Asteroids: New Observations, New Models
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Asteroid Family Identification: History and State of the Art

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Abstract. The history of asteroid families, from their discovery back in 1918, until the present time, is briefly reviewed. Two threads have been followed: on the development of the theories of asteroid motion and the computation of proper elements, and on the methods of classification themselves. Three distinct periods can be distinguished: the first one until mid-1930s, devoted to discovery and first attempts towards understanding of the properties of families; the second one, until early 1980s, characterized by a growing understanding of their importance as key evidence of the collisional evolution; the third one, characterized by an explosion of work and results, comprises the contemporary era. An assessment is given of the state-of-the-art and possible directions for the future effort, focusing on the dynamical studies, and on improvements of classification methods to cope with ever increasing data set.

Keywords. Asteroids, asteroid families, proper elements, hierarchical clustering

1. Introduction

Asteroids were discovered more than 200 years ago (more exactly, the first asteroid has been discovered by Giuseppe Piazzi at the Palermo Observatory, on January 1, 1801). Since then the asteroid science traveled a long journey along which it underwent several ascents correlated with the advances of observational techniques and large increase in the rates of new discoveries, and with breakthroughs in understanding of their role in the solar system dynamical and collisional evolution. The interim standstills were due to the unavoidable temporary waning of interest and lack of new ideas, so that asteroids were at some point even considered an annoying nuisance polluting images of other, "more important" celestial objects, therefore mockingly termed "vermin of the skies"!

The appreciation of asteroids, however, radically changed when it has been recognized that they are among the objects least changed in the course of the solar system evolution. Thus, not only that at present they are considered as clues to the origin and evolution of the solar system as a whole, but their distribution and dynamical and physical properties must be reproduced in full detail by any planetary system model that pretends to assess evolutionary processes with due reliability (Morbidelli, *private communication*).

The asteroid families, on the other hand, have been known for nearly 100 years: families have been discovered by Kyotsugu Hirayama in 1918. Being immediately assumed to be of collisional origin, their importance for the studies of solar system evolution, asteroid composition, chaotic diffusion, and non-gravitational effects has from the very beginning been well understood. Hence, asteroid family membership, origin and properties have always been and still are in the focus of the asteroid science.

In the following, a brief review of the most important achievements of the research on asteroid families is given, from the historical perspective and giving a personal account on the status of the art in the field and some of the most recent new results. It goes without saying that it is not possible to cover here all the aspects of the ongoing research, thus we

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shall restrict ourselves to the discussion of the current situation with determination of the parameters for the classification of asteroids into families, and with current improvements of the methods of classification and the resulting dynamical list of families. On some new achievements in determination of family ages see Milani *et al.*, *this volume*. Note that the present review is intended as a complement to the recent detailed review of the research done in the last 10 years prepared by Nesvorný *et al.* for the Asteroids IV book (Nesvorný *et al.* 2015).

2. Historical Notes

2.1. *Kiyotsugu Hirayama: Family Discovery*

In his celebrated paper "Groups of Asteroids Probably of Common Origin", Hirayama (1918) announced that he has noticed certain features of the distribution of asteroids too conspicuous to be due to a chance. The distributions of orbital inclinations and angles of eccentricity† in three specific narrow intervals of mean motion showed condensations which were, according to him, "surely out of proportion". In addition, he found that objects belonging to these condensations seem to be distributed on a circumference if their poles of orbital planes are projected on the plane ($p = \tan i \sin\Omega, q = \tan i \cos\Omega$), the center of this circumference coinciding with the position of the pole of Jupiter's orbit in the same plane. Similar circular configuration was found for the projections on the ($u = e \sin\varpi, v = e\cos\varpi$) plane‡, with the only difference that the pole of Jupiter's orbit was slightly shifted with respect to the center of the distribution but in the same direction from the origin.

Hirayama readily explained all these features by assuming "the physical relation connecting these asteroids", and using a simple secular perturbation theory to describe the observed distributions. Even if not (yet) using explicitly the notion of proper elements, he is employing the equations with constant amplitudes (radii of the circumferences), frequencies and phases to describe the secular motions and explain the distribution of the asteroids in the groups along the circumferences in the (u, v) and (p, q) planes by differential perturbations. Hirayama even ventured to name the groups he found as Koronis, Eos, and Themis *families*, introducing also the convention to associate to the group the name of the asteroid which was discovered first. Having at his diposal the catalog with 790 asteroid orbits only (*Berliner Jahrbuch* for 1917), he concludes his paper by listing 13 Koronis, 19 Eos and 22 Themis family members.

