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Dimensional synthesis of a 3UPS-PRU parallel robot

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SUMMARY
This paper introduces the methodology of the dimensional synthesis for a 3UPS-PRU parallel robot. The dimensional synthesis of the 3UPS-PRU parallel robot is proposed considering the maximum input velocity of actuating joints as the objective function and constraints on the installation dimension, robot dimension, joint rotation angle and interference. The objective of the dimensional synthesis is to minimize the maximum input velocity of actuating joints when the moving platform translates along the z-axis in the maximum linear velocity and rotates about an arbitrary axis in the maximum angular velocity in the desired workspace. The constraint on the robot dimension is included in the dimensional synthesis of the 3UPS_PRU parallel robot when pursuing the kinematic property to meet the miniaturization principle with the reduced building cost. An example of the dimensional synthesis of a 3UPS-PRU parallel robot is presented with the maximum linear velocity and angular velocity required for the moving platform in the desired workspace.

KEYWORDS: 3UPS-PRU parallel robot, Dimensional synthesis, Kinematics, Maximum input velocity.

1. Introduction

Parallel robots have been successfully used in machine tools,[1–3] motion simulators,[4–6] manipulators[7,8] and in pick-and-place operation.[9–11] A parallel robot should be designed to meet the requirement performance in the application. There are two types of design problems for parallel robots:[12–14] structural synthesis and dimensional synthesis. The structural synthesis is to find possible mechanical structures to fulfill the required motion pattern for the moving platform. The dimensional synthesis determines the appropriate dimension of mechanical structures to meet performance requirements. Once the structural synthesis is completed, the dimensional synthesis is required to achieve an optimal performance.

Dimensional synthesis of parallel robot is usually a non-linear optimization problem. There are five elements considered in the optimization of parallel robots: design variable, objective function, constraint, optimization algorithm and studied objective as follows. (1) Design variable: Structural parameters are usually chosen as design variables for the dimensional synthesis of parallel robots. The number of design variables can reflect the complexity of the optimization problem in a certain degree. When an optimization problem can be described, the number of design variables should be as few as possible in order to reduce the complexity of the optimization problem. Gao[15] and Liu[16] adopted performance atlases in the dimensional synthesis of parallel robot to reduce one design variable. (2) Objective function: When the kinematic performance is pursued, the objective functions adopted in the dimensional synthesis of parallel robots are usually conditioning indices,[17–20] workspace,[21–24] manipulability measures,[25] velocity transmission indices,[26] etc. Dimensional synthesis of parallel robot is a constrained optimization problem with multi-criteria.[26,27] (3) Constraint: The constraints are usually considered in the design, a constraint function should also be included in the dimensional

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synthesis of parallel robot. From the perspective of an optimization design, an optimal solution can be searched quickly by applying an appropriated constraint. When the kinematic performance is pursued, the objective functions used for the dimensional synthesis of parallel robots are often based on the Jacobian matrix. The Jacobian matrix is related to the robot's configuration and it varies non-linearly when the end-effector moves in the workspace.[26] When a robot is magnified or minified in equal proportion, the Jacobian matrix performs a linear change. Therefore, an appropriate constraint should be applied to the dimensional synthesis of parallel robot to search an optimal solution. Huang[19] adopted the constraint on the robot dimension in the dimensional synthesis of parallel robot when pursuing the kinematic property. (4) Optimization algorithm: As there is usually no explicit expression between the objective function and the design variable, the numerical approaches are often used for the non-linear optimization problem. Sequential quadratic programming,[14,19,26] Newton method and its modification[28] and particle swarm optimization[29–32] have been used in the dimensional synthesis of parallel robots. Some of the above-mentioned algorithms are available in MATLAB® Optimization Tool Box. (5) Studied objective: The investigations on the dimensional synthesis of parallel robots focus on the lower mobility parallel robots such as Diamond,[19,20] Delta,[14,21,24,26,31–32] Tricept,[8] TriVariant[33,34] and Gough–Stewart parallel platform.[22,30] However, there is not a universal design method and process that can be used for all the parallel robots. For a specific robot, the dimensional synthesis should be carried out by employing the appropriate objective function and constraints while considering the application, design task and performance requirement.

By taking the maximum input velocity of actuating joints as an objective function, this paper presents a dimensional synthesis of the 3UPS-PRU parallel robot while considering constraints on the installation dimension, robot dimension, joint rotation angle and interference. The paper is organized as follows. The 3UPS-PRU parallel robot is explained in Section 2. A kinematic model used in dimensional synthesis is then provided in Section 3. Dimensional synthesis of the 3UPS-PRU parallel robot is investigated in Section 4. Design example is illustrated in Section 5. Finally, Section 6 gives the conclusions.

