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RICARDO AND RICARDIANS ON THE ORDER OF CULTIVATION

BY
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The Ricardian dynamics are based on the study of the order of cultivation when demand increases. Piero Sraffa criticized David Ricardo for having assumed that the incoming method is defined by a natural order, and stressed that the law of succession of methods is based on a profitability criterion. Then, in the case of intensive cultivation, the question is whether the incoming method is indeed more productive than the one it replaces. Sraffa's argument relies on the positivity of rent. However, there is a flaw in his reasoning, and a failure of the Ricardian dynamics is possible. Post-Sraffian scholars have misunderstood that construction and have substituted a static approach for it. The critiques they address to Sraffa are better understood by returning to Ricardo and Sraffa's own methodology. Fifty years ago, mathematicians rediscovered Ricardo's approach independently and worked out a powerful algorithm inspired by it.

I. INTRODUCTION AND METHODOLOGY

Two centuries ago, the publication of *On the Principles of Political Economy and Taxation* (Ricardo [1817] 1951) was a landmark in the development of political economy. In chapter I, David Ricardo introduced the labor theory of value for industrial commodities. In chapter II, he explained that the theory extends to agricultural commodities, when applied to the marginal methods, which pay no rent. He also introduced what was later called his "magnificent dynamics" (Baumol 1951), which are the logical ground of his political project: the rise in the demand for agricultural goods due to the development of capitalism and the need for more industrial workers requires the cultivation of lands of lower grades. This leads to a general rise of rents and, for a

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given real wage, a long-run fall of the ruling rate of profit. The dramatic loss of incentive for investment and technical progress can be met only by importing foreign corn, thus reducing the level of rents (and also, incidentally, the landlords' incomes).

Piero Sraffa (1960), the faithful modern interpreter of Ricardo's thought, translated that construction in terms of prices of production instead of labor values. Following Ricardo's general plan, he introduced the question of production with lands (chapter XI in part II of *Production of Commodities by Means of Commodities;* hereinafter *PCMC*) after that of industrial commodities. He, however, took distance with some of Ricardo's views and criticized them; for instance, when Ricardo assumed a classification of lands according to natural qualities. But Sraffa's intent was also to criticize "economic theory," i.e., the marginalist school, and he therefore interwove Ricardian themes with more contemporary issues on the determination of prices and capital theory. That critical goal led him to stress in the preface, "No changes in output and (at any rate in Parts I and II) no changes in the proportions in which different means of production are used by an industry are considered" (Sraffa 1960, p. v).

That statement, which is directed against the introduction of a hypothesis on returns and the use of marginal equalities, is, however, incompatible with the study of the dynamics on quantities. The coexistence of Sraffa's two projects (faithfulness to Ricardo and critique of marginalism) becomes a problem when the above quotation is compared with the description of the intensification process in section 88 of chapter XI: "The existence side by side of two methods can be regarded as a phase in the course of a progressive increase of production on the land. The increase takes place through the gradual extension of the method that produces more corn at a higher unit cost, at the expense of the method that produces less" (Sraffa 1960, p. 76). The contradiction opens the door to two interpretations of chapter XI. As we shall see, post-Sraffians have read it in the light of the preface, whereas, we claim, Sraffa himself followed Ricardo's dynamic approach.

In section II, we examine the main features of those Ricardian dynamics, which study the sequence of local transformations of a long-run equilibrium in the presence of scarce lands when demand rises continuously. The law of succession of methods, or, in Ricardo's terms, the order of cultivation, aims at determining the incoming method when a scarcity constraint is met. Three versions of the law are given, including those due to Ricardo and Sraffa. For several reasons, the case of intensive rent plays an important role and sets a bridge between rent theory and capital theory. In section III, we examine the specific problem that Sraffa identified and solved, or thought he had solved. His answer led him to defend too optimistic views (which were also Ricardo's) on the economic behavior of productive systems in the presence of lands. Section IV describes the static approach followed in the most part of the literature inspired by Sraffa's formalization, and section V points at Ricardo's unexpected (and still unrecognized!) legacy in modern applied mathematics.

