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HETEROGENEITY AND REDISTRIBUTION IN FINANCIAL CRISES

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This paper presents a simple model of an economy with heterogeneous agents to show that the redistribution of wealth among such agents can play a significant role in the propagation mechanism of financial crises. In an economy where firms with heterogeneous productivity operate under borrowing constraints, the redistribution reproduces hump-shaped responses for output and labor and procyclicality in observed productivity. In this model, a financial shock generates a persistent and hump-shaped response, whereas a productivity shock does not. Further, the redistribution of wealth significantly amplifies the persistence and hump shape of these responses following a financial shock. This model suggests that redistribution may thus be a key driving force behind the transmission of financial crises.

Keywords: Heterogeneity, Borrowing Constraint, Redistribution of Wealth, Labor Wedge

1. INTRODUCTION

This paper aims to highlight the significant role played by the redistribution of wealth among heterogeneous agents in the propagation of financial crises, including protracted recessions. The model presented implies that the redistribution can amplify a financial shock in such a way that output and labor display conspicuous persistence and hump shapes and that productivity varies procyclically.

Empirical studies of business cycles show that the responses of macroeconomic variables to exogenous shocks are persistent and hump-shaped [see Christiano

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et al. (1999) on monetary policy shocks and Altig et al. (2011) on technology shocks]. Indeed, the reproduction of this persistence and hump-shaped response is one of the goals that motivate business cycle theorists [e.g., Carlstrom and Fuerst (1997); Bernanke et al. (1999)]. In this vein, Reinhart and Rogoff (2009) report the conspicuous persistence of macroeconomic variables after a financial crisis, showing that output continues to decrease for two years, on the average, whereas it decreases for only one year or less as a result of business cycle fluctuations. Kehoe and Prescott (2007) define "great depressions," namely large recessions in the aftermath of financial crises, as time periods during which detrended output per capita decreases for more than 10 years, with the accumulated decrease more than 15% as measured from the beginning of the decade, and 20% during the whole period. According to the Kehoe–Prescott definition, such great depressions broke out globally throughout the twentieth century, such as in the 1970s in New Zealand and Switzerland and in the 1990s in Japan. By analyzing these episodes, Kehoe and Prescott (2007) conclude that the primary driving force of great depressions is a decline in (observed) productivity. Business cycle accounting [BCA; Chari et al. (2007)] compares productivity with other factors that induce business fluctuations. In particular, BCA focuses on four wedges: the efficiency wedge (EW), labor wedge (LW), investment wedge (IW), and government wedge (GW). EW is observed total factor productivity; LW is MRS/MPL, where MRS is the marginal rate of substitution between consumption and leisure and MPL is the marginal product of labor; IW is the wedge between the market interest rate and the stochastic discount factor; and GW is the deadweight loss, which manifests itself as government consumption in a simple real business cycle model. Chari et al. (2007) note that EW and LW are the two primary factors that explain the Great Depression of the 1930s in the United States, whereas Kobayashi and Inaba (2006) stress that the same factors also explain Japan's lost decade of the 1990s. Indeed, a growing body of research on BCA has focused considerable attention on how a deterioration in LW affects large recessions [see Mulligan (2002); Shimer (2009)]. A sharp decline in LW was also observed in the U.S. economy during the global financial crisis of 2008–2010 [Pescatori and Tasci (2011)].

These studies imply that a theoretical framework for the analysis of persistent recessions in the aftermath of financial crises should have the following three features:

i. Persistent and hump-shaped response of macroeconomic variables to exogenous shocks,
ii. EW display of procyclicality, and
iii. LW display of procyclicality.

Based on the foregoing, in this study we present a simple model that successfully reproduces these three features, even though the reproduction of the third point is slightly problematic. The present study employs a heterogeneous-agent economy model with borrowing constraints, where a financial shock reproduces responses in line with the first and second features through the redistribution of wealth among

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heterogeneous agents. In addition, the responses to an aggregate productivity shock display lesser persistence and no hump shape. In our model, LW is procyclical only in response to an extremely large financial shock.

There are two main strands of research on business cycles. The first focuses on standard dynamic stochastic general equilibrium (DSGE) models [Christiano et al. (2005); Smets and Wouters (2003)], in which the individual behavior of a representative agent is persistent and hump-shaped because of nominal rigidities, the habit persistence of consumption, and the specialized adjustment costs of investments. The second centers on "heterogeneous agent models" such as the credit cycle model presented by Kiyotaki and Moore (1997) and the liquidity shock model of Kiyotaki and Moore (2012). Although the individual behavior of each agent in this setting does not display nonlinearity or relatively less persistence, the macroeconomic variables are hump-shaped and strongly persistent as a result of aggregation.

The model used in this study is a heterogeneous agent model. As Cordoba and Ripoll (2004) point out, the hump shape in the model put forward by Kiyotaki and Moore (1997) depends on specific functional forms of utility and production technology and almost disappears under standard functions. Our model is complementary to the model of Kiyotaki and Moore (1997) in that it reproduces a strong hump shape under standard settings. Moreover, it is a simplified version of the one used by Kiyotaki and Moore (2012), and the results derived are similar to their outcomes, which show that a liquidity shock can generate persistence and nonlinearity in the macroeconomic variables in a monetary model. Our model is also complementary to the one of Kiyotaki and Moore (2012) in that it indicates that the redistribution of wealth may play a key role in the propagation mechanism by generating persistence and nonlinearity.[1]

The policy implications that flow from the results of standard DSGE and heterogeneous agent models may differ sharply. The standard DSGE model emphasizes that the cyclical movement of macroeconomic variables is the optimal response to shocks and thus that the stabilization of business cycles per se may not improve welfare significantly.[2] Further, these models note that the objective of macroeconomic policy is not the stabilization of business cycles, but rather, the mitigation of inefficiency due to nominal rigidities. In contrast, our heterogeneous agent model suggests that persistent recessions caused by financial shocks may be intrinsically inefficient because of credit constraints and endogenous wealth distribution, implying that corrective redistribution by government policy could improve aggregate efficiency and thus that the stabilization of aggregate variables per se may significantly improve welfare.

The remainder of this paper is organized as follows. A simple model is presented in Section 2 to demonstrate the basic mechanism of propagation due to the redistribution of wealth. In Section 3, we describe the generalized model, in which capital depreciates partially and both long-term and short-term borrowing is subject to borrowing constraints. In Section 4, we analyze and compare the responses to a financial shock and a productivity shock. Section 5 concludes.

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2. BASIC MODEL

In this section, we present and analyze a simple model that is a version of Kiyotaki's (1998) model with only one modification: we make labor supply endogenous.[3] The model presented is a closed economy with discrete time, inhabited by continua of workers and entrepreneurs. The measure of these continua is one. Workers and entrepreneurs both have the subjective discount factor β (< 1).

