

CAEP/9-agreed certification requirement for the Aeroplane CO₂ Emissions Standard: a comment on ICAO Cir 337

J.E. Green

greens@woburnhc.freeseve.co.uk

J.A. Jupp

Royal Aeronautical Society
Greener by Design Group
London
England

ABSTRACT

The International Civil Aviation Organization (ICAO) Circular Cir 337 is the first step towards ICAO establishing an Aeroplane CO₂ Emissions Standard to form part of Annex 16, Volume III to the Chicago Convention. It describes itself as ‘a work in progress’. This paper reviews Cir 337 against the background of flight physics, the published literature on aircraft fuel burn and CO₂ emissions and the current practices of the aircraft and engine manufacturers and the airline operators. We have taken, as our starting point, the aim of ICAO to reduce the fuel used per revenue tonne-kilometre performed and argue that the Breguet range equation, which captures all the relevant flight physics, should be the basis of the metric system underpinning the standard. Our overall conclusion is that Cir 337 provides an excellent basis for the initial regulation of aviation’s CO₂ emissions and, further in the future, for developing measures to increase the fuel efficiency of the operational side of civil aviation. Our main criticism of the circular in its current form is that it does not address the ICAO goal of reducing *fuel used per revenue tonne-kilometre performed* and makes no reference to payload. This defect could be eliminated simply by omission of the exponent 0.24 of the Reference Geometric Factor (*RGF*) in the formula for the metric given in Chapter 2 (paragraph 2.2) of the circular. Retaining the *RGF* to the power unity in the metric and multiplying it by an appropriate value of the effective floor loading would convert it to what the 37th Assembly of ICAO called for – a statement of fuel used per revenue tonne-kilometre performed. Finally, correlating the amended metric against design range, as determined from the measured specific air range and the key certificated masses, provides a sound scientific basis for an initial regulation to cap passenger aircraft emissions.

Keywords: CO₂ emissions regulation; civil aircraft fuel efficiency; fuel burn metrics; Breguet range equation; Aircraft Design; Air Transport; Performance

NOMENCLATURE

<i>D</i>	drag
<i>E</i>	rate of energy release in engine
<i>EFL</i>	effective floor loading (= MPM/RGF)
<i>FM</i>	fuel mass
<i>FMB</i>	fuel mass burned since start of cruise
<i>g</i>	acceleration due to gravity (9.807m/s^2)
<i>GM</i>	instantaneous aircraft gross mass
<i>L</i>	lift
<i>LCV</i>	lower calorific value of fuel (reference value $43,217\text{kJ/kg}$ in Cir 337)
<i>MFM</i>	maximum fuel mass
<i>MLM</i>	maximum landing mass
<i>MPM</i>	maximum payload mass
<i>MTOM</i>	maximum take-off mass
<i>MZFM</i>	maximum zero-fuel mass
<i>N</i>	number of passengers
<i>OEM</i>	operating empty mass
<i>PM</i>	payload mass
<i>R</i>	range
<i>RGF</i>	reference geometric factor
<i>S</i>	distance along flight path
<i>SAR</i>	specific air range (= distance flown per unit mass of fuel burned in km/kg)
<i>T</i>	thrust
<i>t</i>	time
<i>TFM</i>	trip fuel mass
<i>TOM</i>	take-off mass
<i>V</i>	flight velocity
<i>X</i>	range parameter (= $(LCV/g)\eta L/D$)
<i>ZFM</i>	zero-fuel mass
β_{\min}	minimum reserve fuel mass/ <i>MTOM</i>
ϵ	lost fuel/ <i>MTOM</i>
η	overall propulsion efficiency

Subscripts

<i>AVG</i>	the average ($1/SAR$) at three different values of gross mass
<i>C</i>	at point C on the payload-range diagram, corresponding to tanks full at <i>MTOM</i>
<i>2C</i>	two-class seating
<i>3C</i>	three-class seating
<i>des</i>	at the design point, corresponding to maximum range at maximum payload
<i>mid-cruise</i>	at the half-way point in cruise

1.0 INTRODUCTION

The International Civil Aviation Organization (ICAO), at a High-Level Meeting of Member States⁽¹⁾ in October 2009, adopted a Programme of Action on International Aviation

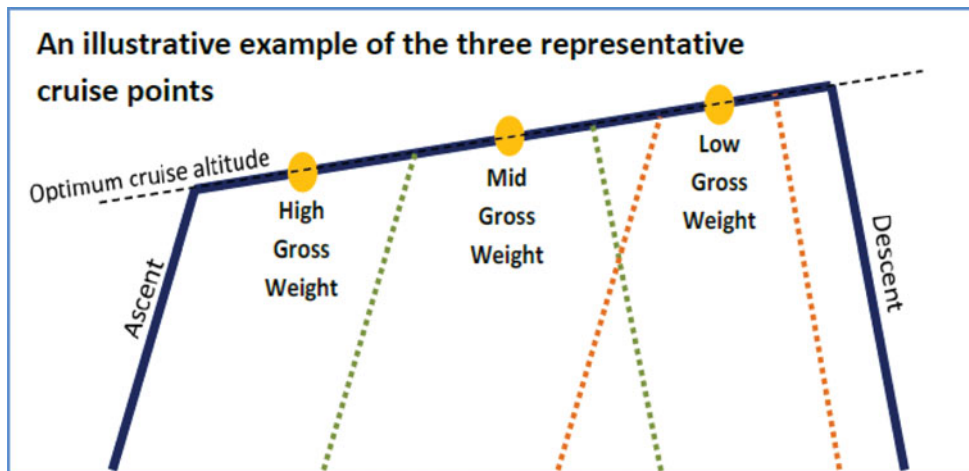


Figure 1. (Colour online) Illustration of ICAO CO₂ metric.

and Climate Change. One of the recommended elements within the programme was the development of a CO₂ emissions standard as part of the range of measures to address greenhouse gas emissions from the air transport system.

At its 37th Assembly, in September-October 2010, ICAO adopted the following resolution:

that States and relevant organizations will work through ICAO to achieve a global annual average fuel efficiency improvement of 2 per cent until 2020 and an aspirational global fuel efficiency improvement rate of 2 per cent per annum from 2021 to 2050, calculated on the basis of volume of fuel used per revenue tonne kilometre performed.

The development of the ICAO Aeroplane CO₂ Emissions Standard will result in a new Annex 16, Volume III to the Chicago Convention.

The work to develop a CO₂ standard has been undertaken by the ICAO Council's Committee on Aviation Environmental Protection (CAEP). It has been in two phases – the outcome of Phase 1 being the agreement at the CAEP/9 meeting in February 2013 of a CO₂ metric system and the development of a mature Annex 16, Volume III CO₂ Standard certification requirement. This was preceded by the issue of a press release in July 2012 accompanied by a fact sheet⁽²⁾. The ongoing work of Phase 2 will address the Standard setting process, which consists of developing the regulatory limit and applicability data required to complete the CO₂ Standard.

The fact sheet outlined the metric system in general terms. It is based on three elements associated with aircraft technology and design:

- Cruise point fuel burn performance;
- Aircraft size; and
- Aircraft weight.

Figures 1 and 2, taken from the fact sheet, illustrate the essential features of the metric.

The metric agreed at CAEP/9 is set out in ICAO Cir 337⁽³⁾. It involves two parameters, Specific Air Range (SAR) and Reference Geometric Factor (RGF), which are a measure of the cabin floor area in square metres (m²). The circular specifies in detail how these two

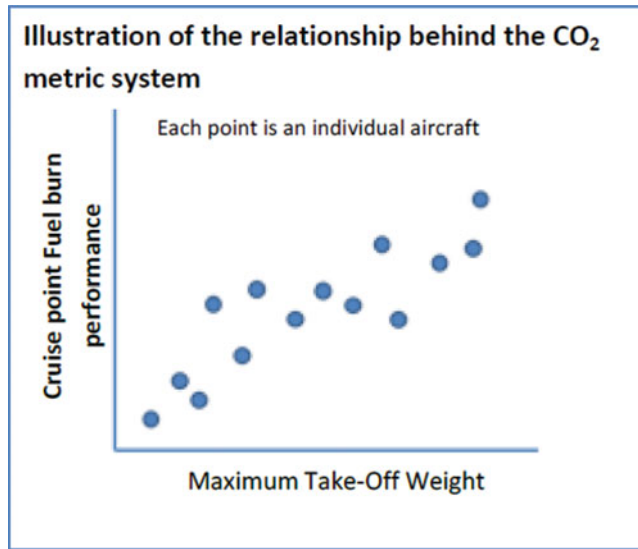


Figure 2. (Colour online) Illustration of the ICAO CO₂ metric.

parameters should be measured and specifies three values of aircraft gross mass at which *SAR* should be determined, with the certification value taken from the average of the reciprocals of the three values of *SAR*.

Cir 337, paragraph 3.2, states, "The intent of this metric system is to equitably reward advances in aeroplane technologies (i.e. structural, propulsion and aerodynamic) that contribute to reductions in aeroplane CO₂ emissions, and to differentiate between aeroplanes with different generations of these technologies". The metric is thus aimed at the airframe and engine manufacturers and the fuel efficiency of the basic aircraft. It does not address the operational factors – seat layout, load factor, range relative to design range – that can have a greater impact on fuel efficiency than variations in the basic aircraft design. These operational factors are important and are primarily the responsibility of individual airlines. Further action by ICAO will be needed to address operational efficiency, but that is outside the scope of Cir 337.

The present paper discusses the metric system set out in Cir 337 against the background of flight physics, the published literature on aircraft fuel burn and CO₂ emissions, and the current practices of the aircraft and engine manufacturers and the airline operators. It concludes that the methods and procedures set out in Cir 337 are well founded and provide a sound basis for future regulation of civil aviation's CO₂ emissions. However, one particular aspect of the formula for the CO₂ metric is open to question, being empirically derived and having the units kilograms/kilometre/metre^{0.48}. The present paper concludes that, with minor adaptation of the formula, a fundamentally based metric can be obtained which makes full use of the procedures set out in Cir 337 and has the generally accepted units kilograms per tonne-kilometre. This metric can be robustly justified by reference to the laws of physics and the publicly available data, historical and current, on jet transport aircraft and would treat all classes and sizes of aircraft fairly. It would also provide a sound and transparent basis for carrying the CO₂ Standard forward into the future, increasing stringency as technology standards advance. Further, it would provide a sound basis for developing a rational approach to the regulation of operational procedures that have a greater impact on fuel efficiency than the details of future aircraft and engine design.

2.0 ENVIRONMENTAL IMPACTS AND SOCIAL BENEFITS OF AVIATION

The three main environmental impacts of civil aviation are noise around airports, air pollution around airports and the contributions of greenhouse gas emissions to climate change. Noise and local air pollution are covered by existing, long-standing ICAO regulations, which, over the years, have become increasingly stringent in line with advances in technology.

Reducing civil aviation's contribution to climate change is a new challenge for ICAO. The contribution comes from three main sources – the emission of CO₂, the emission of oxides of nitrogen at altitude and the creation of contrails and cirrus clouds. The last two of these, though important, are relatively short lived and are thought to be capable of substantial reduction in the foreseeable future. By virtue of its much longer life, CO₂ is generally considered to be potentially the most damaging of the three and it is only CO₂ emission that ICAO is currently addressing. The environmental impact of CO₂ is measured by the mass emitted and, because this mass is directly proportional to the mass of fuel burned, the latter is an equally acceptable measure of environmental impact¹. There is universal acceptance of this proposition.