II. THE LAW OF SUCCESSION

The Ricardian dynamics start from a given long-term equilibrium for a certain demand and consider the effects of a continuous rise of demand on that equilibrium. They are based on three general rules. The first is that, as long as no new scarcity constraint is met, the activity levels of the operated methods can be adapted to changes in demand, with no changes in prices and rents. The second states that, when a scarcity constraint on

some land is met, one new method is introduced, initially at a low activity level. This means that, even if prices and rents rise suddenly, the dynamics are not chaotic: there is no complete reorganization of production and the activity levels always vary smoothly. These first two rules reduce the study of the dynamics to the determination of the incoming marginal method. The law of succession of methods is the third rule, which complements the dynamics. We first examine three statements of that law.

Ricardo's Law

Ricardo assumed that lands can be classified according to a physical criterion of productivity, with land of a lower quality requiring more inputs per quarter of corn. The order of cultivation follows that physical order, and the cost of production rises at each step. Even when that hypothesis is met, Ricardo's answer is incomplete as it does not take into account the alternative possibility of an intensification of cultivation. Quite consistently with his views on the extension of cultivation, Ricardo conceived intensification as the deposit of an additional layer of capital and labor on an already fully cultivated land:

> It often, and, indeed, commonly happens, that before No. 2, 3, 4 or 5, or the inferior lands are cultivated, capital can be employed more productively on those lands which are already in cultivation. It may perhaps be found, that by doubling the original capital employed on No 1, [the produce] may be increased by eighty-five quarters, and that this quantity exceeds what could be obtained by employing the same capital, on land No. 3. In such case, capital will be preferably employed on the old land, and will equally create a rent. (Ricardo [1817] 1951, p. 71)

Ricardo had previously assumed that the same amount of capital and labor produces 100 quarters of corn on land No. 1, ninety quarters on land No. 2, and eighty quarters on land No. 3. The most fertile land 1 is cultivated first, then land 2, and, in that quotation, Ricardo claims that the intensification method on land 1 precedes an extension of cultivation on land 3. Again, the criterion is merely physical and the law of succession is that of maximum product. However, an implicit hypothesis of the reasoning is that the same capital (seeds, labor, manure, etc.) can be used either to extend cultivation on land 3 or as an additional layer for the intensification of cultivation on land 1: it seems that 'capital' in that passage should be understood in value terms rather than in physical terms.

Ricardo re-examined that question in the note attached to the last sentence of chapter II (the same numerical example is used in chapter VI). That passage, which is devoted to a comparison of money rents and corn rents, has rarely attracted attention, but it is instructive to read it while keeping the question of the order of cultivation in mind. All types of rent are considered ("the labor of ten additional men ... on the same or any other land"), and Ricardo makes reference to two ideas: on the one hand, the investment leads to a maximum product among all alternatives; on the other hand, the price of corn rises, but that price is a minimum because of competition among farmers. That remark opens the door to the definition of an order of cultivation that refers to values.

Sraffa's Law

Sraffa criticized Ricardo's views on a ranking of lands rooted in their physical characteristics and stressed that the general criterion on the order of cultivation must be a value criterion, expressed in terms of profitability. Since values (prices and rents) depend on

distribution, the same holds for the order of cultivation, which has no natural character. For extensive cultivation proper, Sraffa assumed that each quality of land is associated with a specific agricultural method, with no a priori physical classification between the agricultural methods used on different lands. Similarly, the case of intensification is characterized by the coexistence of two methods on a fully cultivated land: one of them is more productive per acre, but that method does not need more of any input. The only general rule, stated in section 87 of *PCMC*, is that the more productive method is also more costly when rent is ignored, this being the condition for the coexistence of the two methods with a positive rent (Sraffa 1960, p. 75).

As Sraffa paid specific attention to intensive rent proper, assume that land is homogeneous and fully cultivated. For simplicity, we consider a bisector model, with corn the agricultural good and iron the industrial good. In the present equilibrium, two agricultural methods 1 and 2 and one iron method are operated, method 2 being more productive than method 1. When demand rises, method 2 is progressively substituted for method 1, with no changes in prices and rents.[1] What happens in the borderline case when method 2 only is used and method 1 ceases to be operated, but the demand for corn continues to rise? The price of corn starts rising, as well as rent and the price of iron, because corn enters the production of iron.[2] That general rise continues until some alternative agricultural method 3 becomes profitable: up to that point, there is no incentive to introduce a new method; after it, the profitability of method 3 would exceed the ruling rate of profit. The incoming agricultural method 3 is therefore uniquely determined.