2.1. Workers

A unit mass of workers can save but cannot borrow. Based on Kiyotaki and Moore (2012), we assume that workers provide entrepreneurs with labor, and save and consume to maximize the following Greenwood–Hercowitz–Huffman utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln \left[c_t' - \frac{\omega}{1+\nu} (l_t)^{1+\nu} \right], \tag{1}$$

subject to the budget constraint

$$c_t' + \frac{b_{t+1}'}{r_t} = w_t l_t + b_t', \tag{2}$$

where E_0 denotes the expectation operator at time 0; c_t' denotes consumption; b_{t+1}' is the bond, which is issued at time period t and redeemed at $t+1$; r_t is the gross interest rate; w_t is the real wage rate; and l_t is labor supply. The first-order conditions (FOCs) with respect to consumption and labor indicate that labor supply is given by $l_t = (w_t/\omega)^{1/\nu}$. The FOC with respect to b_{t+1}' is

$$\frac{x_{t+1}}{x_t} \geq \beta r_t, \tag{3}$$

and the nonnegativity constraint is $b_{t+1}' \geq 0$, where $x_t = c_t' - \frac{\omega}{1+\nu}(l_t)^{1+\nu}$. If (3) holds with strict inequality, then $b_{t+1}' = 0$, and workers can be considered to be leading a hand-to-mouth existence as they consume their entire income and do not save. Condition (3) implies that the condition for workers to be hand-to-mouth consumers is

$$\left(\frac{w_{t+1}}{w_t} \right)^{\frac{1+\nu}{\nu}} > \beta r_t, \tag{4}$$

which is satisfied for all the numerical simulations presented in this paper.[4]

2.2. Entrepreneurs

There is a unit mass of entrepreneurs. n entrepreneurs have productivity z and $1-n$ have productivity 1, where $z > 1$. We consider entrepreneurs with productivity z to

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be high-productivity entrepreneurs (high entrepreneurs or high firms hereafter) and those with productivity 1 to be low-productivity entrepreneurs (low entrepreneurs or low firms hereafter). At the end of every period, $(1-\gamma)n$ high entrepreneurs are randomly chosen to become low entrepreneurs in the next period, and $(1-\gamma)n$ low entrepreneurs are randomly chosen to become high entrepreneurs in the next period. The entrepreneur's utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{5}$$

where c_t is the entrepreneur's consumption. An entrepreneur with productivity $A_t \in \{1, z\}$ can produce output y_t from labor l_t and capital k_t by using the following production technology:

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}.$$

We assume that A_t follows a Markov process, which is characterized by $\Pr(z|z) = \gamma$, $\Pr(1|z) = 1-\gamma$, $\Pr(z|1) = (1-\gamma)\frac{n}{1-n}$, $\Pr(1|1) = 1-(1-\gamma)\frac{n}{1-n}$, where $\Pr(a|b)$ denotes the probability of $A_{t+1} = a$ conditional on $A_t = b$. We assume for analytical simplicity that capital stock k_t fully depreciates after the production of output in this baseline model. The budget constraint for an entrepreneur is as follows:

$$c_t + k_{t+1} - \frac{b_{t+1}}{r_t} \leq A_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - b_t, \tag{6}$$

where b_{t+1} is the bond issued in period t and redeemed in period $t+1$. As workers do not save, only entrepreneurs hold the bonds issued by other entrepreneurs. When an entrepreneur purchases these bonds, b_{t+1} can be a negative number. We consider the following assumption pertaining to a lack of commitment.

Assumption 1. An entrepreneur cannot commit to repay debt (b_{t+1}). The creditors (or bondholders) can seize θy_{t+1}, where y_{t+1} is output and $0 < \theta < 1$, if the entrepreneur repudiates his/her debt.

Under this assumption, a share of output (θy_{t+1}) works as collateral for the debt and represents the upper limit of the amount that can be borrowed. Thus, the entrepreneur faces the following borrowing constraint:

$$b_{t+1} \leq \theta A_{t+1} k_{t+1}^{\alpha} l_{t+1}^{1-\alpha}, \tag{7}$$

where l_{t+1} is the labor input at $t+1$, which is decided in period $t+1$. As we see later in equation (8), l_{t+1} is a linear function of k_{t+1}. After the revelation of A_{t+1} at the end of period t, the entrepreneur with productivity A_{t+1} chooses $\{c_t, k_{t+1}, b_{t+1}\}$ in period t and l_{t+1} in period $t+1$ to maximize his/her utility (5), subject to the budget constraint (6) and borrowing constraint (7).

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2.3. Benchmark without Borrowing Constraints

The key determinant of aggregate dynamics in this economy is the entrepreneur's borrowing constraint. Here, we describe the dynamics in the case where there are no borrowing constraints. In this economy, entrepreneurs can borrow an unlimited amount. Thus, whereas high firms purchase all the capital stock and produce output at the equilibrium, low firms purchase the bonds issued by these firms but do not produce output. The market interest rate is equal to the marginal product of capital for high firms.

Given that the capital stock of a high firm is k_{t+1}, the labor demand of this firm, l_{t+1}, is decided by the following equation:

$$l_{t+1} = \arg\max zk_{t+1}^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1} = \left[\frac{(1-\alpha)z}{w_{t+1}}\right]^{\frac{1}{\alpha}} k_{t+1}. \tag{8}$$

Thus, given that the total amount of capital in the economy is K_{t+1}, total labor demand in period $t+1$ is $L_{t+1}^{d} = [\frac{(1-\alpha)z}{w_{t+1}}]^{1/\alpha} K_{t+1}$. As total labor supply is given by $L_{t+1}^{s} = (\frac{w_{t+1}}{\omega})^{1/\nu}$ from the workers' optimization, labor market clearing can be indicated as follows: $w_{t+1} = [(1-\alpha)^{\nu} z^{\nu} \omega^{\alpha} K_{t+1}^{\alpha\nu}]^{1/(\alpha+\nu)}$. The profit of a high firm, $\pi_{t+1} = zk_{t+1}^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1}$, can be written as $\pi(k_{t+1}, K_{t+1}) = r(K_{t+1})k_{t+1}$, where

$$r(K_{t+1}) = aK_{t+1}^{\phi-1},$$

where $a = \alpha(\frac{1-\alpha}{\omega})^{(1-\alpha)/(\alpha+\nu)} z^{(1+\nu)/(\alpha+\nu)}$ and $\phi = \frac{(1+\nu)\alpha}{\alpha+\nu}$.

The dynamics is described as follows. As workers are hand-to-mouth consumers,[5] their consumption is given by the following equation:

$$C_t' = w_t L_t = [(1-\alpha)^{1+\nu} z^{1+\nu} \omega^{\alpha-1} K_t^{(1+\nu)\alpha}]^{\frac{1}{\alpha+\nu}},$$

where L_t is total labor. As the optimization problem for entrepreneurs is the Brock–Mirman model, they consume $1-\beta$ of total wealth, W_t, by the end of period t and save βW_t. Thus,

$$C_t = (1-\beta)W_t, \qquad K_{t+1} = \beta W_t, \qquad W_t = r(K_t)K_t.$$

These equations indicate the evolution of capital stock as follows:

$$K_{t+1} = a\beta K_t^{\phi}. \tag{9}$$

Numerical Example. The equilibrium path from the initial wealth W_0 ($< W^*$) is shown in Figure 1. Figure 1 shows the equilibrium paths of wealth (W_t), output (Y_t), and labor (L_t). The parameter values are $\alpha = 0.75$, $\beta = 0.98$, $\nu = 1$, $\theta = 0.3$, $z = 1.1$, $\gamma = 0.9$, $n = 0.1$, and $\omega = 1$. Because we focus on the qualitative features of the model, we do not calibrate these parameter values. We also consider a large value for α because we interpret output as including the remaining capital

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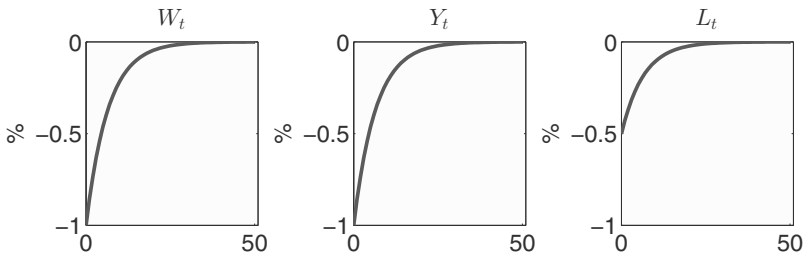

FIGURE 1. Equilibrium path without borrowing constraints. Percentage deviations from the steady state are shown. Convergence time is 31 periods. We judge convergence when the deviation is less than 0.01%.

stock. The initial W_0 is adjusted so that output in period 0 is lower than the steady-state value by 1%. As the variables quickly converge to the steady state, following dynamics similar to that of the Solow model, persistence and hump shape are not observed.

