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VARIABLE ANNUITIES

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ABSTRACT

This paper provides a detailed overview of variable annuities. Consideration is given first to the definition of the term variable annuity. Common terminology used in the variable annuity market is introduced. The current state of the United Kingdom and other international markets is described. Then, by reference to a simplified product, an analysis of customer outcomes, pricing, reserving, risk management and hedging is carried out. The paper ends with a description of current U.K. pensions legislation and how it potentially constrains product development.

KEYWORDS

Variable Annuities; Dynamic Hedging; Static Hedging; Pensions; Drawdown; Guaranteed; Death; Accumulation; Income; Withdrawal; Living; Benefits; GMxB; GMAB; GMDB; GMIB; GMWB; GMLB

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1. Introduction

1.1 This paper provides a detailed analysis of a new product category, variable annuities, which has enjoyed significant international success. We believe that the prospects for the product in the United Kingdom and, indeed, throughout Europe are very favourable.

1.2 The U.K. retail financial services market has gone through a period of very rapid change in recent years. In particular, the share of the market represented by with-profits business has declined sharply over the past ten years, with little prospect of any recovery. With-profits undoubtedly played a very important role in the U.K. financial services market, offering customers an apparently simple product, with the prospects of high investment returns through a balanced portfolio of investments coupled with a range of guarantees. The security offered by guarantees on with-profits contracts was a feature valued by many consumers, but it is largely absent

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from sales in the market today. In this context, variable annuities provide the opportunity to offer attractive guarantees in a transparent manner without the perceived disadvantages of with-profits business.

1.3 The term 'variable annuity' can be applied, potentially, to a wide range of product types and guarantees. In Section 2 we describe the types of guarantee which can be considered to be variable annuity products.

1.4 Products with similar features to variable annuities have been available for some considerable time. Consider, for example, the Report of the Maturity Guarantees Working Party (1980) which considered reserving and other aspects of maturity guarantees offered on unit-linked business. It is interesting to note that the products described in the 1980 paper fall within our definition of variable annuities. So, what is different today?

1.5 Significant advances in hedging techniques have been the key enabler which has allowed product providers, of many types, again to offer guarantees. Interestingly, the 1980 paper considered a form of delta hedging proposed by Fagan (1977), but took the view that it would be impractical. The practical difficulties raised would not apply in the current environment, with a wide range of derivative products available in the market. These hedging techniques allow providers to offer sophisticated guarantees which potentially require significantly less capital and deliver more stable profits. A key topic of the paper is therefore an analysis of these hedging techniques.

1.6 In order to illustrate the features and the management of variable annuities, we have focussed the analysis on a specific product category. This allows us to simplify the analysis which the reader can generalise to other types of product and guarantee. The product chosen, however, is also of great interest in its own right. We have chosen to analyse a retirement income product which allows investment in risky assets, but which provides a minimum guaranteed income for life. We demonstrate the considerable consumer benefits offered by this product design, but we note also the current legislative barriers in the U.K. which make it difficult for product designers to meet fully the needs of consumers.

1.7 In Sections 3 and 4 we describe developments in the U.K. and international markets. In Section 5 we describe a sample product upon which we develop subsequent analysis. Section 6 considers the potential product outcomes from the consumer's perspective. Sections 7 and 8 then consider pricing and reserving. Sections 9 and 10 describe risk management and the hedging of market risk. In Section 11 we consider the impact of current U.K. pensions legislation on product design.

2. Definitions

2.1 As variable annuities are essentially a new product class in the U.K., an industry standard definition does not yet exist. For the reasons set out

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below, we shall define a variable annuity as any unit-linked or managed fund vehicle which offers optional guarantee benefits as a choice for the customer.

2.2 Internationally, the term variable annuity is being derived from the United States life assurance market. It is perhaps not the most helpful of terms in the U.K. context, where the words have particular associations with existing product markets, in particular immediate annuities. We show, in this section, that the definition applies to a wide range of product types.

2.3 In the U.S.A. the National Association of Variable Annuity Writers (NAVA, 2006) define an annuity as: "the liquidation of assets through periodic payments over a specified time, or for as long as the designated individual is still alive." This is a more generalised definition than the one which we might use in the U.K., and associates with compulsory purchase annuities, for example. NAVA go on to say: "Today, an annuity is an insurance contract that can help individuals save for retirement through tax-deferred accumulation of assets, as well as a means of receiving payments, usually during retirement, that are guaranteed to last for a specified period, including the lifetime of the annuitant." Thus, NAVA use the term 'annuity' to cover both immediate and deferred annuities in U.K. parlance.

2.4 NAVA then explain that: "with a *variable* annuity, contract owners are able to choose from a wide range of investment options called sub-accounts, enabling them to direct some assets into investment funds that can help keep pace with inflation, and some into more conservative choices. Sub-accounts are similar to mutual funds that are sold directly to the public in that they invest in stocks, bonds, and money market portfolios." Thus sub-accounts are analogous to unit-linked funds, with either internal or external fund choices. Thus, we can see that variable annuities are similar to unit-linked retirement savings vehicles, such as personal pensions, SIPPS, or unit-linked post-retirement asset decumulation vehicles, such as unit-linked annuities, income drawdown or estate planning bonds in the U.K.

2.5 However, there are some further important characteristics of U.S. variable annuities which distinguish them from traditional U.K. unit-linked business, the most significant of which is the availability of guarantees. Devine *et al.* (2004) coined the term 'the *new* variable annuity' to highlight the introduction of guarantees, available as a rider feature to the overall product. Traditionally, the guarantees were offered as an integral part of the overall product package, but, since about 2000, insurance companies began offering more innovative guarantees, for an explicit price, as an optional choice to the customer.

2.6 The guarantees offered generally fall into four main classes, as described below. Graphical illustrations and practical examples of each can be found in Hanif *et al.* (2007).

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GMDBs Guaranteed Minimum *Death* Benefits, in their simplest form, guarantee a return of the principal invested upon the death of the policyholder. This is a minimum guarantee — if the underlying unit-linked account balance is greater than this, the death benefit would be the account balance. There are also variations to this, e.g. by guaranteeing the principal invested accumulated at a specified minimum roll-up rate, or by periodically locking into (and thereby guaranteeing) the growth in the account balance from time to time (a so-called ratchet).

