Macroeconomic Dynamics, **14** (2010), 29–55. Printed in the United States of America.
doi:10.1017/S1365100509080298

SEMI-NONPARAMETRIC ESTIMATES OF CURRENCY SUBSTITUTION BETWEEN THE CANADIAN DOLLAR AND THE U.S. DOLLAR

APOSTOLOS SERLETIS
University of Calgary

GUOHUA FENG
Monash University

In this paper we investigate the issue of whether a floating currency is the right exchange rate regime for Canada or whether Canada should consider a currency union with the United States. In the context of the framework recently proposed by James L. Swofford, we use a semi-nonparametric flexible functional form—the asymptotically ideal model (AIM), introduced by William A. Barnett and A. Jonas—and pay explicit attention to the theoretical regularity conditions of neoclassical microeconomic theory, following the suggestions of William A. Barnett and William A. Barnett and Meenakshi Pasupathy. Our results indicate that U.S. dollar deposits are complements to domestic (Canadian) monetary assets, suggesting that Canada should continue the current exchange rate regime, allowing the exchange rate to float freely with no intervention in the foreign exchange market by the Bank of Canada.

Keywords: Flexible Functional Forms, Currency Union, Asymptotically Ideal Model (AIM)

1. INTRODUCTION

The exchange rate for the Canadian dollar has attracted a lot of attention in recent years. This attention stems from the long swings in the Canadian dollar per U.S. dollar nominal exchange rate, the recent creation of a single European currency (the euro) to replace the national currencies of member countries of the European monetary union [see, for example, Courchene and Harris (2000) and Grubel (2000)], the trend toward currency unions and dollarization in Latin America and Eastern Europe, and Japan's recent interest in exploring alternative monetary arrangements. The debate in Canada has revolved around exchange rate

Serletis gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada (SSHRCC). Address correspondence to: Apostolos Serletis, Department of Economics, University of Calgary, Calgary, Alberta T2N 1N4, Canada; e-mail: Serletis@ucalgary.ca; URL: http://econ.ucalgary.ca/serletis.htm.

 1365-1005/09

alternatives and particularly around the issue of whether a floating currency is the right exchange rate regime for Canada or whether the exchange rate between the Canadian and U.S. currencies should be fixed, as it was from 1962 to 1970—see, for example, Schembri (2001), Murray and Powell (2002), and Murray et al. (2003).

A floating exchange rate gives Canada the flexibility to have monetary conditions different from those in the United States. A floating currency acts as a shock absorber between the two economies, allowing them to respond differently to external economic shocks (such as fluctuations in world commodity prices) and domestic policy requirements. The costs of a floating currency come in two forms. First and most obviously, there are certain transactions costs that are large when the number of cross-border and financial transactions is large, as is Canada's case with the United States. A further cost is the fact that exchange rates fluctuate wildly in comparison with goods prices (in fact, almost as wildly as stock prices), although the effects of exchange rate volatility on macroeconomic quantities are difficult to demonstrate.

In investigating the policy implications of currency substitution, İmrohoroğlu (1994) used a dynamic equilibrium (money-in-the-utility-function) model of the Canadian economy and Hansen's (1982) generalized method of moments (GMM) estimation procedure to estimate the degree of currency substitution between the Canadian dollar and the U.S. dollar. He reported an estimate of the elasticity of currency substitution of 0.3037 and argued that U.S. dollar deposits in Canada are weak substitutes for the domestic Canadian dollar. More recently, Serletis and Pinno (2007), using İmrohoroğlu's (1994) model, recent monetary data adjusted for take-overs and acquisitions [as discussed in Kottaras (2003)], and an econometric methodology slightly different from the one used by İmrohoroğlu (1994), argued that the elasticity of currency substitution is lower than that reported by İmrohoroğlu (1994). Clearly, this has implications for the theory of optimum currency areas and can be used to evaluate the desirability of a monetary union between Canada and the United States.

The İmrohoroğlu (1994) and Serletis and Pinno (2007) studies employ the general equilibrium approach and use a constant-elasticity-of-substitution (CES) production function that restricts the elasticity of substitution between domestic currency and foreign currency to be constant over time. Although this assumption makes the general equilibrium model much easier to compute, it is very restrictive. As is well known, the elasticity of substitution between domestic currency and foreign currency is sensitive to many factors, such as exchange rate regime changes, financial innovations, and changes in monetary policies, and thus is very volatile over time. For example, in İmrohoroğlu's (1994) study, the sample period is from 1974 to 1990, a very volatile period covering the 1981–1982 recession, the rapid financial innovation, and the monetary regime change in 1982. Thus, it is unlikely that the elasticity of substitution between Canadian currency and U.S. currency is constant over this sample period. In other words, with a restrictive

CES production function, the estimates of the elasticity of currency substitution will be inaccurate and may even be misleading.

Moreover, the İmrohoroğlu (1994) study suffers from the problem of assuming that one of the relevant choice variables for the representative agent is a simple sum monetary aggregate. In particular, the domestic (Canadian) money measure used in İmrohoroğlu (1994) is simple sum M1, which is actually a simple sum aggregate of currency outside banks, personal checking accounts, and current accounts. The use of a simple-sum monetary aggregate is implicitly based on the assumption that the components of the monetary aggregate are perfect substitutes to each other. However, this assumption has been theoretically and empirically shown to be unrealistic by many studies in the monetary aggregation literature—see, for example, Barnett (1980) and Serletis and Rangel-Ruiz (2005). The inappropriate use of simple-sum monetary aggregates may result in inaccurate measures of the actual quantities of the monetary products, which in turn lead to inaccurate estimates of the elasticities of currency substitution.

Motivated by these considerations, in this paper we extend the literature by investigating the issue of whether a floating currency is the right exchange rate regime for Canada or whether we should consider a currency union with the United States, using the framework recently proposed by Swofford (2000, 2005). He focuses on the requirements that economic theory places on optimizing behavior for any area to have a common currency and sets forth the microeconomic foundations for an optimum currency area. In particular, Swofford (2000, 2005) defines an optimum currency area as a region in which economic agents treat the same asset or assets as money across countries—that is, there is a common currency that is a strong substitute for domestic currency. This new definition has microeconomic content, as it is explicitly based on neoclassical microeconomic theory and existing aggregation theory. It requires that the common currency be an asset in the agent's optimizing function and be a strong substitute to the domestic currency.

In following Swofford (2000, 2005), we postulate a parametric reciprocal indirect utility function for the representative economic agent and fit the derived demand functions to observed data. The estimated demand functions are then used to estimate price and substitution elasticities, which are used to evaluate whether a currency union should be formed between Canada and the United States—see Barnett and Serletis (2008) for more details regarding the demand systems approach to the demand for liquid assets. Although our methodology, like that of İmrohoroğlu (1994) and Serletis and Pinno (2007), is far removed from the usual criteria used to establish an optimum currency area [see, for example, Mundell (1961), McKinnon (1963), and Canzoneri and Rogers (1990)], we follow Swofford (2000, 2005) and Serletis and Rangel-Ruiz (2005) and assume that a low degree of currency substitution is consistent with monetary independence and a high one with an optimum currency area.

As Varian (1982, p. 945) puts it, the parametric approach to applied demand analysis "will be satisfactory only when the postulated parametric forms are good approximations to the true demand functions." We tackle this problem by using a semi-nonparametric flexible functional form—the asymptotically ideal model (AIM), introduced by Barnett and Jonas (1983) and employed and explained in Barnett and Yue (1988) and Serletis and Shahmoradi (2005, 2008) among others. Another semi-nonparametric flexible functional form is the Fourier, introduced by Gallant (1982). The AIM is based on a multivariate Müntz–Szatz series expansion, whereas the Fourier flexible functional form is based on a Fourier series expansion. Both functional forms are globally flexible in the sense that they are capable of approximating the underlying aggregator function at every point in the function's domain by increasing the order of the expansion, and thus have more flexibility than most locally flexible functional forms, which theoretically can attain flexibility only at a single point or over an infinitesimally small region—see, for example, Serletis and Shahmoradi (2007).

We approximate the representative agent's indirect utility function by the AIM reciprocal indirect utility function and estimate elasticities of substitution between domestic and foreign currency deposits, consistent with microeconomic utility maximization principles. We pay explicit attention to the theoretical regularity conditions (of positivity, monotonicity, and curvature) of the AIM reciprocal indirect utility function. We argue that unless economic regularity is attained by luck, flexible functional forms should always be estimated subject to regularity, as suggested by Barnett (2002) and Barnett and Pasupathy (2003). In fact, we follow Gallant and Golub (1984), Serletis and Shahmoradi (2005, 2008), and Feng and Serletis (2008, 2009) and treat the curvature property as a maintained hypothesis and build it into the model being estimated.