2.4. Dynamics of the Economy with Binding Borrowing Constraints

In an economy with borrowing constraints (7), high firms cannot issue a sufficient amount of bonds, and as a result, they cannot use all the capital stock in the economy. When the borrowing constraint is sufficiently tight, low firms also use capital and produce output by utilizing their inferior technology. This situation gives rise to the particular type of equilibrium analyzed next. As low firms are typically indifferent between purchasing capital and purchasing the bonds issued by high firms, returns from investment at k_{t+1} and b_{t+1} should be identical at equilibrium in this case. Thus,

$$r_t = \alpha \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}. \tag{10}$$

Next, we calculate the rate of return on investment for high entrepreneurs. The profit of a high entrepreneur who has k_{t+1} units of capital stock is expressed as follows:

$$\pi(k_{t+1}, w_{t+1}) = \max_{l} z k_{t+1}^{\alpha} l^{1-\alpha} - w_{t+1} l = \alpha z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} k_{t+1}.$$

Suppose the wealth of a high entrepreneur is a_t. He/she consumes c_t and invests $a_t - c_t$ in capital stock. Additionally, he/she can issue bonds to increase his/her capital stock to k_{t+1}. The entrepreneur chooses $\{b_{t+1}, k_{t+1}\}$ to maximize $a_{t+1} = \pi(k_{t+1}, w_{t+1}) - b_{t+1}$. The optimization problem can thus be written as follows:

$$\max_{k_{t+1}, b_{t+1}} \pi(k_{t+1}, w_{t+1}) - b_{t+1},$$

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$$\text{subject to} \quad \begin{cases} k_{t+1} = a_t - c_t + \frac{b_{t+1}}{r_t}, \\ b_{t+1} \leq \theta z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha}. \end{cases}$$

The solution for this problem, combined with (10), implies the following dynamics of $\{k_{t+1}, a_{t+1}\}$:

$$k_{t+1} = \hat{k}(a_t - c_t), \tag{11}$$

$$a_{t+1} = R_t(a_t - c_t),$$

where

$$\hat{k} = \frac{1}{1 - \frac{\theta}{\alpha} z^{1/\alpha}}, \tag{12}$$

$$R_t = \left(1 - \frac{\theta}{\alpha}\right) z^{1/\alpha} r_t \hat{k}. \tag{13}$$

The reduced form of the optimization problem for a high entrepreneur is as follows:

$$\max_{c_t} E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s},$$

$$\text{subject to} \quad a_{t+1} = R_t(a_t - c_t).$$

The reduced form of the problem for a low entrepreneur is as follows:

$$\max_{c_t} E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s},$$

$$\text{subject to} \quad a_{t+1} = r_t(a_t - c_t).$$

These optimizations are the same as those in the Brock–Mirman model, where entrepreneurs consume $(1 - \beta)a_t$ and save βa_t. If the wealth of entrepreneur i is denoted by a_{it}, where $i \in [0, 1]$, the total wealth in this economy is defined as follows:

$$W_t = \int_0^1 a_{it} di.$$

Furthermore, we denote the wealth owned by high entrepreneurs by $s_t W_t$, where $s_t \in [0, 1]$ is the share of their wealth. The aggregate dynamics of the economy are decided by the evolution of the two state variables $\{s_t, W_t\}$, as shown in the following. First, the total consumption of entrepreneurs is expressed as follows:

$$C_t = (1 - \beta) W_t.$$

As workers are hand-to-mouth consumers,[6] total capital stock is as follows:

$$K_{t+1} = \beta W_t.$$

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We denote the total capital stock used by high entrepreneurs as $\hat{K}_{t+1}$ and that by low entrepreneurs as $K'_{t+1} = K_{t+1} - \hat{K}_{t+1}$. Equation (11) implies that

$$\hat{K}_{t+1} = \tilde{k}(s_t)\beta W_t,$$
$$K'_{t+1} = (1 - \tilde{k}(s_t))\beta W_t,$$

where $\tilde{k}(s_t) = \min\{1, \hat{k}s_t\}$. Similarly, we denote the total labor used by high entrepreneurs as $\hat{L}_{t+1}$ and that by low entrepreneurs as L'_{t+1}:

$$\hat{L}_{t+1} = z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} \hat{K}_{t+1},$$
$$L'_{t+1} = \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} K'_{t+1}.$$

Labor market clearing ($L_{t+1} = \hat{L}_{t+1} + L'_{t+1}$) determines the following wage rate:

$$w_{t+1} = \left[1 + (z^{\frac{1}{\alpha}} - 1)\tilde{k}(s_t)\right]^{\frac{\alpha v}{\alpha+v}} (1-\alpha)^{\frac{v}{\alpha+v}} \omega^{\frac{\alpha}{\alpha+v}} (\beta W_t)^{\frac{\alpha v}{\alpha+v}}. \tag{14}$$

The market interest rate is determined as a function of (s_t, W_t), according to (10), and R_t is given by (13).[7] The total output produced by high entrepreneurs is $\hat{Y}_{t+1} = z\hat{K}^{\alpha}_{t+1}\hat{L}^{1-\alpha}_{t+1}$ and that by low entrepreneurs is $Y'_{t+1} = (K'_{t+1})^{\alpha}(L'_{t+1})^{1-\alpha}$. Finally, we can decide the evolution of (s_{t+1}, W_{t+1}). As the rate of return for high firms is R_t and that for low firms is r_t, wealth in the next period, W_{t+1}, evolves based on the following expression:

$$W_{t+1} = [R_t s_t + r_t(1 - s_t)]\beta W_t. \tag{15}$$

High entrepreneurs in period t obtain $R_t s_t \beta W_t$ in the next period and survive with probability γ. Thus, the total wealth of surviving high entrepreneurs is $\gamma R_t s_t \beta W_t$. As the per capita wealth of a low entrepreneur is $\frac{(1-s_t)r_t\beta W_t}{1-n}$, the total wealth of high entrepreneurs who were previously low entrepreneurs is $(1-\gamma)n \times \frac{(1-s_t)r_t\beta W_t}{1-n}$. Thus, the share of wealth owned by high entrepreneurs in period $t+1$ is given by the following:

$$s_{t+1}W_{t+1} = \left[\gamma R_t s_t + (1-\gamma)\frac{n}{1-n}(1-s_t)r_t\right]\beta W_t. \tag{16}$$