GMABs Guaranteed Minimum *Accumulation* Benefits are similar to GMDBs, except that, instead of the guarantees being contingent on the death of the insured, they typically bite on specified policy anniversaries, or between specified dates, if the policy is still in force. Variations of this guarantee class again include minimum roll-up rates (accumulation rates in U.S. parlance), ratchets (see above) and resets, which enable the customer to secure a new GMAB on the expiry of the first GMAB.

GMIBs Guaranteed Minimum *Income* Benefits guarantee a minimum income stream (typically in the form of a life annuity) from a specified future point in time. The amount of the guaranteed minimum income benefit may be fixed in absolute terms at outset, or it could be expressed as a percentage of the premiums invested by the policyholder, or some function of the account balance at the annuitisation. The customer loses access to the unit-linked fund value if he or she annuitises and converts to income.

GMWBs Guaranteed Minimum *Withdrawal* Benefits guarantee a minimum income stream through regular withdrawals from the account balance. Fixed term GMWBs usually guarantee a return of the principal through this means, e.g. by guaranteeing that withdrawals of 5% of the original investment can be made for 20 years, the original investment is guaranteed to be returned through these instalments. More recently, however, GMWBs have been offered for the life of the policyholder. Thus, the policyholder is guaranteed a minimum amount with which to meet living expenses (the amount of the withdrawal benefit), even if the account value runs out. Variations, such as ratchets on the GMWB, are also possible. GMWBs differ from GMIBs, in that the remaining fund is paid to the estate of the deceased on death.

2.7 As a result of the variety of types of guarantee offered, the term

'GMxB' is commonly used to refer to variable annuities. The term 'guaranteed living benefits' is used to refer to GMIB and GMWB products which offer benefits for the lifetime of the policyholder.

2.8 Variable annuities in the U.S.A. have traditionally attracted valuable tax concessions in the retirement market (with a tax deferral approach similar to our U.K. pension arrangements). Nevertheless, the concept of offering different types of guarantees on managed funds is also applicable to other market segments.

2.9 In summary, in the U.K. context, we take the term variable annuity to mean any unit-linked or managed fund product offering either accumulation or decumulation benefits (frequently, but not necessarily, for retirement purposes), but that also offers at least one optional guarantee, typically of the types described above.

3. The U.K. Market Context

3.1 *Background*

3.1.1 The U.K. market is ripe for new solutions to meet consumers' needs for financial guarantees. The traditional bedrocks of defined benefit pension schemes and 'with-profits' have both faded in recent times, and have left U.K. consumers holding much more risk than they were traditionally comfortable with.

3.1.2 Worthy of particular focus is the '50+' age group. Thanks to the post War baby boom and the prevalence of 1960s births, this is expected to be the only growing customer segment for the next 30 years, according to Mercer Oliver Wyman (2006).

3.1.3 For older generations, a traditional annuity continues to offer a valuable guarantee once purchased. However, given that annuities are backed by fixed-interest assets, they are a less obvious fit for today's retiree who might expect to live for another 30+ years. It could be argued that such a long-term horizon means that it is better to invest in riskier assets. Consumers also perceive annuities to be poor value, as a result of their continued increasing costs (due to longevity improvements, lower bond returns and, arguably, the increasing emphasis on risk-based pricing).

3.1.4 Income drawdown (which allows a consumer to withdraw income whilst still investing in a defined contribution pension) provides a way to invest in risky assets, but offers no protection of income level from severe market falls.

3.1.5 With this background, several companies have launched variable annuity products in 2007 for defined contribution pensions (and also on investment bond products). The main feature of these guarantees has been to provide a minimum income, but with the potential to continue to benefit from stock market growth. The product designs are an import from other

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markets, where they have enjoyed considerable success — notably the U.S.A. and Japan.

3.2 *The Pensions Market Opportunity*

3.2.1 The most recently launched guarantees are offered as part of an income drawdown pension, and, typically, are sold through intermediaries with pensions expertise.

3.2.2 The concept is to offer consumers a 'third way' to spend their pension pots, which sits somewhere between the existing options in terms of risk and reward, as illustrated in Figure 3.2.2.

3.2.3 With-profits and unit-linked annuities already offer customers a way to pass on longevity risk to an insurer, whilst retaining some exposure to asset growth, and, in the case of with-profits annuities, offering guarantees. However, sales of these products have been limited. This is perhaps because the risk profile of these solutions is often very close to that of income drawdown.

3.2.4 Supporters of the new guarantees will argue that they give a more efficient trade-off between risk and return than with-profits or unit-linked annuities, or, indeed, than a combination of conventional annuities and income drawdown.

3.2.5 Looking in more detail at the two main choices available to those with defined contribution pensions, both have downsides:

A conventional annuity provides a guaranteed income for life, but:
— buying an annuity means giving up access to potential investment growth;
— once bought, the product cannot be altered; and

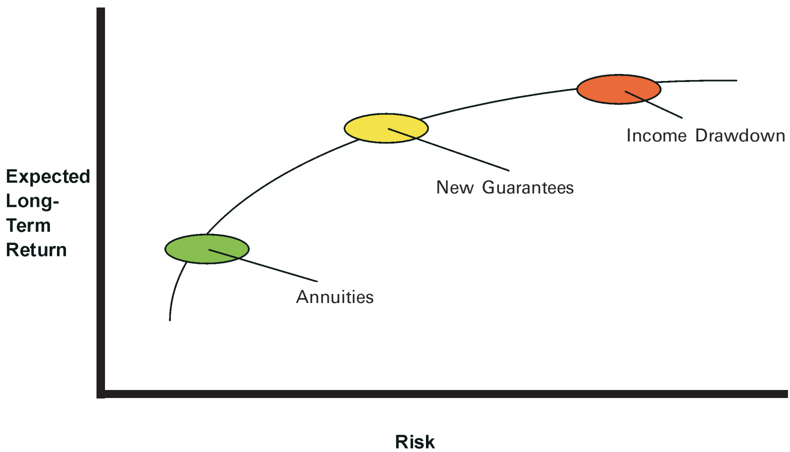

Figure 3.2.2. Variable annuity risk and reward

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— there is no remaining fund on death (although some spouse's benefits or a guarantee period can be purchased in return for a lower starting income).