The rest of the paper is organized as follows. In the next section we briefly discuss İmrohoroğlu's (1994) dynamic equilibrium (money-in-the-utility-function) model of the Canadian economy. We discuss İmrohoroğlu's (1994) main result as well as recent empirical evidence by Serletis and Pinno (2007), using İmrohoroğlu's (1994) model, recent monetary data, and an econometric methodology slightly different from the one used by İmrohoroğlu (1994). In Section 3, we present the semi-nonparametric approach to applied demand analysis, and in Section 4, we discuss computational considerations. In Section 5, we estimate the AIM model, assess the results in terms of their consistency with optimizing behavior, and explore the economic significance of the results. The last section summarizes and concludes the paper.

2. THE GMM APPROACH

İmrohoroğlu (1994) considers an economy made up of a large number of infinitely lived identical agents. At the beginning of each period, the representative domestic agent decides how much to consume, c_t, how much to hold in the form of domestic balances, m_t, and foreign balances, m_t^*, and how much to save in the form of an

https://doi.org/10.1017/S1365100509080298 Published online by Cambridge University Press

internationally traded bond, b_t^*. He assumes that money services are produced using a combination of domestic and foreign real balances in a constant elasticity of substitution (CES) aggregator function,

$$x_t = f(h_t, h_t^*) = \left[\alpha\left(\frac{m_t}{p_t}\right)^{-\rho} + (1-\alpha)\left(\frac{m_t^*}{p_t^*}\right)^{-\rho}\right]^{-1/\rho}, \quad (1)$$

where $0 < \alpha < 1$, $-1 < \rho < \infty$, $\rho \neq 0$, and h_t $(= m_t/p_t)$ and h_t^* $(= m_t^*/p_t^*)$ denote domestic and foreign real money balances, respectively. In the liquidity aggregator function (1), the elasticity of substitution is given by $1/(1+\rho)$; α and $(1-\alpha)$ denote the shares of domestic and foreign real balances (respectively) in the production of money services. Aggregator functions such as (1) were pioneered by Chetty (1969) and have been used by Husted and Rush (1984) and Poterba and Rotemberg (1987), among others.

İmrohoroğlu (1994) assumes that the representative consumer's preferences are given by

$$u\left(c_t, \frac{m_t}{p_t}, \frac{m_t^*}{p_t^*}\right) = \frac{\left(c_t^{\sigma} x_t^{1-\sigma}\right)^{1-\psi} - 1}{1-\psi}, \quad (2)$$

where x_t is the liquidity aggregate given by equation (1). This utility function exhibits constant relative risk aversion in an aggregate of consumption and liquidity services. With these preferences and the liquidity aggregator function (1), the Euler equations for an interior solution are given [see the second case presented in İmrohoroğlu (1994) for details regarding the derivations] by

$$\beta(1+r_t^*)\left(\frac{c_{t+1}}{c_t}\right)^{\phi-1}\left[\alpha\left(\frac{h_{t+1}}{h_{t+1}^*}\right)^{-\rho} + (1-\alpha)\right]^{b} \times\left[\alpha\left(\frac{h_t}{h_t^*}\right)^{-\rho} + (1-\alpha)\right]^{-b}\left(\frac{h_{t+1}^*}{h_t^*}\right)^{-\rho b} - 1 = \varepsilon_{1,t+1}, \quad (3)$$

$$\beta\sigma\left(\frac{c_{t+1}}{c_t}\right)^{\phi-1}\left[\alpha\left(\frac{h_t}{h_t^*}\right)^{-\rho} + (1-\alpha)\right]^{-b}\left[\alpha\left(\frac{h_{t+1}}{h_{t+1}^*}\right)^{-\rho} + (1-\alpha)\right]^{b} \times\left(\frac{h_{t+1}^*}{h_t^*}\right)^{-\rho b}\left(\frac{p_t}{p_{t+1}}\right) + \alpha(1-\sigma)\left[\alpha\left(\frac{h_t}{h_t^*}\right)^{-\rho} + (1-\alpha)\right]^{-1} \times\left(\frac{h_t}{h_t^*}\right)^{-\rho-1}\left(\frac{h_t^*}{c_t}\right)^{-1} - \sigma = \varepsilon_{2,t+1}, \quad (4)$$

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$$\alpha\left[\alpha\left(\frac{h_t}{h_t^*}\right)^{-\rho}+(1-\alpha)\right]^b\left(\frac{h_t}{h_t^*}\right)^{-\rho-1}-(1-\alpha)\left[\alpha\left(\frac{h_t}{h_t^*}\right)^{-\rho}+(1-\alpha)\right]^b$$
$$-\alpha\beta\left(\frac{c_{t+1}}{c_t}\right)^{\phi-1}\left[\alpha\left(\frac{h_{t+1}}{h_t^*}\right)^{-\rho}+(1-\alpha)\left(\frac{h_{t+1}^*}{h_t^*}\right)^{-\rho}\right]^b$$
$$\times\left(\frac{h_t}{h_t^*}\right)^{-\rho-1}\left(\frac{p_t}{p_{t+1}}\right)\left(\frac{e_{t+1}}{e_t}\right)+(1-\alpha)\,\beta\left(\frac{c_{t+1}}{c_t}\right)^{a-1}$$
$$\times\left[\alpha\left(\frac{h_{t+1}}{h_t^*}\right)^{-\rho}+(1-\alpha)\left(\frac{h_{t+1}^*}{h_t^*}\right)^{-\rho}\right]^b\left(\frac{p_t}{p_{t+1}}\right)=\varepsilon_{3,t+1}, \qquad \textbf{(5)}$$

where $\beta \in (0, 1)$ is the subjective discount factor, r_t^* denotes the realized real interest rate on b_t^*, e_t is the nominal exchange rate (note that purchasing power parity is not imposed), $\phi = (1-\psi)\sigma$, $b = -(1-\sigma)(1-\psi)/\rho$, and $\varepsilon_{j,t+1}$ for $j = 1, 2, 3$ are the Euler equation errors.

İmrohoroğlu (1994), using Hansen's (1982) GMM estimation procedure and quarterly data (over the period from January, 1974 to June, 1990) estimates the Euler equations (3)–(5) and reports an estimate of the elasticity of currency substitution, $1/(1+\rho)$, of 0.3037. Based on this low estimate of the elasticity of currency substitution, İmrohoroğlu (1994) argues that U.S. dollar deposits in Canada are weak substitutes for the domestic Canadian dollar.

More recently, Serletis and Pinno (2007), using İmrohoroğlu's (1994) model and recent quarterly data (over the period from 1981:1 to 2003:1), report a GMM estimate of the elasticity of currency substitution of 0.249, a bit lower than the estimate of 0.3037 reported by İmrohoroğlu (1994). Based on this estimate, Serletis and Pinno (2007) conclude that "Canada should continue the current exchange rate regime (allowing the exchange rate to float freely with no intervention in the foreign exchange market by the Bank of Canada) as well as the current monetary policy regime (of inflation targeting)."

As noted in the Introduction, the above general equilibrium approach suffers from two serious problems that may lead to inaccurate or even misleading estimates of currency substitution. First, as can be seen from equation (1), the specification of the production function as a CES aggregator function restricts the elasticity of currency substitution to be constant over time and equal to $1/(1+\rho)$. For a variable that is volatile over time, such as the elasticity of currency substitution, such a restrictive production function specification is certainly not satisfactory. Second, in both equations (1) and (2), the use of one domestic monetary aggregate (m_t), rather than more disaggregated measures, also implies that the substitution between the different components of m_t has been ignored. Again, this will lead to mismeasured or even misleading estimates of elasticity of currency substitution when substitution between the different components of m_t is less than perfect.

In what follows, we investigate the robustness of these results to an alternative modeling procedure, based on the use of recent state-of-the-art advances in microeconometrics. In particular, we follow Swofford (2000, 2005) and Serletis and Rangel-Ruiz (2005) in postulating a parametric reciprocal indirect utility function for the representative economic agent and fitting the derived demand system to observed data. We treat the curvature property as a maintained hypothesis and build it into the model being estimated, following Gallant and Golub (1984), Serletis and Shahmoradi (2005, 2008), and Feng and Serletis (2008, 2009). We obtain parameter estimates that are consistent with theoretical regularity at every data point in the sample and present elasticities of substitution between domestic and foreign currency deposits, which are then used to evaluate whether a currency union should be formed between Canada and the United States.

3. THE SEMI-NONPARAMETRIC APPROACH

Assuming that financial decisions are weakly separable from consumption decisions, we assume that the representative consumer faces the problem

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} = m, \tag{6}$$

where $f(\boldsymbol{x})$ is the direct utility function, which is a continuous, twice differentiable, positive, nondecreasing, and quasiconcave function—see Diewert (1974). $\boldsymbol{x} = (x_1, \ldots, x_{10})$ is the vector of monetary asset quantities, included in the Bank of Canada's M3 monetary aggregate and described in Table 1. $\boldsymbol{p} = (p_1, \ldots, p_{10})$ is the corresponding vector of monetary asset user costs, and m is the expenditure on the services of monetary assets. Essentially, the utility maximization problem (6) is just the second stage of the Barnett et al. (1992) two-stage utility maximization problem, where $f(\boldsymbol{x})$ serves as a subutility function.