We take a numerical example to illustrate the working of the law (its data are arbitrary). Let the rate of profit be $r = 10\%$ and the present prices and rent associated with the $n = 3$ operated methods be $p_c = 1$, $p_i = 1$, $\rho = 2$, with labor as numéraire. The corn method 2 and the iron method are written:

$$\text{corn method 2: } 6\,\text{qr. corn} + 8\,\text{t. iron} + 4.6\,\text{labor} + 1\,\text{acre} \rightarrow 22\,\text{qr. corn}$$

$$\text{iron method: } 4\,\text{qr. corn} + 2\,\text{t. iron} + 4.4\,\text{labor} \rightarrow 11\,\text{t. iron}$$

In the present equilibrium, the two prices and the rent per acre are determined by the three methods. After the elimination of the corn method 1, the corn method 2 and the iron method are still operated and their attached value equations continue to hold:

$$(1+10\%)(6p_c + 8p_i) + 4.6 + \rho = 22\,p_c$$

$$(1+10\%)(4p_c + 2p_i) + 4.4 = 11p_i$$

These two equations with three unknowns leave one degree of freedom $\lambda = \Delta p_c > 0$: their general solution is

$$p_c = 1 + \lambda,\ p_i = 1 + 0.5\lambda,\ \rho = 2 + 11\lambda$$

[1]The statement that prices and rents do not change when an agricultural method is progressively replaced by another lies on an implicit constant-returns hypothesis, otherwise the technical coefficients and the value equations would vary during that process. A hypothesis on returns is unavoidable in a dynamic approach.
[2]Ricardo denied the latter influence in the *Essay* ([1815] 1951) but changed his views soon after.

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The incoming method is the first non-operated agricultural method that becomes profitable when the price of corn (or, alternatively, the level of rent) rises, and this determines also the level of λ and the new values. Hence, Sraffa's law of succession as stated in section 88 of *PCMC*: "As soon as the former method has extended to the whole area, the rent rises to the point where a third method … can be introduced to take the place of the method which has just been superseded" (Sraffa 1960, p. 76).

The General Law

Sraffa's law of succession can be extended to extensive and intensive cultivation, to an arbitrary number of lands and agricultural goods, and even to the joint production of agricultural and industrial commodities. Let there be n commodities and m varieties of lands. In the present equilibrium, $m + n$ methods are operated and, for a given rate of profit, they determine the n prices and the m rents. Let us stop for a moment on that statement, because it seems contradictory with the fact that, in the case of extension of cultivation, more and more methods are operated. The two points of view are compatible and depend on the convention one follows. In the representation we adopt here, the price equation associated with any agricultural method always includes rent. But, when a land is not fully cultivated, we consider that fallowing is an agricultural method that operates on a part of land, and its attached price equation is that the rent is zero ($\rho_i = 0$). With that convention, the number of operated methods is indeed constant and equal to $m + n$. To claim that a land becomes fully cultivated means that the activity level of the fallowing method vanishes, thus leaving room for another method, whose price-and-rent equation will replace equation $\rho_i = 0$ in the next equilibrium.

The operated methods, their attached equations, and the values remain the same as long as the change in demand meets no new scarcity constraint. A scarcity constraint is reached when the activity level of some operated method vanishes. That rule also works for extensive cultivation, since claiming that some land is fully cultivated amounts to claiming that the activity level of the fallowing method becomes zero. In all cases, the price-and-rent equation of the corresponding method disappears and one is left at the breaking point with $m + n - 1$ equations, thus leaving one degree of freedom. The price of the commodity hit by the scarcity constraint starts rising, and the rises in all other prices and rents follow proportionally. Hence the general law:

Law of succession. When demand increases, the outgoing method is determined by a physical criterion. The incoming method is the first non-operated method, which becomes profitable when the price of the scarce commodity rises (taking into account the effect of that rise on other prices and on rents).

III. RENT AND CAPITAL THEORY

The general law of succession is an extension of Sraffa's law and coincides with it when applied to intensive cultivation. There is, however, another subtle difference: Sraffa refers to a minimum rise of rent, whereas the general law refers to a minimum rise of corn. As these two magnitudes are proportional, the two statements are formally equivalent, but Sraffa's wording sounds strange in view of Ricardo's stress in chapter

II that the level of the rent is the effect of the price of corn, and not its cause. Why does Sraffa proceed as if rent was the leading autonomous variable?