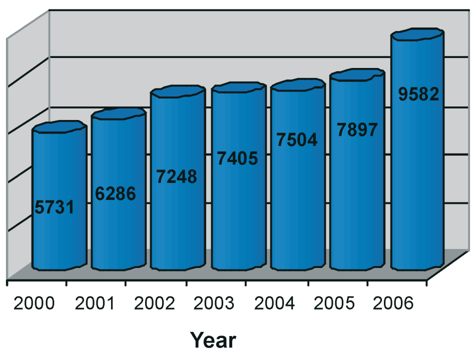
Income drawdown offers investment growth potential, flexibility and a return of the accumulated fund on death, but:

— the fund value is exposed to investment risk and is impacted by government income limits; for example, if the fund value halves, then, effectively, so does the amount of income which the customer will be allowed to take; and
— the policyholder is exposed to longevity risk, and may have to reduce income substantially if he or she lives longer than expected.

3.2.6 Considerably more customers buy annuities than invest in income drawdown, not least because the minimum fund size for income drawdown (typically between £50,000 and £100,000) is high relative to the average pension pot. Of those who bought an annuity in 2006, 67% had a purchase price of less than £50,000. The average purchase price for annuity sales in 2006 was £26,346, whilst for income drawdown it was £104,224 (Source: ABI/Mintel data).

3.2.7 This is further illustrated when we compare the two by total value invested. It can be seen from Figure 3.2.7 that annuities are the dominant choice.

3.2.8 One reason why annuities are so dominant over income drawdown is suggested by Figure 3.2.7b. Through to 2002, assets invested in drawdown were growing steadily. However, after equity market falls around this time, consumers (and advisors) became very nervous about investing new money in



Source: ABI data

Figure 3.2.7a. Annuities total new premiums (£ million)

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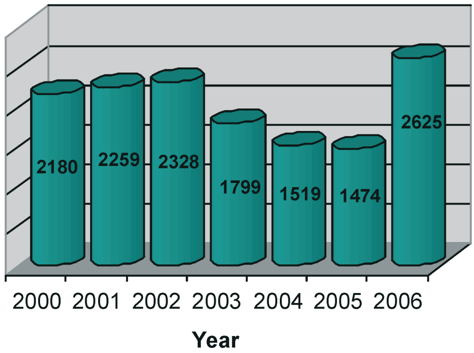

Source: ABI data

Figure 3.2.7b. Income drawdown total new premiums (£ million)

a retirement income solution with downside risk. This is despite all the advantages of flexibility, growth potential and higher death benefits that income drawdown has over annuities, and this underlines the ongoing desire for guarantees in the U.K. retirement income market.

3.2.9 After several years of much stronger investment performance, income drawdown sales bounced back to 21.5% of the retirement income market in 2006 (Source: ABI data). Renewed advisor confidence in income drawdown looks set to develop further as annuity rates remain at historical lows. Adding guarantees into the mix may be the catalyst which turns steady increases into explosive growth for this market.

3.3 *Key Market Challenges*

3.3.1 One of the hardest challenges for providers wishing to offer new variable annuity products will be to ensure that their design meets customer needs whilst accommodating the existing complex legislative framework (such as the maximum and minimum income limits for drawdown). This is discussed further in Section 11.

3.3.2 Other significant challenges include communicating the value for money of the new guarantees in the U.K.'s price-sensitive market and balancing simplicity with the desire to compete with the most attractive features.

3.4 *Other Market Opportunities*

3.4.1 This paper focuses primarily on the use of guarantees in the retirement income market, as we see this as the most likely area of initial market penetration.

3.4.2 There are, however, other areas in the U.K. where variable

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annuities could offer significant potential following the demise of alternative forms of guarantee; for example, the group pension and single-premium investment bond markets. For some of these markets different product designs, which focus guarantees more on the accumulation (rather than the income) phase of life, will be suitable.

4. International Markets

4.1 *Introduction*

In this section we consider the development of variable annuities in a number of major international markets.

4.2 *U.S.A.*

4.2.1 Variable annuities have existed in the U.S.A. since the 1950s. NAVA(2006) report that the first variable annuity was issued in 1952 by the TIAA — CREF (Teachers Insurance And Annuity Association — College Retirement Equities Fund), for use in college and qualified university retirement plans. The first guarantee attaching to variable annuities was a guaranteed minimum death benefit, introduced in 1980. The first guaranteed minimum income benefit followed in 1996, and it was not until 2000 that the first guaranteed minimum accumulation and withdrawal benefits were introduced.

4.2.2 Figure 4.2.2 illustrates the general growth curve which the U.S. industry experienced after guarantees were introduced. Note that, in the U.S.A., the vast majority of variable annuity business is by single premium, and the published figures generally include so-called 1035 Exchange transactions from existing (either fixed or variable) annuity contracts. The growth illustrated in Figure 4.2.2 was very rapid during the 1990s — no doubt related to the stock market and the 'tech' boom which occurred in the same period. The stock market declines in 2000 to 2002 led to a temporary fall in sales as well as a drying up of the reinsurance market for certain guarantees, as a result of the substantial losses incurred by many reinsurance companies at the time. Many of these reinsurers were not actively hedging their risks. The industry has since embraced modern financial risk management techniques to continue to offer guarantees, and has generally prospered since.

4.2.3 In 1999 variable annuity sales exceeded \$100bn, and in 2000 variable annuity assets under management exceeded \$1 trillion. At the time of writing, 2007, whole year figures are not publicly available. However, Q3 YTD sales were very strong at \$134bn, and the year end figures are estimated to be approximately \$180bn.

4.2.4 Table 4.2.4 summarises the relative popularity of the various types of variable annuities in the U.S.A.

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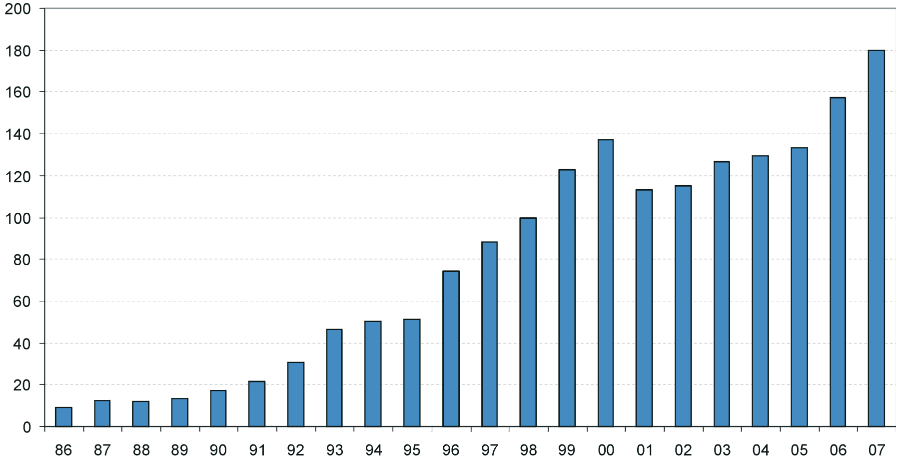

Source: Nava (2006), Milliman research

Figure 4.2.2. U.S. variable annuity sales (US$ billions)

Table 4.2.4. Relative popularity of types of variable annuity in U.S.A.