2. 3UPS-PRU Parallel Robot

The 3UPS-PRU parallel robot is shown in Fig. 1. There are three external identical limbs and one central limb between the base platform and the moving platform. For each external limb, it is composed of a universal, prismatic and spherical joint. The central limb includes a prismatic, revolute and universal joint. All the limbs are driven by the prismatic joint. The central limb is fixed on the base platform. Due to constraints of the central limb and three external limbs, the moving platform has one translational and three rotational degrees of freedom.

3. Kinematics

3.1. Position analysis

As shown in Fig. 2, the closed-loop vector equation associated with the ith limb is

$$q_0 \boldsymbol{w}_0 + \boldsymbol{a}_i = \boldsymbol{b}_i + q_i \boldsymbol{w}_i \quad i = 1, 2, 3 \tag{1}$$

where q_0 and q_i denote joint variables of the central limb and the ith limb, $\boldsymbol{w}_0$ and $\boldsymbol{w}_i$ are unit vectors along the central limb and the ith limb, $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ denote vectors A_0A_i and B_0B_i, respectively. $q_0\boldsymbol{w}_0$ also represents the position vector of the moving platform.

The joint variable of the ith limb can be obtained as follows:

$$q_i = \sqrt{(q_0 \boldsymbol{w}_0 + \boldsymbol{a}_i - \boldsymbol{b}_i)^T (q_0 \boldsymbol{w}_0 + \boldsymbol{a}_i - \boldsymbol{b}_i)} \tag{2}$$

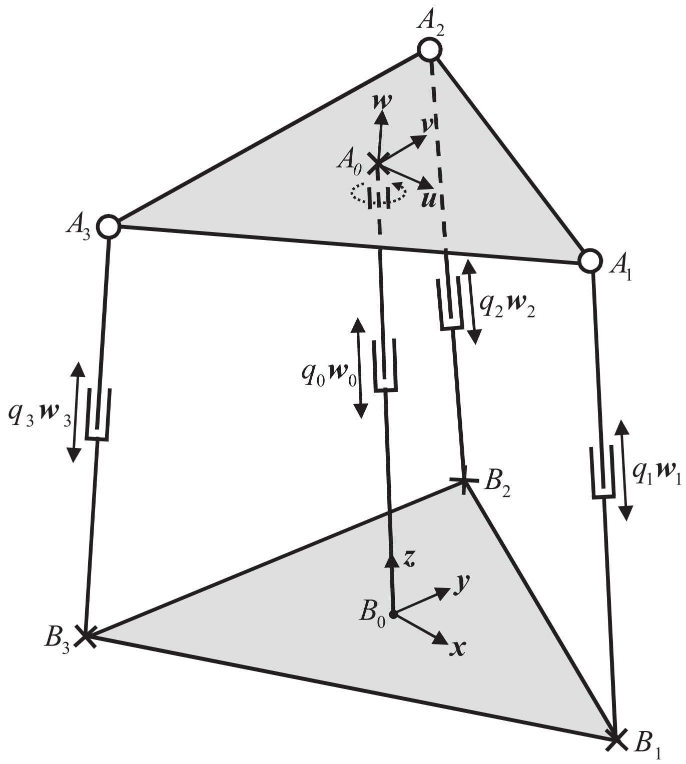

Fig. 1. The UPS-PRU parallel robot.

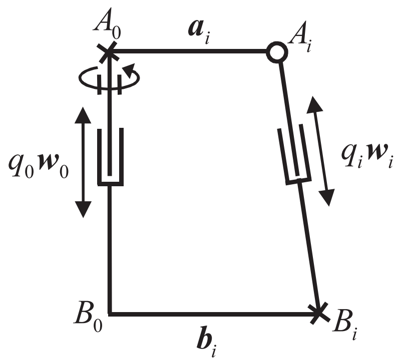

Fig. 2. Vector diagram of the ith limb.

3.2. *Velocity analysis*

Taking the derivative of Eq. (1) with respect to time, we get

$$\dot{q}_0 \mathbf{w}_0 + \boldsymbol{\omega} \times \boldsymbol{a}_i = \dot{q}_i \boldsymbol{w}_i + \boldsymbol{\omega}_i \times q_i \boldsymbol{w}_i \tag{3}$$

where $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_i$ are the angular velocity vector of the moving platform and the angular velocity vector of the ith limb, respectively.