There is slightly more scope for debate about quantifying the social benefit of civil aviation, but there is general agreement among authors that it should be the product of distance travelled and what is transported. In its 1999 report, *Aviation and the Global Atmosphere*⁽⁴⁾, the Intergovernmental Panel on Climate Change used fuel burned (in kilograms) per passenger-kilometre as a metric (see e.g. Table 8.3 of the report). In the paper⁽⁵⁾ discussing the feasibility of achieving carbon-neutral growth by 2020, presented by the People's Republic of China to the 37th session of the ICAO Assembly, the measure adopted was Revenue Tonne-Kilometres (RTK).

Thus, whether we are considering a single flight, the operation of a fleet of one type of aircraft or the operation of the world fleet, we can express the ratio of environmental cost to social benefit as the ratio of total fuel burned (kilograms) to total payload-range (tonne-kilometres). This is a metric that has been widely used by authors considering the efficiency and environmental impact of aviation. It relates readily to the output of scientific and engineering studies aimed at reducing aircraft CO₂ emissions. Its value, expressed as kilograms of fuel burned per available tonne-kilometre (kg/ATK) to be calculated at the maximum payload maximum range condition, was adopted by the Independent Experts in their report to CAEP⁽⁶⁾. It is stated implicitly in the ICAO resolution at its 37th Assembly, and this paper starts from the premise that kilograms of fuel burned per tonne-kilometre of payload-range is the appropriate measure of the environmental cost-benefit ratio².

3.0 PAYLOAD-RANGE DIAGRAM

The payload-range diagram is used by all manufacturers as one way of defining the capabilities of an aircraft. [Figure 3](#) shows a typical³ payload-range diagram for a long-range wide-body aircraft. The top line AB in the diagram represents the aircraft operating at maximum payload,

¹ In the ICAO circular, kerosene is the only fuel considered. If, however, the fuel were to contain a component derived from biomass sources, an allowance for this would have to be included.

² We follow Cir 337 in adopting kilogram as the unit of mass in the analysis in the paper and introducing a scaling factor of 1,000 where appropriate to give fuel burn per unit payload-range in kilograms per tonne-kilometre.

³ A particular engine-airframe combination may have more than one maximum payload and more than one certified *MTOM*. [Figure 3](#) is an example of one such diagram for a large wide-body aircraft.

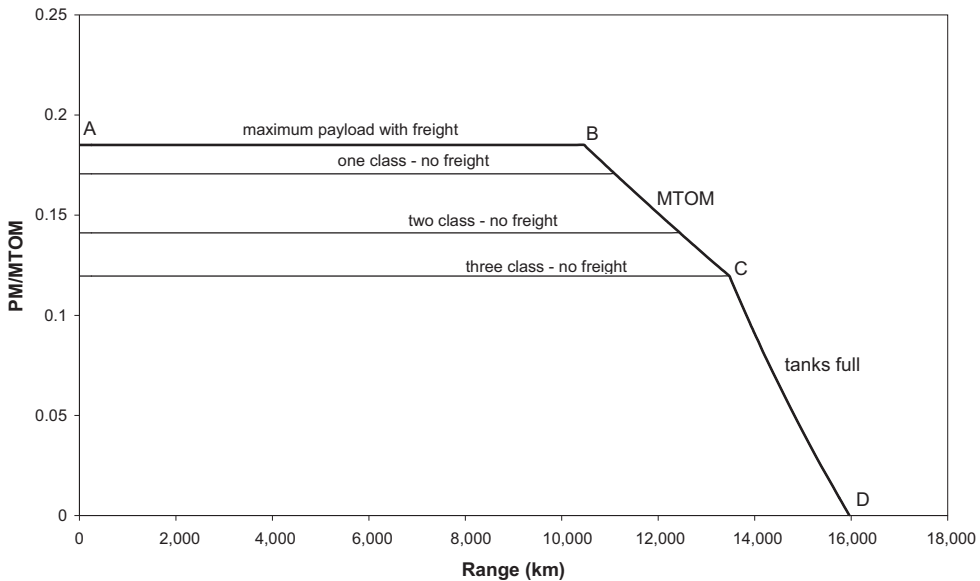


Figure 3. Payload-range diagram for typical wide-body airliner.

with the fuel load being increased progressively as range is increased until, at point B, the total mass of the aircraft plus payload and fuel reaches its certified maximum value (*MTOM*).

In this paper, point B, corresponding to maximum range at maximum payload, is taken as the aircraft design point. This is the point giving the range and payload for which the ICAO Independent Experts⁽⁶⁾ defined fuel burn per tonne-kilometre.

Between points B and C, the aircraft is operating at its maximum take-off mass, with the fuel load progressively increased to achieve the increased range and payload correspondingly reduced. Over this part of the diagram, the effects of the increase in fuel burn and reduction in payload more than offset the increase in range and the ratio of fuel burn to payload-range increases steeply. At point C (not necessarily coincident with the maximum three-class no-freight range), the fuel load reaches the maximum for the aircraft, limited by tank capacity, and range can be increased further only by reducing payload. Beyond point C, the ratio of fuel burn to payload-range increases even more steeply with range, reaching infinity at point D, the maximum ferry range of the aircraft with zero payload.

Along line BC, the ratio of fuel burn to payload-range is at a minimum at point B, with a maximum combined payload of passengers and freight. The intermediate horizontal lines in Fig. 3 typically represent the payloads of the aircraft operating without freight and with one-, two- or three-class seating. As seating density is reduced, the ratio of fuel burn to payload-range increases appreciably.

4.0 FLIGHT PHYSICS

4.1 Specific air range

The CAEP circular prescribes *SAR* as the quantity to be measured in flight to determine the performance of the aircraft. It is defined as the distance in kilometres covered per kilogram of fuel burned at specified measuring conditions. The circular uses its reciprocal $1/SAR$ as part

of the metric. It is possible (see Appendix Equation (A2)) to write an identity:

$$\frac{1}{SAR} = \frac{GM}{(LCV/g)\eta L/D} = \frac{GM}{X} \text{ kg/km}, \quad \dots (1)$$

where GM is the gross mass of the aircraft in kilograms, LCV is the lower calorific value of the fuel, g is the acceleration due to gravity, η is the overall propulsion efficiency, and L/D is the lift-to-drag ratio of the aircraft. The quantities η and L/D are dimensionless, so the product X of the three quantities has the same dimension as LCV/g , which has the dimension length. Both LCV/g and X can be expressed in kilometres (for kerosine LCV/g is about 4,400 km and for a typical modern aircraft X is slightly above 30,000 km).

Equation (1) is an exact statement of the physics. It provides a value for the combined propulsive-aerodynamic efficiency $\eta L/D$, which is the key performance measure. It also shows that, for a particular aircraft at a particular flight condition, the value of $1/SAR$, the rate of fuel burn per kilometre, is proportional, at any particular instant, only to the instantaneous gross mass of the aircraft. In the context of the procedures set out in Cir 337, this is an important fact.

4.2 Breguet range equation

4.2.1 Trip fuel burn per unit payload-range

At its most fuel-efficient, an aircraft operates in the cruise-climb mode at a constant Mach number and lift coefficient and consequently at constant values of η , L/D and X . Cir 337 (para 2.5.1) expects SAR to be measured at this optimum condition. Starting from Equation (1), integration of the rate of fuel burn over the full length of a flight with X constant (see Appendix) results in the classic Breguet range equation, which can be arranged (Appendix Equation (A12)) to give an expression for fuel burned on a trip divided by payload-range for the trip:

$$\frac{1,000}{R} \frac{TFM}{PM} = \frac{1,000}{X} \left(1 + \frac{OEM}{PM} \right) \frac{1.015 - \exp(-R/X)}{(\exp(-R/X) - 0.046)(R/X)} \text{ kg/tonne-km}, \quad \dots (2)$$

where R is the range in kilometres and TFM , PM and OEM are the respective masses in kilograms of the fuel for the trip, the payload and the operating empty mass of the aircraft, with the scaling factor of 1,000 introduced to give fuel burn per unit payload-range in the preferred units. Like Equation (1), Equation (2) is essentially an exact statement of the physics, its only empiricism being the change from 1.0 to 1.015 in the last term to allow for the additional fuel used in climbing to cruise altitude and Mach number and the inclusion of a small⁴ term (0.046) to allow for the minimum reserve fuel (see Poll⁽⁷⁾).

This equation sets out the quantities influencing fuel burn over which the manufacturer and operator have some control. The calorific value of the fuel is essentially fixed and the other two quantities in X , propulsion efficiency and lift-to-drag ratio, which have increased substantially over the past 40 years, are now increasing more slowly as technology advances. In the illustrative analysis that follows, a constant value of 30,700 km has been taken for X , corresponding to values of η and L/D typical of the current generation of civil aircraft ($\eta L/D = 7.0$). The other factors affecting the ratio of fuel burn to payload-range are the range

⁴ Though small, this is of the order of one third of the payload for the longest range aircraft.

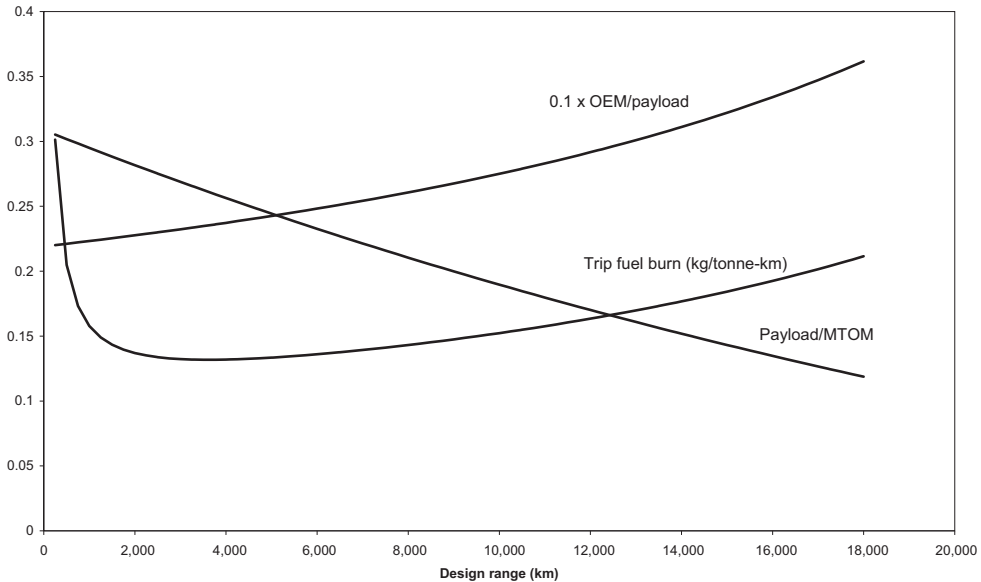


Figure 4. Variation of key ratios with design range.

itself and the ratio of empty mass to payload. The latter depends on the design range of the aircraft, cabin seating layout and the structural efficiency of the design, and here we follow Poll⁽⁷⁾, Equation (A17) in our Appendix.

The importance of design range as a controlling influence is illustrated in Fig. 4. For a given standard of aerodynamic, structural and propulsion technology, fuel burn per unit payload-range at the design point is determined by design range, through its effect on structural mass and fuel load. Figure 4 shows the growth in the ratio of empty mass to payload and the decline in the ratio of payload to *MTOM* as design range increases. It also shows that trip fuel burn per unit payload-range at the design point has a minimum at a range around 3,500 km.