We use quarterly data over the period from 1982:1 to 2006:4. Under the assumption of a representative consumer, all quantities were deflated by the consumer price index (CANSIM II series V735319) and divided by Canadian population 15 years old and older (CANSIM II series V158980) to give per capita real monetary assets, that is, $\boldsymbol{x} = (x_1, \ldots, x_{10})$. The user costs of the monetary assets have been calculated, for $j = 1, \ldots, 10$, using

$$p_j = p^* \frac{R - r_j}{1 + R}, \tag{7}$$

where r_j is the yield on the jth asset, R is the yield on the benchmark asset, and p^* is the true cost of living index—see Barnett (1978) for more details. In equation (7), p_j denotes the discounted interest foregone by holding a dollar's worth of the jth asset.

With regards to the benchmark rate, R, we follow Serletis and Rangel-Ruiz (2005) and construct a benchmark interest rate series by selecting (in each period) the highest available interest rate from the interest rates r_j, $j = 1, \ldots, 10$, as well

TABLE 1. Monetary assets used in the monetary aggregates

Monetary aggregate and components	CANSIM II series
M1	
1. Currency outside banks	V37173
2. Personal checking accounts	V36844
3. Current accounts	V36845
M1+ = M1 and the following assets:	
4. Personal chequable savings deposits	V36815
5. Nonpersonal chequable notice deposits	V36827
M1++ = M1 and the following assets:	
6. Personal nonchequable savings deposits	V36818
7. Nonpersonal nonchequable notice deposits	V36828
M2 = M1++ and the following assets:	
8. Personal fixed-term savings deposits	V36823
M3 = M2 and the following assets:	
9. Nonpersonal term deposits	V36830
10. Foreign currency deposits	V36876

Source: Serletis and Rangel-Ruiz (2005).

as the rate on Canadian bonds with a term to maturity of over 10 years (CANSIM II series V122487), the rate on 5-year guaranteed investment certificates (CANSIM II series V122551), the rate on long-term corporate bonds (CANSIM II series V122518), and the rate on medium-term corporate bonds (CANSIM II series V122519).

In the calculation of r_j, $j = 1, \ldots, 10$, in (7), we also follow Serletis and Rangel-Ruiz (2005). In particular, for demand deposits (that is, personal checking accounts and current accounts) we calculate the implicit rate of return, as in Klein (1974) and Startz (1979), using the formula

$$r_D = (1 - \kappa) r_A,$$

where r_A is the interest rate on an alternative asset and κ is an estimate of the maximum required reserve ratio. Here r_A is taken to be the interest rate on 3- to 5-year government of Canada bonds and κ is constructed from both the primary and secondary reserve ratios against demand deposits over the sample period.

The interest rate on personal chequable savings deposits is taken to be the rate on personal chequable savings deposits from 1974 to September 1982 and the interest rate on daily interest chequing accounts in excess of \$5,000 (DICA 5K+) from October 1982 to 1999. For the interest rate on personal nonchequable savings deposits, we use the rate on personal nonchequable savings deposits from 1974 to December 1986, the rate on DISA 25 from January 1987 to January 1988, and the average of DISA 25 and DISA 75 over the period from February 1988 to 1999.

The rate on 90-day personal fixed-term deposits is used as a proxy for the interest rate on both nonpersonal chequable notice deposits and nonpersonal nonchequable notice deposits. Finally, the 5-year term deposit rate is used as the interest rate on personal fixed-term savings deposits, the prime rate as the interest rate on nonpersonal term deposits, and the 3-month Eurodollar deposit rate in London, which is closely linked to the wholesale deposit rate in Canada, as a proxy for the interest rate on foreign currency deposits. All these data series for the construction of r_j, $j = 1, \dots, 10$, are obtained from Statistics Canada CANSIM II—for more details, see Serletis and Rangel-Ruiz (2005).

3.1. The Divisia Monetary Aggregator

Because flexible functional forms are parameter-intensive, we need to rationalize the estimation to a small set of monetary asset demand equations by imposing a separable structure on preferences. Using the same Canadian monetary assets as in this study (but over a shorter period), Serletis and Rangel-Ruiz (2005) searched for all possible separable groupings of assets and used the Divisia monetary aggregator to construct monetary subaggregates consistent with a representative economic agent maximizing a separable utility function. They found that the imposition of the following separable structure on preferences is most reasonable when the substitution between domestic monetary aggregates and foreign currency deposits is allowed for

$$f(\boldsymbol{x}) = f\left[f_1\left(x_3, x_5, x_7\right), f_2\left(x_1, x_2, x_4, x_6, x_8\right), x_{10}\right], \tag{8}$$

where the subaggregator functions $f_1(x_3, x_5, x_7)$ and $f_2(x_1, x_2, x_4, x_6, x_8)$ provide subaggregate measures of monetary services and will be thought of as Divisia quantity indices. Assuming the same separable structure of preferences as in Serletis and Rangel-Ruiz (2005), we rewrite (8) as

$$f(\boldsymbol{x}) = f\left(Q_1, Q_2, x_{10}\right),$$

where $Q_1 = f_1(x_3, x_5, x_7)$ and $Q_2 = f_2(x_1, x_2, x_4, x_6, x_8)$.

Compared with the simple sum approach used by the Bank of Canada, the advantage of the Divisia monetary aggregator is that it allows for less than perfect substitutability among the relevant monetary components. Corresponding to each of the two quantity indices, Q_1 and Q_2, there exist Divisia price indices, which we denote as P_1 and P_2. The demand system in the next section is estimated using data on Q_1, Q_2, and x_{10} and the corresponding Divisia price indices for Q_1 and Q_2, P_1 and P_2, and the user cost of x_{10}, p_{10}. The Divisia price indices P_1 and P_2 are computed making use of Fisher's weak factor reversal test. The test states that the product of the values of the price and quantity indices should be equal to the ratio of total expenditure in the two periods.

3.2. The AIM Reciprocal Indirect Utility Function

Because the direct utility function specified in the maximization problem (6) satisfies continuity, positiveness, monotonicity (nondecreasing), and quasiconcavity, it can be completely characterized by a reciprocal indirect utility function—see Diewert (1974). In this study, we assume that the reciprocal indirect utility function takes on the AIM [see Barnett and Jones (1983)] with three assets ($n = 3$),

$$h(\boldsymbol{v}) = a_0 + \sum_{k=1}^{K}\sum_{i=1}^{3} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{K}\sum_{m=1}^{K}\left[\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)}\right]$$
$$+ \sum_{k=1}^{K}\sum_{m=1}^{K}\sum_{g=1}^{K}\left[\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{h=1}^{3} a_{ijhkmg} v_i^{\lambda(k)} v_j^{\lambda(m)} v_h^{\lambda(g)}\right], \quad \textbf{(9)}$$

where $\boldsymbol{v}$ is the income normalized price vector, that is, $\boldsymbol{v} = \boldsymbol{p}/m$; $h(\boldsymbol{v})$ denotes the reciprocal indirect utility function; $\lambda(z) = 2^{-z}$ for $z = \{k, m, g\}$ is the exponent set; and a_{ik}, a_{ijkm}, and a_{ijhkmg}, for all $i, j, h = 1, 2, 3$, are the parameters to be estimated. The number of parameters is reduced by deleting the diagonal elements of the parameter arrays so that $i \neq j$, $j \neq h$, and $i \neq h$. This does not alter the span of the model's approximation.

Diewert (1974) shows that the reciprocal indirect utility function has to satisfy the same regularity conditions as the direct utility function, i.e., continuity, positiveness, monotonicity (nondecreasing), and quasiconcavity. Once the reciprocal indirect utility function, $h(\boldsymbol{v})$, satisfies these four regularity conditions, its corresponding direct utility function can be constructed as follows:

$$f(\boldsymbol{x}) = \min_{\boldsymbol{v}}\left\{\frac{1}{h(\boldsymbol{v})} : \boldsymbol{v}'\boldsymbol{x} \leq 1, \boldsymbol{v} \geq 0\right\}.$$

In other words, $f(\boldsymbol{x})$ and $h(\boldsymbol{v})$ are dual to each other only when both of them satisfy continuity, positiveness, monotonicity (nondecreasing), and quasiconcavity. Clearly, $h(\boldsymbol{v})$, specified in (9), satisfies continuity automatically by construction. However, positiveness, monotonicity, and quasiconcavity of the AIM reciprocal indirect utility function have to be checked empirically.