The Productivity Issue

The application of the law of succession sets no difficulty when cultivation is extended to another land. In the case of intensive cultivation, however, the very fact that the law is based on a value criterion shows that the study of the Ricardian dynamics remains incomplete because of the following potential contradiction: the change of method comes from the necessity to produce more corn, but it is still unclear whether the incoming method, which is selected for its profitability, is indeed more productive than the one it replaces and solves the scarcity problem. The question, which is that of the coincidence between profitability and productivity, is close to the one set by capital theory: a fall of the rate of profit lets us expect the introduction of a more capital-intensive method, with a higher product per worker. Is that always the case? Sraffa's most famous contribution to capital theory is that the answer is negative. Either the same holds for the intensification process, or a specific argument must be given. Sraffa was aware of the parallel between intensive rent theory and capital theory and, in his concise style, provided the argument to solve that difficulty. In section 87, the condition for the coexistence of two methods on the same land is stated as follows: "They must satisfy the economic condition of not giving rise to a negative rent: which implies that the method that produces more corn per acre should show a higher cost per unit of product, the cost being calculated at the ruling levels of the rate of profits, wages and prices" (but rent excluded) (Sraffa 1960, p. 75).

In the initial equilibrium, let methods 1 and 2 operate jointly on the same fully cultivated land, with a positive rent, method 2 being more productive. When demand increases, the area devoted to method 1 is reduced and eventually vanishes. What can be said of method 3, which replaces it? On the one hand, method 3 is more costly than method 2 (that is why method 2 was operated before it); on the other hand, by the law of minimum rise of rent as stated by Sraffa, the coexistence of methods 2 and 3 goes with a higher positive rent. In section 88, Sraffa thus concludes that method 3 is more productive than method 2: "[T]he rent rises to the point where a third method *which produces still more corn at a still higher unit cost* can be introduced to take the place of the method which has just been superseded" (Sraffa 1960, p.76; emphasis added).

To sum up, it is the positivity of rent that guarantees the coincidence of productivity and profitability and excludes the difficulties met in capital theory: the incoming method is able to meet the rise in demand.

Sraffa's Mistake

The existence of a flaw in Sraffa's reasoning can be shown by completing the numerical example given in the above section on Sraffa's law. Let a third agricultural method on the same land be

$$6\,\text{qr.corn} + 4\,\text{t.iron} + 8\,\text{labor} + 1\,\text{acre} \rightarrow 20\,\text{qr.corn}$$

Method 3 is not profitable at the initial prices, but its profitability improves when prices and rent vary in the direction $\Delta p_c = \lambda$, $\Delta \rho = 11\lambda$, $\Delta p_i = 0.5\lambda$. It reaches the ruling

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rate of profit 10% for $\lambda = 5$ and may well win the race for profitability among all non-operated methods. The coexistence of methods 2 and 3 is then ensured with a positive rent, but the physical data show that the incoming method 3 produces less corn per acre than method 2. As the introduction and progressive substitution of method 3 for method 2 would lead to a fall of the net product, the Ricardian dynamics fail. If, in an attempt to overcome that difficulty, the law of succession was applied only to methods more productive than method 2, the value of λ would be higher and method 3 would yield extra profits.

The mistake in Sraffa's argument is that it mixes the properties associated with two price-and-rent vectors: for the initial price vector, method 2 is cheaper than method 3, and that is why it is operated at an early stage; but for the final price vector, method 2 becomes more costly when the cost is calculated as indicated by Sraffa, i.e., positive rent excluded. That reversal explains why a mechanical application of the law of succession might lead to the introduction of a less productive method.