4.2.2 Rate of fuel burn at mid-cruise per unit payload-range

Cir 337 sets out a procedure for determining the rate of fuel burn in cruise as $(1/SAR)_{AVG}$, the average of three values of $(1/SAR)$ measured at three specified aircraft gross masses. Knowledge of the payload is required to determine rate of fuel burn per unit payload range, but Cir 337 does not provide for this. However, the Breguet range equation yields an expression (Appendix Equation (A15)) for the rate of fuel burn per unit payload-range at the cruise half-way point, which is similar in general form to the expression for trip fuel per unit payload-range, Equation (2):

$$\frac{1,000}{PM} \frac{1}{SAR_{\text{mid-cruise}}} = \frac{1,000}{X} \left(1 + \frac{OEM}{PM} \right) \frac{\exp(-R/2X)}{\exp(-R/X) - 0.046} \text{ kg/tonne-km} \quad \dots (3)$$

The variation with design range of the two measures of fuel efficiency, fuel burn per unit payload-range over a trip and rate of fuel burn per unit payload-range at mid-cruise, is shown in Fig. 5.

The fuel burn per unit payload-range over the trip is higher than the fuel burn at mid-cruise, because integration of the fuel used over the cruise gives an average rate of fuel burn higher

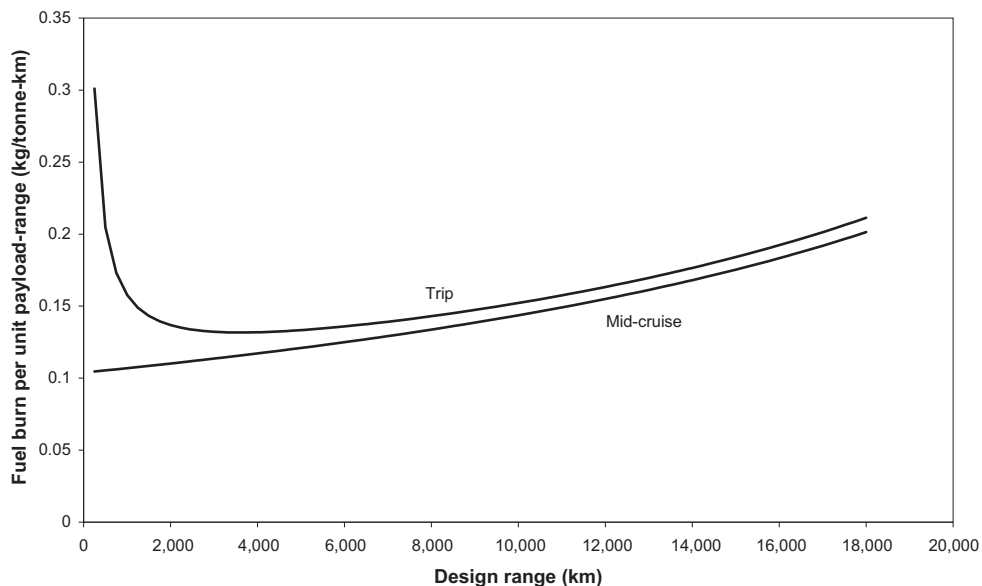


Figure 5. Rates of fuel burn per unit payload-range over a trip and at mid-cruise.

than the rate at mid-cruise. In addition, the increase in trip fuel burn at low design ranges, which gives rise to the minimum in the curve and contributes to the difference between the curves at medium ranges, is the result of the lost fuel used in climbing to cruise altitude and Mach number and an allowance for departure and arrival routings, minus the savings on descent.

It is clear from Equations (2) and (3) that these two expressions for fuel burn per unit payload-range show the same dependency on the combined aerodynamic-propulsive efficiency X and on the structural efficiency PM/OEM . They differ only in the functions of R/X they contain and are related by:

$$\frac{TFM}{R} = \frac{1}{SAR} \frac{1.015 - \exp(-R/X)}{\exp(-R/2X) (R/X)} \text{ kg/km} \quad \dots (4)$$

This again is an essentially exact result derived from the Breguet range equation. Its potential value is that it provides a means of reading across from a measurement of SAR in mid-cruise to obtain the trip fuel burn per unit payload-range not only at the design range but at other significant off-design conditions which may need to be considered in future ICAO regulations.

5.0 ILLUSTRATIVE PERFORMANCE CALCULATIONS

5.1 Basis of calculations

The fuel burn of civil aircraft is an important competitive issue and a commercially sensitive matter. The approach taken in this paper is to perform illustrative calculations of fuel burn per unit payload-range for a number of aircraft which have the same payload-range diagrams

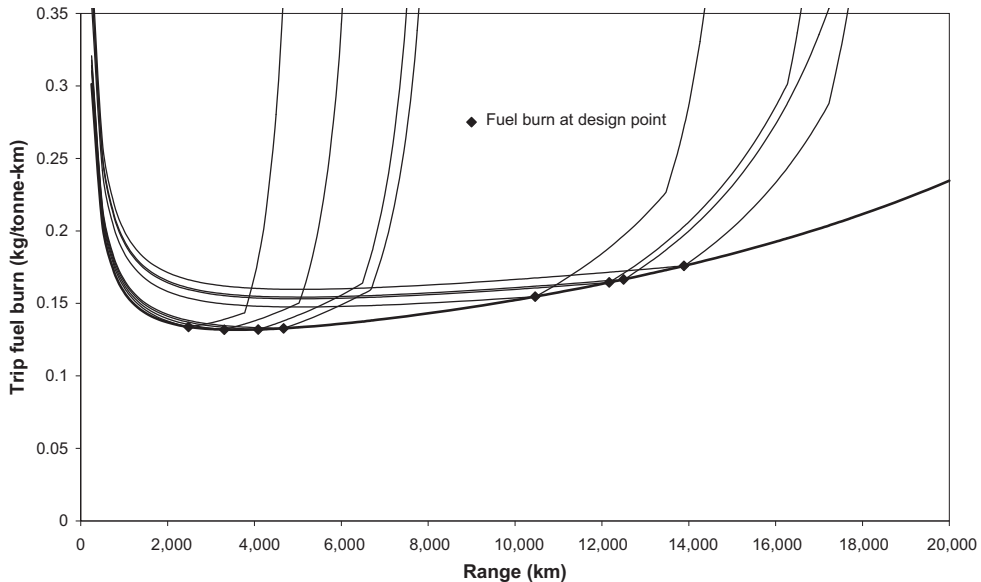


Figure 6. Variation with range of trip fuel burn per unit payload-range.

as selected current types. The calculations use Equation (2) above, with the range parameter X taken as a constant 30,700 km. The empty weight has been determined as a function of maximum take-off mass and payload following Küchemann⁽⁸⁾, as reformulated more recently by Poll⁽⁷⁾. Equation (19) of his paper is reproduced here in the Appendix as Equation (A17). With these assumptions, the calculations can only be illustrative. Even so, the likely difference between the calculations and the actual performance of current types with similar payload-range diagrams is relatively small compared to the variations due to other factors influencing fuel burn. It does not change the main conclusions.

Figure 6 shows the fuel burn per unit payload-range for eight illustrative aircraft with the same payload-range diagrams as eight aircraft currently in airline and business service. The group includes three long-range wide-body aircraft, three short-medium range single-aisle aircraft and two business jets, one small and short range, the other larger and long range. The smaller is well above the lower *MTOM* limit of applicability of the ICAO circular. The vertical axis is the fuel burned on the trip in kilograms per tonne-kilometre of payload-range and the horizontal axis is the range in kilometres. The lowest curve, which forms an envelope to the others, is the design range curve, showing the variation with design range of point B on the payload-range diagram. The symbols show the design points of the eight illustrative aircraft.

5.2 Effect of design range and operating range

As noted above, the design range curve has a minimum at around 3,500 km and this fact has been used in several studies of the effect of design range⁽⁹⁻¹¹⁾. The three short-medium range types and the small business jet have fuel burns very close to this minimum value. For the illustrative aircraft with the longest design range, nearly 14,000 km, the fuel burn per unit payload-range at the design point is some 30% greater than that of the aircraft with the

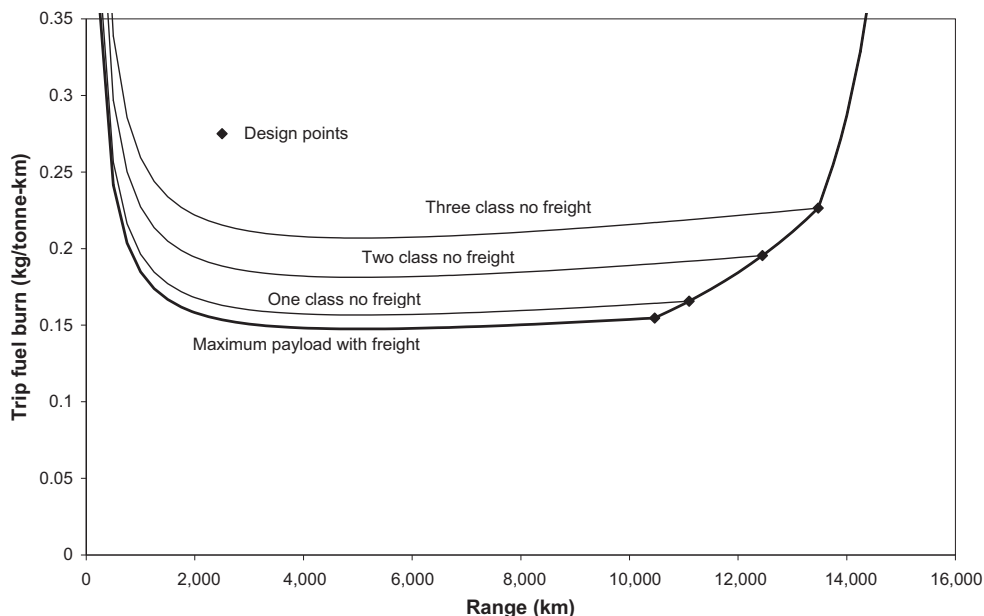


Figure 7. Variation with seating layout of trip fuel per unit payload-range.

shortest design range⁵. The controlling parameter, the design range, is not featured in the metric proposed by CAEP.

Figure 6 shows the upward kink in the fuel burn curves at the design point, point B on the payload-range diagram, beyond which the trip fuel burn begins to increase rapidly as range is increased. Beyond the second kink in the curves, corresponding to point C on the payload-range diagram at which the maximum fuel load, limited by tank capacity, is reached, the curves begin to climb even more steeply. At point C, for the illustrative aircraft with the longest design range, the fuel burn per unit payload-range is more than twice that of the shortest range aircraft at its design point.

5.3 Effect of cabin layout

The typical payload-range diagram in Fig. 3 shows four alternative payloads: the maximum payload, corresponding to an unspecified combination of passengers and freight; a full single-class passenger payload with no freight; a full two-class payload; and a full three-class payload. Figure 7 shows the fuel burn for an illustrative aircraft with this payload-range diagram. The symbols show the design range points for these various arrangements. The full three-class arrangement at its design range burns nearly 50% more fuel per tonne-kilometre than the fully loaded aircraft at its design range and 70% more fuel per tonne-kilometre than the shortest range aircraft at its design point. The effect of cabin layout, including current and future developments, such as the increasing use of four-class layouts on long-haul flights, is outside the scope of the metric proposed by CAEP but is clearly important. It is an operational question with, as Figs 6 and 7 show, an impact on CO₂ emissions similar in magnitude to operation at ranges appreciably longer than the design range. These are matters for the

⁵ As discussed in the Appendix, the tendency for X to increase with design range will reduce this difference.

operators rather than the manufacturers and will require further regulation, probably involving some form of incentive, perhaps in the form of market-based measures.