	2004	2005	2006	H1 2007
Proportion of all variable annuities purchased that have the choice of a guarantee				
GMWB (term or lifetime)	n/a	n/a	85%	86%
GMIB	n/a	n/a	67%	68%
GMAB	n/a	n/a	57%	59%
Total	87%	89%	93%	94%
Proportion of customers electing for a guarantee where they have the choice				
GMWB (term or lifetime)	24%	29%	37%	40%
GMIB	25%	24%	23%	22%
GMAB	7%	7%	5%	4%

Source: Third Annual Milliman Survey on Variable Annuity GLB Market Dynamics (2007)[1]

4.2.5 While, traditionally, the guaranteed minimum income benefit was the most important form of guarantee, it is clear from the above statistics that this has now been overtaken by the GMWB as the retirement vehicle of choice. Although election rates for GMIBs have been falling steadily, the Milliman (2006) survey also reported that: "there is still a strong core of companies and producers that remain committed to this feature."

[1] The universe of survey participants consisted of 20 carriers, representing approximately 65% of the variable annuity market, 14 of whom were ranked in the top 20, based on new variable annuity sales (according to VARDS).

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4.2.6 Extensive market research and data on the U.S. variable annuity industry are available from Variable Annuity Research Data Services (VARDS). The 2006 VARDS rankings of the major variable annuity issuers (measured by assets) is given in Table 4.2.6.

4.2.7 The VARDS report also publishes various other statistics, such as the breakdown of asset mixes underlying variable annuity funds, sales by distribution channel, and market segments. Some interesting statistics sourced from VARDS are:

— Approximately 55% of the variable annuity industry is invested in equities.
— Approximately 34% of sales are through captive agents, with 31% through independent agents (similar to independent financial advisers (IFAs) in the U.K.). Stockbrokers accounted for 20% of sales (variable annuities are considered securities in the U.S.A.), while banks accounted for 14% of sales. Only 1% of sales were through direct response, which is very much lower than in the managed fund industry generally. We learn from this that variable annuities can be customised to be attractive in most of the main distribution outlets.
— Of those purchasing variable annuities, 24% are college graduates and 22% are post-graduates. The remainder comprise high school graduates (29%), those with vocational training (5%), or those currently in education or who did not complete any formal education programme (20%). In short, variable annuities attract the educated, but also sell well across the population. Different companies may have different distribution channels or product features customised to their respective target markets.

4.2.8 There have been various analyst reports documenting the success of variable annuities, and suggesting reasons for their success. An in-depth analysis of the U.S. market can be found in Abkemeier *et al.* (2006). One of the more recent reports, with a global focus, by Hanif *et al.* (2007), put forward the following reasons for the popularity of variable annuities:

— *Equity exposure*. There are potentially high returns through exposure to real assets, and, in particular, the equity market upside.
— *Longevity protection*. The product is well suited to the needs of the pre and post retirement market segments.
— *Transparency and flexibility*. Customers and financial advisors value the transparency of explicit charges for guarantees and the ability to customise these guarantees to suit particular needs, such as income planning or inheritance.
— *Profitability and capital efficiency*. Hanif *et al.* (2007) note that, from an insurance company's perspective, variable annuities are relatively profitable and capital efficient under 'economic capital' measures (e.g. Solvency II), provided that the guarantee is hedged and the rider charge

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Table 4.2.6. Leading variable annuity writers in the U.S.A.

Rank 12/06	Rank 12/05	Issuer	Assets at 31 December 2005	Assets at 31 December 2006	Market share	Dollar change	Percent change
Top 25 variable annuity assets by issuer							
1	1	TIAA-CREF Life Insurance Company	324,933.0	356,531.1	26.28%	31,598.1	9.72%
2	2	Hartford Life Insurance Company	93,011.0	104,665.0	7.71%	11,654.0	12.53%
3	3	MetLife	85,111.7	95,898.0	7.07%	10,786.3	12.67%
4	5	AXA Financial/MONY	64,476.1	77,626.5	5.72%	13,150.4	20.40%
5	4	AIG Sunamerica	74,056.6	76,165.9	5.61%	2,109.3	2.85%
6	6	Prudential/American Skandia/Allstate	59,014.2	71,622.7	5.28%	12,608.5	21.37%
7	7	Lincoln National Life Insurance Company	57,593.9	68,098.3	5.02%	10,504.4	18.24%
8	8	ING Group	55,327.7	63,750.4	4.70%	8.422.7	15.22%
9	10	John Hancock Life Insurance	38,433.0	48,174.7	3.55%	9,741.7	25.35%
10	11	Ameriprise Financial	35,495.4	47,291.2	3.49%	11,795.8	33.23%
11	12	Pacific Life Insurance Company	32,049.6	46,023.6	3.39%	13,974.0	43.60%
12	9	Nationwide Life Insurance Company	39,375.5	42,950.3	3.17%	3,574.8	9.08%
13	13	AEGON/Transamerica	27,922.1	29,720.4	2.19%	1,798.3	6.44%
14	14	Jackson National Life Insurance Company	18,929.3	26,110.3	1.92%	7,181.0	37.94%
15	15	Allianz Life Insurance Company	17,245.2	21,177.5	1.56%	3,932.3	22.80%
16	17	New York Life Insurance & Annuity Corporation	13,433.4	17,419.0	1.28%	3,985.6	29.67%
17	16	Sun Life Assurance Co of Canada	15,862.8	16,626.0	1.23%	763.2	4.81%
18	22	Allmerica Financial Life Insurance & Annuity Co	8,937.4	16,418.0	1.21%	7,480.6	83.70%
19	18	Fidelity Investment Life Insurance	13,012.9	14,767.1	1.09%	1,754.2	13.48%
20	19	Thrivent Financial	12,553.4	13,764.1	1.01%	1,210.7	9.64%
21	20	Massachusetts Mutual Life Insurance	12,267.9	12,702.6	0.94%	434.7	3.54%
22	21	Genworth Financial	9,343.3	10,188.4	0.75%	845.1	9.04%
23	23	Northwestern Mutual Life Insurance Co	8,927.8	10,149.4	0.75%	1,221.6	13.68%
24	24	Merrill Lynch Life Insurance Company	8,878.4	8,378.0	0.62%	−500.4	−5.64%
25	26	Kemper Investors Life Insurance Co	7,043.2	7,541.2	0.56%	498.0	7.07%
Other	Other		54,051.7	52,940.6	3.90%	−1,111.1	−2.06%
		Industry Totals	1,187,286.5	1,356,700.3	100.00%	169,413.8	14.27%

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for guarantees is sufficient to finance the cost of the hedging programme.
- *External factors*. The most important of these are uncertainties relating to future inflation combined with an ageing population and an increasing burden on the state pension provision.