Taking the dot product of both sides of Eq. (3) with $\boldsymbol{w}_i$, we obtain

$$\dot{q}_i = \dot{q}_0 \boldsymbol{w}_i^T \boldsymbol{w}_0 + (\boldsymbol{a}_i \times \boldsymbol{w}_i)^T \boldsymbol{\omega}\, i = 1, 2, 3 \tag{4}$$

By rewriting Eq. (4) in the matrix form:

$$\dot{\boldsymbol{q}} = \boldsymbol{J}\dot{\boldsymbol{x}} \tag{5}$$

where

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{q}_0 & \dot{q}_1 & \dot{q}_2 & \dot{q}_3 \end{bmatrix}^T \tag{6}$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} v \\ \boldsymbol{\omega} \end{bmatrix} \tag{7}$$

$\dot{\boldsymbol{x}}$ is the velocity vector of the moving platform.

$$v = \dot{q}_0 \tag{8}$$

v is the linear velocity of the moving platform.

$$\boldsymbol{J} = \begin{bmatrix} 1 & \boldsymbol{0}_{1\times 3} \\ \boldsymbol{w}_1^T \boldsymbol{w}_0 & (\boldsymbol{a}_1 \times \boldsymbol{w}_1)^T \\ \boldsymbol{w}_2^T \boldsymbol{w}_0 & (\boldsymbol{a}_2 \times \boldsymbol{w}_2)^T \\ \boldsymbol{w}_3^T \boldsymbol{w}_0 & (\boldsymbol{a}_3 \times \boldsymbol{w}_3)^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_0 \\ \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \boldsymbol{J}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_{e0} & \boldsymbol{J}_{e1} & \boldsymbol{J}_{e2} & \boldsymbol{J}_{e3} \end{bmatrix} \tag{9}$$

$\boldsymbol{J}$ is the Jacobian matrix which maps the velocity vector of the moving platform into the joint velocity vector.

4. Dimensional Synthesis

When a design task is assigned, the dimensional synthesis is to determine the structural parameters of the 3UPS_PRU parallel robot in order to achieve an optimal performance. It is usually a non-linear optimization problem.

4.1. Deign task

For the 3UPS_PRU parallel robot, the translational range along the z-axis is given as h, and the rotational range about the x-axis, y-axis and z-axis are assigned as $\pm\phi_{x\max}$, $\pm\phi_{y\max}$ and $\pm\phi_{z\max}$, respectively. The desired workspace W_d of the 3UPS_PRU parallel robot can be described as follows:

$$\begin{cases} H - \frac{h}{2} \le q_0 \le H + \frac{h}{2} \\ -\phi_{x\max} \le \phi_x \le \phi_{x\max} \\ -\phi_{y\max} \le \phi_y \le \phi_{y\max} \\ -\phi_{z\max} \le \phi_z \le \phi_{z\max} \end{cases} \tag{10}$$

where H is the distance from the central point of the desired workspace to the central point of the base platform.

The 3UPS_PRU parallel robot is assigned to fulfill operations in the desired workspace with the required maximum linear velocity $v_{\max}$ and with the required maximum angular velocity $\omega_{\max}$ at the same time.

4.2. Deign variables

The structure of the 3UPS_PRU parallel robot can be described using parameters: r_a, r_b, l_a and l_b, where r_a is the radius of the moving platform, r_b is the radius of the base platform, l_a is the length of the push rod connected with the moving platform and l_b is the length of the sleeve connected with the base platform. r_a and r_b should be big enough to mount the servomotors and joints. The objective of the dimensional synthesis is to achieve an optimal kinematic performance in the desired workspace, not in the whole reachable workspace. A design variable should be assigned to describe the position relationship between the desired workspace and whole reachable workspace. Here, the distance between the central point of the desired workspace and the central point of the base platform is adopted to represent the relationship. l_a and l_b place restrictions on the maximum length and the minimum length of the telescopic rod, respectively. Inertial parameters of the telescopic rod take no effect on results of the dimensional synthesis of the 3UPS_PRU parallel robot while considering kinematic property. Therefore, the design variable for the dimensional synthesis of the 3UPS_PRU parallel robot is as follows:

$$\boldsymbol{X}_d = \begin{bmatrix} r_a & r_b & H \end{bmatrix}^T \tag{11}$$

4.3. Objective function

The velocity transmission index is adopted as the objective function for the dimensional synthesis of the 3UPS_PRU parallel robot. From Eqs. (5) and (9), the maximum input velocity can be obtained when the moving platform translates along the z-axis in the maximum linear velocity $v_{\max}$

$$f_{t\max} = |v_{\max}| \, \|\boldsymbol{J}_{e0}\|_\infty \tag{12}$$

where $\|\cdot\|_\infty$ is the ∞−norm of the vector.