As can be appreciated from the above short summary, Hirayama's work, due to his extraordinary scientific skills and intuition, is packed with results, original insights and interpretations, opening a new, fascinating and important field of research. And all this in a paper of just 3 pages in lengths, including tables and figures. Remarkable indeed!

In the subsequent years Hirayama published two more papers in which he proposed four more families and six doubtful ones (Hirayama 1919), and identified 16 out of the 120 asteroids newly discovered in the period from 1914 to 1919 as members of the already known families (Hirayama 1920).

From the theoretical point of view, the most important in the series of Hirayama's papers was the one published in 1922 (Hirayama 1922)¶. Hirayama here for the first time mentions the preceding attempts to detect groups of asteroids by Kirkwood (1876, 1890, 1891), Tisserand (1891), and Mascart (1899, 1902), who were able to identify groups

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† angle of eccentricity φ is defined as $\sin\varphi = e$

‡ note that throughout the present text, the notation used is as in the original papers

¶ after NASA Astrophysics Data System; Hirayama in his later papers, as well as the other authors, quote 1923 as the year of publication of this paper

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of two or three asteroids at best, and warns against such results because of the considerable probability of accidental coincidence. He attributes failure of these attempts to the "use of actual orbits ... for comparison, whereas these orbits are varied remarkably by the action of the planets", and claims that such attempts could be successful only if the separation has occurred very recently (we know now that he was perfectly right, use of the instantaneous, osculating elements to identify families is limited to the very youngest ones). Pursuing the same reasoning, Hirayama claims that some kind of *invariable elements* must be used to identify families originated at remoter age.

Next, Hirayama gives a detailed account of the Lagrangian linear secular perturbation theory, arriving to the well-known solutions consisting of *free and forced oscillations*:

$$u = e \sin \varpi = \nu_0 \sin (g_0 t + \beta_0) + \sum_r \nu_r \sin (g_r t + \beta_r);$$

$$v = e \cos \varpi = \nu_0 \cos (g_0 t + \beta_0) + \sum_r \nu_r \cos (g_r t + \beta_r).$$

for the eccentricity, longitude of perihelion couple, and similar for the inclination, longitude of node one. Giving the geometrical interpretation of the obtained equations, Hirayama states that, considering u, v as a set of rectangular coordinates, the point (u, v) makes a circular motion about the moving center $(u_0 = \sum_r \nu_r \sin (g_r t + \beta_r), v_0 = \sum_r \nu_r \cos (g_r t + \beta_r))$ with the constant radius ν_0 and uniform angular velocity g_0. Following Laplace (*Œuvres*, XI, pp. 423 and 428), he calls the amplitude of free oscillation ν_0 *the proper eccentricity*, and similarly, μ_0, appearing in the other pair of solutions, *the proper inclination*. Referring to the Laplace-Poisson's theorem on the invariability of the major axis of the planetary orbits, he completes the list of invariable elements:

$$a,\ \nu_0,\ \mu_0,\ \alpha_0,\ \beta_0\,,$$

where α_0, β_0 are the phase angles of the free oscillations at the epoch, that is *proper longitude of node* and *proper longitude of perihelion*. Hirayama concludes: "Now, among these five elements, the two angular elements α_0, β_0, depend on the epoch chosen, and their initial values cannot be determined so long as the time of the separation is not known, whereas the other three can be determined without a knowledge of the elapsed time. Hence, taking these three elements as the invariable elements, if there may be noticed a group of the asteroids with the elements approximately common to all, and if at the same time, their number is *sufficiently large* to be insured from the effect of chance, then we can conclude that they have probably originated from the *breaking up* of a single asteroid".