4.3 *Japan*

4.3.1 The Japanese variable annuity industry is described by Ino (2006) as young, rapidly growing, and undergoing a fundamental change in regulation.

4.3.2 Variable annuities have a relatively short history in Japan. As a result of financial deregulation permitting the sale of variable annuities, ING Life started selling variable annuities in April 1999, with the first products providing relatively basic GMDBs. Mitsui Life followed, by introducing a variable annuity which provided a GMAB, which guaranteed that the ultimate annuity payout would not fall below a pre-set level, regardless of the underlying investment performance of the variable annuity. AIG, Sony Life, and other companies began selling variable annuities through their traditional insurance distribution, but with limited success.

4.3.3 Hartford Life, a leader in the U.S. variable annuity market, entered Japan in December 2000. Hartford was unique among new market entrants in focusing on variable annuity products sold through stockbrokers. Hartford's strategy was successful; they had significantly more sales than companies distributing through their traditional insurance sales channels. With further deregulation in October 2002, banks were allowed to sell annuity products. This significantly boosted the potential for guaranteed products. Figure 4.3.3 shows the dramatic growth in variable annuity assets.

4.3.4 Ino (2006) explains this growth as being due to a number of factors, as follows:
- *Demographic trends*. Japan is one of the most rapidly ageing societies. The average issue age of variable annuities in Japan is over 65 years (even 70 years at some companies). Demand for annuity and savings products will continue as the baby boom generation reaches retirement age over the next several years.
- *The economic environment*. Interest rates in Japan have been extremely low for the past decade. The ten-year treasury yield has been as low as 1.5%, and bank deposits have generally credited less than 0.1%. Consumers are looking for a more attractive investment vehicle.
- *Concern for risk*. Japanese people, especially older people, are conservative investors. They do not like losing money by investing in stocks, even though the expected return may be much higher than on bank deposits. Variable annuities with minimum guarantee features are welcomed by these conservative investors.

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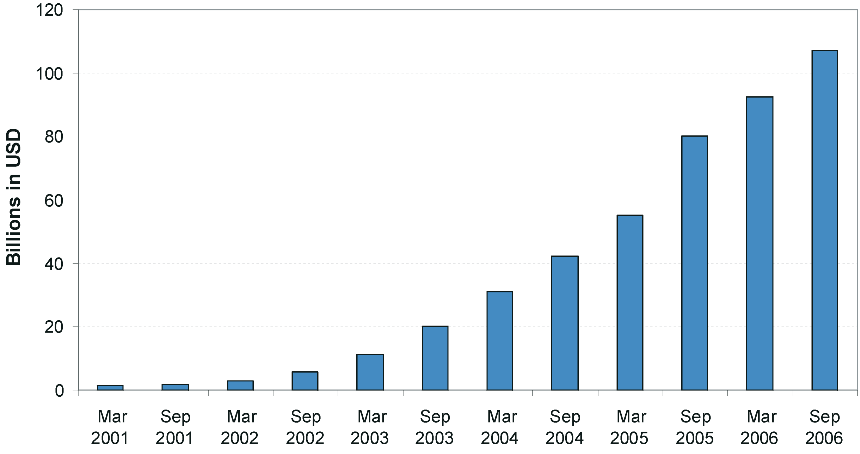

Source: Milliman, Morgan Stanley research, Hanif *et al.* (2007); Hoken Mainichi Shinbun for prior years; 1US$ = 100JPY for simplicity

Figure 4.3.3. Japanese variable annuity net assets

— *The savings culture*. The savings rate is higher in Japan than in the U.S.A. The financial assets of the individual sector are worth about US$14 trillion, of which 50% is in bank deposits. Variable annuity assets, as of September 2005, were still only 1% of bank deposits, and so have considerable potential for growth.
— *Deregulation*. Deregulation of the bank channel accelerated annuity sales. In addition, changes to the government guarantee of banking deposits have provided a further incentive to diversify away from traditional bank deposits into other guarantee vehicles.

4.3.5 Table 4.3.5 shows the major variable annuity writers ranked by assets, as of September 2006.

4.3.6 It is evident from this table that the top three companies, which account for a 50% market share, are affiliates of foreign-based companies. Ino (2006) puts forward several reasons why this is the case. These are related to their experience from the U.S.A., along with an existing global brand, and the opening of the bank channel enabling a rapid growth in distribution.

4.4 *Other Asian and Pacific Basin Markets*

4.4.1 Korea is the third largest insurance market in Asia (after Japan and China), and currently the second largest variable annuity market in Asia

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Table 4.3.5. Japanese variable annuity writers

Company	Foreign/ domestic	VA assets ($m)	Market share (%)	Policy count
Japan variable annuity assets at 30 September 2006				
Hartford	Foreign	28,342	26.3%	405,000
Mitsui Sumitomo Metlife	Foreign	15,063	14.0%	167,000
ING	Foreign	14,629	13.6%	240,000
Tokyo Marine	Domestic	11,987	11.1%	359,993
Sumitomo Life	Domestic	10,559	9.8%	183,000
Manulife	Foreign	7,374	6.8%	125,000
Mitsui Life	Domestic	4,919	4.6%	160,000
AIG (ALICO Japan)	Foreign	4,883	4.5%	72,000
T&D Financial	Domestic	4,141	3.8%	54,000
Dai-chi	Domestic	2,304	2.1%	54,434
Nippon	Domestic	2,034	1.9%	29,888
AXA	Foreign	1,080	1.0%	35,279
Others		575	0.5%	23,686
Total		107,890	100.0%	1,909,280

Source: Milliman, Morgan Stanley Research, Hanif *et al.* (2007)

It is dominated by three domestic companies (Samsung, Korea Life and Kyobo) and foreign companies (Allianz, Metlife, Prudential, ING and AIG). Initially, variable-annuity-styled products enabled foreign companies to differentiate and gain market share in the high net worth segment. However, the domestic companies (notably Samsung) have responded, and they are now successfully selling across market segments and distribution channels.