6.0 THE CAEP CO₂ METRIC

6.1 The basic formula

The CO₂ metric agreed by CAEP/9 and set out in Cir 337 involves two quantities. The *SAR* is the distance in kilometres covered per kilogram of fuel consumed when the aircraft is flying at its optimum cruise condition as specified in the circular. The *RGF* is the cabin floor area in square metres, measured as prescribed in the circular.

The circular simply states (Chapter 2, paragraph 2.2), “The metric shall be defined in terms of $(1/SAR)_{AVG}/RGF^{0.24}$, where $(1/SAR)_{AVG}$ is the average of the $1/SAR$ values established at each of the three reference masses defined in 2.3”. No analysis is presented to explain or justify this formula and it is not clear if there is a difference in meaning between ‘shall be defined in terms of’ and ‘shall be’. In this paper, it is taken to mean ‘shall be’.

6.2 Reference aircraft masses

Cir 337 specifies that the $1/SAR$ value shall be determined at each of three reference aircraft masses:

- a) high gross mass: 92% *MTOM*
- b) mid gross mass: Average of high gross mass and low gross mass
- c) low gross mass: $(0.45 \times MTOM) + (0.63 \times MTOM^{0.924})$

Figure 8 shows the variation of these three reference gross masses with *MTOM*. The rationale for introducing three reference masses, to represent the aircraft at different stages of its cruise for typical payloads and missions, is set out in paragraph 3.3 of the circular. (The low gross mass is below the Maximum Landing Mass (MLM) for an aircraft operating with maximum payload, which is the mission taken as reference by the ICAO Independent Advisers⁽⁶⁾). The equation for the metric involves $(1/SAR)_{AVG}$, which is the average of the three values of $1/SAR$ (i.e. rate of fuel burn in kilograms per kilometre) measured at the three reference masses appropriate to the *MTOM* of the aircraft⁶.

As stated in paragraph 2.5.1 of the circular, the reference conditions ‘are generally to be expected to be the combination of airspeed and altitude that results in the highest *SAR* value, which is usually at the maximum range cruise Mach number at the optimum altitude’. These conditions will yield the optimum combined propulsion and aerodynamic efficiency $\eta L/D$, which will be effectively constant and give a constant value for the range parameter X . Hence, from Equation (1), the instantaneous value of $1/SAR$ throughout cruise will be directly proportional to the instantaneous aircraft Gross Mass (*GM*). It is, thus, simple arithmetic to show that the average of the three measurements of $1/SAR$ will be the value of $1/SAR$ at the mid gross mass. Besides providing a check on consistency and accuracy, the value of measuring *SAR* at three masses is to obtain data at three altitudes and hence at three Reynolds

⁶ There is a problem with dimensions here. The high gross mass has the dimension kilograms; the other two have no legitimate dimensions. The actual gross mass at which *SAR* is to be measured at the mid and low masses is presumably the numerical value, in kilograms, obtained by evaluating the term in brackets in Equation (5) with *MTOM* in kilograms.

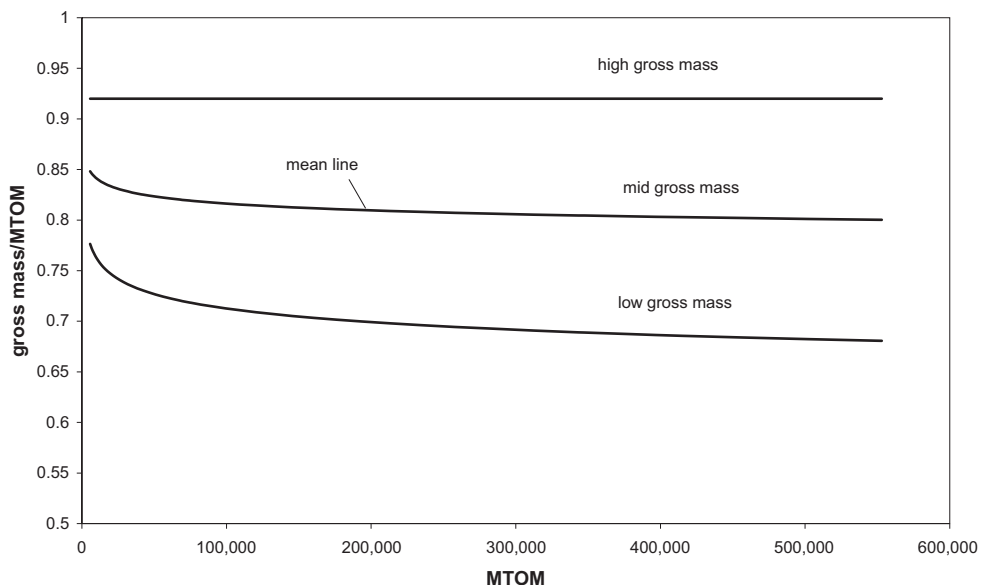


Figure 8. Variation with MTOM of the three CAEP reference aircraft masses.

numbers. The effect of Reynolds numbers is small, however, and likely to be lost in the noise. If the effect is neglected, the average value of SAR^7 is given by:

$$\frac{1}{SAR_{AVG}} = \frac{1}{X} (0.685 MTOM + 0.315 MTOM^{0.924}) \quad \dots (5)$$

An important conclusion from Equation (1) is that, since the lower calorific value of the fuel is required to be known with precision, an accurate measurement of SAR , obtained at an accurately known aircraft gross mass, will enable the determination of an accurate value of the combined propulsion and aerodynamic efficiency of the aircraft-engine combination, $\eta L/D$. In reality, it is not important to measure SAR at a precise gross mass, only to know precisely the gross mass at which SAR is measured. Accurate knowledge of $\eta L/D$ is important in determining all measures of fuel burn per unit payload-range discussed in this paper.

It is worth noting that SAR_{AVG} and SAR at the cruise mid-point are related (see Equation (A6)) by:

$$\frac{1}{SAR_{mid-cruise}} = \frac{1}{SAR_{AVG}} \frac{0.985 \exp(-R/2X)}{0.685 + 0.315 MTOM^{-0.076}} \quad \dots (6)$$

This relationship enables a read-across, via Equations (3) and (4), from SAR_{AVG} determined following the procedures of Cir 337 to the fuel burn per unit payload-range at the cruise mid-point and also for the trip. As set out in the Appendix, all other aspects of fuel

⁷ Equation (5) gives values of gross mass, and hence of SAR_{AVG} , lower than those given by the Breguet range equation at the cruise mid-point at the design range, Equation (3), by about 5% for the wide-bodies in Fig. 6 and about 10% for the single-aisle aircraft.

burn illustrated in Figs 6 and 7 can then be determined from the Breguet range equation, with the precision entailed in the use of the equation.

6.3 Reference geometric factor

The CAEP circular introduces the concept of Reference Geometric Factor (*RGF*) as a measure of the aircraft cabin size. It is defined as the cabin floor area in square metres determined by rules set out in Circular 337 and is explained in the following terms:

“In some aeroplane designs, there are instances where changes in aeroplane size may not reflect changes in aeroplane weight such as when an aeroplane is a stretched version of an existing aeroplane design. To better account for such instances, not to mention the wide varieties of aeroplane types and the technologies they employ, an adjustment factor was used to represent aeroplane size.This improved the performance of the CO₂ metric system, making it fairer and better able to account for different aeroplane designs”

Despite the clear statement in the resolution at the 37th ICAO Assembly that fuel efficiency should be calculated on the basis of volume of fuel used per revenue tonne-kilometre performed, there is no reference whatever to payload in Cir 337. Even so, *RGF* happens to have the merit of being a fair indicator of maximum passenger payload with single-class seating. From an analysis of the current fleet, Poll^(12, Fig. 1) showed that the average floor area required for a passenger in a maximum density, single-class cabin layout is close to 0.65 m². This value includes allowances for aisles, toilets, galleys and cabin crew areas. The passenger capacity of the 44 Boeing and Airbus aircraft included in Poll’s study all fall within roughly ±15% of this average. Given that the service entry dates of these 44 aircraft span 40 years, over which passenger accommodation standards have varied somewhat, a fair amount of scatter in the data is to be expected. Some variation is also to be expected, and can be discerned in the data, between single-aisle and twin-aisle aircraft and also between single- and double-deck aircraft.

If we stay with Poll’s analysis and take the average mass of a single-class passenger plus baggage as 95 kg, we obtain:

$$\text{maximum total single-class passenger payload} = 146 \times RGF \text{ (kg)} \quad \dots (7)$$

So, although Cir 337 makes no reference to payload, in specifying *RGF* as one of the two parameters of the metric, it provides the basis for an engineering approximation to the maximum single-class passenger-only payload of an aircraft, *MPM*, as:

$$MPM = 150 RGF \text{ kg} = 0.15 RGF \text{ tonne} \quad \dots (8)$$

This value could reasonably be used for estimates of fuel burn per unit payload-range.

6.4 Units of the metric

If we accept Equation (8) as an engineering approximation to a ‘standard’ maximum passenger-only payload, it follows that at this condition:

$$\text{mid-cruise fuel burn per unit payload-range} = \frac{1}{SAR_{\text{mid-cruise}}} \times \frac{1}{0.15 RGF} \text{ kg/tonne-km} \quad \dots (9)$$

is an estimate of minimum fuel burn per unit payload-range in cruise that represents the flight physics correctly and is a statement of the ratio of environmental impact to social benefit.

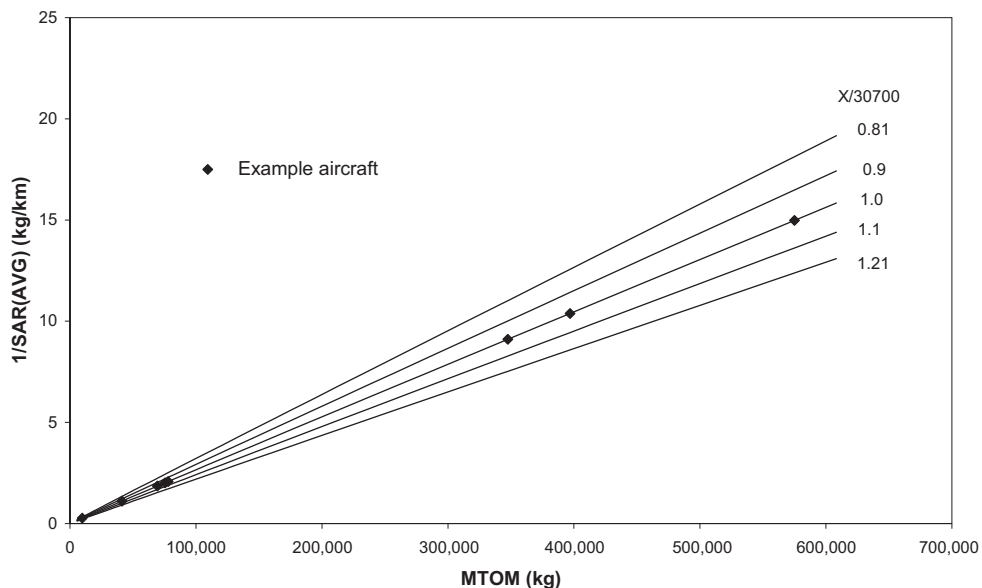


Figure 9. Variation of fuel burn with $MTOM$ and X .

Undoubtedly, within engineering accuracy, this is true. If CAEP had proposed this as the metric, the present paper might not have been written, with the proviso that, as noted under Equation (5), the CAEP $(1/SAR)_{AVE}$ is significantly lower than $1/SAR$ at mid-cruise with a maximum payload.

However, the metric of Cir 337, $(1/SAR)_{AVG}/RGF^{0.24}$, has the units kilograms/kilometre/metre^{-0.48}. It does not represent fuel used per tonne-kilometre performed and has been derived empirically.