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Assuming that $\tilde{k}(s_t) = \hat{k}s_t < 1$ always holds, (15) and (16) imply the following law of motion for the state variables:

$$W_{t+1} = \left[1 + (z^{1/\alpha} - 1)\hat{k}s_t\right] r_t \beta W_t, \tag{17}$$

$$s_{t+1} = \frac{\gamma\left(1 - \frac{\theta}{\alpha}\right) z^{1/\alpha}\hat{k}s_t + (1-\gamma)\frac{n}{1-n}(1-s_t)}{1 + \left(z^{\frac{1}{\alpha}} - 1\right)\hat{k}s_t}. \tag{18}$$

This completes the description of the dynamics. In the numerical simulation, we calculate EW and LW based on the BCA method:

- EW:

$$EW_t = \frac{(\hat{Y}_t + Y_t')}{K_t^{\alpha} L_t^{1-\alpha}},$$

- LW:

$$LW_t = \frac{w_t}{(1-\alpha)\frac{(\hat{Y}_t + Y_t')}{L_t}}.$$

We now consider the dynamic response of the economy to deviations in the two state variables, (W_t, s_t). A change in s_t mostly represents the redistribution of wealth. In this economy, high firms are borrowers and low firms are lenders. Redistribution, namely a change in s_t, is a change in the net worth of borrowers. The effect of such changes on the aggregate dynamics has been analyzed by Bernanke and Gertler (1989) and Bernanke et al. (1999). As Bernanke and Gertler (1989) point out, redistribution can represent "debt deflation," because an unexpected decline in prices transfers wealth from borrowers to lenders. Similarly, the emergence and collapse of an asset-price bubble could be an example of such redistribution in an economy where assets are used as collateral, because asset-price bubbles increase the loan amount and the subsequent bubble collapse decreases the net worth of borrowers.

We set the parameters at the same values as those in Section 2.3: $\alpha = 0.75$, $\beta = 0.98$, $\nu = 1$, $\theta = 0.3$, $z = 1.1$, $\gamma = 0.9$, $n = 0.1$, $\omega = 1$. We assume that the economy is initially at the steady-state equilibrium. The solid line in Figure 2 represents the response to a decrease in W_0. The value of W_0 is chosen so that output in period 0 decreases by 1% from the steady state. Similarly, the solid line in Figure 3 shows the response to a decrease in s_0. The value of s_0 is chosen so that output in period 0 decreases by 1% from the steady state.

Figure 2 shows that the dynamic response of the economy to a W_t shock is qualitatively the same as that in the case without borrowing constraints (Figure 1). In this case, s_t and the two wedges (EW_t and LW_t) are invariant over time.

Figure 3 shows that the response to a s_t shock displays remarkable persistence compared with that to a W_t shock: the convergence time in Figure 3 is twice as long as that in Figure 2. Figure 3 shows that the responses of output (which is proportional to consumption and investment) and labor are conspicuously

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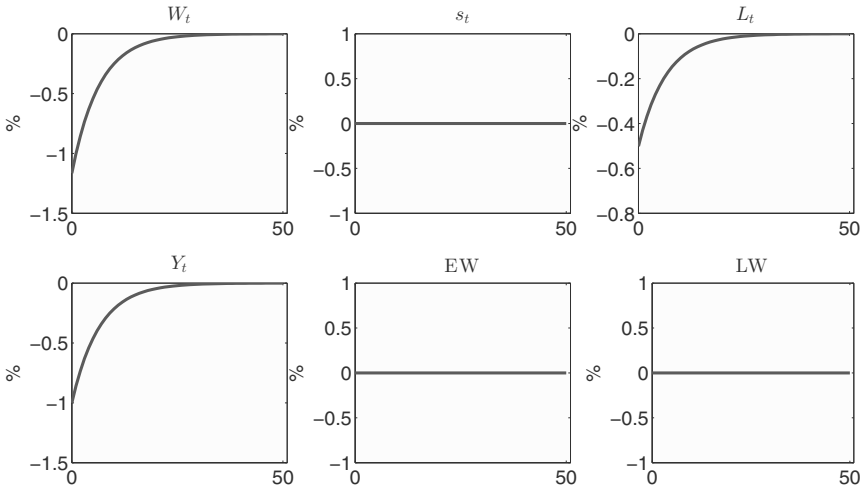

FIGURE 2. Response to a W_t shock. Percentage deviations from the steady state are shown. Convergence time is 31 periods. We judge convergence when the deviation is less than 0.01%.

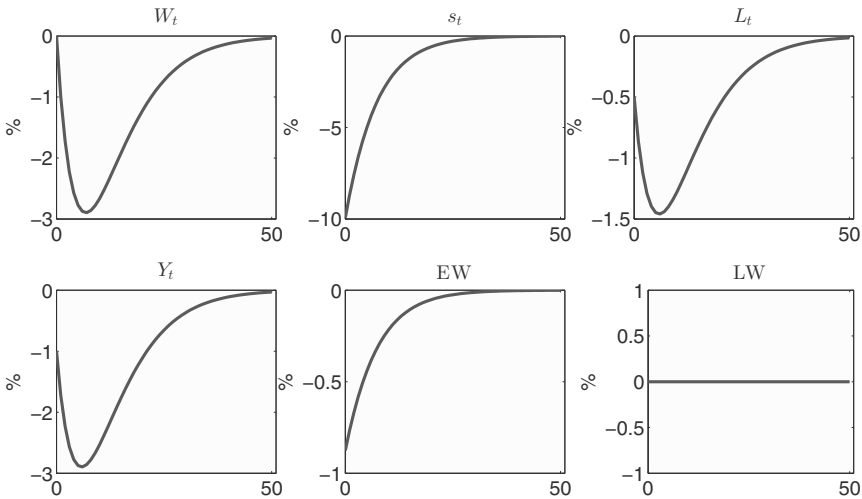

FIGURE 3. Response to a s_t shock. Percentage deviations from the steady state are shown. Convergence time is 60 periods. We judge convergence when the deviation is less than 0.01%.

hump-shaped. It is quite rare for a model with such a simple structure to reproduce strong hump-shaped responses. In contrast, the response to the redistribution of wealth in Carlstrom and Fuerst (1997), Bernanke and Gertler (1989), and Bernanke et al. (1999) is weakly hump-shaped.

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The intuition behind the results shown in Figures 2 and 3 can be understood from (17) and (18). As equation (18) shows, a change in W_t does not affect s_t, and the only effect is a proportional change in the capital stock for high as well as low firms. Therefore, the effect of a W_t shock in the basic model with borrowing constraints is the same as that without borrowing constraints.

The strongly persistent and hump-shaped response to a s_t shock is generated from the dependence of W_t on s_t. Equation (18) indicates that s_t monotonically converges from the initial value, s_0, to the steady-state value, s^*. In contrast, equations (10), (14), and (17) imply that

$$W_{t+1} = \alpha \left(\frac{1-\alpha}{\alpha+\nu}\right)^{\frac{1-\alpha}{\alpha+\nu}} \{[1 + (z^{\frac{1}{\alpha}} - 1)\hat{k}s_t]\beta W_t\}^{\phi}. \tag{19}$$

Because $0 < \phi < 1$, the gross rate of wealth growth (W_{t+1}/W_t) is increasing in s_t and decreasing in W_t. When s_t deviates sufficiently from s^*, the negative effect of s_t on the growth of wealth is dominant and W_{t+1} deviates further from W^*, whereas when s_t goes back sufficiently close to s^*, the positive effect of W_t on the growth of wealth becomes dominant and W_{t+1} starts converging to W^*. In this way, W_t diverges from W^* immediately after the redistribution and starts converging to W^* several periods later.[8] This fact explains the presence of the persistent and hump-shaped response. The response of EW_t is procyclical and consistent with the observations of a long recession such as the Great Depression in the United States.