4.4.2 Other launches in the region have included Taiwan and Hong Kong (where Manulife recently launched a GMWB for life product), as well as Australia, where Axa launched during 2007.

4.4.3 It is relevant to note that all the developing variable annuity markets also have relatively developed futures and derivatives markets to facilitate modern financial risk management of these products.

4.5 *Europe*

4.5.1 Variable annuities are now also spreading across Europe. Some of the more significant and high profile launches have been Axa's launches in France, Germany, Spain, Italy and Belgium, as well as ING's launches in Spain, Hungary and Poland, Generali's recent (December 2007) launch in Italy and Ergo's recent (February 2008) launch in Germany. This is in addition to the various launches by Aegon (Scottish Equitable), Hartford, Metlife and Lincoln in the U.K., as well as Axa's inheritance tax bond in the Isle of Man. It is understood that there is considerable interest across Europe, and the working party believes that there will be further launches in the coming months in different parts of Europe.

4.5.2 Hanif *et al.* (2007) provide some detailed analysis of the market

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segmentation and growth opportunities in the main European markets. Importantly, they provide an analysis which indicates why variable annuities fulfil a need (real asset exposure and longevity protection) generally not well served by existing products in the retirement market segments. Consistent with this, it is noted that, in Northern Europe, the focus is on GMWB and GMIB guarantees for the retirement market. This is in contrast to Southern Europe (Spain and Italy), as well as to Eastern Europe (Hungary and Poland), where the focus has been on the more traditional accumulation and death guarantees (GMABs and GMDBs). In these markets, there is generally an attractive state pension system in place, and investment in the less developed retirement savings market segment is seen as a longer-term investment.

5. Sample Product Design and Features

5.1 *Introduction*

5.1.1 In the later sections of the paper, we carry out a range of quantitative analyses to illustrate pricing, customer outcomes, reserving and hedge effectiveness. In carrying out this modelling work, we have considered, throughout, an illustrative product design. We have chosen a unit-linked pensions contract with an optional guaranteed minimum withdrawal benefit (GMWB), which provides the customer with a guaranteed minimum income for life. Under this contract the customer benefits from the investment return on his accumulated fund. Income is paid out from the contract by the cancellation of units. The guarantee is such that, should the fund be depleted either through poor investment returns or through unexpectedly long life the insurer will continue to pay a guaranteed minimum level of income.

5.1.2 To keep the design relatively simple, we have ignored many of the features included in product launches in the U.K. so far — for example, extra death benefit guarantees or a range of different fund choices, each with a guarantee. We have also ignored, for simplicity, the possible impact of the current HM Revenue and Customs (HMRC) income limits and features which may be incorporated in the product design to optimise the customer proposition when these limits apply. Further examples of features which we have omitted for simplicity are given in Section 5.5.

5.2 *Sample Product Design*

5.2.1 Our sample pension product, for which the rider can be selected, allows investment in only one unit-linked fund, which retains a constant proportion of 60% in equities (a FTSE All Share total return tracker) and 40% in fixed interest (a zero coupon bond fund, duration ten years). The annual management charge for the pension is 1% p.a., and advisers take an additional 0.5% p.a. in fund-based commission, bringing the total management charge to 1.5% p.a.

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5.2.2 A customer investing in this pension can choose to switch on the rider on their 65th birthday in return for increasing their fund management charge by 0.75% p.a. The total charge would then be 2.25% p.a. rather than 1.5% p.a. We assume, in this paper, that the guarantee is selected by a male for the entire plan value of £100,000.

5.2.3 The guarantee locks in a minimum income for life of 5% of the 'guarantee base' — initially the plan value at the point when the rider is selected. So, in this case, the guarantee base starts at £100,000, and the income level starts at £5,000 p.a.

5.2.4 The customer can then withdraw this amount of income each year, even if the pension value has depleted to zero.

5.2.5 Each year, up to the policyholder's 75th birthday, the guarantee base will step up (or 'ratchet') to the current fund value, if this is higher, resulting in a higher income level. For example, if the pension plan value increases to £110,000 by age 66, the customer's guarantee base is reset at this level, and the guaranteed income becomes £5,500 p.a. (i.e. 5% of £110,000). This step up is subject to a maximum increase in any one year of 15%. Whilst the ratchet, at first sight, appears to be a complex feature in our simplified product design, it is a common and popular feature of variable annuity products, and it also offers a benefit to the provider of reducing exposure to a selective lapse risk.

5.2.6 The guarantee base (and hence the income level) does not reduce if the fund value falls.

5.2.7 The guarantee ends when the customer chooses to switch off the rider or dies. A customer who chooses to end the rider will stop paying the additional 0.75% p.a. charge, and can continue to take income as supported by the pension or cash-in to buy an annuity. On death, the remaining pension value is used to provide pension benefits for the customer's spouse/dependant(s) and/or a lump sum (less tax) to the beneficiaries.

5.3 *Lapse Assumptions*

5.3.1 The chance of the customer lapsing the rider at any point depends on how their funds are performing; they are less likely to turn off the guarantee if it is heavily in-the-money after poor market returns, and more likely to turn it off if the fund has performed strongly.

5.3.2 Therefore, we have constructed a dynamic lapse assumption, where the level of assumed lapses varies according to the 'in-the-moneyness' of the rider. The 'in-the-moneyness' is determined by considering the relative level of the guaranteed income to the annuity which could be bought on the open market. The lapse rate is assumed to vary between 2% p.a. and 6% p.a., according to the extent to which the guarantee is in or out of the money.

5.3.3 This is illustrated in Figure 5.3.3, where the lapse rate '$W(y)$' depends on in-the-moneyness 'y'.

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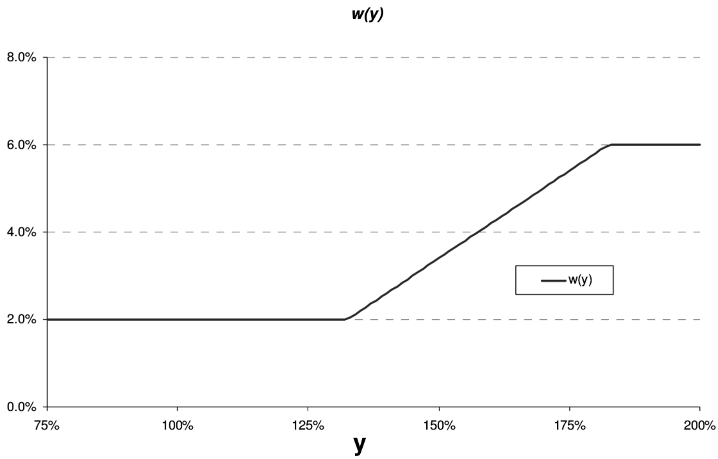

Figure 5.3.3. Dynamic lapse function

5.4 *Mortality Assumptions*

The probability of death in any year was based on the RMC00 tables, medium cohort, with a minimum improvement rate of 1% p.a.