6.5 $MTOM$ as aircraft design parameter

Paragraph 3.5 of Cir 337 states, “The overall design of the aircraft is represented in the CO₂ metric system by the certified $MTOM$. This accounts for the majority of aircraft design features which allow an aircraft type to meet its market demand”. We understand that the intention is to correlate the chosen metric against $MTOM$ as well as any chosen regulatory capping value. This is discussed further below.

Paragraph 3.3 of the circular states, “The fuel efficiency performance of an aeroplane is represented by $1/Specific\ Air\ Range\ (SAR)$ ”. As noted in Section 6.2, Equation (5) is an exact relationship between $1/SAR_{AVG}$, $MTOM$ and the range parameter $X (= (LCV/g)\eta L/D)$, its particular form being determined by the specification of the reference masses.

Figure 9 shows the variation with $MTOM$ of $1/SAR_{AVG}$, with $1/SAR_{AVG}$ as specified in Cir 337, for five different values of X . These are spaced at 10% geometric intervals with the middle line, $X = 30,700$ km, representing the technology standard of current aircraft with a combined aerodynamic and propulsive efficiency $\eta L/D$ of 7.0. The eight example aircraft are defined by their values of $MTOM$ and are plotted on the line for $\eta L/D = 7.0$, $X = 30,700$ km. Advances in aerodynamic and propulsive efficiencies will move aircraft down the page to higher values of X .

Plotting values of $1/SAR_{AVG}$ and $MTOM$ for a particular aircraft on a chart of the form of Fig. 9 enables its value of X to be determined precisely. This is not a correlation but a

statement of flight physics. Plotting values for a range of aircraft will produce a degree of scatter because of variations in X , as shown schematically in Fig. 2.

Dividing $1/SAR_{AVG}$ by $RGF^{0.24}$ to produce the metric appears to be a device for reducing the scatter. If two aircraft have the same $MTOM$ and the same standards of aerodynamic and propulsion technology, the aircraft with the greater floor area, RGF , will also have the greater fuselage wetted area and drag and, hence, the lower values of L/D and X . It will plot further up the page. Scaling both values of $1/SAR_{AVG}$ down by dividing by RGF to a small power will reduce the difference between the two aircraft. For aircraft of a single family but different lengths of fuselage, and therefore different values of X , this kind of scaling can reduce the scatter due to variations in X . We conjecture that the exponent 0.24 has been chosen to minimise overall scatter across the range of aircraft used in the study. This is a correlation. If we are correct in this supposition, and if a non-dimensional form had been chosen for the scaling factor by, say, dividing floor area by gross wing area, a correlation that reduces the scatter across aircraft families without compromising dimensional integrity might have been found.

6.6 Fuel used per revenue tonne-kilometre performed

The declared aim of ICAO is to increase global fuel efficiency ‘calculated on the basis of volume of fuel used per revenue tonne-kilometre performed’. *The CAEP circular does not speak to this question directly and neither Fig. 9 nor a plot of the metric against MTOM enables it to be determined.* The measurement of $1/SAR$ gives the fuel used per kilometre, but there is no reference to mass in the circular other than the maximum certificated take-off mass $MTOM$ and the three reference aircraft masses at which SAR is measured. Without a measure of the payload, Cir 337 falls short of the goal set for it by the 37th Assembly.

Fuel used per revenue tonne-kilometre performed is given by the Breguet range equation, Equations (2) and (3). These set out clearly its dependence on the three determining parameters – the range parameter X , the ratio R/X of range to this parameter, and the term $(1 + OEM/PM)$, which can also be written as ZFM/PM . As noted above, the value of X can be determined precisely from Equation (5) by the measurement of SAR as specified in Cir 337. This leaves the ratio of zero-fuel mass to payload as the remaining fundamental quantity needed to determine fuel used per revenue tonne-kilometre over a specified range.

The factors that influence ZFM/PM are the efficiency of the structural design, the materials used in construction and, above all, the design range. Figure 4 shows the steady growth of OEM/PM with design range. Poll⁽¹²⁾, in a study of the masses of 44 different Boeing and Airbus aircraft types, extrapolated values of ZFM/PM back to zero design range for each type, in order to eliminate the influence of design range, for the case of maximum single-class passenger-only payload. This provided a measure of the efficiency of structural design for each type. He found, shown in Figs 6 and 7 of the paper, that there was no discernible trend of the ‘zero-range’ values of ZFM/PM at the maximum payload condition, either with the size of aircraft or with the date of entry into service.

The latter result was considered surprising, given the increased use of lightweight materials in recent years. However, in a subsequent paper, Poll⁽¹³⁾ showed that the lack of any reduction in this quantity reflected a fundamental feature of aircraft design optimisation. Potential weight-saving measures, either through more advanced design and manufacturing methods or

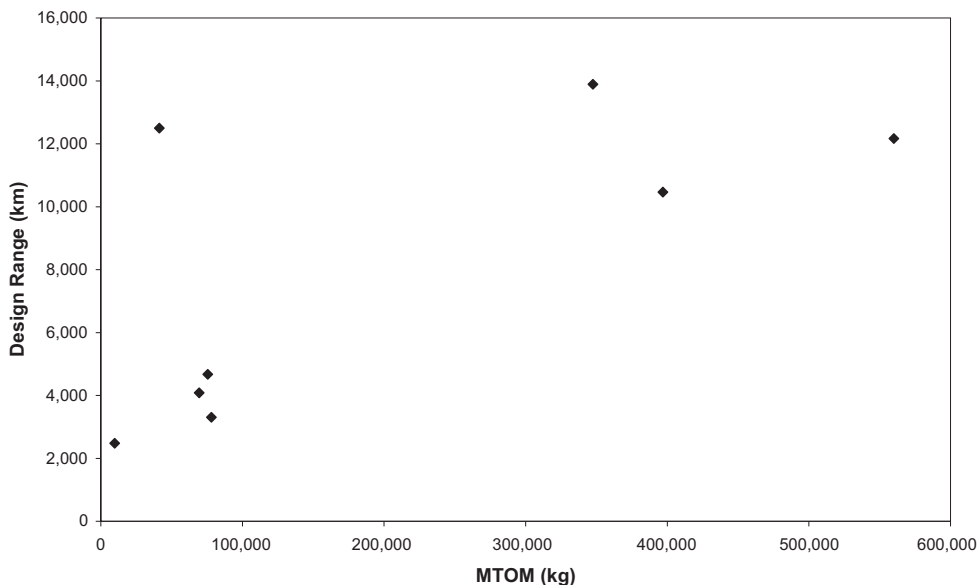


Figure 10. Map of design range against *MTOM* for eight example aircraft types.

the use of lighter materials, are better used to improve aerodynamic and propulsion efficiencies at constant weight (through increased aspect and bypass ratios) than to reduce weight.

The fact that advances in structures and materials have been, and will continue to be, manifest as an increase in X rather than a reduction in ZFM/PM increases the importance of design range as the primary determinant of this mass ratio and of the fuel used per revenue tonne-kilometre performed.

6.7 Design range as aircraft design parameter

There is, therefore, a powerful case for adopting design range rather than *MTOM* as the basic parameter to characterise aircraft design. The variation of key ratios shown in Fig. 4 makes clear the fundamental importance of design range, and the Breguet range equation makes clear the dependence on it of fuel efficiency at the design point. There is a general trend for *MTOM* to increase with increasing design range, but there are exceptions to this: the long-range business jet in our example aircraft is one; the proposed^(14,15) Large Aircraft for Short Ranges (LASR) is another. The LASR concept was proposed specifically because, for an aircraft with a 300-passenger payload, redesigning the aircraft for a reduced range of 2,800 km rather than 11,800 km was calculated to reduce Maximum Zero-Fuel Mass (*MZFM*) by 20% for the same payload and, despite a reduction in L/D , to reduce fuel burn per passenger-kilometre by 13% when operated over the same range. At their design ranges, the original long-range aircraft had a fuel burn per passenger-kilometre some 26% higher than the LASR. There would be the same difference between the LASR and a long-range design that was a 90%-scale version of the original with a payload of approximately 240 passengers and the same *MTOM* as the LASR.

Figure 10 shows the design range plotted against *MTOM* for the eight example aircraft types used in earlier figures. From the spread of the points, it is obvious that the two parameters are

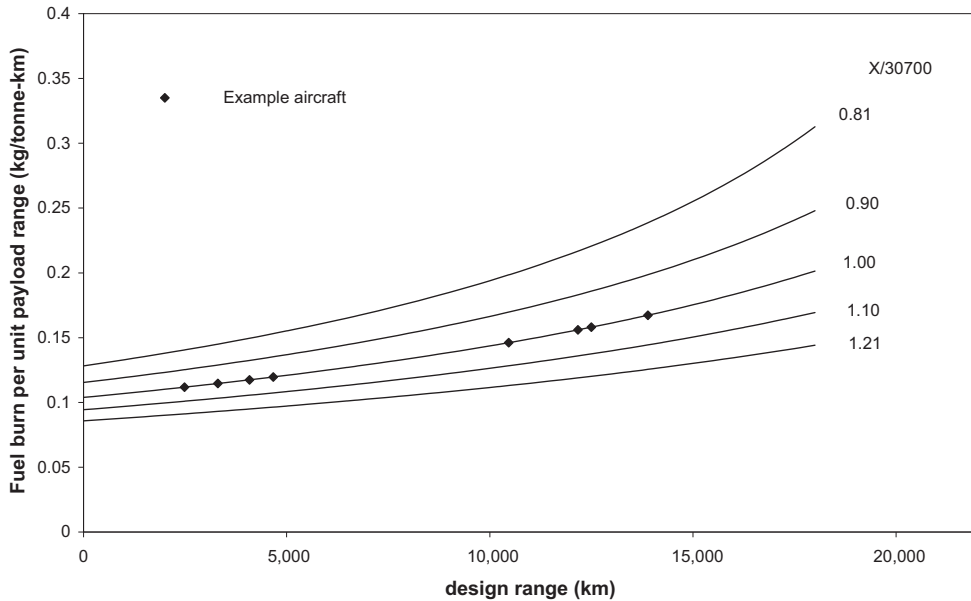


Figure 11. Variation of mid-cruise rate of fuel burn with design range and X .

not well correlated, and we submit that, in the present context, $MTOM$ is not an appropriate parameter for characterising overall aircraft design.

Taking now the design range as the appropriate aircraft design parameter, Fig. 11 shows the rate of fuel burn per unit payload-range as given by the Breguet range equation, Equation (3), at the cruise half-way point. The term $(1 + OEM/PM)$ in Equation (3) has been evaluated from Poll's correlation⁽⁷⁾ as described in Section 5.1. As in Fig. 9, the curves are for values of X at 10% intervals and the diamonds show fuel burn per unit payload-range for the eight illustrative aircraft at their design ranges for a value of 30,700 km for X .

The curves in Fig. 11 combine the Breguet range equation applied at mid-cruise with an empirical expression due to Poll⁽⁷⁾ for the ratio of maximum zero-fuel mass to maximum payload $1 + OEM/MPM (= MZFM/MPM)$. In Fig. 12, the solid triangles represent fuel burn values in which the Poll expression in Equation (3) has been replaced by the ratio of certificated $MZFM$ to maximum payload MPM approximated by $MPM = 0.2RGF$. These are estimates for the eight example aircraft types of the fuel burn per unit payload-range given by the Breguet range equation with maximum payload taken as proportional to RGF . Taking the constant of proportionality as 0.2 tonne/m² rather than 0.15 reflects the average of the actual values of the mass ratio and the fact that the actual maximum payloads included freight and were some 33% greater than the maximum single-class passenger-only payload. The curve is for $X = 30,700$ km. The scatter of the triangular points reflects scatter in the payload mass per square metre.