Note that the original model in Kiyotaki (1998), in which labor supply is fixed, does not display a hump-shaped response to a redistribution shock, whereas the current model does. The finding of this paper is thus that the endogeneity of the labor input can significantly amplify the shock.

3. GENERALIZED MODEL

3.1. Setting

The assumption of complete capital stock depreciation in one period enables us to describe the dynamics of the model by using two equations, (17) and (18), and thus understand the workings of the model analytically. This assumption is unrealistic, however.

To confirm whether our results hold in a general setting, we modify our model in such a way that capital depreciates partially. In the generalized model, we set the depreciation rate of capital, δ, as 0.03 and consider that a share of the remaining capital stock works as the collateral for borrowing. We use more standard values for the parameters: $\alpha = 0.3,\ \beta = 0.98,\ \delta = 0.03,\ \gamma = 0.99,\ \nu = 1,\ \theta = 0.3,\ z = 1.1,\ n = 0.1,\ \omega = 1$. In reality, firms are constrained not only when borrowing long-term loans or issuing long-term bonds to finance capital expenditure, but also when borrowing short-term loans to finance working capital expenditures such as wage payments and the purchase of material and intermediate goods.

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The borrowing constraints on the latter type of loans have been comprehensively analyzed by recent studies [e.g., Mendoza (2010), Jermann and Quadrini (2012), Kobayashi et al. (2012)]. These authors have shown that the borrowing constraint on short-term loans, especially for wage payments, can generate fluctuations in LW [Kobayashi et al. (2012)]. Moreover, LW is time-invariant in the models presented in the previous section, as there is no friction in the demand and supply of labor inputs. In this section, we numerically check whether a deterioration in LW during a recession can be reproduced in our model by introducing borrowing constraints on short-term loans.

We assume that the optimization problem for an entrepreneur is given by

$$\max E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}, \tag{20}$$

$$\text{subject to} \qquad c_t + k_{t+1} - \frac{b_{t+1}}{r_t} \leq y_t - w_t l_t - b_t + (1-\delta)k_t \tag{21}$$

$$\text{and} \qquad w_{t+1} l_{t+1} + b_{t+1} \leq \theta k_{t+1}. \tag{22}$$

The second constraint is the borrowing constraint, which depicts that the entrepreneur cannot pay wages to his/her workers before the completion of production. The entrepreneur is obligated to pay these wages and redeem the bonds issued in the previous period after production. In other words, workers who worked for the entrepreneur in period $t+1$ have a claim over $w_{t+1}l_{t+1}$ after the completion of production in period $t+1$. However, there is a possibility that the entrepreneur absconds without paying wages and redeeming the bonds (limited commitment). We assume here that creditors may seize θk_t in the case of default, meaning that the entrepreneur has an upper limit of debt, which is given by (22). This constraint affects the borrowing twice. In period $t+1$, given the prefixed amount of b_{t+1}, the constraint (22) limits the short-term loan $w_{t+1}l_{t+1}$. In period t, given the (expected) amount of $l_{t+1} = l_{t+1}(b_{t+1})$ to be chosen at $t+1$, (22) limits the amount of borrowing, b_{t+1}. Therefore, in the modified model discussed in this section, an entrepreneur chooses $\{c_t, k_{t+1}, b_{t+1}, l_{t+1}\}$ to maximize (20), subject to the budget constraint (21) and borrowing constraint (22).

3.2. Dynamics

Similar arguments in the case of the equilibrium at which high and low firms both operate imply that the market interest rate is given by

$$r_t = \alpha \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta. \tag{23}$$

In this case, the high entrepreneur's optimization is solved as follows.[9] The variables $\{c_t, k_{t+1}, b_{t+1}, l_{t+1}\}$ are determined backward. First, l_{t+1} is chosen in period

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$t+1$, given the other three variables; second, k_{t+1} and b_{t+1} are chosen in period t, given the (expected) value of l_{t+1} and the prefixed value of c_t; and third, c_t is chosen in period t, given the (expected) values of the other three variables.

The first step for high firms involves solving

$$\max_{l_{t+1}} z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1},$$
$$\text{s. t.} \quad w_{t+1} l_{t+1} + b_{t+1} \leq \theta k_{t+1}. \qquad (\mu)$$

We denote the Lagrange multiplier by μ and will solve this soon. The FOC indicates that labor demand is

$$l_{t+1} = l(\mu,\ k_{t+1}) \equiv \left[\frac{(1-\alpha)z}{(1+\mu)w_{t+1}}\right]^{\frac{1}{\alpha}} k_{t+1}.$$

The second problem is written as follows, given labor demand $l(\mu,\ k_{t+1})$, consumption c_t, and wealth a_t:

$$\max_{\mu, k_{t+1}, b_{t+1}} a_{t+1},$$
$$\text{s. t.} \begin{cases} k_{t+1} \leq a_t - c_t + \frac{b_{t+1}}{r_t}, \\ b_{t+1} \leq \theta k_{t+1} - w_{t+1} l_{t+1}, \\ a_t \equiv z k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - b_t + (1-\delta) k_t. \end{cases}$$

This problem is further simplified to the following for the variable $x \equiv [\frac{1-\alpha}{(1+\mu)w_{t+1}}]^{(1/\alpha)}$:

$$\max_x R_t(x) \times \{a_t - c_t\},$$
$$\text{where } R_t(x) \equiv \frac{z^{1/\alpha} x^{1-\alpha} + 1 - \delta - \theta}{1 - \frac{\theta}{r_t} + \frac{w_{t+1}}{r_t} x z^{1/\alpha}}.$$

The FOC is

$$(1-\alpha) x^{-\alpha} (r_t - \theta + z^{1/\alpha} x_t w_{t+1}) = w_{t+1} \{z^{1/\alpha} x^{1-\alpha} + 1 - \theta - \delta\}. \qquad \textbf{(24)}$$

The third problem, or the consumption choice by the high entrepreneur, is $\max_{c_t} E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}$, subject to $a_{t+1} = R_t(a_t - c_t)$, whereas the problem for the low entrepreneur is also $\max_{c_t} E_t \sum_{s=0}^{\infty} \beta^s \ln c_{t+s}$, subject to $a_{t+1} = r_t(a_t - c_t)$, both of which are from the Brock–Mirman model. The solution is $c_t = \beta a_t$.