5.5 *Common Product Features Not Modelled*

We have omitted, for simplicity of presentation, the following features which might normally be included in a product offered in the marketplace. Some of the features could have a material impact on the modelling results:

— allowing the guarantee to be selected at a range of ages with differing withdrawal levels (increasing with age to reflect the shorter expected payment period);
— allowing the customer to switch on the guarantee for only part of their pension, with the option to increase the proportion guaranteed over time;
— a wider range of unit-linked fund choices;
— guaranteed death benefits;
— giving advisers a choice of commission levels;
— the possibility of charging for the guarantee as a percentage of the guarantee base rather than of the fund value; some providers believe this is a better method to align more closely the interests of customer and provider; and
— other product features to reduce the likelihood of customer anti-selection on the provider — such as loyalty bonuses or exit penalties.

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6. Customer Outcomes

6.1 *Introduction*

In this section we present a stochastic analysis of the customer proposition for the example product and discuss some consequences of treating customers fairly (TCF). Analysis has been undertaken using the Barrie & Hibbert economic scenario generator (ESG). The ESG is a collection of carefully calibrated stochastic models which provide projections of interest rates, inflation rates and asset returns over various timescales. The analysis presented here is based on the June 2007 best estimate calibration of the ESG, which includes an arithmetic equity risk premium over the short-term risk-free interest rates of 4% p.a. and an equity volatility of 20% p.a. Results are based on 1,000 simulations over 40 years using an annual time step.

6.2 *Example Scenarios*

6.2.1 We start by showing the behaviour of the variable annuity in two selected scenarios from amongst the stochastic projections, in order to illustrate the behaviour of the product. These scenarios represent roughly the tenth and 90th percentiles of the distributions for cumulative equity returns over the projection period. For comparison, we also show the performance of an income drawdown product with the same underlying asset mix and where the same level of income is taken (until the fund runs out), i.e. we ignore any Government Actuary's Department (GAD) income limits. The first example scenario, which we have called Scenario A (in fact it is Simulation 38 from the original 1,000), shows poorer investment returns than Scenario B (Simulation 81), so that the guarantee is more likely to bite in Scenario A.

6.2.2 For each example scenario we show a number of figures.

6.2.3 Figure 6.2.3 shows the raw asset return indices in each example simulation, together with the annualised returns over five or ten year periods. Thus, for example, we can see that Scenario A is characterised by a relatively poor equity return in the early years (especially the first year), while Scenario B is characterised by a very strong initial equity performance. The figure also shows the cumulative return on the fund underlying the variable annuity or drawdown product net of fees. Note that, as expected, the fund returns generally fall between the equity and bond indices, since each is a combination of 60% equities and 40% bonds, but that the drawdown return exceeds the variable annuity return, since the latter includes an additional charge for the guarantee provided.

6.2.4 Figure 6.2.4 shows the combined effect of the net fund returns and income withdrawals from the fund. As explained, we assume that income from the variable annuity product is taken at the guaranteed level, i.e. 5% of the guarantee base, which is also shown in Figure 6.2.4. In Scenario A the variable annuity fund runs out, so that the guarantee starts to cost the insurer for any policies still in force at age 83, i.e. after 18 years. In Scenario B the

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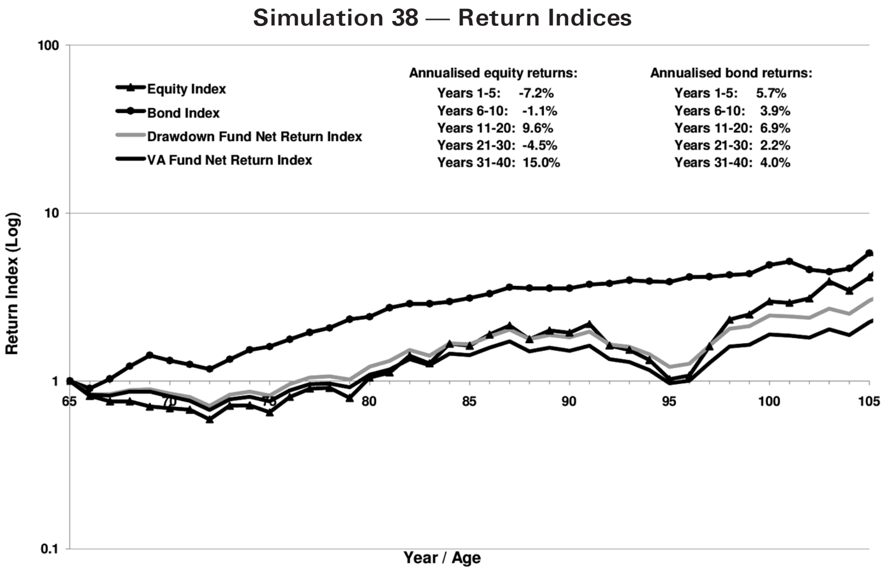