IV. THE STATIC APPROACH

Two methodologically distinct approaches of the problem of lands can be defined depending on whether one follows the preface or chapter XI of *PCMC*. The above approach is faithful to chapter XI, which is itself in line with Ricardo's views. A few studies apart (Montani 1975; Erreygers 1990 and 1995; Bidard 2010)—which, however, failed to notice their methodological specificity—the post-Sraffian literature initiated by Alberto Quadrio Curzio (1966) has developed the alternative approach, or static approach (e.g., Abraham-Frois and Berrebi 1980; Klimovsky 1981; Saucier 1981; D'Agata 1983; Salvadori 1986; Bidard 1987 and 2004; Schefold 1989; Freni 1991; Kurz and Salvadori 1995). In that conception, the demand basket is given and the main question concerns the existence and the uniqueness of a long-run equilibrium in the presence of lands. The two approaches are easily distinguished: in the static problem, even the basic property—that the operated methods remain unchanged as long as no new scarcity constraint is met—plays no role and is only mentioned as a historical tribute to Ricardo. There is no room for a reflection on the law of succession, and no link is established between rent theory and capital theory.

Post-Sraffian studies have, however, established new and valuable results. Many of them are based on numerical examples showing that some statements drawn from chapter XI of *PCMC* cannot be sustained. Sraffa saw no difficulties, except for the standard system, in the cases of extensive rent proper and intensive rent proper, and suggested in section 89 that "the main type of complication arises from the multiplicity of agricultural products" (Sraffa 1960, p. 76). The general lesson of post-Sraffian literature is that this optimism is ill-founded. We propose to reinterpret these results in light of the dynamic approach (see Appendix B in Bidard 2014 for full analytical details).

(i) Type of equilibrium. Philippe Saucier (1984) noticed that, in a model with corn as an agricultural product and iron as the industrial product, an equilibrium can be made of one corn method and two iron methods, thus providing a counter-example to Sraffa's statement in section 89 that the number of operated methods

in agriculture is equal to that of lands plus that of agricultural products. That phenomenon is easily explained: let the original equilibrium be of the usual type. When a scarcity constraint is met, the price of corn starts rising and a rise in iron follows. The alternative iron methods that save corn participate in the run for profitability and one of them may win the race. Since more corn is available for final demand, the new equilibrium sees the coexistence of one corn method and two iron methods, with the progressive substitution of the new iron method for the old one.

(ii) Existence. The set of equations describing an equilibrium position for a given demand vector is a variant of the usual Lippi model, and the existence of a solution is guaranteed if the demand vector d belongs to some domain D (Salvadori 1986). How is that problem dealt with in a dynamic approach? The incoming method as characterized by the law of succession exists if the direction of the change in the price-and-rent vector improves the profitability of at least one non-operated method. That condition defines the same domain D.

(iii) Uniqueness. Assume that, when the demand for corn increases, there is no problem up to 1,000 quarters, but that the incoming method at that point is less productive than the method it would replace and would reduce production to 600 quarters: the Ricardian dynamics fail. However, the insertion of that method does define a new equilibrium and two equilibria are thus found for $d = 800$ qrs. Therefore, uniqueness requires that, for any change, productivity goes with profitability. That condition, which is also sufficient, can be given an algebraic form (Erreygers 1990, 1995) as a coincidence of signs: it is met if two determinants have the same relative sign as two other determinants. The first two determinants refer to matrices $B - A$ before and after the change (with the input matrix A including the characteristics of production with lands), i.e., to physical net products, whereas the other two determinants refer to matrices $B - (1+r)A$, i.e., to values and profitability.

(iv) Oddity. Antonio D'Agata (1983) showed that multiple equilibria can exist for a given level of demand, and this may happen even in a simple corn model with homogeneous land (Freni 1991). That result does not fit with Sraffa's conclusions on the behavior of a productive system with lands. The dynamic approach allows us to go further: let the demand vector $d(t)$ move along a curve drawn in the domain D, which starts from 0 and comes back to 0 without crossing itself. Every time point d is reached, a new equilibrium is found. In the absence of U-turns on the curve, each point d is reached once. U-turns (or "antitone moves," in the mathematical literature) occur when the dynamics fail, but, since $d(t)$ cruises on the whole curve, the algorithm reaches a given demand basket d an odd number of times, the exceptional positions corresponding to U-turns apart.