In the rest of the paper, Hirayama first gives his results of the computation of proper elements, based on the Stockwell's work and considering the actions of all perturbing planets, including a catalog of proper elements for 933 asteroids known at the time. Next he presents results of the search for families and lists identified members of five, in his opinion, most reliable families: Themis (with 25 members), Eos (23), Coronis¶ (15), Maria (13), and Flora (53). He also demonstrates how the use of proper elements and the inclusion of perturbations from all the planets in their computation has improved the compactness, and thus the reliability of the identified family, compares the size (via the mean magnitude) of the family members with respect to the corresponding mean for all the objects in the catalog, and estimates the percentage of the objects belonging to

¶ Note the change of the name, from Koronis in the paper from 1918 to Coronis in this one

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the reliable families to 14%, while allowing this percentage to be as high as 30% if all the uncertain groups are taken into account.‖

In conclusion, Hirayama emphasizes the importance of the determination of the family ages, stating that this is not easy to achieve, and even mentions the possibility of successive separations occurring within the same family, which would, according to him, render the age determination hopeless. His final remark discusses the terminological issue: according to him the word *family* is the most suitable for a class of objects which are deemed as originating from a single body. The word *group*, on the other hand, is appropriate for a class of asteroids such as Trojans, or Hildas.

If we were not aware of the time of writing of this paper, its contents and conclusions could have been easily confused for some contemporary review on the considered topics.

Hirayama published yet another important paper devoted to families (Hirayama 1927)††. He recomputed the invariable elements for all the asteroids known at the time, providing a catalog with 1025 proper element records, and identifies a total of 32 asteroids as belonging to Themis family, 27 to Eos, 20 to Coronis, 14 to Maria, and 63 to Flora. Finally, he proposes also Phocaea as probable family with 11 members.

Most of the paper is, however, devoted to the discussion of the unisotropy of the identified families, measured by the mean deviation of the classification parameters: mean daily motion, n, proper eccentricity, ν_0, and proper inclination, μ_o. Finding these deviations to be very different in absolute and relative sense, he states that families show different "degree of tenuity", with the Flora family being "more than six times as rare as the Coronis family". He, thus, concludes that "the peculiarities in the deviations of the invariable elements, seem to show that the 'explosion' by which the original asteroid was broken, had not been isotropic, i.e. the relative velocities with which the fragments flew away from the position of the original asteroid had not been symmetrical in all directions".

He next proceeds to elaborate on these anisotropies in great detail by using the equations relating the variations of elements with the components of the relative velocities‡‡, mentioning even such a finesse that this velocity is not a true relative velocity at the moment of "explosion", but the effective relative velocity reduced by the attractions of the other fragments on the departure!

The final Hirayama's paper on families has been published in 1933 (Hirayama 1933). There he only lists the additional members of the five original families, thus indicating that these are the only real ones (see Carusi & Valsecchi (1982)).

2.2. *Post-Hirayama Developments: Brouwer et al.*

Nearly two decades elapsed from the publication of the final Hirayama's paper on families, until Brouwer (1951) has ventured to redo the analysis of asteroid families.

Brouwer made use of the improved secular theory of motion of major planets by Brouwer & van Woerkom (1950), and the resulting induced (forced) oscillations, to compute the proper elements for a total of 1537 asteroids. The improvement consisted in using of the more accurate planetary masses and in accounting for the effects of the principal second-order terms due to the mutual actions of Jupiter and Saturn, which most significantly affected the eccentricities and perihelia in the inner part of the asteroid belt. In the enclosed catalog Brouwer also listed the sum of the longitudes of the proper perihelion and node, $\pi_1 + \theta_1$, which is a constant in the linear secular perturbation theory.

‖ recent family classifications find this percentage to be close to 25%.

†† after NASA Astrophysics Data System; later authors quote 1928 as the year of publication of this paper

‡‡ a form of nowadays often used Gauss' equations

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Brouwer's comprehensive analysis of the identified families confirmed the existence of families Themis, Eos, Coronis, Maria and Phocaea, enlarged somewhat for the newly numbered asteroids, but generally identical with Hirayama's original classification. Flora family, on the contrary, has been divided into four separate families, using also the sum of the proper angles as an auxiliary parameter. In addition, Brouwer found 19 groups, with, as a rule, five or more members, which exhibit concentration not only of proper elements, but of the proper angles as well. He presents them with "some diffidence", admitting that some of them "will be rejected or will require considerable modification if better criteria should become available".

At the end, Brouwer discusses Kuiper's hypothesis on the collisional origin of asteroid families, and concludes that the evidence presented in this paper is favorable to the collision theory, but only in a qualitative way.