By applying Diewert's (1974) modified Roy's identity,

$$s_j(\boldsymbol{v}) = \frac{v_j \partial h(\boldsymbol{v})/\partial v_j}{\sum_{j=1}^{n} v_j \partial h(\boldsymbol{v})/\partial v_j}, \quad \textbf{(10)}$$

to (9), we obtain the AIM(K) demand system, where $s_j = p_j x_j/\boldsymbol{p}'\boldsymbol{x} = v_j x_j$. In what follows, to simplify the estimation problems and deal with computational difficulties in the large parameter space, we assume that $K = 2$ and thus use the

https://doi.org/10.1017/S1365100509080298 Published online by Cambridge University Press

AIM(2) model—see Serletis (2007) or Serletis and Shahmoradi (2008) for details regarding the AIM(1), AIM(2), and AIM(3) models.

For $K = 2$, equation (9) becomes

$$h_{K=2}(\boldsymbol{v}) = a_0 + \sum_{k=1}^{2}\sum_{i=1}^{3} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{2}\sum_{m=1}^{2}\left[\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)}\right] + \sum_{k=1}^{2}\sum_{m=1}^{2}\sum_{g=1}^{2}\left[\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{h=1}^{3} a_{ijhkmg} v_i^{\lambda(k)} v_j^{\lambda(m)} v_h^{\lambda(g)}\right]. \quad \textbf{(11)}$$

To avoid the extensive multiple subscripting in the coefficients a_{ijhkmg}, we follow Barnett and Yue (1988) and reparameterize by stacking the coefficients as they appear in (11) into a single vector of parameters, $\boldsymbol{b} = (b_0, \ldots, b_{26})'$, containing the 27 coefficients in (11), as follows [since $z = 1, 2$, so that $\lambda(1) = 1/2$ and $\lambda(2) = 1/4$, for $z = \{k, m, g\}$]:

$$\begin{aligned} h_{K=2}(\boldsymbol{v}) = {}& b_0 + b_1 v_1^{1/2} + b_2 v_2^{1/2} + b_3 v_3^{1/2} + b_4 v_1^{1/4} + b_5 v_2^{1/4} + b_6 v_3^{1/4} \\ & + b_7 v_1^{1/2} v_2^{1/2} + b_8 v_1^{1/2} v_2^{1/4} + b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} + b_{11} v_1^{1/2} v_3^{1/2} \\ & + b_{12} v_1^{1/2} v_3^{1/4} + b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} + b_{15} v_2^{1/2} v_3^{1/2} \\ & + b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\ & + b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} \\ & + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} \\ & + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}. \end{aligned} \quad \textbf{(12)}$$

Applying the modified version of Roy's identity, (10), to (12) we obtain the AIM(2) demand system,

$$\begin{aligned} s_1 = {}& \big(2b_1 v_1^{1/2} + b_4 v_1^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + 2b_8 v_1^{1/2} v_2^{1/4} + b_9 v_1^{1/4} v_2^{1/2} \\ & + b_{10} v_1^{1/4} v_2^{1/4} + 2b_{11} v_1^{1/2} v_3^{1/2} + 2b_{12} v_1^{1/2} v_3^{1/4} + b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} \\ & + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} + b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} \\ & + 2b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + 2b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} \\ & + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}\big)/D, \end{aligned} \quad \textbf{(13)}$$

$$\begin{aligned} s_2 = {}& \big(2b_2 v_2^{1/2} + b_5 v_2^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + b_8 v_1^{1/2} v_2^{1/4} + 2b_9 v_1^{1/4} v_2^{1/2} \\ & + b_{10} v_1^{1/4} v_2^{1/4} + 2b_{15} v_2^{1/2} v_3^{1/2} + 2b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} \\ & + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} + 2b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} \end{aligned}$$

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$$+ 2b_{22}v_1^{1/2}v_2^{1/2}v_3^{1/4} + b_{23}v_1^{1/2}v_2^{1/4}v_3^{1/4} + 2b_{24}v_1^{1/4}v_2^{1/2}v_3^{1/4} + b_{25}v_1^{1/4}v_2^{1/4}v_3^{1/2} + b_{26}v_1^{1/4}v_2^{1/4}v_3^{1/4}\big)/D, \tag{14}$$

$$s_3 = \big(2b_3v_3^{1/2} + b_6v_4^{1/4} + 2b_{11}v_1^{1/2}v_3^{1/2} + b_{12}v_1^{1/2}v_3^{1/4} + 2b_{13}v_1^{1/4}v_3^{1/2} + b_{14}v_1^{1/4}v_2^{1/4} + 2b_{15}v_1^{1/2}v_3^{1/2} + b_{16}v_1^{1/2}v_3^{1/4} + 2b_{17}v_2^{1/4}v_3^{1/2} + b_{18}v_2^{1/4}v_3^{1/4} + 2b_{19}v_1^{1/2}v_2^{1/2}v_3^{1/2} + 2b_{20}v_1^{1/4}v_2^{1/2}v_3^{1/2} + 2b_{21}v_1^{1/2}v_2^{1/4}v_3^{1/2} + b_{22}v_1^{1/2}v_2^{1/2}v_3^{1/4} + b_{23}v_1^{1/2}v_2^{1/4}v_3^{1/4} + b_{24}v_1^{1/4}v_2^{1/2}v_3^{1/4} + 2b_{25}v_1^{1/4}v_2^{1/4}v_3^{1/2} + b_{26}v_1^{1/4}v_2^{1/4}v_3^{1/4}\big)/D, \tag{15}$$

where D is the sum of the numerators in equations (13)–(15). It is to be noted that b_0 is not estimated and that $b_3 = 1 - b_1 - b_2$, so that there are 25 free parameters. Further, we need not estimate s_3, because from s_1 and s_2 we can compute $s_3 = 1 - s_1 - s_2$; see Barnett and Yue (1988).

3.3. Elasticities of Substitution

There are five elasticities that can be used to assess the substitutability/complementarity relationship between monetary assets and classify assets as complements or substitutes—see Blackorby and Russell (1989) and Davis and Gauger (1996) for more details. These five elasticities are the Hicksian demand elasticity,

$$\eta_{ij}^h = \frac{\partial \ln x_i^h}{\partial \ln p_j}, \tag{16}$$

the Allen elasticity of substitution (AES),

$$\sigma_{ij}^a = \sigma_{ji}^a = \frac{\eta_{ij}^h}{s_j}, \tag{17}$$

the Morishima elasticity of substitution (MES),

$$\sigma_{ij}^m = \frac{\partial \ln\left(x_i^h/x_j^h\right)}{\partial \ln(p_j/p_i)} = \eta_{ij}^h - \eta_{jj}^h = \left(\sigma_{ij}^a - \sigma_{jj}^a\right)s_j, \tag{18}$$

the Marshallian demand elasticity,

$$\eta_{ij}^m = \frac{\partial \ln x_i^m}{\partial \ln p_j} = \eta_{ij}^h - s_jE_i, \tag{19}$$

and the Mundlak (1968) elasticity of substitution (UES),

$$\sigma_{ij}^u = \frac{\partial \ln\left(x_i^m/x_j^m\right)}{\partial \ln(p_j/p_i)} = \eta_{ij}^m - \eta_{jj}^m = \sigma_{ij}^m + s_j(E_j - E_i), \tag{20}$$

where E_j denotes the income elasticity of the jth asset. It is to be noted that in equations (18) and (20), we hold the price of asset i fixed and examine how changes in the price of asset j affect the quantity ratio x_i/x_j.

These five elasticities can be classified into two groups, according to whether utility or expenditure is held constant. In particular, the first three elasticities (the Hicksian demand elasticity, the Allen elasticity of substitution, and the Morishima elasticity of substitution) are given under the assumption that utility is held constant, whereas the last two (the Marshallian demand elasticity and the Mundlak elasticity of substitution) are given under the assumption that expenditure is held constant. We thus call the first three, the Hicksian demand elasticity and the Allen and Morishima elasticities of substitution, "net substitution" elasticities, in the sense that they do not account for income effects (i.e., utility is held constant). In contrast, we call the last two, the Marshallian demand elasticity and the Mundlak elasticity of substitution, "gross substitution" elasticities, because they allow for income effects (i.e., expenditure is held constant).

These five elasticities can also be classified into two groups, depending on whether the interest is in the impact on quantity levels or on quantity ratios. In particular, (16), (17), and (19) measure the percentage change in quantity from a percentage price change, whereas (18) and (20) measure the percentage change in the quantity ratio from a percentage price change. In fact, Davis and Gauger (1996) call the first group "one-asset one-price" elasticities of substitution and the second group "two-asset one-price" elasticities of substitution. It should be noted that both the AES and Hicksian demand elasticity are "one-asset one-price" elasticities of substitution. Moreover, the Allen elasticity of substitution, as shown in (17), is the Hicksian demand elasticity divided by the cost share s_j. Due to this close relationship between the Hicksian demand elasticity and the Allen elasticity of substitution, reporting both is not necessary.