The aggregate dynamics is similar to that presented in Section 2.4. Total consumption by entrepreneurs is $C_t = (1-\beta) W_t$ and total capital stock is

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$K_{t+1} = \beta W_t$. Thus,

$$\hat{K}_{t+1} = \frac{\beta s_t W_t}{1 - \frac{\theta - z^{1/\alpha} x_t w_{t+1}}{r_t}},$$

$$K'_{t+1} = \beta W_t - \hat{K}_{t+1}.$$

Here, we assume that $\hat{K}_{t+1} < K_{t+1}$ always holds.[10] Labor demands are as follows:

$$\hat{L}_{t+1} = z^{\frac{1}{\alpha}} x_t \hat{K}_{t+1}, \quad L'_{t+1} = \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} K'_{t+1}.$$

High firms produce $\hat{Y}_{t+1} = z^{1/\alpha} x_t^{1-\alpha} \hat{K}_{t+1}$ and low firms produce $Y'_{t+1} = (\frac{1-\alpha}{w_{t+1}})^{(1-\alpha)/\alpha} K'_{t+1}$. Labor market clearing ($L_{t+1} = \hat{L}_{t+1} + L'_{t+1}$) implies that

$$\left(\frac{w_{t+1}}{\omega}\right)^{\frac{1}{\nu}} = z^{\frac{1}{\alpha}} x_t \hat{K}_{t+1} + \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} K'_{t+1}. \tag{25}$$

The variables $\{w_{t+1},\ x_t,\ r_t\}$ can be determined numerically as functions of (s_t, W_t) using equations (23), (24), and (25). The trend of the state variables (s_t, W_t) is governed by

$$W_{t+1} = [R_t s_t + (1 - s_t) r_t]\, \beta W_t,$$

$$s_{t+1} = \left[\gamma R_t s_t + (1-\gamma)\frac{n}{1-n}(1-s_t) r_t\right] \frac{\beta W_t}{W_{t+1}}.$$

This completes the description of the dynamics.[11]

3.3. Numerical Calculation

Here, we use the parameter values given in Section 3.1, which make the borrowing constraint binding in the steady state. As expected, the responses of output and labor to a decrease in W_0 are not hump-shaped.[12] Figure 4 shows the dynamics when the initial value of the wealth share, s_0, is not equal to s^*, where s^* is the steady-state value. The value of s_0 is chosen so that output in period 0 is lower by 1% than the steady-state value. Output and labor display conspicuously hump-shaped and persistent responses. An interesting result is that LW_t varies over time. In other words, LW *improves* in response to a decrease in output. This result is the opposite of that observed in previous empirical studies.[13]

Our results indicate that reproducing procyclicality in LW remains a significant challenge for macroeconomics research [see, for example, Pescatori and Tasci (2011), Arellano et al. (2012)].[14]

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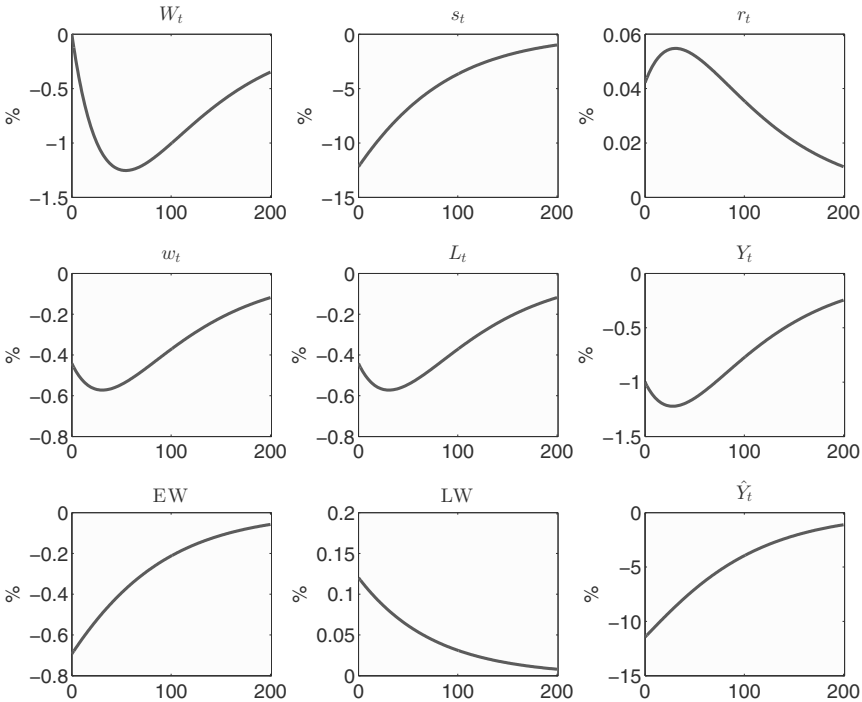

FIGURE 4. Dynamics with an initial s_0 smaller than the steady-state value. Percentage deviations from the initial steady state are shown. Convergence time is 444 periods. We judge convergence when the deviation is less than 0.01%.

4. PROPAGATION OF FINANCIAL AND PRODUCTIVITY SHOCKS

In this section, we show numerically the response of the economy to exogenous shocks (i.e., productivity shocks and financial shocks). It is shown in this model that the redistribution of wealth among heterogeneous agents is crucial to generating hump-shaped and persistent responses to financial shocks.

4.1. Responses with Binding Borrowing Constraints

We assume the same parameter values as in Section 3. Note that as $\theta = 0.3$, the borrowing constraint for high firms is always binding in our numerical simulation.

Productivity shocks. The solid lines in Figure 5 show the response of the economy to a productivity shock (z_t, ζ_t), where ζ_t is the productivity of low firms, the value of which is 1 in the steady state. We assume that the economy was initially in the steady state and that the shock lowers the initial values of (z_0, ζ_0) below their steady-state values by the same ratio, in such a way that output at time 0 is lower than its steady-state value by 1%. We assume that (z_t, ζ_t) evolves by

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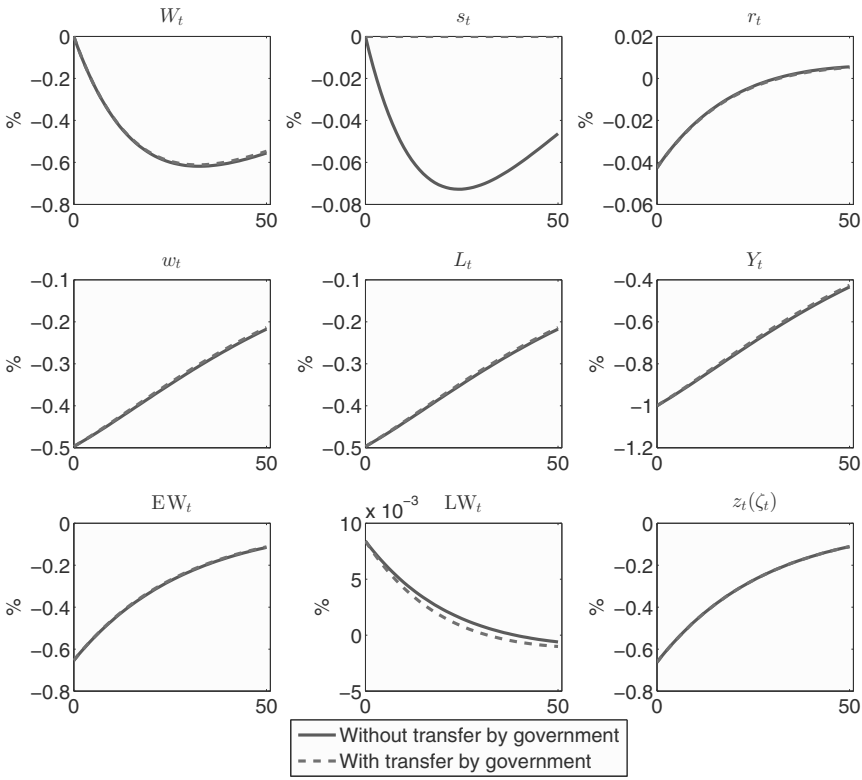

FIGURE 5. Response to a macro technology shock (z_t and ζ_t shocks). Percentage deviations from the steady state are shown. Convergence times are 196 periods (without transfer by government) and 206 periods (with transfer by government). We judge convergence when the deviation is less than 0.01%.