When the moving platform of the 3UPS_PRU parallel robot rotates about the x-axis, y-axis and z-axis in the unit angular velocity, respectively, the maximum input velocity of the actuating joints is as follows:

$$f_{rx\max} = \|\boldsymbol{J}_{e1}\|_\infty \tag{13}$$

$$f_{ry\max} = \|\boldsymbol{J}_{e2}\|_\infty \tag{14}$$

$$f_{rz\max} = \|\boldsymbol{J}_{e3}\|_\infty \tag{15}$$

If the moving platform rotates about an arbitrary axis in the maximum angular velocity, $\boldsymbol{\omega}_{\max}$ can be described as

$$\boldsymbol{\omega}_{\max} = \lambda_1 \omega_{\max} \boldsymbol{u}_x + \lambda_2 \omega_{\max} \boldsymbol{u}_y + \lambda_3 \omega_{\max} \boldsymbol{u}_z \tag{16}$$

where

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1 \tag{17}$$

$\boldsymbol{u}_x$, $\boldsymbol{u}_y$ and $\boldsymbol{u}_z$ denote the unit vectors along the x-axis, y-axis and z-axis, respectively.

Form Eqs. (5), (9) and (16), when the moving platform rotates about an arbitrary axis in the maximum angular velocity $\boldsymbol{\omega}_{\max}$, the maximum input velocity of the actuating joints can be obtained based on the theory of matrix norm and inequality as follows:

$$\begin{aligned} \left\|\dot{\boldsymbol{q}}_{r\omega_{\max}}\right\|_\infty &= \| |\lambda_1 \omega_{\max} \boldsymbol{J}_{e1}| + |\lambda_2 \omega_{\max} \boldsymbol{J}_{e2}| + |\lambda_3 \omega_{\max} \boldsymbol{J}_{e3}| \|_\infty \\ &\le (|\lambda_1 \omega_{\max}| \, \|\boldsymbol{J}_{e1}\|_\infty + |\lambda_2 \omega_{\max}| \, \|\boldsymbol{J}_{e2}\|_\infty + |\lambda_3 \omega_{\max}| \, \|\boldsymbol{J}_{e3}\|_\infty) \\ &\le \sqrt{3} \max \left(|\omega_{\max}| \, \|\boldsymbol{J}_{e1}\|_\infty, |\omega_{\max}| \, \|\boldsymbol{J}_{e2}\|_\infty, |\omega_{\max}| \, \|\boldsymbol{J}_{e3}\|_\infty\right) \\ &= \sqrt{3} \, |\omega_{\max}| \max \left(\|\boldsymbol{J}_{e1}\|_\infty, \|\boldsymbol{J}_{e2}\|_\infty, \|\boldsymbol{J}_{e3}\|_\infty\right) = f_{r\max} = \left\|\dot{\boldsymbol{q}}_{r\max}\right\|_\infty \end{aligned} \tag{18}$$

where $|\cdot|$ is the absolute value.

When the moving platform translates along the z-axis in the maximum linear velocity $v_{\max}$ and rotates about an arbitrary axis in the maximum angular velocity $\boldsymbol{\omega}_{\max}$ at the same time, the maximum input velocity of the actuating joints can be obtained as

$$\begin{aligned} \left\|\dot{\boldsymbol{q}}_{tr}\right\|_\infty &= \| |v_{\max}| \, |\boldsymbol{J}_{e0}| + |\lambda_1 \omega_{\max} \boldsymbol{J}_{e1}| + |\lambda_2 \omega_{\max} \boldsymbol{J}_{e2}| + |\lambda_3 \omega_{\max} \boldsymbol{J}_{e3}| \|_\infty \\ &\le (|v_{\max}| \, \|\boldsymbol{J}_{e0}\|_\infty + |\lambda_1 \omega_{\max}| \, \|\boldsymbol{J}_{e1}\|_\infty + |\lambda_2 \omega_{\max}| \, \|\boldsymbol{J}_{e2}\|_\infty + |\lambda_3 \omega_{\max}| \, \|\boldsymbol{J}_{e3}\|_\infty) \\ &\le \left(|v_{\max}| \, \|\boldsymbol{J}_{e0}\|_\infty + \sqrt{3} \max \left(|\omega_{\max}| \, \|\boldsymbol{J}_{e1}\|_\infty, |\omega_{\max}| \, \|\boldsymbol{J}_{e2}\|_\infty, |\omega_{\max}| \, \|\boldsymbol{J}_{e3}\|_\infty\right)\right) \\ &= |v_{\max}| \, \|\boldsymbol{J}_{e0}\|_\infty + \sqrt{3} \, |\omega_{\max}| \max \left(\|\boldsymbol{J}_{e1}\|_\infty, \|\boldsymbol{J}_{e2}\|_\infty, \|\boldsymbol{J}_{e3}\|_\infty\right) \\ &= f_{t\max} + f_{r\max} = f_{\max} = \left\|\dot{\boldsymbol{q}}_{\max}\right\|_\infty \end{aligned} \tag{19}$$