The open squares in Fig. 12 are the values of the Cir 337 metric, calculated for the eight example aircraft and scaled by a factor of 1/8 to fit on the chart. The plot shows that, while the proposed metric may improve correlation of fuel burn in kilograms per kilometre of itself, it appears not to be a potential means of regulating fuel used per tonne-kilometre performed.

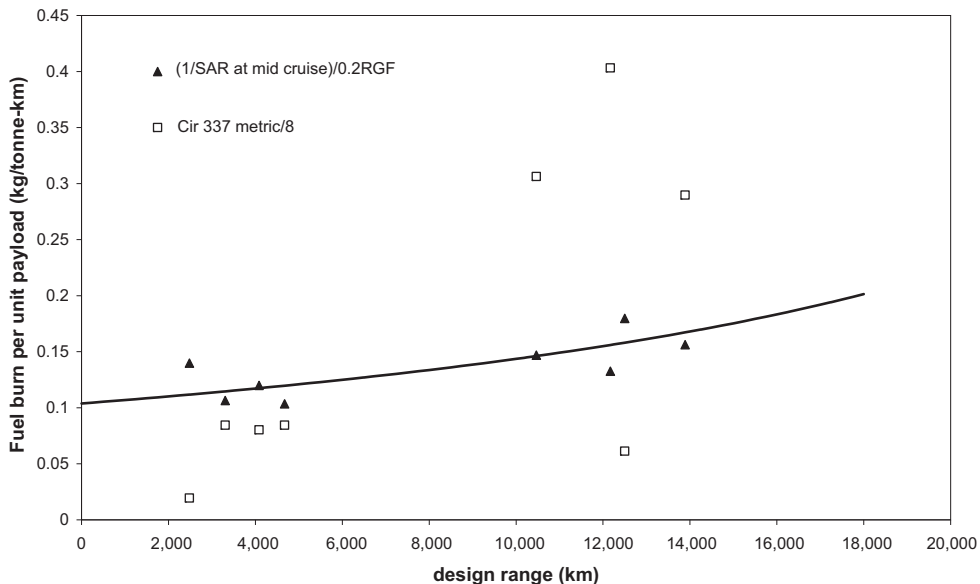


Figure 12. Fuel burn per tonne-kilometres at mid-cruise.

6.8 Specifying design range and payload

If design range is to be used in the certification process, it must be capable of being defined objectively and robustly by the certifying authority. Cir 337 provides the basis for this in specifying the measurement of SAR at a known gross mass, which enables the combined aerodynamic and propulsive efficiency and the range parameter X to be determined precisely from Equation (5).

The design range of the aircraft, i.e. the maximum range at maximum take-off mass and maximum payload, is then determined by the maximum fuel load at these conditions, $MTOM - MZFM$, by $MTOM$ and by X . The Breguet range equation gives (Appendix Equation (A19)):

$$R_{des} = X \ln \left(\frac{(1 - \epsilon) MTOM}{MZFM + \beta_{min} MTOM} \right), \quad \dots (10)$$

where ϵ represents the fuel lost in climbing to cruise altitude and Mach number, and β_{min} is the minimum reserve fuel fraction. Poll⁽⁷⁾ suggests values of 0.015 for ϵ and 0.045 for β_{min} , but for any specific aircraft, we can write:

$$\beta_{min} = \frac{MLM - MZFM}{MTOM} \quad \dots (11)$$

This leaves Equation (10) as an effectively exact definition of design range, subject only to the small uncertainty in the value adopted for ϵ .

Determining maximum payload for the purpose of certifying CO₂ emission is less straightforward. Equations (2) and (3), which give the trip fuel and mid-cruise fuel burn in kilograms per tonne-kilometre both include the term $(1 + OEM/PM)$, which, at the design point, is $MZFM/MPM$. While $MZFM$ is a certificated quantity, MPM is specified in the

manufacturer's payload-range diagram and is determined by $OEM (= MZFM - MPM)$, which is not a certificated quantity and can vary with customer requirements. One possibility is to develop a standardised process for defining OEM and hence MPM for the purpose of certifying CO_2 emissions. An alternative is to use a relation of the form of Equation (8) to determine the maximum one-class passenger payload from RGF , measured following the procedures set out in Cir 337. However, it is not self-evident that 'maximum single-class passenger-only' is the most appropriate measure of payload. Therefore, we suggest introducing 'effective floor loading' EFL , which is the ratio of payload to floor area (PM/RGF) in tonnes per square metre, as a factor that can characterise the combination of passengers and freight loading considered appropriate for regulation purposes. The quantities specified in Cir 337 can then be determined as proposed in the circular and combined to give:

$$\text{Fuel burn per unit payload-range} = \frac{1}{SAR_{AVG}} \frac{1}{RGF \cdot EFL} \quad \dots (12)$$

Determination of an appropriate value of EFL for regulation purposes should draw upon current aircraft design and operating practices rather than on the historic data analysed by Poll. In this context, it is worth noting that in August 2015, Boeing revealed⁽¹⁸⁾ changes to the generic range performance of its current aircraft family to update its standard rule for passenger/luggage weight, increasing it by 15lb per passenger to reflect changes in passengers and seating practice over the past 20 years.

6.9 A work in progress

In defining its purpose, Cir 337 states:

2.2 The material contained in this circular is based on a document that is a work in progress, and CAEP has identified future CAEP/10 work where further discussions, analyses, and decisions would help reinforce the Annex 16, Volume III CO_2 Standard certification requirement and facilitate its subsequent implementation. As a result, additional amendments are expected over the next three years to the Annex 16, Volume III CO_2 Standard certification requirement text contained within this circular.

Encouraged by the last sentence, we suggest that work to reinforce the Annex 16, Volume III CO_2 Standard certification requirement – the setting of the Standard, and subsequent amendments to increase stringency as technology advances – should be based on the flight physics embodied in the Breguet range equation. In particular, the controlling variable against which fuel burn performance is plotted should be design range rather than $MTOM$. It would be quite possible to define a regulatory cap which is a function of design range and which would have a much firmer scientific basis than one based on $MTOM$, which is what we understand is currently proposed.

From the data base of the fuel burn of existing aircraft, which presumably includes values of SAR at defined gross masses and certificated values of $MTOM$, $MZFM$ and MLM , design range can be determined from Equations (10) and (11). In order to meet the aim of regulating fuel burn per payload tonne-kilometre, a form of Equation (12) could be introduced, with an appropriate constant of proportionality EFL , to measure maximum payload. The distribution of the actual performance of current types on a chart of the metric $(1/SAR)/RGF$ against the design range would provide a basis for determining the first regulatory limit and a basis for future increases in stringency. Such a chart would enable account to be taken of the increase in propulsion efficiency and lift-to-drag ratio with increasing design range that is a feature of current aircraft designs that is ignored in the simplified calculations of this paper.

7.0 CONCLUDING REMARKS

At the 37th Assembly of ICAO in 2010, it was resolved ‘to achieve a global annual average fuel efficiency improvement of 2 per cent.... calculated on the basis of volume of fuel used per revenue tonne kilometre performed’. The instrument to pursue this goal will be the development of the ICAO Aeroplane CO₂ Emissions Standard for a new Annex 16, Volume III of the Chicago Convention. ICAO Cir 337 sets out the certification requirement for this Standard, agreed at CAEP 9, but still described as ‘a work in progress’.

In this paper, we have reviewed Cir 337 against the background of flight physics, the published literature on aircraft fuel burn and CO₂ emissions, and the current practices of the aircraft and engine manufacturers and the airline operators. We have taken as our starting point the aim of ICAO to reduce the fuel used per tonne-kilometre performed. Consequently, we believe that the metric to be incorporated in an ICAO CO₂ Emissions Standard should be rigorously based on fuel burned per unit payload-range, expressed in kilograms per tonne-kilometre.

The cornerstone of our analysis is the Breguet range equation. This is an absolutely robust, virtually exact equation from which, like the Second Law of Thermodynamics, there is no escape. It sets out, transparently, the influence of the four variables that determine fuel burn per unit payload-range. Two of these, propulsion efficiency and lift-to-drag ratio, are the responsibility of the aircraft and engine manufacturers. The third, the ratio of the aircraft empty mass to payload, is the combined responsibility of the airframe manufacturer and the airline operator. The fourth, the range over which the aircraft is flown on any particular day, is the responsibility of the operator.

Cir 337 addresses only the fuel efficiency of the basic aircraft design. That is as it should be. The ICAO regulation is to provide a CO₂ standard applicable to new, and possibly also to ‘in production,’ aircraft types and is aimed clearly at the aircraft manufacturers. It should, therefore, address only those factors that the manufacturer can influence. For an aircraft designed to carry passengers, most important among these are passenger capacity and design range. These are set by the manufacturer after discussion with airline customers but are essentially the manufacturer’s decision. For a particular aircraft, the design range can be determined, precisely and unambiguously, from the measurement of *SAR* as specified in Cir 337 and the key certificated aircraft masses. Because of its influence both on the mission weight (including fuel load) and on the aircraft mass empty, and hence, on fuel burn per unit payload-range, design range should be incorporated explicitly in the metric system.

There appears to be some agreement that the Standard should apply to the fuel efficiency of an aircraft operating at its most efficient, which is at its design range, the range at maximum take-off mass and maximum payload (but, as noted in 6.2, the currently proposed reference masses exclude this possibility). Two possibilities of specifying fuel burn per unit payload-range are considered, the total fuel used on a trip divided by the payload and range, and the rate of fuel burn per kilometre in mid-cruise divided by the payload. Whilst total trip fuel per tonne-kilometre is the more widely used, rate of fuel burn in mid-cruise is also an acceptable way of characterising the fuel efficiency of the basic design. The Breguet range equation provides expressions for both, [Equations \(2\)](#) and [\(3\)](#) above.

Operational variables – seating arrangements, load factors, range relative to design range – all have, potentially, a greater influence than aircraft design on fuel burn per unit payload-range. [Figure 6](#) shows the increase in trip fuel burn on stages longer than the design range and [Fig. 7](#) shows the increase in fuel burn per unit payload caused by multi-class seating. These are determined by the operators and are outside the scope of the envisaged ICAO regulation.

They must be addressed by subsequent measures, possibly market-based. The Breguet range equation provides a natural and robust framework for formulating such measures. The relevant equations are set out in the Appendix to this paper. The measurement of in-cruise fuel burn as specified in Cir 337 together with other defined aircraft data will provide all the information needed.

The Breguet range equation shows clearly that a fundamental aircraft design parameter is the design range. Whilst, for a given technology standard, fuel burn increases in direct proportion to *MTOM*; it is design range, through its effect both on the increased weight due to the fuel carried and on the ratio of aircraft empty mass to payload, that determines fuel burn per unit payload capability. The paper illustrates the anomalous results of using *MTOM*, and we believe that, in the context of the ICAO CO₂ Standard, the case for adopting design range as the correlating parameter is scientifically irrefutable.

Our overall conclusion is that Cir 337 could provide an excellent basis for the initial regulation of aviation's CO₂ emissions. So far as we can judge, the measurements specified and the processes for capturing them are satisfactory in every respect. Used in conjunction with the certificated quantities *MTOM* and *MZFM* and the Breguet range equation, they could offer a robust and transparent basis for regulating the CO₂ emissions of future civil aircraft designs and, further in the future, of developing measures to increase the fuel efficiency of the operational side of civil aviation.

