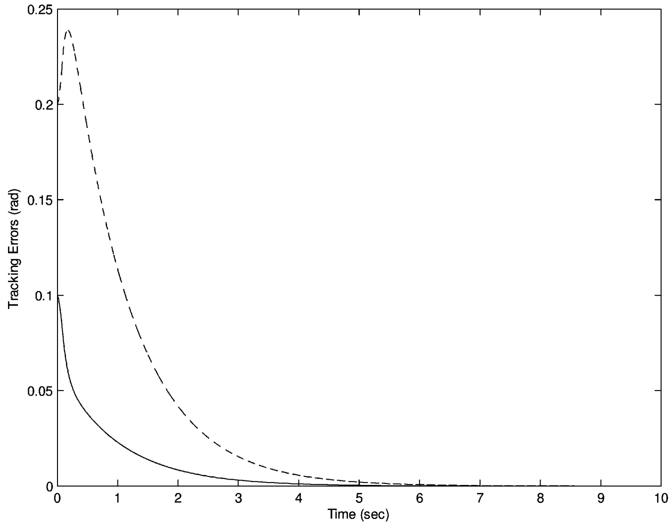

Fig. 1. Position tracking errors versus time – e_1 solid, e_2 dashed.

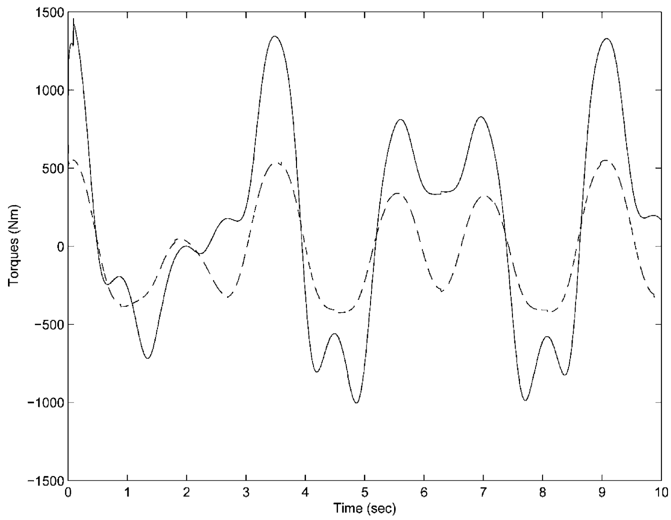

Fig. 2. Control torques versus time – u_1 solid, u_2 dashed.

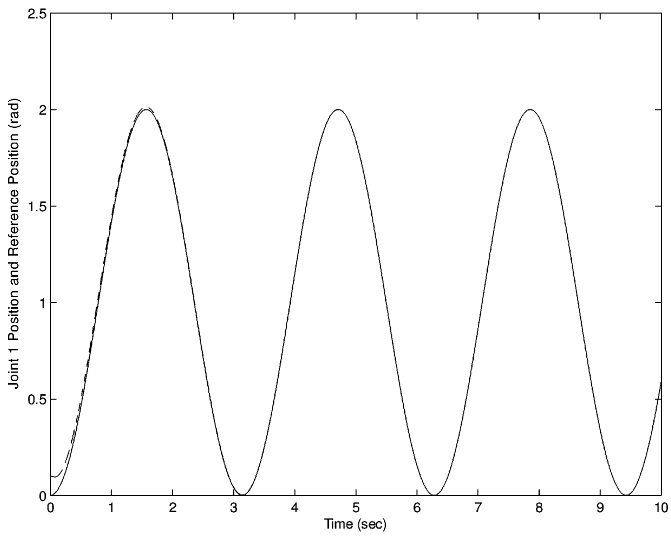

Fig. 3. Joint 1 position and reference position versus time – reference solid, actual dashed.

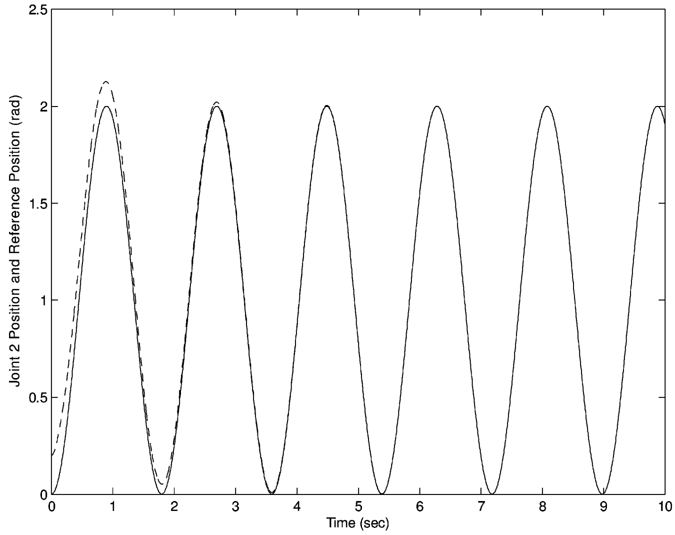

Fig. 4. Joint 2 position and reference position versus time – reference solid, actual dashed.

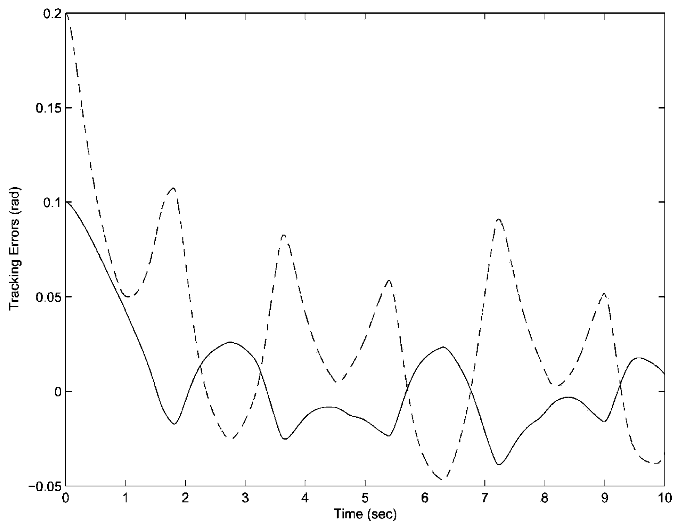

Fig. 5. Tracking errors *without* friction compensation – e_1 solid, e_2 dashed.

6. Conclusions

We considered the problem of position tracking for multiple DOF robotic manipulators with hysteretic friction in the joints. We proposed and presented a continuous observer/controller that estimates the hysteresis state and drives the position tracking error to zero. We proved that the closed loop system's combined tracking and observation error converges to zero globally exponentially.

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