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Considering the velocity transmission, the objective function of the dimensional synthesis of the 3UPS_PRU parallel robot is adopted as

$$f(\boldsymbol{X}_d) = f_{\max} \to \min \tag{20}$$

The physical meaning of the objective function is the maximum input velocity of actuating joints when the moving platform translates along the z-axis in the maximum linear velocity $v_{\max}$ and rotates about an arbitrary axis in the maximum angular velocity $\boldsymbol{\omega}_{\max}$.

4.4. Constraints

For the dimensional synthesis of the 3UPS_PRU parallel robot, a set of appropriate constraints regarding the prototype building and assembly condition should be considered. The base platform and moving platform should be big enough to mount the servomotors and joints. In general, the base platform is bigger than the moving platform since the installation dimension of the servomotors is bigger than that of the joints. The studied 3UPS_PRU parallel mechanism is used to build a hyper redundant robot. Therefore, the constraint on the maximum value of the base platform should also be considered. Constraints on dimensions of the base platform and moving platform can be described as

$$r_b \geq r_a \tag{21}$$

$$r_b \geq r_{b\min} = 0.100\,\text{m} \tag{22}$$

$$r_a \geq r_{a\min} = 0.100\,\text{m} \tag{23}$$

$$r_b \leq r_{b\max} = 0.22\,\text{m} \tag{24}$$

The constraint on the ratio of the stroke of the UPS limb and the PRU limb to the minimum lengths should be considered.[33,34] This is because the span between two support bearings of the limb should be large enough to provide a sufficient stiffness. The constraint can be described as

$$\mu = \frac{q_{i\max} - q_{i\min}}{q_{i\min}} \leq \mu_0 \tag{25}$$

where μ_0 is the maximum allowable value of μ. 0.6 $\sim$ 0.8 is recommended while considering the miniaturization principle of the 3UPS_PRU parallel robot and the specification of the commercial telescopic rod. For the dimensional synthesis of the 3UPS_PRU parallel robot, $\mu_0 = 0.66$ is adopted. In order to ensure the installation, the minimum length of the telescopic rod should be bigger than the minimum installation dimension. The constraint is, therefore, as follows:

$$q_{i\min} \geq s_{\min} \tag{26}$$

where $s_{\min}$ is the minimum installation dimension. Considering the computation and specification of the commercial telescopic rod, $s_{\min} = 0.270\,\text{m}$ is adopted in the dimensional synthesis of the 3UPS_PRU parallel robot.

The limit on the rotation angle of the spherical joint should be set by

$$\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0}) \leq \theta_{s\max} = \frac{\pi}{9} \tag{27}$$

where $\theta_{s\max}$ is the maximum allowable rotation angle of the spherical joint, $\boldsymbol{w}_{i0}$ is the unit vector of the ith limb on the initial configuration

$$\boldsymbol{w}_{i0} = \frac{\overrightarrow{B_i A_i}}{|B_i A_i|} s.t. \begin{cases} \phi_x = \phi_y = \phi_z = 0 \\ q_0 = H \end{cases} \tag{28}$$

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The constraint on the rotation angle of the universal joint should be considered as

$$\begin{cases} \phi_{x\max} \le \theta_{u\max} = \frac{\pi}{4.5} \\ \phi_{y\max} \le \theta_{u\max} = \frac{\pi}{4.5} \end{cases} \tag{29}$$

where $\theta_{u\max}$ is the maximum allowable rotation angle of the universal joint.

In order to avoid the interference between the limb and the base platform/or moving platform, constraints should be set by

$$\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0) \ge \theta_{b\min} = \frac{\pi}{12} i = 1, 2, 3 \tag{30}$$

$$\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}) \ge \theta_{a\min} = \frac{\pi}{12} i = 0, 1, 2, 3 \tag{31}$$

Considering the miniaturization principle and building cost of the 3UPS_PRU parallel robot, the constraint on the robot dimension should be considered in the dimensional synthesis when pursuing the kinematic property. The corresponding constraint should be set by

$$\eta_{1\min} \le \eta_1 = \frac{h + 2r_a \sin\phi_{x\max}}{H} \le \eta_{1\max} \tag{32}$$

For the dimensional synthesis of the 3UPS_PRU parallel robot, $\eta_{1\min} = 0.375$ and $\eta_{1\max} = 0.50$ would be reasonable choices.