After Brouwer's work, again nearly two decades elapsed before another new important contribution was made. It was due to Arnold (1969) who pioneered objective, computer-aided methods of asteroid family classification. He still used Brouwer's proper elements to first estimate an assumed uniform density of the main belt asteroids in the phase space of proper elements, and then look for the concentrations with significantly higher density than the reference one. The computer code was of a "particle-in-the-box" kind, based on a Poisson probability to assess the statistical significance of the concentration. The families were formed by overlapping of the neighboring boxes. At the end, to get results with maximum reliability, identified families were subjected to various tests and comparisons, including the comparison with families obtained by applying the identical classification procedure to the synthetic random distributions. It has to be noted, however, that Arnold, in spite of all the sophisticated tools he made use of, still in some cases allowed a manual intervention, using "intuition" and tolerating some less reliable results.

Arnold confirmed the existence and membership of all the original Hirayama families (but, following Brouwer, with Flora divided in four families), as well as of a fraction of Brouwer's groups, but changing, combining and dividing the others (including two by Anders (1964)), even rejecting some of them entirely. Adding to this conglomerate several groups discovered by himself, he increased the total number of families to 37.

It is interesting to note that Arnold identified also a few "jet streams", proposed by Alfven (1969) as groups characterized by concentration of all five proper elements $(a_p, e_p, \sin i_p, \varpi_p, \Omega_p)$, but warned that their statistical significance is much less than that of the families, and that they do not fit the family collisional origin hypothesis, which he considers the most probable one.

In the same year Williams (1969) developed a new, higher-degree theory of secular perturbations, which solves the averaged equations of motion by combining analytical and numerical techniques and enables computation of proper elements. Since the computations involved with this solution are quite demanding, Williams published the results only later (Williams 1979). For the family classification Williams used pairs of stereo-projections of proper element distributions, defining by eye the candidate families, and subjecting them to a Poisson-type probability test. Let us mention here only that he identified a large number of small families, with, e.g., Flora family divided into 10 smaller ones. Using locations of the linear secular resonances he was the first to determine, and the associated gaps in the distribution of asteroids in the phase space, he claimed that Phocaea is not a family but the island of objects surrounded by resonances.

In the same period, a few other asteroid family classifications were published (Lindblad & Sothworth 1971, Carusi & Massaro 1978, Kozai 1979), using different asteroid samples, different classification methods, proper elements coming from different theories, and this

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only created a considerable confusion. The embarrassing situation has been summarized and thoroughly analyzed in Carusi & Valsecchi (1982).

Let us conclude this brief historical overview by stating that 1970's and 1980's represent a period of rapidly growing interest and activity in asteroid research. As mentioned above, this was due to the recognition of significance of asteroid studies for the studies of the origin and evolution of the solar system as a whole.

Let us just mention the Palomar-Leiden survey, the predecessor of many later observational surveys, the advent of new observational techniques (CCD) and facilities resulting in an unprecedented growth of asteroid discovery rates and accuracy of asteroid positions and orbits, photometric, polarimetric and spectral reflectance observations to assess asteroid physical properties and furnish the taxonomy, laboratory experiments with hypervelocity impacts and subsequent development of corresponding simulations and models, the first dedicated database TRIAD (Tucson Revised Index of Asteroid Data), followed later by many more of general or specialized kind, like PDS, AstDyS, etc., conference series, like Asteroids, starting in 1971 in Tucson, or Asteroids, Comets, Meteors, commencing in 1983 with Uppsala meetings, and so on.

In the following we shall, for the sake of brevity, directly connect the subsequent developments with the current status-of-the-art in the field, thus abandoning the hitherto employed purely chronological approach.

3. Later Developments and State of the Art

3.1. *Classification Parameters*

Different parameters have been used over time for asteroid family classification (see Knežević *et al.* 2002). Some of them are either more primitive "relatives" of contemporary proper elements, the others are parameters adapted to the specific dynamics, like in the case of secular, (see Morbidelli 1993), or mean motion resonant asteroids (e.g. Brož *et al.* 2011). The common property of all these parameters was that they shared the property of being "proper", that is nearly constant over long time spans.

The three proper elements commonly used nowadays for the asteroid family classification are, in essence, those proposed already by Hirayama (see Section 2.1): the proper semimajor axis, a_p, the proper eccentricity, e_p, and the sine of proper inclination, $\sin i_p$. They are obtained from the instantaneous osculating orbital elements by removing the planetary perturbations of short and long periods. The procedure is based on the averaging over the full cycle of each perturbing term, and has to be applied separately for the short and long periodic perturbations. The remaining two proper elements: the proper longitude of perihelion, ϖ_p, and the proper longitude of node, Ω_p, being epoch-dependent are not used for the classification, but rather for the determination of frequencies and for various dynamical studies.