Our main criticism of the circular in its current form is that it does *not* correctly address the ICAO goal of reducing fuel used per revenue tonne-kilometre performed and makes no direct reference to payload. However, regarding the latter, we believe that the factor *RGF*, as a measure of available cabin floor area, is an acceptable surrogate for the relevant maximum payload – i.e. the maximum number of passengers that could be carried. The main defect of the proposal could be eliminated simply by removing, in the current document, the exponent 0.24 of *RGF* in the formula for the metric given in Chapter 2 (paragraph 2.2). Retaining *RGF* to the power unity in the metric and multiplying it by an appropriate value of the *EFL* converts it to what the 37th Assembly called for, a statement of fuel used per revenue tonne-kilometre performed.

Finally, correlating the amended metric against design range, as determined from the measured *SAR* and the key certificated masses, provides a sound scientific basis for an initial regulation to cap passenger aircraft emissions.

ACKNOWLEDGEMENT

We are grateful to Professor Ian Poll, on whose published work we have drawn heavily, for many discussions which have influenced the preparation of this paper.

APPENDIX

BREGUET RANGE EQUATION AND MASS RELATIONSHIPS

A1. Preamble

In this appendix, we derive the key equations used in the paper. Most have been derived by other authors and are collected here primarily for ease of reference and consistency of

notation. An important source is Poll^(6,11) who derives many of the key relationships in non-dimensional form. In this paper, we derive the equations in the same dimensional form as used by ICAO, with fuel burn expressed as kilograms of fuel burned per tonne-kilometre of payload-range.

The analysis here of the Breguet range equation follows Küchemann⁽⁸⁾, who worked with a calorific value of the fuel H^8 , defined as energy per unit weight and expressed in kilometres, and with aircraft, payload and fuel weights in kilograms. Green^(14,15) followed Küchemann and introduced the range parameter:

$$X = H\eta L/D, \quad \dots (A1)$$

where η is overall propulsion efficiency and L/D is the lift-to-drag ratio. In his Breguet range equation, the important variable is the ratio R/X of the actual range R to this parameter. He also dealt in weights.

More recently, Poll has dealt in non-dimensional quantities, expressing the fuel efficiency as the Energy to Revenue Work ratio (ETRW) and working with a non-dimensional range, $R/((LCV/g)\eta L/D)$, but for which he has also used the symbol X . It is important to be aware of this difference in notation and not to be confused by it. Poll deals in masses rather than weights. This paper also deals in masses and draws on Poll's study of aircraft masses⁽¹²⁾. The difference between weight and mass is not significant in the context of the paper. However, masses in the paper and in this appendix are expressed in kilograms, whilst fuel burn per unit payload-range is expressed in kilograms per tonne-kilometre, requiring the introduction of a factor of 1,000 at appropriate points in the analysis.

A2. The Breguet range equation

For an aircraft cruising at constant speed V and effectively at constant altitude, with an instantaneous gross mass GM , lift L , drag D and engine thrust T , the balance of forces on it are:

$$L = gGM \quad \text{and} \quad T = D,$$

and the rate at which the engines are doing work is:

$$TV = g \frac{GM}{L/D} V = g \frac{GM}{L/D} \frac{dS}{dt},$$

where S is distance along the flight path and t is time. This is equal to the rate at which energy is available from the engines burning fuel:

$$E = LCV\eta \frac{dFMB}{dt},$$

where LCV is lower calorific value, η is overall propulsion efficiency and FMB is the mass of fuel burned since the start of cruise. Equating the rate of work against drag to the rate of consuming energy gives:

$$LCV\eta \frac{dFMB}{dt} = g \frac{GM}{L/D} \frac{dS}{dt},$$

⁸ In the transition from weights to masses which follows, H is replaced by (LCV/g) .

which can be re-arranged as:

$$\frac{dFMB}{dS} = \frac{GM}{(LCV/g)\eta L/D} = \frac{GM}{X} = \frac{1}{SAR}, \quad \dots (A2)$$

where SAR is the specific air range in kilometres per kilogram and GM and X are local, instantaneous values in kilograms and kilometres, respectively.

Consider now an aircraft operating at its most economical condition, flying at a constant Mach number at which $\eta L/D$ is a maximum and is constant throughout the cruise. This will be in a cruise-climb mode, but in this mode the term in the force balance equation arising from the rate of climb can be neglected. Writing GM_1 as the gross mass of the aircraft at the start of cruise, we can write the instantaneous gross mass as $GM = GM_1 - FMB(t)$, and, equating the rate of work against drag with the rate of consuming energy, we obtain:

$$LCV\eta \frac{dFMB}{dt} = g \frac{GM_1 - FMB}{L/D} \frac{dS}{dt} \text{ or } \frac{dFMB}{GM_1 - FMB} = \frac{dS}{(LCV/g)\eta L/D} = \frac{dS}{X},$$

which, on integration over a range R , gives:

$$R = X \ln \frac{GM_1}{GM_1 - FMB}, \text{ which yields } FMB_{\text{cruise}} = GM_1 \left(1 - \exp\left(-\frac{R}{X}\right) \right) \quad \dots (A3)$$

This is the basic Breguet range equation. It is an exact expression for the mass of fuel (FMB) burned in cruise over a range R by an aircraft operating at constant $\eta L/D$, with a gross mass GM_1 at the start of cruise.

In mid-cruise, at $S = R/2$, the mass of fuel burned is:

$$FMB_{\text{mid-cruise}} = GM_1 \left(1 - \exp\left(-\frac{R}{2X}\right) \right) \quad \dots (A4)$$

A3. Aircraft masses and fuel burn per unit payload-range

The gross mass of an aircraft can be written:

$$GM = OEM + PM + FM, \quad \dots (A5)$$

where OEM is the operating empty mass, PM is the mass of the payload (passengers plus freight) and FM is the mass of the fuel.

FMB_{cruise} is the mass of fuel that would be used in cruise on a great-circle flight at maximum $\eta L/D$, i.e. at optimum cruise Mach number and altitude, between the origin and destination airports, as given by Equation (A3). Then, for the trip fuel mass TFM – the fuel actually consumed between take-off and landing – we follow Poll⁽⁷⁾ and write:

$$TFM = FMB_{\text{cruise}} + \varepsilon TOM, \quad \dots (A6)$$

where TOM is the mass at take-off and εTOM is the ‘lost fuel’ associated with climbing to cruise Mach number and altitude. Poll proposes that ε be assumed constant, with a value of 0.015 for current aircraft and operations. At take-off, the minimum possible fuel load is the trip fuel plus the reserve fuel. Poll⁽⁷⁾ proposes that the reserve fuel be written βTOM and

argues that it is an acceptable approximation to take β as a constant with a minimum value of 0.045. We may then write:

$$GM_1 = (1 - \varepsilon) TOM, \quad \dots (A7)$$

and then, from Equations (A3) and (A6):

$$TFM = \left(1 - (1 - \varepsilon) \exp\left(-\frac{R}{X}\right)\right) TOM \quad \dots (A8)$$

Minimum total fuel mass at take-off is then:

$$FM = \left(1 - (1 - \varepsilon) \exp\left(-\frac{R}{X}\right) + \beta_{\min}\right) TOM, \quad \dots (A9)$$

and we can also write the zero-mass fuel ZFM as:

$$ZFM = TOM - FM = \left((1 - \varepsilon) \exp\left(-\frac{R}{X}\right) - \beta_{\min}\right) TOM \quad \dots (A10)$$

For the trip, the fuel burn per unit payload-range expressed in kilograms per tonne-kilometre is:

$$\begin{aligned} \frac{1,000}{R} \frac{TFM}{PM} &= \frac{1,000}{R} \frac{TOM}{PM} \left(1 - (1 - \varepsilon) \exp\left(-\frac{R}{X}\right)\right) \\ &= \frac{1,000}{X} \frac{TOM}{PM} \frac{(1 - (1 - \varepsilon) \exp(-R/X))}{R/X}, \quad \dots (A11) \end{aligned}$$

whence substituting for TOM from Equation (A10), we have:

$$\frac{1,000}{R} \frac{TFM}{PM} = \frac{1,000}{X} \frac{ZFM}{PM} \frac{(1 - (1 - \varepsilon) \exp(-R/X))}{((1 - \varepsilon) \exp(-R/X) - \beta_{\min})(R/X)}$$

or

$$\frac{1,000}{R} \frac{TFM}{PM} = \frac{1,000}{X} \left(1 + \frac{OEM}{PM}\right) \frac{(1 - (1 - \varepsilon) \exp(-R/X))}{((1 - \varepsilon) \exp(-R/X) - \beta_{\min})(R/X)}$$

With values of 0.015 for ε and 0.045 for β_{\min} , this yields, to a first degree:

$$\frac{1,000}{R} \frac{TFM}{PM} = \frac{1,000}{X} \left(1 + \frac{OEM}{PM}\right) \frac{1.015 - \exp(-R/X)}{(\exp(-R/X) - 0.046)(R/X)} \text{kg/tonne-km} \quad \dots (A12)$$

In this form, the equation separates the combined propulsive-aerodynamic efficiency X , the structural efficiency PM/OEM and the effect of non-dimensional range R/X .

In mid-cruise, the mass of the aircraft is given by:

$$GM_{\text{mid-cruise}} = GM_1 - MFB_{\text{mid-cruise}} = TOM (1 - \varepsilon) \exp\left(-\frac{R}{2X}\right),$$

the rate of fuel burn is:

$$\frac{1}{SAR_{\text{mid-cruise}}} = \frac{GM_{\text{mid-cruise}}}{X} = \frac{TOM(1-\varepsilon)}{X} \exp\left(-\frac{R}{2X}\right) \text{ kg/km}, \quad \dots (\text{A13})$$

and the rate of fuel burn per unit payload is:

$$\frac{1}{PM} \frac{1}{SAR_{\text{mid-cruise}}} = \frac{GM_{\text{mid-cruise}}}{X} \frac{1}{PM} = \frac{1}{X} \frac{TOM}{PM} (1-\varepsilon) \exp\left(-\frac{R}{2X}\right) \text{ kg/tonne-km}, \quad \dots (\text{A14})$$

whence substituting for TOM from Equation (A10) and subsequently introducing the factor of 1,000, we arrive at:

$$\begin{aligned} \frac{1}{PM} \frac{1}{SAR_{\text{mid-cruise}}} &= \frac{GM_{\text{mid-cruise}}}{X} \frac{1}{PM} \text{ kg/tonne-km} \\ &= \frac{1,000}{X} \left(1 + \frac{OEM}{PM}\right) \frac{\exp(-R/2X)}{\exp(-R/X) - \beta_{\text{min}}/(1-\varepsilon)} \\ &= \frac{1,000}{X} \left(1 + \frac{OEM}{PM}\right) \frac{\exp(-R/2X)}{\exp(-R/X) - 0.046} \text{ kg/tonne-km}, \end{aligned} \quad \dots (\text{A15})$$

where ε and β_{min} have the values 0.015 and 0.045, proposed by Poll.

It is important to note that the term TOM/PM in Equations (11) and (A14) can be eliminated to give a relationship between the two expressions of fuel burn per unit payload-range:

$$\frac{1}{R} \frac{TFM}{PM} = \frac{1}{PM} \frac{1}{SAR_{\text{mid-cruise}}} \frac{1/(1-\varepsilon) - \exp(-R/X)}{\exp(-R/2X) (R/X)} \quad \dots (\text{A16})$$

Apart from the inclusion of the empirical factor ε for the fuel lost in climbing to cruise altitude and Mach number, this is an exact statement of the physics. It enables a measurement of SAR at mid-cruise, as specified in Cir 337, to be used via Equation (6) to determine the trip fuel per unit payload-range for the variety of operating conditions discussed below.