4.5. Optimization implementation

The optimum dimensional synthesis of the 3UPS_PRU parallel robot can be formulated as the following non-linear constrained optimization problem subject to Eqs. (21)–(27) and Eqs. (29)–(32):

$$\underset{X_d \in R^3}{f(X_d)} \to \min \tag{33}$$

The non-linear constrained optimization problem can be solved by the Sequential Quadratic Programming algorithm available in MATLAB® Optimization Tool Box. The objective function and constraints can be calculated as follows:

$$f_{\max} = \max(f(X_d)) \to \min s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{34}$$

$$\mu = \frac{\max(q_i) - \min(q_i)}{\min(q_i)} \le \mu_0 \quad s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{35}$$

$$\min(q_i) \ge s_{\min} \quad s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{36}$$

$$\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) \le \theta_{s\max} \quad s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{37}$$

$$\min\left(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0)\right) \ge \theta_{b\min}\ i = 1, 2, 3 \quad s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{38}$$

$$\min\left(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w})\right) \ge \theta_{a\min}\ i = 0, 1, 2, 3 \quad s.t.(z, \theta_x, \theta_y, \theta_z) \in W_d \tag{39}$$

where z,θ_x,θ_y and θ_z are the pose of the moving platform, W_d is the desired workspace which is meshed with $K \times L \times M \times N$ nodes. The objective function and constraints can be calculated, respectively, when the moving platform is on each node of the meshed workspace, then the maximum objective function and minimum constraint/ or maximum constraint can be searched in the exhaustive way.

Table I. Dimensional synthesis of the 3UPS_PRU parallel robot subject to $\eta_{1\,\min} = 0.375$.

	Initial value		
Design task	Design variables	Constraints	Objective function
$v_{\max} = 0.2\,\mathrm{m/s}$ $\omega_{\max} = 0.4\,\mathrm{rad/s}$	$r_a = 0.105$ m $r_b = 0.205$ m $H = 0.318$m	Satisfy the constraints Eqs. (21)–(27) and Eqs. (29)–(32), and $\min(q_i) = 0.271973\,\mathrm{m}$ $\max(q_i) = 0.423637$ m $\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) = 0.301022\,\mathrm{rad}$ $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0)) = 0.997124\,\mathrm{rad}$ $\min(\frac{\pi}{2} - \arccos(\mathbf{w}_i^T \mathbf{w})) = 0.350358\mathrm{rad}$	$f_{\max} = 0.272347\,\mathrm{m/s}$
Optimization results			
Design variables		Constraints	Objective function
$r_a = 0.100000\,\mathrm{m}$ $r_b = 0.185624\,\mathrm{m}$ $H = 0.320000\,\mathrm{m}$		Satisfy the constraints Eqs. (21)–(27) and Eqs. (29)–(32), and $\min(q_i) = 0.270000\,\mathrm{m}$ $\max(q_i) = 0.416802\,\mathrm{m}$ $\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) = 0.284804\,\mathrm{rad}$ $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0)) = 1.055277\,\mathrm{rad}$ $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w})) = 0.409646\,\mathrm{rad}$ $r_a = r_{a\min},\ q_{i\min} = s_{\min},\ \eta_1 = \eta_{1\min}$	$f_{\max} = 0.268808\,\mathrm{m/s}$

5. Design Example

In this section, the dimensional synthesis of a 3UPS_PRU parallel robot is presented when the design task is assigned. The translational range of the moving platform along the z-axis is assigned as $h = 0.02\,\mathrm{m}$. The rotational ranges about the x-axis, y-axis and z-axis are assigned as $\pm\phi_{x\max} = \pm\frac{\pi}{6}$, $\pm\phi_{y\max} = \pm\frac{\pi}{6}$ and $\pm\phi_{z\max} = \pm\frac{\pi}{6}$, respectively. In the desired workspace, the moving platform translates along the z-axis in the maximum linear velocity $v_{\max} = 0.2\,\mathrm{m/s}$ and rotates about an arbitrary axis in the maximum angular velocity $\omega_{\max} = 0.4$ rad/s at the same time.