Although either of the above two groups of elasticity measures can be used to stratify assets as substitutes or complements, in general they will yield different stratification sets. Thus, the choice of the appropriate elasticity measure is very important. In our particular case, we are interested primarily in the substitution elasticities when the user cost of foreign currency deposits changes. Clearly, when the user cost of foreign currency deposits changes (holding all other things constant), the household will end up on a different indifference curve; in other words, utility is no longer constant. This means that the appropriate elasticity measure in this study should be a gross substitution elasticity (either the Marshallian demand elasticity or the Mundlak elasticity of substitution) instead of a net substitution elasticity (such as the Hicksian demand elasticity or the Allen and Morishima elasticities of substitution). However, many studies in the money demand literature have ignored this important difference between gross substitution and net substitution, and report estimates of Hicksian demand elasticities or the Allen and Morishima elasticities of substitution.

We now turn to the calculation of the five elasticities. Diewert (1974, p. 125) proved in general that the Allen elasticities of substitution can be computed from the formula

$$\sigma_{ij}^A = \frac{\left[\sum_{k=1}^n v_k h_k\right] h_{ij}}{h_i h_j} - \frac{\sum_{k=1}^n v_k h_{jk}}{h_j} - \frac{\sum_{k=1}^n v_k h_{ik}}{h_i} + \frac{\sum_{m=1}^n \sum_{k=1}^n v_k h_{km} v_m}{\sum_{k=1}^n v_k h_k}, \tag{21}$$

where v_k is the income normalized price, $v_k = p_k/m$, and h is the reciprocal indirect utility function, $h(\boldsymbol{v})$, as defined in (12). It can be easily shown that (21) is the same formula as that for the Allen elasticities of substitution used in Barnett and Yue (1988). Barnett and Yue (1988) proved in general that the income elasticities can be computed from

$$E_i = \frac{\partial s_i}{\partial m}\frac{m}{s_i} + 1. \tag{22}$$

Once the Allen elasticities of substitution, σ_{ij}^A, and the income elasticities, E_i, are calculated, the other four elasticities—the Hicksian demand elasticity, the Morishima elasticity of substitution, the Marshallian demand elasticity, and the Mundlak elasticity of substitution—can be readily obtained using equations (16) and (18)–(20).

4. SEMI-NONPARAMETRIC ESTIMATION

The demand system (13)–(14) can be written as

$$\boldsymbol{s}_t = \boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \boldsymbol{\epsilon}_t, \tag{23}$$

with an error term, $\boldsymbol{\epsilon}_t$, appended. In (23), $\boldsymbol{s} = (s_1, s_2)'$, $\boldsymbol{\psi}(\boldsymbol{v}, \boldsymbol{\theta}) = (\psi_1(\boldsymbol{v}, \boldsymbol{\theta}), \psi_2(\boldsymbol{v}, \boldsymbol{\theta}))'$, $\boldsymbol{\theta}$ is the parameter vector to be estimated, and $\psi_i\ (\boldsymbol{v}, \boldsymbol{\theta})$ is given by the right-hand side of each of (13) and (14).

Autocorrelation in the disturbances in consumer demand systems is a common result and has mostly been dealt with by assuming a first-order autoregressive process—see, for example, Ewis and Fisher (1984), Serletis and Robb (1986), Serletis (1987, 1988), Fisher and Fleissig (1994), Fleissig and Swofford (1996), Fisher and Flessing (1997), Fleissig (1997), Fleissig and Serletis (2002), and Drake and Fleissig (2004). We follow this general practice and correct for the serial correlation problem by allowing the possibility of a first-order autoregressive process in the error terms of equation (23),

$$\boldsymbol{\epsilon}_t = \boldsymbol{R}\boldsymbol{\epsilon}_{t-1} + \boldsymbol{u}_t, \tag{24}$$

where $\boldsymbol{R} = [R_{ij}]$ is a matrix of unknown parameters and $\boldsymbol{u}_t$ is a nonautocorrelated vector disturbance term with constant covariance matrix. In this case, estimates of the parameters can be obtained by using a result developed by Berndt and Savin (1975). They showed that if one assumes no autocorrelation across equations (i.e.,

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$\boldsymbol{R}$ is diagonal), the autocorrelation coefficients for each equation must be identical, say ρ. Formally,

$$\boldsymbol{R} = \rho \boldsymbol{I}_2, \tag{25}$$

where $\boldsymbol{I}_2$ is a 2×2 identity matrix. Consequently, by writing equation (23) for period $t-1$, multiplying by ρ, and subtracting from (23), we can estimate stochastic budget share equations given by

$$\boldsymbol{s}_t = \boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \rho \boldsymbol{s}_{t-1} - \rho \boldsymbol{\psi}(\boldsymbol{v}_{t-1}, \boldsymbol{\theta}) + \boldsymbol{u}_t. \tag{26}$$

In this paper, we follow Gallant and Golub (1984) and use optimization methods to estimate (26). Moreover, instead of using a residual-based simple objective function as Gallant and Golub (1984) do, we specify the objective function as a log likelihood function. There are two concentrated log likelihood functions that can be used to estimate the demand system described by (23)–(25) [or more compactly by (26)]—see Beach and MacKinnon (1979) for more details. The first is

$$\ln L_c\,(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho\,) = \text{constant} - \frac{(T-1)}{2} \times \log \left| \sum_{t=2}^{T} (\boldsymbol{u}_t - \rho \boldsymbol{u}_{t-1})\,(\boldsymbol{u}_t - \rho \boldsymbol{u}_{t-1})' \right|, \tag{27}$$

and the second is

$$\ln L\,(\boldsymbol{s} \,|\, \boldsymbol{\theta}, \rho) = \text{constant} + \frac{(n-1)}{2} \log(1-\rho^2) - \frac{T}{2} \times \log \left| (1-\rho^2)\boldsymbol{u}_1 \boldsymbol{u}_1' + \sum_{t=2}^{T} (\boldsymbol{u}_t - \rho \boldsymbol{u}_{t-1})\,(\boldsymbol{u}_t - \rho \boldsymbol{u}_{t-1})' \right|. \tag{28}$$

In equation (27), $\ln L_c(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$ is referred to by Beach and MacKinnon (1979) as "the conventional concentrated maximum likelihood function," whereas in equation (28), $\ln L(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$ is referred to as "the full concentrated maximum likelihood function." Compared with $\ln L_c(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$, $\ln L(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$ has two major advantages. First, it enforces a stationarity condition on the error process. In particular, it incorporates the term $(n-1)\log(1-\rho^2)/2$, which goes to minus infinity as $|\rho| \to 1$, and thus a priori restricts the error process to be stationary. Second, $\ln L(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$ makes full use of all available information, as can be seen from the term $(1-\rho^2)\boldsymbol{u}_1\boldsymbol{u}_1'$ in the determinant expression in (28), whereas the conventional concentrated maximum likelihood function ignores the information contained in the first observation. Although the asymptotic properties are the same, the estimates from the two log likelihood functions can differ sharply—see Beach and MacKinnon (1979). Therefore, we choose to use the concentrated maximum likelihood function, $\ln L(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$, as our objective function in this paper.

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In maximizing (28), we use the TOMLAB/NPSOL tool box with MATLAB—see http://tomlab.biz/products/npsol. NPSOL uses a sequential quadratic programming algorithm and is suitable for both unconstrained and constrained optimization of smooth (that is, at least twice continuously differentiable) nonlinear functions. We first run an unconstrained optimization using (28). Because results in nonlinear optimization are sensitive to the initial parameter values, to achieve global convergence, we randomly generated 500 sets of initial parameter values and chose the starting $\boldsymbol{\theta}$ that led to the highest value of the objective function. We also check the regularity conditions of positivity, monotonicity, and curvature.

In cases where the curvature conditions are not satisfied at all observations, we follow Gallant and Golub (1984) and Feng and Serletis (2008, 2009) and use nonlinear constrained optimization to impose curvature. Curvature requires that the principal minors of the bordered Hessian determinant $|\boldsymbol{H}|$ alternate in sign—see, for example, Morey (1986). The principal minors of the bordered Hessian matrix $\boldsymbol{H}$ are given by

$$\boldsymbol{H}_i = \begin{vmatrix} 0 & h_1 & \cdots & h_i \\ h_1 & h_{11} & \cdots & h_{1i} \\ \vdots & \vdots & \cdots & \vdots \\ h_i & h_{i1} & \cdots & h_{ii} \end{vmatrix} > 0 \text{ if } i \text{ is even} \quad \text{and} \quad <0 \text{ if } i \text{ is odd},$$

where $h_n = \partial h(\boldsymbol{v})/\partial v_n$ and $h_{ij} = \partial^2 h(\boldsymbol{v})/\partial v_i \partial v_j$.