The alternative classification parameters, like the proper frequencies (n,g,s), where n is the asteroid mean-motion and g, s are the secular frequencies of the perihelion and node, respectively, have been proposed by Carruba & Michtchenko (2007). Even the osculating elements themselves proved to be useful for identification and characterization of asteroid families, in particular the young ones. Due to the smallness of the escape velocity of collisional fragments with respect to their orbital velocity, the orbital elements of fragments immediately after the family-forming event tend to be fairly similar. As proposed by Nesvorný *et al.* (2006), if the family is very young, it should appear as a cluster in the five-dimensional space of osculating elements, a, e, I, ϖ, Ω. "Very young

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family" in this context means $\leqslant 1$ My, which is the typical time before slow angles ϖ and Ω of family members become dispersed by differential perturbations.

As of recently the sizeable WISE catalog of asteroid albedos†, and the SDSS catalog of asteroid colors‡ have become available. Attempts to improve the asteroid family classification using these data were based either on their post-classification consideration as auxiliary information to confirm/reject candidate dynamical families and identify the interlopers, or on their direct use as additional genuine parameters to find families in the extended classification parameter space. Examples of the multidimensional family classification in terms of proper elements, albedos and colors can be found in, e.g., Parker *et al.* (2008), and Carruba *et al.* (2013); in the latter paper, for instance, authors made use of different metrics in extended dimensional space, up to the six-dimensional one (three proper elements, two color indices and albedo). The families obtained by multi-parameter classifications are presumably more reliable, as they are combining dynamical and physical properties of asteroids, but, as pointed out by Milani *et al.* (2014), in this way a large amount of information contained in the dynamical data for smaller asteroids is wasted because of the lack of physical observations of faint objects and/or because of their inferior accuracy. Some recent searches in (a, H) space, where H is the absolute magnitude, looking for families dispersed in eccentricity and inclination, resulted in a proposed identification of the "new Polana family" (Walsh *et al.* 2013), but its statistical significance seems to be difficult to prove (see also Milani *et al.*, *this volume*).

Apart from the above mentioned theories to compute proper elements (Hirayama 1922, Brouwer 1951, Williams 1969), there are at present two general theories that can be used to compute proper elements: analytical and synthetic. The analytical theory (Milani & Knežević 1990, 1994) is very efficient in terms of the computation of proper elements, but applicable only to asteroids of low to moderate eccentricities and inclinations. Together with the improvements of the classification methods (see below), this theory in early 1990's enabled a major break-through in the field; in particular, by providing the significantly more accurate proper elements and frequencies than previously available, it gave rise to a more reliable family classification, to the first determination of the location of nonlinear secular resonances and estimate of their effects, to better understanding of the long-term dynamics of asteroids, etc. Let us note that this theory has been based on the previous work by Hori (1966), Yuasa (1973), and Knežević (1989).

The synthetic theory (Knežević & Milani 2000) involves much more time consuming computations that the analytical one, but can be used for all asteroids and gives by a factor of more than 3 better results. A direct predecessor to this theory is the work by Milani (1993). This is nowadays a "standard" theory to compute the best available proper elements distributed via AstDyS service (Knežević & Milani 2003).

In addition to the above theories intended for mass production of proper elements for asteroids in the main belt, various specially adapted theories were developed for specific asteroid populations: the semi-analytical theory for high-inclination and high-eccentricity objects (Lemaitre & Morbidelli 1994), the theories for Trojans (c.f. Milani 1993, Beaugé & Roig 2001) and Hildas (c.f. Schubart 1991), etc.

There were only a few changes in the past period regarding all these theories (see, c.f., Carruba 2010 for a modification in computing the proper frequencies in the framework of synthetic theory), the main improvement being in terms of the efficiency of computation, and in a rapid growth of catalogs of asteroid proper elements.

† http://wise2.ipac.caltech.edu/staff/bauer/NEOWISE_pass1
‡ http://www.astro.washington.edu/users/ivezic/sdssmoc/

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The current state of the art regarding the proper elements as primary asteroid family classification parameters can be summarized as follows. On the AstDyS site¶, as of September 2015, there are 18 catalogs containing proper elements and their uncertainties for about 510,000 numbered and multiopposition objects, including main belt asteroids, Trojans, and TNOs, computed by means of the above mentioned theories using customized dynamical models.