$z_t = \rho z_{t-1} + (1-\rho)z^*$ and $\zeta_t = \rho\zeta_{t-1} + (1-\rho)\zeta^*$, with $\rho = 0.965$, for $t \geq 1$. As expected, there is no hump shape in the path of either output or labor. The dotted lines in Figure 5 show the case with the presence of a transfer policy, in which the government makes a lump-sum transfer between high and low firms in such a way that $s_t = s^*$ for all t. This figure shows that there is no significant difference between the solid lines and dotted lines. We can thus summarize the effects of the productivity shock as follows:

- The productivity shock does not generate a hump-shaped response.
- Wealth share does not play a significant role in amplifying the effect of the productivity shock.

Here, we note that a weakly hump-shaped response may be generated by the productivity shock for a specific set of parameter values. The fact that the productivity shock can generate a hump-shaped response is sometimes found in business cycle

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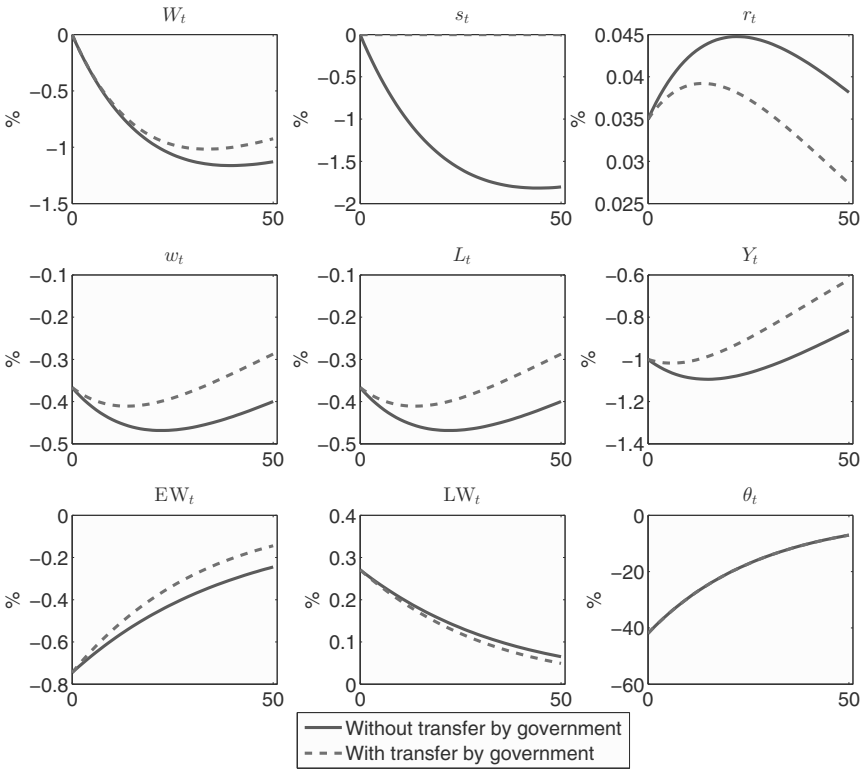

FIGURE 6. Response to a financial shock (θ shock). Percentage deviations from the initial steady state are shown. Convergence times are 365 periods (without transfer by government) and 228 periods (with transfer by government). We judge convergence when the deviation is less than 0.01%.

models, although it is not typically emphasized in the literature. For example, Shirai (2014) shows that a simple real business cycle (RBC) model with standard parameter values displays a weakly hump-shaped response to a productivity shock when the shock is sufficiently persistent ($\rho = 0.99$). Shirai's result is notable in that it shows that a simple RBC model without habit persistence or adjustment costs of investment can display a hump-shaped response.[15]

Financial shocks. The solid lines in Figure 6 show the response of the economy to a shock on θ_t, which is called a "financial shock" in Jermann and Quadrini (2012). We assume that the economy was initially in the steady state and that the shock lowers the initial values of θ_0 below their steady-state values, in such a way that output at time 0 is lower than its steady-state value by 1%. We assume that θ_t evolves by $\theta_t = \rho\theta_{t-1} + (1-\rho)\theta^*$, with $\rho = 0.965$, for $t \geq 1$. Output and labor show conspicuously persistent and hump-shaped responses to the financial

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shock. Output hits the bottom of the hump at $t = 16$ when Y_t is lower than Y^* by 1.1%. In contrast, labor hits the bottom of the hump at $t = 23$ when L_t is lower than L^* by 0.47%. The dotted lines in Figure 6 show the case with the transfer policy, in which the government makes a lump-sum transfer between high firms and low firms in such a way that $s_t = s^*$ for all t. The dotted lines imply that the humps and persistence are significantly weakened under the transfer policy. We can summarize the effects of the financial shock as follows:

- The financial shock generates a conspicuously persistent and hump-shaped response.
- Wealth redistribution, namely changes in s_t, amplifies the persistence and hump shape in output and labor significantly.

These results imply that the persistent recessions seen in the aftermath of financial crises may be explained by financial shocks (i.e., shocks to θ) and that the redistribution of wealth is the key factor that amplifies the persistence and hump shape in output and labor. Yet the response of LW is countercyclical in our simulation, which is contrary to that observed in previous financial crises.

In the next subsection, we therefore consider the conditions that generate procyclical LW.

4.2. Response to an Extremely Large Financial Shock

An extremely large financial shock, causing a large change in the parameter θ, can reproduce procyclical LW. If the collateral ratio θ changes so that the borrowing constraint (22), which was nonbinding before, becomes binding after the shock hits, output declines and LW deteriorates to a large extent in response to the shock.

Figure 7 shows the result of the following numerical experiment. Initially, the parameter is sufficiently large, say, $\theta = 1$, and the economy is in the steady state where (22) does not bind.[16] In this initial steady state, there is no LW, that is, $\text{LW}_t = \text{MRS}/\text{MPL} = 1$, and the share of wealth owned by high firms satisfies $s = n$. Suppose a large financial shock hits the economy and θ changes permanently to 0.3. As the borrowing constraint becomes tighter and permanently binding, output declines and LW deteriorates. The movements in output, labor, EW, and LW shown in Figure 7 are qualitatively similar to the movements of these variables during the Great Depression reported by Chari et al. (2007). Another interesting feature of this result is that the wealth share s_t increases significantly and permanently as a result of the large financial shock: the initial steady-state value of s_t is $n = 0.1$, whereas it converges to the new steady-state value, 0.504. This result seems to be consistent with the argument put forward by Piketty (2014), which states that permanent redistribution from unproductive to productive agents is associated with chronic recession in the aftermath of a financial crisis.[17]