V. RICARDO AND MODERN MATHEMATICS

In the static approach, the search for a long-term equilibrium consists in solving a system of inequalities with complementarity relationships, for a given demand basket d. The difficulty to solve such linear complementarity problems (LCP) can be illustrated by an example. Consider a homogeneous land with ten barley methods and ten corn

methods. When demand is high and land is fully cultivated, there are two types of equilibria: those with two barley methods and one corn method (450 combinations), and those with one barley method and two corn methods (450 combinations), hence 900 systems of price equations. Assuming we have the patience to calculate the 900 price-and-rent vectors, it turns out that only a very small proportion of these are acceptable: most vectors admit negative components or, when they are positive, some alternative method yields extra profits. As LCPs are very common in many branches of science, the complexity of the search for an effective solution was a mathematical challenge, eventually solved in the sixties (Lemke 1965; Cottle et al. 1992). Two efficient algorithms were then worked out, the first being inspired by the simplex algorithm. We pay attention to the second, called the "parametric Lemke algorithm," which admits a simple interpretation. Its principle, when applied to the rent problem (Carlton Lemke was not aware of that application!), is to consider the demand vector $d = d(t)$ as a parameter and to follow the successive transforms of the solution obtained for a certain vector d_0 into a solution for another vector d_1. This was precisely Ricardo's idea, and the main two properties on which the algorithm relies are those mentioned by Ricardo: there are no changes in prices and rents as long as no new scarcity constraint is met, and only one method is changed when a new physical constraint appears. Thanks to the second property, the present equilibrium is used as a ladder to determine the next equilibrium, and this explains the efficiency of the algorithm. The only missing step of the ladder is determined by a minimum rule that coincides with the law of succession. It thus turns out that, in order to solve a static problem, mathematicians have independently rediscovered Ricardo's dynamic approach a century and half after the *Principles* (for his creative works on the LCP algorithms, Lemke received in 1978 the John von Neumann Theory Prize, jointly with John Nash). The Ricardo–Lemke algorithm is a powerful tool that is used today in many technical devices.

A numerical example illustrates the three approaches we have mentioned to the land problem. Let there be a homogeneous land with total area 100 acres and three corn methods:

$$\text{method 1: } 60\,\text{qrs. corn} + 4\,\text{labor} + 1\,\text{acre} \rightarrow 70\,\text{qrs. corn}$$

$$\text{method 2: } 10\,\text{qrs. corn} + 6\,\text{labor} + 1\,\text{acre} \rightarrow 16\,\text{qrs. corn}$$

$$\text{method 3: } 10\,\text{qrs. corn} + 30\,\text{labor} + 1\,\text{acre} \rightarrow 24\,\text{qrs. corn}$$

The rate of profit is $r = 10\%$ and labor is chosen as numéraire.

(i) The Ricardian dynamics. When demand is low and rent is zero, method 1 is the cheapest ($p = 1$, $\rho = 0$) and is operated. That method can produce up to 1,000 quarters in terms of net product. When land is fully cultivated, the price and rent become $(1+\Delta p, \Delta\rho)$ with $(1+10\%)60(1 + \Delta p) + 4 + \Delta\rho = 70(1 + \Delta p)$, or $\Delta\rho = 4\Delta p$. When $\Delta p > 0$ increases, method 2 is the first to reach the ruling rate of profit for $\Delta p = 1$ (then $p = 2$ and $\rho = 4$). The rent is positive, but method 2 is less productive than method 1 and the Ricardian dynamics stop.

(ii) The parametric Lemke algorithm. The algorithm is identical to the Ricardian dynamics except that it does not care about a temporary fall in production. Method 2 is therefore introduced and replaces method 1 progressively. The net

product falls to 600 quarters when land is fully cultivated by method 2. The change in prices and rents that occurs at this stage ($\Delta\rho = 5\Delta p$) leads to the introduction of method 3 for $\Delta p = 1$ (then $p = 3$ and $\rho = 9$). The net product can increase up to 1,400 quarters. At that point, the changes in values do not improve the profitability of any non-operated method (because 1,400 is outside the existence domain D) and the algorithm stops.

When, starting from low levels, demand rises, it turns out that the level of 1,000 quarters is not the ceiling of competitive equilibria. But crossing that level means a sudden switch from a first equilibrium where land is fully cultivated by method 1 to another, for which methods 2 and 3 are each operated on fifty acres: that physical discontinuity does not fit with Ricardo's views on the dynamics of cultivation.

(iii) The static approach. Enumerating and studying all possible combinations of methods is a caveman's strategy, which works only when the number of methods is very small. Here, six combinations of methods, viz. method 1 alone, {2}, {3}, {1, 2}, {1, 3}, {2, 3} are conceivable. For each of them, let us calculate the associated values. The acceptable combinations are {1}, {1, 2}, and {2, 3}, which sustain demands $d \in [0, 1000]$, $d \in [600, 1000]$, and $d \in [600, 1400]$, respectively.