The accuracy of the synthetic proper elements for the numbered main belt objects is typically $\sigma a < 0.0003$ au, $\sigma e < 0.001$ and $\sigma \sin i < 0.001$. This roughly corresponds to 15 m/s in terms of the relative velocity, which is good enough for asteroid family classification purposes.

3.2. *Classification Methods*

Failure to explain some important physical properties of asteroid families available in the beginning of 1980's, led to recognition that "better proper elements computation and classification methods" are urgently needed to improve the situation with poor and ambiguous definition of families (Zappalà *et al.* 1984). As already pointed out above, this somewhat later resulted in the new theories to compute asteroid proper elements, and in the improvements of the classification methods, which enabled a major break-through in the field. The real turning point, for what concerns the classification methods, represents the advent of Hierarchical Clustering Method (HCM) introduced by Zappalà *et al.* (1990), and subsequently improved in Zappalà *et al.* (1994), and Zappalà *et al.* (1995). Using an ever increasing catalog of analytical proper elements by Milani & Knežević, Zappalà and his collaborators in this series of papers for the first time presented a reliably defined asteroid families, which served, for several years to come, as the basis for most of the research done in the field.

Another method proposed at about the same time (Bendjoya *et al.* 1991) was the Wawelet Analysis Method (WAM). However, as pointed out by Bendjoya and Zappalà (2002), the HCM and WAM were giving results in a very good mutual agreement and of comparable reliability and robustness. Thus, only HCM, being conceptually simpler and easier to use, survived until now and continued to be used in almost all recent classifications.

The original HCM, as introduced by Zappalà *et al.* (1990), underwent various modifications, improvements and adaptations in the meantime, but the core of the method (search for the nearest neighbor, choice of the metrics, concept of quasi-random level, stalactite diagrams) remained basically unchanged. The selection is based on a couple of parameters: the minimum number of members, N_{min}, required for a group to be considered candidate family, and the cutoff distance, d_c, which, expressed in terms of the chosen classification metrics, defines the distance below which the adjacent objects are considered to belong to the same concentration; the choice of d_c is based on the concept of the Quasi-Random Level (QRL), that is of the distance at which in a random population of objects, mimicking the real distribution of asteroids in the phase space of classification parameters, one can still find chance groupings of N_{min} members.

To refine the choice of the cutoff distance and adjust it to each individual family, Nesvorný *et al.* (2005) proposed a method that also makes use of the distances in terms of a chosen metrics computed in a HCM sense, but represents them in the form of a cumulative number frequency distribution with respect to the cutoff velocity. This way of representing the concentration of objects clearly reflects the accumulation of fragments around the parent body with the increase of the cutoff velocity, and is quite effective in determining the reliable limiting distance and family boundaries.

¶ http://hamilton.dm.unipi.it/astdys

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Nowadays, the classification of asteroids into families becomes increasingly challenging as the asteroid data sets continue to grow at an unprecedented rate. To cope with such an increase and with an already large volume of data, the most recent, state of the art improvement in the field has been proposed by Milani *et al.* (2014). In several respects this proposal intends to modify the long standing paradigms: (i) it introduces the new, dynamic classification by combining the classical HCM with an automated procedure to attribute members to existing families; in this way the general validity of the classification can be maintained for many years, without the need to always repeat the entire procedure (in practice, the general classification can be revised only once in a while, but being automatically updated every time the data set is significantly increased). (ii) it accounts for the difference between "dynamical families" identified by number density contrasts in the phase space of proper elements, and the "collisional families" defined as outcomes of a single collisional events, and dismisses the assumption of one-to-one correspondence between them. (iii) it makes use of the proper elements first, thus defining the dynamical families, and then employs information from absolute magnitudes, albedos and colors as either confirmation or rejection.

The procedure consists of several steps described in full detail in Milani *et al.* (2014) and Knežević *et al.* (2014). In brief, it begins with segmentation of the problem to handle the very large amount of data in the most efficient and accurate manner (see Table 2 in Milani *et al.* (2014)). By applying HCM in the central, most populous zones of the main belt, the "core families" are obtained, consisting of brightest objects and representing the inner skeletons of larger families (see Fig. 1 in Milani *et al.* 2014); in the zones with less objects (Hungarias, zones beyond 2:1 mean motion resonance with Jupiter, and high-inclination zones) families are identified in a usual manner - i.e. by the direct application of the HCM procedure, without use of the multistep approach.