The result of the dimensional synthesis of the 3UPS_PRU parallel robot subject to $\eta_{1\min} = 0.375$ is presented in Table I.

Results of the dimensional synthesis of the 3UPS_PRU parallel robot subject to different constraints $\eta_{1\min}$ are presented in Table II. It is shown that constraints of Eqs. (32) and (26) have no effect on the result of the dimensional synthesis when $\eta_{1\min} \leq 0.36$. From Eq. (32) and results of the dimensional synthesis shown in Table II, H will be big when $\eta_{1\min}$ becomes small, and H will be small when $\eta_{1\min}$ becomes big. $\eta_{1\min}$ should not be too big otherwise H will be small and the constraint on the $r_{b\max}$ should be cancelled in order to satisfy the minimum installation dimension for the telescopic rod. $0.37 \leq \eta_{1\min} \leq 0.38$ would be a reasonable choice from the computation and analysis. $\eta_{1\min} = 0.375$ is adopted in the dimensional synthesis of the 3UPS_PRU parallel robot.

From Table I, when the moving platform translates along the z-axis in the maximum linear velocity $v_{\max} = 0.2\,\mathrm{m/s}$ and rotates about an arbitrary axis in the maximum angular velocity $\omega_{\max} = 0.4\,\mathrm{rad/s}$, results of the dimensional synthesis of the 3UPS_PRU parallel robot subject to $\eta_{1\min} = 0.375$ are

$$r_a = 0.100000\,\mathrm{m},\quad r_b = 0.185624\,\mathrm{m},\quad H = 0.320000\,\mathrm{m},\quad f_{\max} = 0.268808\,\mathrm{m/s}$$

The parameters are rounded as $r_a = 0.100$ m, $r_b = 0.186$ m and $H = 0.320$ m.

Distributions of the maximum input velocity of actuating joints and constraints are shown in Fig. 3 when the moving platform translates along the z-axis in the maximum linear velocity $v_{\max} = 0.2$ m/s and rotates about an arbitrary axis in the maximum angular velocity $\omega_{\max} = 0.4$ rad/s, where $\phi_z = 0.1$ rad, and the upper layer, the middle layer and the bottom layer correspond to $z = 0.330$ m, $z = 0.320$ m and $z = 0.310$ m, respectively.

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Table II. Dimensional synthesis of the 3UPS_PRU parallel robot subject to different constraints of $\eta_{1\min}$.

Constraint	Optimization results: Design variables	Constraints	Objective function
$\eta_{1\min} = 0.38$	$r_a = 0.100000$m $r_b = 0.194820$m $H = 0.315789$m	Satisfy the constraints Eqs. (21)–(27) and Eqs. (29)–(32), and $\min(q_i) = 0.270000$ m $\max(q_i) = 0.415813$ m $\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) = 0.288110$ rad $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0)) = 1.021980$ rad $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w})) = 0.373303$ rad $r_a = r_{a\min}, q_{i\min} = s_{\min}, \eta_1 = \eta_{1\min}$	$f_{\max} = 0.268887$ m/s
$\eta_{1\min} = 0.37$	$r_a = 0.100000$ m $r_b = 0.175091$ m $H = 0.324324$m	Satisfy the constraints Eqs. (21)–(27) and Eqs. (29)–(32), and $\min(q_i) = 0.270000$ m $\max(q_i) = 0.417758$ m $\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) = 0.280973$ rad $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0)) = 1.092410$ rad $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w})) = 0.450626$ rad $r_a = r_{a\min},\ q_{i\min} = s_{\min},\ \eta_1 = \eta_{1\min}$	$f_{\max} = 0.268653$ m/s
$\eta_{1\min} = 0.36$	$r_a = 0.100000$m $r_b = 0.171123$m $H = 0.331706$m	Satisfy the constraints Eqs. (21)–(27) and Eqs. (29)–(32), and $\min(q_i) = 0.275530$ m $\max(q_i) = 0.423684$ m $\max(\arccos(\boldsymbol{w}_i^T \boldsymbol{w}_{i0})) = 0.273756$ rad $\min(\frac{\pi}{2} - \arccos(\mathbf{w}_i^T \mathbf{w}_0)) = 1.115259$rad $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w})) = 0.473816$ rad $r_a = r_{a\min}$	$f_{\max} = 0.268554$ m/s

6. Conclusions

Using the maximum input velocity of actuating joints as the objective function, the dimensional synthesis of a 3UPS_PRU parallel robot is implemented with constraints on the installation dimension, robot dimension, joint rotation angle and interference. The conclusions are drawn as follows:

1. When the moving platform translates along the z-axis in the maximum linear velocity $v_{\max}$ and rotates about an arbitrary axis in the maximum angular velocity $\boldsymbol{\omega}_{\max}$, the maximum input velocity of actuating joints is achieved based on the theory of matrix norm and inequality. The maximum input velocity index is adopted as the objective function for the dimensional synthesis of the 3UPS_PRU parallel robot. The objective of the dimensional synthesis is to minimize the maximum input velocity of actuating joints when the moving platform translates along the z-axis in the maximum linear velocity $v_{\max}$ and rotates about an arbitrary axis in the maximum angular velocity $\boldsymbol{\omega}_{\max}$ in the desired workspace.
2. Constraints on the installation dimension, robot dimension, joint rotation range and interference are considered in the dimensional synthesis. The constraint on the robot dimension is included in the dimensional synthesis of the 3UPS_PRU parallel robot when pursuing the kinematic property for the miniaturization principle and reduced building cost. The constraints on the dimension are important factors and should be determined by the building cost, overall dimension and desired performance of the 3UPS_PRU parallel robot.
3. The proposed methodology could be useful for the dimensional synthesis of other parallel robots with one translational and three rotational degrees of freedom.

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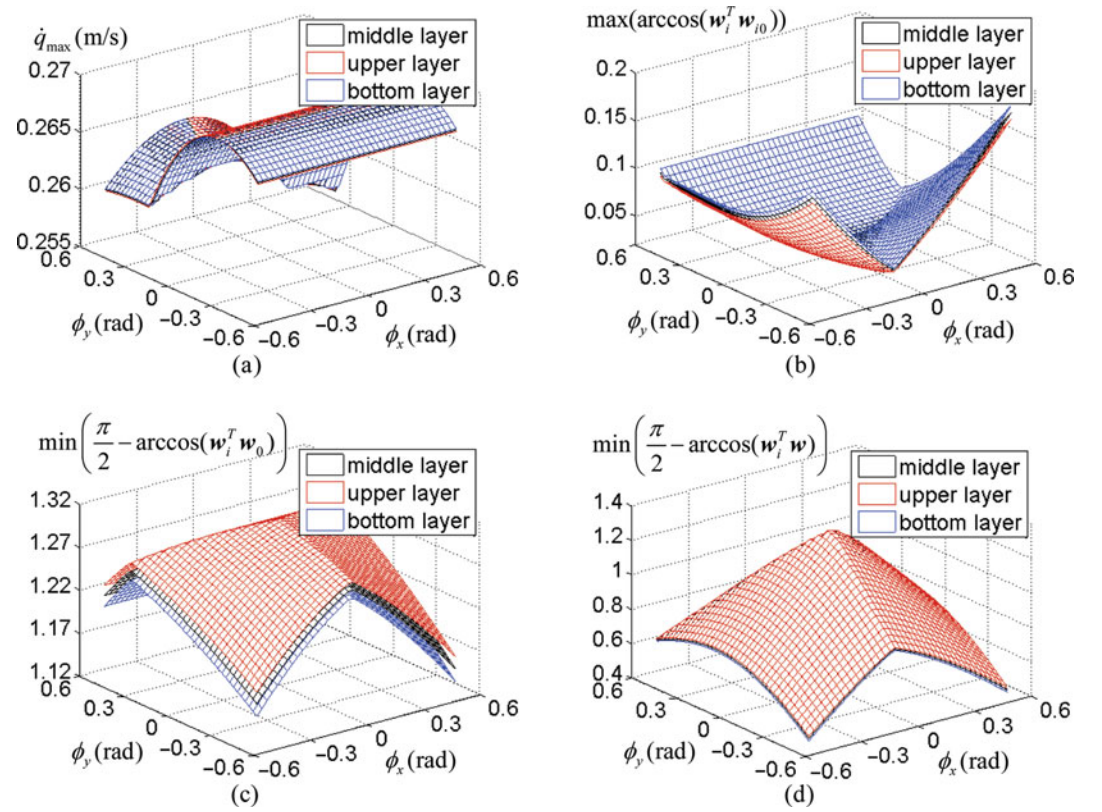

Fig. 3. Distributions of (a) the maximum input velocity of the actuating joints, (b) $\max(\arccos(\mathbf{w}_i^T \mathbf{w}_{i0}))$, (c) $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}_0))$ and (d) $\min(\frac{\pi}{2} - \arccos(\boldsymbol{w}_i^T \boldsymbol{w}))$ when the moving platform is on the plane associated with the upper, middle and bottom layers of the desired workspace.

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