VI. CONCLUSION

We have examined the Ricardian dynamics in the presence of lands and showed that they play an effective analytical role in the *Principles* and in Sraffa's reconstruction of Ricardian theory. The order of cultivation, more generally the law of succession of methods, determines in a unique way the sequence of transformations of a long-term equilibrium into another when demand rises. Sraffa, who stressed that such a law is based on a value criterion, was aware that, in the case of intensification of cultivation, it must moreover be checked that the incoming method does improve productivity: he thought that the positivity of rent suffices to guarantee the coincidence between profitability and productivity. This is not always the case, and 'paradoxes' very similar to those met in capital theory may occur. In the present case, they result in a failure of the Ricardian dynamics, which also explains why the behavior of productive systems with lands may differ significantly from those of single-product systems without lands. The post-Sraffian literature on rent has misunderstood Sraffa's methodology, which, being faithful to Ricardo's, is not in line with the "emphatic warning" of the preface of *PCMC*.

REFERENCES

Abraham-Frois, Gilbert, and Edmond Berrebi. 1980. *Rentes, Raretés, Surprofits*. Paris: Economica.

Baumol, William J. 1951. *Economic Dynamics*. New York: Macmillan.

Bidard, Christian. 2004. *Prices, Reproduction, Scarcity*. Cambridge: Cambridge University Press.

———. 2010. "The Dynamics of Intensive Cultivation." *Cambridge Journal of Economics* 34 (6): 1097–1104.

———. 2014. "The Ricardian Rent Theory: An Overview." *Centro Sraffa Working Papers* 8.

Bidard, Christian, ed. 1987. *La Rente*. Paris: Economica.

Cottle, Richard W., Jong-Shi Pang, and Richard E. Stone. 1992. *The Linear Complementarity Problem*. New York: Academic Press.

D'Agata, Antonio. 1983. "The Existence and Unicity of Cost-Minimizing Systems in Intensive Rent Theory." *Metroeconomica* 35: 147–158.

Erreygers, Guido. 1990. "Terre, Rente et Choix de Techniques." PhD thesis. Two volumes. Mimeo. University Paris X-Nanterre.

———. 1995. "On the Uniqueness of Cost-Minimizing Techniques." *The Manchester School* 63: 145–166.

Freni, Giuseppe. 1991. "Capitale tecnico nei modelli dinamici ricardiani." *Studi Economici* 44: 141–159.

Klimovsky, Edith. 1981. "La rente dans la théorie de la valeur et des prix." PhD thesis. Mimeo. University of Paris X-Nanterre.

Kurz, Heinz D., and Neri Salvadori. 1995. *Theory of Production*. Cambridge: Cambridge University Press.

Lemke, Carlton E. 1965. "Bimatrix Equilibrium Points and Mathematical Programming." *Management Science* 11: 681–689.

Montani, Guido. 1975. "Scarce Natural Resources and Income Distribution." *Metroeconomica* 27: 68–101.

Quadrio Curzio, Alberto. 1966. *Rendita e distribuzione in un modello economico plurisettorale*. Milano: Giuffrè.

Ricardo, David. [1815] 1951. *An Essay on the Influence of a Low Price of Corn on the Profits of Stocks*. In Piero Sraffa, ed., *The Works and Correspondence of David Ricardo*. Volume IV. Cambridge: Cambridge University Press, pp. 9–41.

———. [1817] 1951. *On the Principles of Political Economy and Taxation*. In Piero Sraffa, ed., *The Works and Correspondence of David Ricardo*. Volume I. Cambridge: Cambridge University Press.

Salvadori, Neri. 1986. "Land and Choice of Techniques within the Sraffa Framework." *Australian Economic Papers* 25: 94–105.

Saucier, Philippe. 1981. "Le choix des techniques en situation de limitation des ressources." PhD thesis. Mimeo. University Paris II.

Schefold, Bertram. 1989. *Mr. Sraffa on Joint Production and Other Essays*. London: Unwin Hyman.

Sraffa, Piero. 1960. *Production of Commodities by Means of Commodities*. Cambridge: Cambridge University Press.