Thus, our constrained optimization problem becomes

$$\max_{\{\boldsymbol{\theta},\rho\}} \ln L\,(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$$

subject to

$$\boldsymbol{H}_i > 0 \text{ if } i \text{ is even} \quad \text{and} \quad <0 \text{ if } i \text{ is odd}.$$

With the constrained optimization method, we can impose the curvature restrictions at any arbitrary set of points—at a single data point, over a region of data points, or fully (at every data point in the sample).

5. EMPIRICAL EVIDENCE

5.1. Economic Regularity

In Table 2 we present a summary of the results from the AIM(2) model in terms of parameter estimates and theoretical regularity violations when the model is estimated without the curvature conditions imposed and with the curvature conditions imposed. Clearly, the unconstrained model satisfies positiveness and monotonicity at all sample observations when curvature is not imposed—see column 1 of Table 2. However, it violates curvature at all 100 observations when curvature conditions are not imposed. The theoretical regularity conditions are checked as in Serletis and Shahmoradi (2005). In particular, positivity is checked by direct computation of the values of the estimated budget shares, monotonicity is checked by direct

TABLE 2. AIM(2) parameter estimates

Parameter	Unconstrained	Curvature imposed
b_1	−0.3479	0.6679
b_2	−0.4472	3.4861
b_4	−0.0912	1.2450
b_5	−0.1829	1.7965
β_6	−0.6183	2.4915
β_7	−21.8467	2.5461
β_8	4.2612	42.1125
β_9	7.1188	−18.1267
β_{10}	−0.2689	−15.5411
β_{11}	16.8651	0.8558
β_{12}	−3.0665	10.0353
β_{13}	−11.5546	9.9124
β_{14}	3.3167	−10.7386
β_{15}	10.0325	−18.6303
β_{16}	−0.5424	−5.1822
β_{17}	−10.9303	14.6625
β_{18}	3.2894	−6.1592
β_{19}	−42.5618	−40.2815
β_{20}	−30.9353	5.3103
β_{21}	−76.8977	−41.2967
β_{22}	102.0576	7.9679
β_{23}	−1.6300	−28.8517
β_{24}	−22.5227	56.4638
β_{25}	63.7583	−7.8984
β_{26}	−13.2083	13.1700
ρ	0.9939	0.9987
$\ln L(\boldsymbol{s} \mid \boldsymbol{\theta}, \rho)$	701.523	692.100
Positiveness violations	0	0
Monotonicity violations	0	0
Curvature violations	100	0

computation of the values of the first gradient vector of the estimated indirect utility function, and curvature is checked by performing a Cholesky factorization of the Slutsky matrix and checking whether the Cholesky values are nonpositive.

The violation of the curvature conditions by the unconstrained model implies that the construction of a corresponding direct utility function from the unconstrained AIM reciprocal indirect utility function is impossible. This can be seen from (9), which holds only when all three theoretical regularity conditions (positiveness, monotonicity, and quasi-concavity) are satisfied. As Barnett (2002, p. 199) put it, without satisfaction of all three theoretical regularity conditions, "the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become

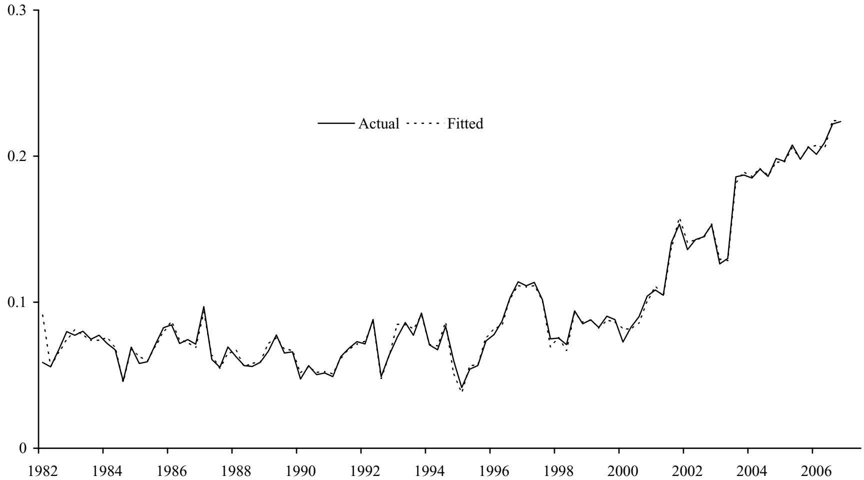

FIGURE 1. Actual and fitted shares for Q1.

invalid." The failure of the duality between the unconstrained AIM reciprocal indirect utility function and the direct utility function also implies that the elasticities of substitution between the assets obtained from the unconstrained model will be invalid.

Motivated by these issues, we impose the curvature restriction fully (that is, at every data point in the sample), using methods suggested by Gallant and Golub (1984)—see also Serletis and Shahmoradi (2005) and Feng and Serletis (2008, 2009) for details regarding the method for imposing the curvature restriction. In particular, we maximize (28) with the curvature constraint imposed fully and report the results in column 2 of Table 2. As can be seen in column 2 of Table 2, (full) imposition of the quasiconcavity constraint has a significant impact on the model, because we obtain parameter estimates that are consistent with all three theoretical regularity restrictions (positiveness, monotonicity, and curvature) at every data point in the sample and also eliminate the serial correlation problem, although in an atheoretic way.

In Table 2 we also report the log likelihood values for both the unconstrained and constrained models. By comparing these log likelihood values, we see that the imposition of the curvature constraint has not had much influence on the flexibility of the AIM model. In particular, the log likelihood value has been decreased slightly from 701.523 to 692.100. To see the performance of the constrained AIM model more closely, we plot the actual and fitted shares in Figures 1 and 2. As can be seen, the three fitted shares resemble their corresponding actual shares so well that it is hard to visually distinguish them from each other. Based on this evidence, we argue that the constrained AIM model used in this paper can guarantee inference consistent with the theory, without compromising much of the flexibility of the functional form.

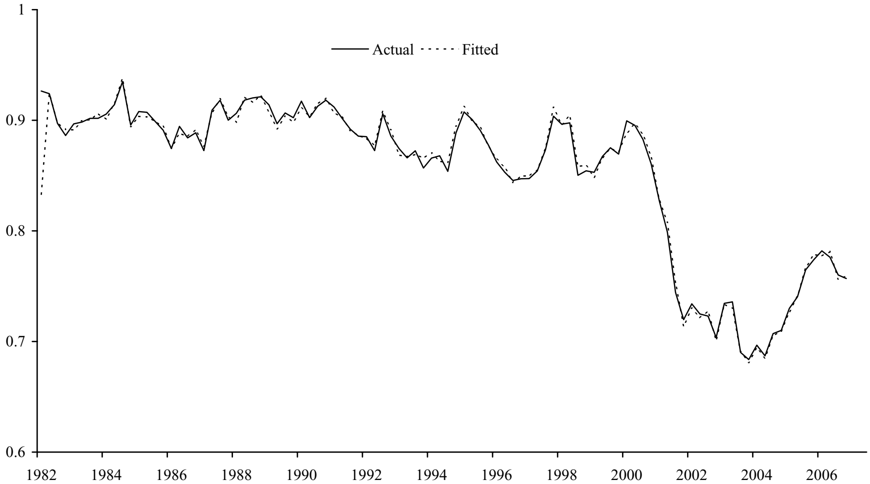

FIGURE 2. Actual and fitted shares for Q2.

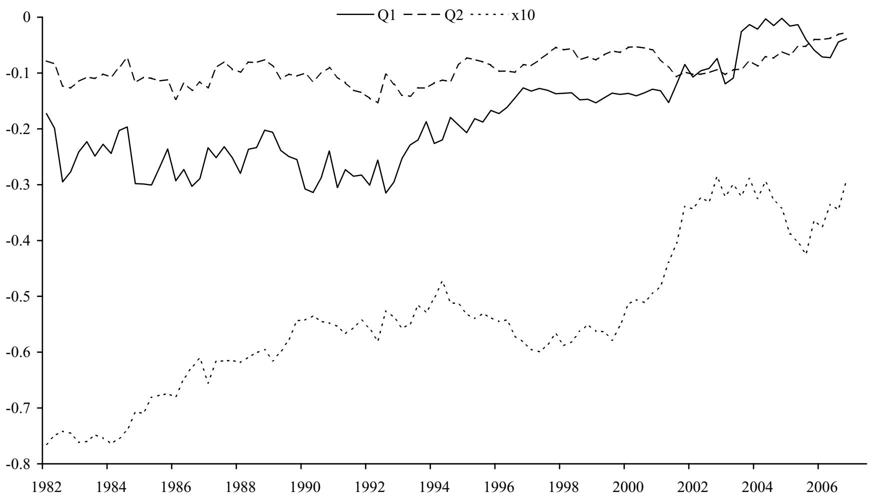

FIGURE 3. Hicksian own-price elasticities.