The two expressions of fuel burn per unit payload-range, Equations (A12) and (A15), are functions of the variable R/X and the term $(1 + OEM/PM) = ZFM/PM$, which Poll⁽⁷⁾ has shown empirically also to be a function of R/X . In the notation of this paper, his Equation (19), for maximum values of payload and, hence, of zero-fuel mass, becomes:

$$\left(1 + \frac{OEM}{MPM}\right) = \left(\frac{(1-\varepsilon)\exp(-R/X) - \beta_{\text{min}}}{0.435((1-\varepsilon)\exp(-R/X) - \beta_{\text{min}}) - 0.1}\right) \quad \dots (\text{A17})$$

Equations (A12) and (A15), combined separately with Equation (A17), provide the basis for drawing the curves in the paper. If the quantities ε and β are taken as constant, the only variables determining the fuel burn per unit payload-range, whichever way specified, are the range parameter X and the range R .

Although $X (= (LCV/g)\eta L/D)$ has been taken as a constant in drawing the curves here, the product $\eta L/D$ does in fact increase with design range. This is partly because the fuselage wetted area becomes a smaller proportion of total wetted area as design range increases, partly because increasing fuel efficiency takes higher priority in the optimisation of long-range designs. Green⁽¹⁶⁾ made an approximate allowance for this effect in comparing the performance of generic long-range and medium-range aircraft in the first Greener by Design technology report, but it merits more careful study. That is outside the scope of the present paper; but in the longer term, we suggest that an assessment of the effect of design range on the combined aerodynamic-propulsive efficiency $\eta L/D$ of modern aircraft should play a part in future ICAO regulation to reduce CO₂ emission.

A4. Design range

Green⁽¹⁷⁾, Poll⁽⁷⁾ and the CAEP Independent Advisors⁽⁶⁾ took the design range to be the range at maximum take-off mass and maximum payload (point B on the payload-range diagram, Fig. 3).

For the assessment of specific aircraft, Equations (A12) or (A15) can be used without reference to the empirical Equation (A17). The three certified masses, Maximum Take-Off Mass (*MTOM*), Maximum Landing Mass (*MLM*) and Maximum Zero-Fuel Mass (*MZFM*) provide almost all that is needed.

Being a relatively small number, β_{\min} can be assigned the value of 0.045, suggested by Poll, or can be determined as:

$$\beta_{\min} = \frac{MLM - MZFM}{MTOM} \quad \dots (A18)$$

From Equation (A10), we can then derive:

$$R_{\text{des}} = X \ln \left(\frac{(1 - \varepsilon) MTOM}{MZFM + \beta_{\min} MTOM} \right), \quad \dots (A19)$$

where the design range R_{des} is defined as point B on the payload-range diagram. It is worth emphasising that this is also an exact equation, apart from the small empirical term ε which accounts for the lost fuel used in climbing to cruise.

Since *SAR* will be determined at a specific gross mass under the procedures specified in Cir 337, it follows from Equation (A2) that X can be determined exactly, within the accuracy limits of the measurements. Design range can thus be determined unambiguously from Equation (A19) without reference to the manufacturer's payload-range diagram.

The trip fuel mass at the design point is then given by Equation (A8) as:

$$TFM_{\text{des}} = \left(1 - (1 - \varepsilon) \exp \left(-\frac{R_{\text{des}}}{X} \right) \right) MTOM \quad \dots (A20)$$

A5. Maximum payload

Sometimes the maximum payload mass (*MPM*) can be taken from the manufacturer's payload-range diagram. However, payload-range diagrams are not always published; and for any given aircraft, with *MZFM* and *MTOM* as certified values, there is scope for appreciable variation in *OEM*, and hence, in *MPM* ($= MZFM - OEM$), by variation in the internal equipment specified by the operator to meet customer requirements. For the purposes of certification, a

protocol would be needed for individual aircraft to determine the breakdown of *MZFM* into its two components.

An alternative approach is to follow the ICAO circular, which introduces the concept of reference geometric factor (*RGF*) as a measure of the aircraft cabin size. It is defined as the cabin floor area in square metres determined by rules set out in Cir 337. This has the merit of being a fair indicator of maximum passenger payload with single-class seating. Poll, from an analysis of the current fleet, has shown (Ref. 11, Fig. 1) that the average floor area required for a passenger in a maximum density, single-class cabin layout is close to 0.65 m². This value includes allowances for aisles, toilets, galleys and cabin crew areas. Taking the average mass of a passenger plus baggage as 95 kg leads to a value for the maximum total single-class passenger-only payload of approximately 0.15 *RGF* tonne. For regulation purposes, however, it would be preferable to define the constant of proportionality as an effective floor loading (*EFL*), in tonnes per square metre, to be determined from an assessment of data for current types. The maximum payload can then be written:

$$MPM = RGF.EFL \quad \dots (A21)$$

At the design point, the term given by the empirical Equation (A17) can then be replaced in Equations (A12) and (A15) by:

$$\left(1 + \frac{OEM}{MPM}\right) = \frac{MZFM}{MPM}, \quad \dots (A22)$$

where *MPM* is given by Equation (A21), and Equations (A12) and (A15) then respectively give the fuel burn per unit payload-range based on total fuel used on the trip or rate of fuel burn at mid-cruise.

A6. Flights beyond design range

The line BC of the payload-range diagram (Fig. 3) represents an aircraft operating at maximum take-off mass and progressively trading payload for fuel. This process reaches a limit at point C when the fuel tanks are full and the fuel mass is at its maximum *MF_M*; along the line CD, fuel mass remains at its maximum, and range beyond point C is increased further by reducing payload.

Operation along line BC

At constant take-off mass (*MTOM*), the increase in the mass of trip fuel with range beyond *R_{des}* can be derived from Equation (A8):

$$TFM = TFM_{des} + (1 - \varepsilon) \left(\exp\left(-\frac{R_{des}}{X}\right) - \exp\left(-\frac{R}{X}\right) \right) MTOM, \quad \dots (A23)$$

and since the total mass of fuel and payload is constant:

$$PM = MPM - (1 + \varepsilon) \left(\exp\left(-\frac{R_{des}}{X}\right) - \exp\left(-\frac{R}{X}\right) \right) MTOM \quad \dots (A24)$$

If trip fuel burn per unit payload-range *TFM/PMR_{des}* at the design condition is determined from Equation (A16), using the measurements specified in Cir 337 and

Equation (6) to determine fuel burn at mid-cruise, with design range obtained from Equation (A19) and maximum payload from Equation (A21), with the aid of Equations (A23) and (A24), we then obtain:

$$\frac{1}{R} \frac{TFM}{PM} = \frac{1}{R_{des}} \frac{TFM}{PM} \frac{TFM}{TFM_{des}} \frac{MPM}{PM} \quad \dots (A25)$$

Operation along line CD

The range R_C at point C on the payload-range diagram, the maximum range possible at $MTOM$, is determined by the capacity of the fuel tanks, MFM , and can be derived from Equation (A9) as:

$$\frac{R_C}{X} = \ln \left(\frac{(1 - \epsilon)}{1 + \beta_{min} - MFM/MTOM} \right) \quad \dots (A26)$$

With increase in range beyond R_C , take-off mass falls as payload is reduced and, from Equation (A9):

$$TOM = \frac{MFM}{1 - (1 - \epsilon) \exp(-R/X) + \beta_{min}} \quad \dots (A27)$$

Then, combining Equations (A24) and (A27):

$$\begin{aligned} PM &= PM_C - (MTOM - TOM) \\ &= MPM - (1 + \epsilon) \left(\exp\left(-\frac{R_{des}}{X}\right) - \exp\left(-\frac{R_C}{X}\right) \right) MTOM \\ &\quad - \left(MTOM - \frac{MFM}{1 - (1 - \epsilon) \exp(-R/X) + \beta_{min}} \right) \end{aligned} \quad \dots (A28)$$

and from Equations (A8), (A9) and (A27):

$$TFM = MFM - \frac{\beta_{min} MFM}{1 - (1 - \epsilon) \exp(-R/X) + \beta_{min}} \quad \dots (A29)$$

As with operation along the line BC, trip fuel burn per unit payload-range along CD is given by Equation (A25), using Equations (A16), (A20), (A21), (A28) and (A29) with the procedures of Cir 337 to determine fuel burn at mid-cruise:

$$\frac{1}{R} \frac{TFM}{PM} = \frac{1}{R_{des}} \frac{TFM}{PM} \frac{TFM}{TFM_{des}} \frac{MPM}{PM} \quad \dots (A30)$$

A7. Operations with two- and three-class seating

As noted above, Poll^(11, Fig. 1), from an analysis of the current fleet, has shown that the average floor area required for a passenger in a maximum density, single-class cabin layout is close to 0.65m². This value includes allowances for aisles, toilets, galleys and cabin crew areas. He

defines the passenger number in such a layout as N_{des} , which, using the notation of Cir 337, can be written as:

$$N_{\text{des}} = \frac{RGF}{0.65} \quad \dots \text{(A31)}$$

From his analysis of two-class and three-class layouts, he similarly suggests:

$$N_{2C} \approx \frac{0.82 RGF}{0.65} \quad \text{and} \quad N_{3C} \approx \frac{0.65 RGF}{0.65} \quad \dots \text{(A32)}$$

If the mass of a passenger plus baggage is taken as 95 kg, the same for all classes, Equation (A32) leads to:

$$MPM_{2C} = 0.12 RGF \text{ tonne}, \quad \text{and} \quad MPC_{3C} = 0.095 RGF \text{ tonne} \quad \dots \text{(A33)}$$

It would be appropriate to review these figures in the light of current practices, which for multi-class seating have changed appreciably⁽¹⁸⁾, notably in the mass of seating and on-board passenger facilities.

A8. Summary

The procedures set out in Cir 337 enable all the important quantities discussed in this Appendix to be determined with an understood level of precision. Points 1 to 5 below relate to certification of the basic aircraft and engine combination as envisaged in Cir 337, and points 6 and 7 relate to operational factors, which we expect ICAO to address at some future date.

1. Measurement of SAR at a known gross mass (GM) enables the range parameter X to be determined from Equation (A2) to the same precision as SAR and GM .
2. Equation (A19) enables the design range R_{des} , point B on the payload-range diagram, to be determined unambiguously from X and the certified values of $MTOM$ and $MZFM$.
3. The RGF proposed in Cir 337 provides a means, in Equation (A21), for assigning a value to the maximum payload for the purposes of CO_2 regulation, with the value of the constant EFL set to allow the payload to include a certain amount of freight, if that is considered appropriate.
4. Dividing the value of $(1/SAR)_{AVG}$ derived from the measurements specified in Cir 337 by the payload given by Equation (A21) yields one value of cruise fuel burn per unit payload-range. Using Equation (6), this can be converted into a value for fuel burn per unit payload-range at the cruise midpoint.
5. In turn, this can be converted into the total trip fuel burn per unit payload-range using Equation (A16). Evaluated at the design range, i.e. at maximum take-off mass and maximum payload, this is arguably a more suitable measure than fuel burn at mid-cruise.
6. For operations beyond the design range, Equation (A25) gives the trip fuel burn per unit payload-range as a function of range along the line BC (at maximum take-off mass) and Equation (A30) gives the same along the line CD (fuel tanks full).
7. For aircraft with multi-class seating, Equation (A33) can be used with Equation (A22) to give $(1 + OEM/MPM)$ which can be introduced into Equations (A12) and (A15) to obtain fuel burn per unit payload-range either at mid-cruise or at the design range.

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