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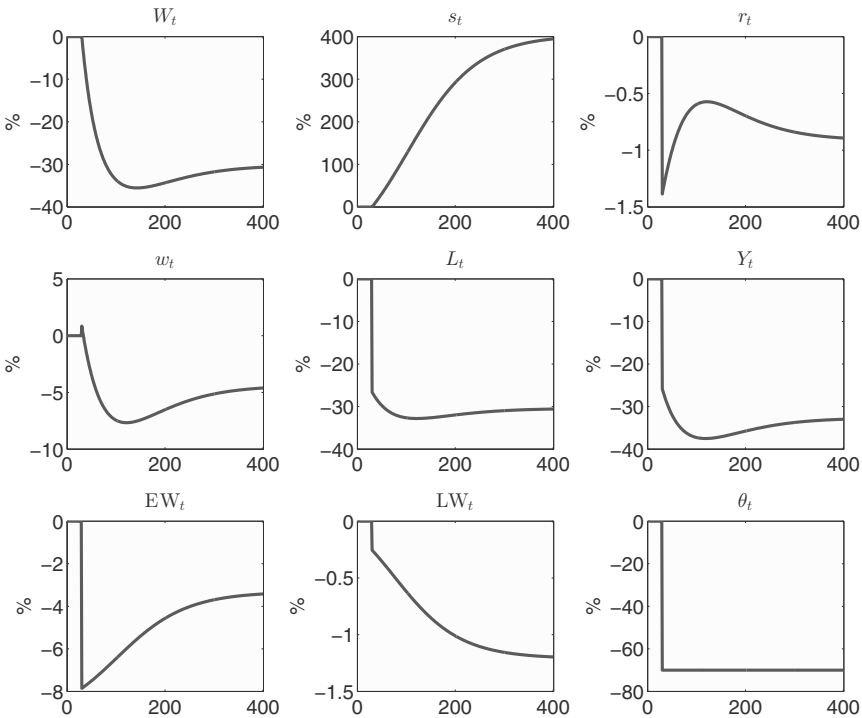

FIGURE 7. Response to large financial shocks. Percentage deviations from the initial steady state are shown. Convergence time is 658 periods. We judge convergence when the deviation is less than 0.01%.

5. CONCLUSION

By using a simple heterogeneous-agent economy model with borrowing constraints, this study showed that the redistribution of wealth could reproduce persistence and nonlinearity in output and labor, traits that are typically observed in the aftermath of financial crises. In this model, because the borrowing constraint limits the accumulation of wealth, changes in the aggregate amount of wealth are persistent. As a result, the macroeconomic variables become persistent as well. The hump shape in the macroeconomic variables is caused by the nonlinear effect of the share of wealth on the aggregate amount of wealth. This model thus suggests that such redistribution could be one of the key driving forces behind the transmission of financial shocks in the aftermath of financial crises. The numerical simulation showed that productivity shocks do not generate a hump-shaped response in the economy, whereas financial shocks do, with these responses heavily amplified by the redistribution of wealth between high and low firms.

However, one limitation of our model is that it does not successfully reproduce the procyclicality of LW in response to small financial and productivity shocks;

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it reproduces procyclical LW only when the financial shock is extremely large. Therefore, it is still a challenge for macroeconomic research to construct a well-developed model that can explain the procyclicality of LW. Future research in this field could explore this aspect in greater detail.

NOTES

1. One noticeable difference in the model settings is that although the productivity of an agent randomly changes in every period in Kiyotaki and Moore (2012), it follows a Markov process and thus is persistent in our model.

2. This statement may not be rigorously true for Smets and Wouters (2003), in which the habit involves a consumption externality.

3. Our model can also be considered to be a simplified version of those presented by Buera and Nicolini (2014) and Kiyotaki and Moore (2012).

4. It is shown that in the steady state with binding borrowing constraints, the market interest rate satisfies $\beta r < 1$. The gross interest rate r is strictly less than β^{-1} because the supply of bonds is smaller than that in the first best and therefore the bond price is higher.

5. As (4) is not satisfied at the equilibrium where no borrowing constraint exists, it is not possible to assess whether workers are hand-to-mouth consumers from the workers' optimization. However, here we have made this assumption for simplicity.

6. We assume that (4) holds and justify later in the numerical experiment that (4) actually holds on the equilibrium path.

7. Note that in the case where $\tilde{k}(s_t) = 1$, low entrepreneurs do not produce output. Thus, $R_t = z^{1/\alpha} r_t$ and the market rate is equal to R_t.

8. Persistence and hump shape in the aggregate variables are sensitive to the parameter values. For example, when $\alpha = 0.4$ in the basic model, the persistence and hump shape disappear in response to a change in s_t.

9. This analysis is a slight modification of that in Kiyotaki (1998).

10. The dynamics in the case where $\hat{K}_{t+1} = K_{t+1}$ is the same as that in the basic model in Section 2.3, because when $\hat{K}_{t+1} = K_{t+1}$, the borrowing constraint is not binding in either model.

11. Note that we assume that

$$\frac{s_t}{1 - \frac{\theta - z^{1/\alpha} x_t w_{t+1}}{r_t}} < 1.$$

The dynamics in the case where this inequality does not hold is described in the Appendix.

12. The numerical result for a W_t shock is not reported in this paper. The result is available from the authors upon request.

13. The intuition behind the improvement in LW can be described as follows. The redistribution of wealth from high firms to low firms exogenously decreases intertemporal debt (b_{t+1}) and capital ($\hat{K}_t$) for high firms and thus lowers labor demand by high firms. As a result, total labor demand decreases and the wage rate decreases, too. There are two competing effects on LW: (i) decreases in both the labor demand of high firms and the wage rate relax the borrowing constraint and improve LW and (ii) a decrease in the net worth of high firms due to the redistribution tightens the borrowing constraint and deteriorates LW. We confirm that the total effect improves LW in this numerical example.

14. One referee suggested that a model of borrowing-constrained workers à la Campbell and Hercowitz (2009, 2011) could reproduce procyclical LW. As our focus in this paper is on the borrowing constraint on firms, we leave this line of research for a future study.

15. Romer (2011) describes how, under the assumption of full depreciation of capital, the RBC model is reduced to the Brock–Mirman model and thus shows a hump-shaped response to a productivity shock.

16. The steady state is similar to the one in Section 2.3, in which only high firms produce output and low firms purchase the bonds issued by high firms.

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17. Another notable result is that the interest rate is lowered in response to the financial shock because the borrowing constraint is tightened. A persistently low interest rate has been observed in Japan since the 1990s, that is, after the bursting of the asset-price bubble. Our result implies that a financial shock may thus explain the performance of the Japanese economy in the 1990s.

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APPENDIX: DYNAMICS OF THE GENERALIZED MODEL WHEN S_T IS LARGE

Here, we describe the calculation method for the generalized model introduced in Section 3 in the case where

$$\frac{s_t}{1-\frac{\theta - z^{1/\alpha} x_t w_{t+1}}{r_t}} \geq 1.$$

If this inequality holds, then $\hat{K}_{t+1} = K_{t+1}$ and $K'_{t+1} = 0$. In this case, the variables are calculated using the following procedures. The wage is determined by

$$w_{t+1} = [(1-\alpha)z]^{\frac{v}{\alpha+v}} \omega^{\frac{\alpha}{\alpha+v}} (\beta W_t)^{\frac{\alpha v}{\alpha+v}}.$$

The market interest rate is depicted as

$$R_t = \alpha z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta.$$

The evolution of the state variables is governed by

$$W_{t+1} = R_t \beta W_t,$$

$$s_{t+1} = \gamma s_t + (1-\gamma)\frac{n}{1-n}(1-s_t).$$

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