5.2. Elasticities

We start by presenting the Hicksian own-price elasticities in Figure 3. These elasticities are negative at all data points. When the Hicksian own-price elasticities, η_{ii}^h, are negative and the income elasticities, E_i, are positive, the Marshallian own-price elasticities, η_{ii}^m, must be negative [see equation (18)], which is confirmed by our empirical results, as can be seen in Figure 4. The negativeness of the own-price elasticities theoretically validates the use of our AIM reciprocal indirect utility

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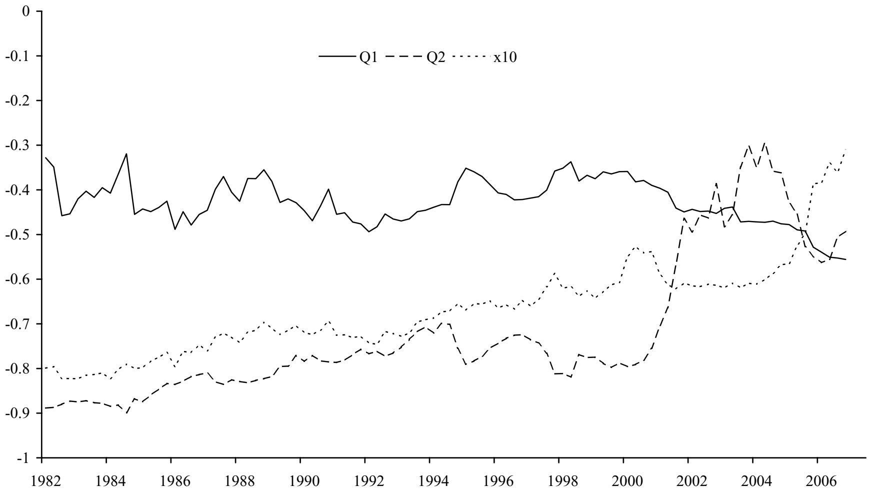

FIGURE 4. Marshallian own-price elasticities.

function. It should be noted that, while we report both Hicksian and Marshallian own-price elasticities, the Marshallian elasticities incorporate both the substitution and the income effects of a price change. From Figure 4, the Marshallian own-price elasticity for Q_1 is small in absolute value (around -0.4) but rather stable over time, whereas the other two own-price elasticities are quite large in absolute value (-0.89 for Q_2 and -0.80 for x_{10}) at the beginning of the sample period but decline in absolute value to -0.49 and -0.31, respectively, at the end of the sample period. This implies that whereas the sensitivity of Q_1 to its own price has been very stable over time, the sensitivities of Q_2 and x_{10} to their respective prices have declined.

We then present the income elasticities in Figure 5. The income elasticities are all positive ($E_i > 0$), with Q_1 being a luxury asset and Q_2 being a normal asset. As can be seen from Figure 5, compared with the stability of the income elasticities of the other assets, the income elasticity for U.S. dollar deposits is rather volatile over time due to the high volatility in its user cost, suggesting that U.S. dollar deposits have been both a luxury and a normal asset over the sample period. Moreover, the nonzero income elasticities imply that the use of gross substitution elasticities (the Marshallian demand elasticity and the Mundlak elasticity of substitution) is appropriate in this study, whereas net substitution elasticities (the Hicksian demand elasticity and the Allen and Morishima elasticities of substitution) are not.

Although we argue that the Morishima elasticity is not the appropriate measure of elasticity of substitution in our study, we present the Morishima elasticities of substitution in Figure 6, for the sake of comparison of the results from this study with those from previous studies. Clearly, foreign currency is indeed a Morishima substitute to the two domestic assets. In particular, σ_{31}^m, σ_{13}^m, σ_{32}^m, and

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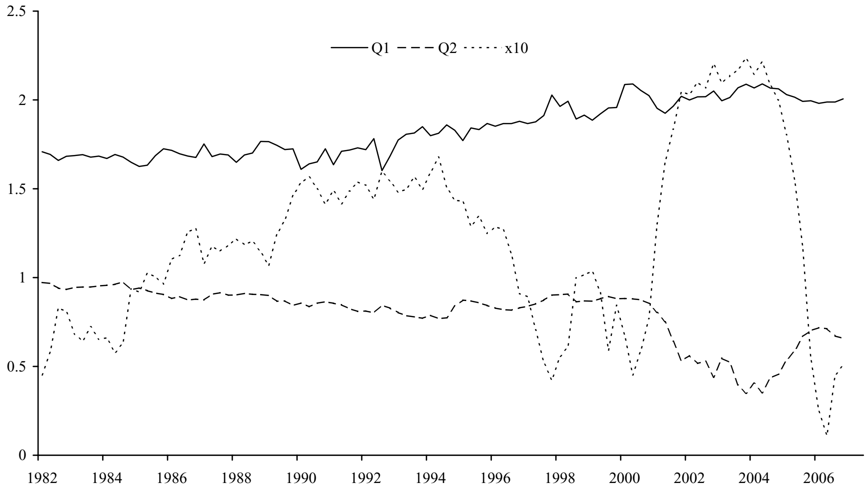

FIGURE 5. Income elasticities.

FIGURE 6. Morishima elasticities of substitution.

σ_{23}^m are all positive through the sample period, with the only exception being σ_{31}^m, which exhibits very slight Morishima complementarity (close to zero) for the period from July 2003 to January 2005. On average, the four Morishima elasticities of substitution, σ_{31}^m, σ_{13}^m, σ_{32}^m, and σ_{23}^m, are 0.1803, 0.5419, 0.6337, and 0.6054, respectively. These results are roughly comparable to those reported by Serletis and Rangel-Ruiz (2005) and İmrohoroğlu (1994). In fact, employing a Fourier indirect utility function with no curvature imposed and a shorter sample

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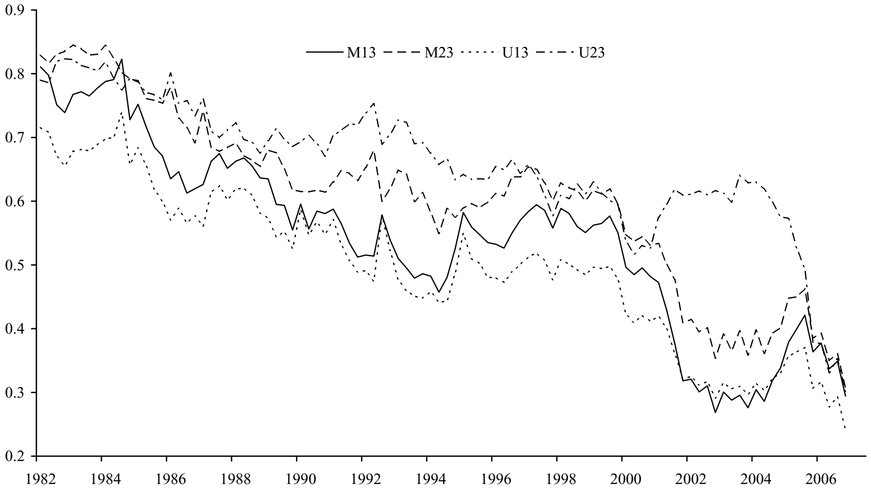

FIGURE 7. Mundlak and Morishima elasticities of substitution.

period, Serletis and Rangel-Ruiz (2005) find that the six Morishima elasticities of substitution range from −0.064 to 0.225. Also, as already noted, İmrohoroğlu (1994) reports an estimate of the elasticity of substitution between the Canadian dollar and the U.S. dollar of about 0.3. Further, the two elasticities of substitution between Q_2 and U.S. dollar deposits, σ_{32}^{m} and σ_{23}^{m}, are larger than those between Q_1 and U.S. dollar deposits, σ_{31}^{m} and σ_{13}^{m}, suggesting that nonpersonal assets are more (Morishima) substitutable with U.S. dollar deposits than personal assets are.

We now turn to a discussion of the Mundlak elasticities of substitution, which allow for both substitution and income effects. The Mundlak and Morishima elasticities, σ_{13}^{u} and σ_{23}^{u} and σ_{13}^{m} and σ_{23}^{m}, are presented in Figure 7. Roughly speaking, the Mundlak elasticities of substitution and their corresponding Morishima elasticities of substitution are of comparable magnitude and of similar time patterns, due to the small term $s_j(E_j - E_i)$—see equation (20). However, despite this similarity in magnitude and time pattern, the differences between the Mundlak and Morishima elasticities of substitution are significant, as evidenced in Figure 7. These differences in magnitude and time pattern imply that the Mundlak elasticities of substitution cannot be replaced by the Morishima elasticities of substitution in the assessment of currency substitution, as some previous studies did.