The next step of the procedure in the central zones is the classification of faint asteroids not used in the first step; these are attached by means of an automated procedure to the previously established family cores. Only single links are allowed for this attachment, which means that the families are extended in the absolute magnitude/size dimension, but not much in proper elements space, especially not in the proper semimajor axis .

Then, the "intermediate background" asteroids only, defined as the set of all the objects not attributed to any family in the previous steps, are used as an input to the next step. Families identified at this step are formed by the population of asteroids left after removing from the proper elements data set the already identified family members. These can either be fully independent new families having no relation with the families identified previously, or they may be found to overlap already identified extended cores and form "satellite families".

A single-link attribution, by means of the same automated procedure, to all the families identified in the previous steps, is then repeated. A small number of asteroids with double classification is typically found in this step; they belong to "intersections". The multiple intersections between particular families indicate that such families are candidates for merging.

The final step of the procedure of asteroid family classification is motivated by the rapid growth of the proper elements database; it consists of an automatic update of the current family classification, that is of repeating the attribution of asteroids to the existing families every time the catalog of synthetic proper elements is updated. What is repeated is only the single-link attribution step, thus the lists of core families members, of members of smaller families, and also the list of already effectuated mergers are kept unchanged.

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Once the above procedure is fully implemented, the classification maintenance requires certain operations to be performed:

- Regularly: repeat the attribution of asteroids to the existing families every time the catalog of synthetic proper elements is updated.
- Occasionaly: reconsider the list of small families, confirm some of them as statistically significant, discard others as statistical flukes, decide on pending mergers if intersections increase enough or otherwise new data give enough reason for such a decision.
- Seldom: reset the whole classification.

See the paper by Milani *et al.* in this volume for the results of the recent classification update, and for the demonstration of how classification maintenance actually works.

4. Concluding Remarks

The business of asteroid family classification, as one can appreciate from the above brief historical description, is quite a tricky one. Let us conclude this review with an example which nicely demonstrates how this apparently straightforward task can produce the confusing results, changing dramatically over time and from one author to another. We are, of course, speaking of the case of the Flora family, which even nowadays gives rise to controversies from the very existence of this family, to its uniqueness, origin, extent and membership.

In the preceding text we have already mentioned Flora family/ies several times: it has been identified as a single family by Hirayama (1922), who confirmed it also later in Hirayama (1927) but warning against remarkably large dispersion of its members. Next, it is divided in four families in Brouwer (1951), and continues like this in Arnold (1969). Lindblad & Sothworth (1971), Carusi & Massaro (1978), and Kozai (1979) consider it a single family, but Williams (1979) divides it in as many as 10 smaller families. In Zappalà *et al.* (1990) Flora family is again divided into four (sub)families, in Zappalà *et al.* (1994) it is considered as a single clan†, and, finally, in Zappalà *et al.* (1995), as a single family. In a 2012 release of the classification which makes part of the Planetary Data System Small Bodies‡, Nesvorný lists Flora as a single family, and as such it is analyzed in detail by Dykhuis *et al.* (2014). Most recently, however, Milani *et al.* (2014) do not find it at all.

What could be the cause of such a remarkable difference among various authors regarding this particular group/family of asteroids? There are at least two technical problems which may have affected the judgment of various authors, that is the vicinity of the strong $g - g_6$ linear secular resonance that renders determination of proper elements less accurate, and an already mentioned large dispersion of the bright members which could have contributed to the confusion with various divisions of the group in different subgroups.

A tentative explanation could be that indeed there is a Flora family, but consisting only of a handful of large asteroids. Namely, assuming that the collisional event took place some Gy ago, and adopting a typical Yarkovsky drift of km-sized fragments on the order of ~ 0.001 au/My, one can conclude that all the smaller members of the family should have left the parent body's vicinity a long time ago (many actually ending up in the $g - g_6$ resonance). They have been replaced by the small bodies coming from neighboring regions and by fragments from other collisions, thus, what is nowadays recognized as

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† Clans are defined as statistically significant groupings for which unequivocal membership definition and/or separation between subfamilies is impossible.

‡ http://sbn.psi.edu/pds/resource/nesvornyfam.html

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the "Flora family", is actually a conglomerate of the bodies of different origin. Only a few largest bodies belong to the original family (thus identified as such in the early classifications), with the rest not at all representing a typical homogeneous concentration of bodies originated in a single breakup of a large parent asteroid.

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