Let us focus on the Mundlak elasticities of substitution when the user cost of U.S. dollar deposits changes, σ_{13}^{u} and σ_{23}^{u}, shown in Figure 7. Let us consider first the Mundlak elasticity of substitution between Q_1 and x_{10}, σ_{13}^{u}, which represents the percentage change in the ratio of the monetary subaggregate Q_1 to x_{10}, Q_1/x_{10}, when the relative price p_{10}/P_1 is changed by changing p_{10} and holding P_1 constant. Clearly, σ_{13}^{u} is positive over the sample period, with the average being 0.4963, implying that the demand for Q_1 always increases relative to that for x_{10} when the

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TABLE 3. Average demand elasticities

Marshallian demand elasticities		Hicksian demand elasticities	
η_{11}^m	−0.4273	η_{11}^h	−0.1821
η_{12}^m	−1.2463	η_{12}^h	0.1704
η_{13}^m	−0.1715	η_{13}^h	0.0020
η_{21}^m	−0.0753	η_{21}^h	0.0232
η_{22}^m	−0.7189	η_{22}^h	−0.0921
η_{23}^m	−0.0084	η_{23}^h	0.0655
η_{31}^m	−0.1669	η_{31}^h	−0.0017
η_{32}^m	−0.3834	η_{32}^h	0.5416
η_{33}^m	−0.6678	η_{33}^h	−0.5399

rental price of U.S. dollar deposits increases. In other words, Q_1 and x_{10} are always Mundlak substitutes for each other when p_{10} changes. Further, as can be seen from Figure 6, the relative demand for Q_1 has declined over the sample period when the rental price of U.S. dollar deposits, p_{10}, has increased. As for the Mundlak elasticity of substitution between Q_2 and x_{10}, σ_{23}^u, it is also positive, with an average of 0.6594. Therefore, Q_2 and x_{10} are also always Mundlak substitutes when p_{10} changes. We also find that σ_{13}^u and σ_{23}^u have magnitudes comparable to those of domestic asset substitution. In particular, σ_{13}^u and σ_{23}^u are on average 0.4963 and 0.6594, compared with σ_{21}^u and σ_{12}^u being 0.3520 and −0.5274, respectively.

Although we found Mundlak substitutability between Q_1, Q_2, and foreign currency deposits, x_{10}, when the rental price of foreign currency deposits, p_{10}, changed, and that the magnitude of the substitutability was comparable to domestic substitution, we need to also examine the Marshallian demand elasticities before we jump into any conclusions regarding the possibility of a North America currency union. This is because the two elasticity measures look at the substitutability/complementarity relation among the monetary assets from different perspectives. In particular, as we already noted, the Mundlak elasticity of substitution shows the change in the demand for asset i relative to asset j when the price of asset j changes, whereas the Marshallian demand elasticity shows the change in the demand for asset i when the price of asset j changes.

We report the Marshallian demand elasticities at the data mean in the first column of Table 3. Interestingly, instead of finding currency substitution, we find that the two domestic monetary subaggregates are slight gross complements to U.S. dollar deposits. In particular, we find that $\eta_{13}^m < 0$, $\eta_{23}^m < 0$, $\eta_{31}^m < 0$, and $\eta_{32}^m < 0$ at the data means. This result is consistent with the finding of Serletis and Rangel-Ruiz (2005) that the three monetary assets are complementary when the Marshallian elasticity of substitution is used. An examination of the Hicksian cross elasticities reveals that η_{13}^h, η_{31}^h, and η_{32}^h are positive and η_{23}^h is slightly negative (see column 2 of Table 3), implying that the two domestic monetary subaggregates and U.S. dollar deposits are net Hicksian substitutes. The different signs of the Marshallian and Hicksian cross elasticities imply that gross complementarities between the two domestic monetary subaggregates and U.S. dollar deposits are actually caused by the positive income elasticities. In terms of magnitude, we find that the two Marshallian demand elasticities when the rental price of x_{10} changes (η_{13}^m and η_{23}^m) are very small. In particular, compared with η_{11}^m and η_{12}^m (−0.4273 and −1.2463, respectively), η_{13}^m is much smaller (−0.1715); and compared with η_{21}^m and η_{22}^m (−0.0753 and −0.7189, respectively), η_{23}^m is much smaller (−0.0084). Clearly, in terms of Marshallian demand elasticities, U.S. dollar deposits are not a substitute for the two domestic monetary subaggregates, Q_1 and Q_2.

We can also calculate the change in the sum of the Q_1 and Q_2 monetary subaggregates when there is a 1% change in z, where $z = P_1, P_2, p_{10}$, as follows:

$$\phi_z = \frac{\triangle(Q_1 + Q_2)}{(\triangle z/z)} = \eta_{1z}^m \times Q_1 + \eta_{2z}^m \times Q_2.$$

In this equation, ϕ_z measures the impact on the domestic monetary subaggregates from a 1% rise in z, $z = P_1, P_2, p_{10}$. Several results emerge. First, results (not shown here but available on request) show that all the three ϕ's, ϕ_{P_1}, ϕ_{P_2}, and $\phi_{p_{10}}$, are very volatile. The high volatility in the ϕ's suggests that the response of the demand for domestic monetary assets to a 1% rise in rental prices can be very different over time. Second, compared with the impact on the domestic monetary subaggregates from a 1% rise in P_2, ϕ_{P_2}, the impact on the domestic monetary subaggregates from a 1% rise in P_1 or p_{10} (that is, ϕ_{P_1} or $\phi_{p_{10}}$) is very small. In particular, we find that 1% rise in P_2 will decrease the sum of Q_1 and Q_2 by 296.35; a 1% rise in P_1 will decrease the sum of Q_1 and Q_2 by 44.17; and a 1% rise in p_{10} will decrease the sum of Q_1 and Q_2 by 24.45. In other words, $\phi_{p_{10}}$ is 1/12.12 of ϕ_{P_2} and 1/1.81 of ϕ_{P_1}. The small magnitude of the impact of the user cost of foreign currency deposits on Canadian domestic monetary assets implies that Canadian monetary policy will be affected by foreign economic variables to a very small degree, and thus cannot lead to loss of monetary independence onto instability in the demand for monetary asset demand functions in Canada.

6. CONCLUSION

We have used recent advances in microeconometrics to investigate currency substitution between Canada and the United States. In doing so, we used the globally flexible AIM model to approximate the unknown reciprocal indirect utility function and estimated income and price elasticities as well as the elasticities of substitution based on the AIM demand system. We also distinguished between net substitution elasticities (such as the Hicksian demand elasticity and the Allen and Morishima elasticities of substitution) and gross substitution elasticities (such as the Marshallian demand elasticity and the Mundlak elasticity of substitution) and argued that the Marshallian demand elasticity and the Mundlak elasticity of substitution are the right measures to use to evaluate the issue of currency substitution in the AIM reciprocal indirect utility function framework.

Based on a multivariate Müntz–Szatz series expansion, the AIM model is globally flexible in the sense that it is capable of approximating the underlying aggregator function at every point in the function's domain by increasing the order of the expansion, and thus has more flexibility than the CES functional form used by İmrohoroğlu (1994) and Serletis and Pinno (2007) and locally flexible functional forms that theoretically can attain flexibility only at a single point or over an infinitesimally small region. The much greater flexibility of the AIM model allows it to capture the temporal variation of the elasticity of substitution between domestic and foreign currency with much better accuracy. In estimating the AIM demand system, we imposed the curvature condition on the parameters of the AIM reciprocal indirect utility function, using the nonlinear constrained optimization approach adopted from Gallant and Golub (1984). We have argued that inference based on flexible functional forms is virtually worthless unless all three theoretical regularity conditions (positiveness, monotonicity, and curvature) are satisfied, because violations of theoretical regularity violate the maintained hypothesis and invalidate the duality theory that produces the estimated model.

Our results, based on the Mundlak elasticities of substitution, indicate low substitutability between monetary assets (a very common result in the literature), implying that simple-sum monetary aggregates are incorrect or biased. We have also taken an optimum currency area approach to the problem of whether a floating currency is the right exchange rate regime for Canada or Canada should consider a currency union with the United States. Although our methodology is far removed from the usual criteria used to establish an optimum currency area [see, for example, Mundell (1961), McKinnon (1963), and Canzoneri and Rogers (1990)], we have followed Swofford (2000, 2005) and Serletis and Rangel-Ruiz (2005) and assumed that a low degree of currency substitution is consistent with monetary independence and a high one with an optimum currency area.

The results based on the Marshallian demand elasticities show that U.S. dollar deposits are complements to domestic monetary assets. This is consistent with Murray and Powell's (2002, p. 11) conclusion that "there is no evidence that Canadians have lost faith in their currency and are beginning to adopt the U.S. dollar." Based on this evidence, we conclude that Canada should continue the current exchange rate regime (allowing the exchange rate to float freely with no intervention in the foreign exchange market by the Bank of Canada) as well as the current monetary policy regime (of inflation targeting).

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