Figure 6.2.3a. Asset and fund return indices, Scenario A

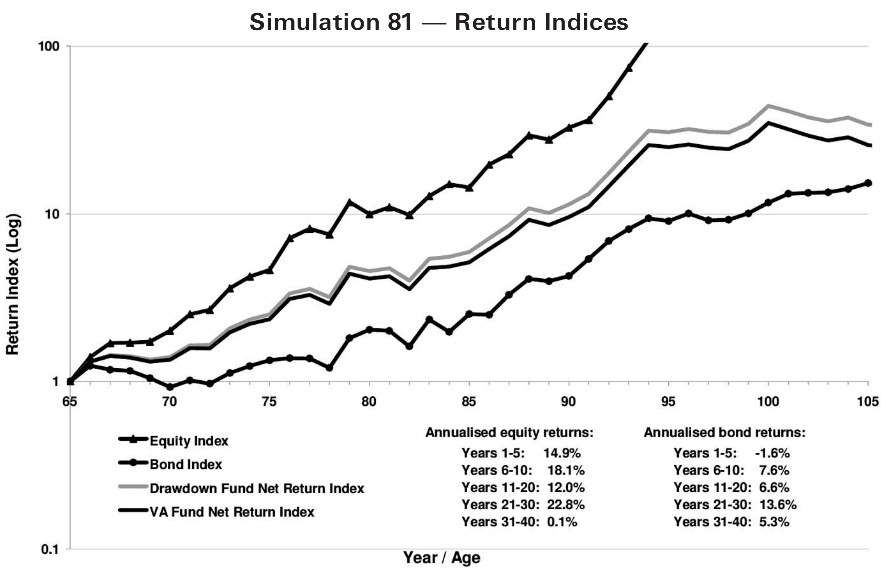

Figure 6.2.3b. Asset and fund return indices, Scenario B

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guarantee base increases in the early years, reflecting the good fund performance. In both cases, the same level of income is taken from the drawdown product as would be taken from the variable annuity product where sufficient funds exist. Note that, in all cases, the drawdown fund exceeds the variable annuity fund because of the higher charges on the latter. However, it is worth pointing out that this effect holds true because we are using the same asset mix for the two products. In practice, the existence of the guarantee with the variable annuity product may mean that the customer is able to choose an underlying fund with a higher expected return resulting from a riskier asset mix. In this situation, the increase in the expected return may be more than sufficient to offset the extra cost of the guarantee.

6.2.5 Figure 6.2.5 shows, explicitly, the level of income from each product in the example simulations, together with the combined effect of assumed lapses and deaths on the probability of the variable annuity policy still being in force in each year — obviously it is only in cases where the policy is still in force that the guarantee actually imposes a cost on the insurer. Note that the dynamic lapse assumption which we have used for the analysis means that there is a small difference between the in-force probabilities in the two scenarios.

6.2.6 In Scenario A, the variable annuity income can only be matched for the drawdown product until age 83, after which a reduced income is available at age 84, and zero thereafter (to re-iterate, this analysis is for comparison purposes only, and we have ignored the effect of GAD limits on the actual income which could be taken in practice). Note that, in this case, the insurer would actually be paying out on the variable annuity guarantees from age 82, due to the effect of higher charges explained earlier.

6.2.7 Of course, when comparing the two products it is not just the income which they provide which is relevant. On death, each example product is assumed to pay the remaining fund value. As explained earlier, in all cases, except where the drawdown fund has fallen to zero, the value of the drawdown fund will be larger than that of the variable annuity fund. However, again as explained earlier, if the existence of the income guarantee means that the customer chooses a fund with, say, 20% more equities than he or she otherwise would do, then, on average, this should be more than sufficient to offset the guarantee charge in practice.

6.2.8 Having spent some time looking at two example scenarios, we now go on to look at the probability distributions for each of the variable annuity and comparison drawdown products.

6.3 *Probability Distributions*

6.3.1 Figure 6.3.1 shows the simulated probability distributions across all the scenarios for the fund size for each of the variable annuity and drawdown products, in each case assuming that the policy remains in force. The figures show the percentiles of the distribution of fund values through time.

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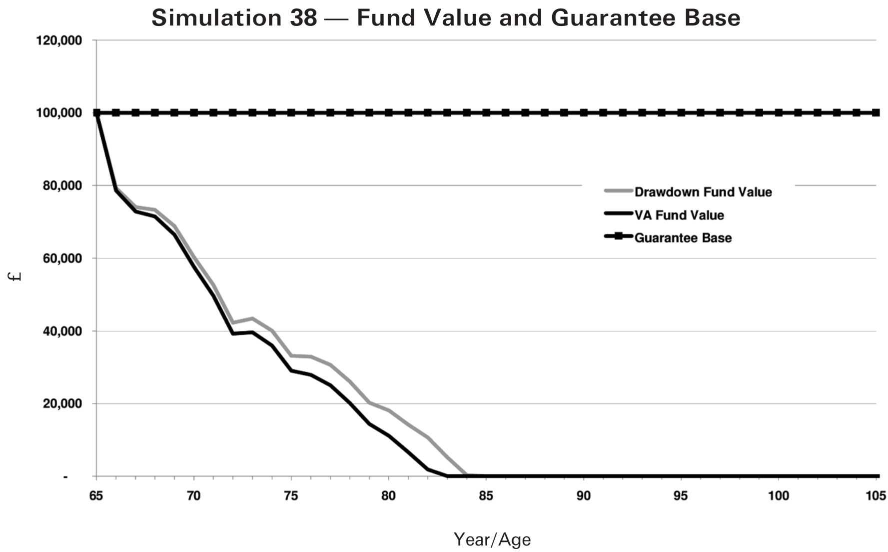

Figure 6.2.4a. Fund value and guarantee base, Scenario A

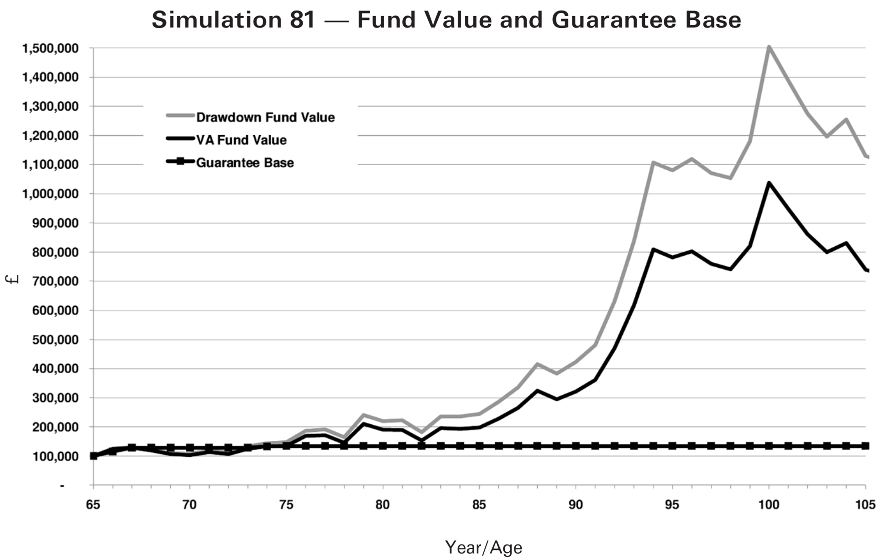

Figure 6.2.4b. Fund value and guarantee base, Scenario B

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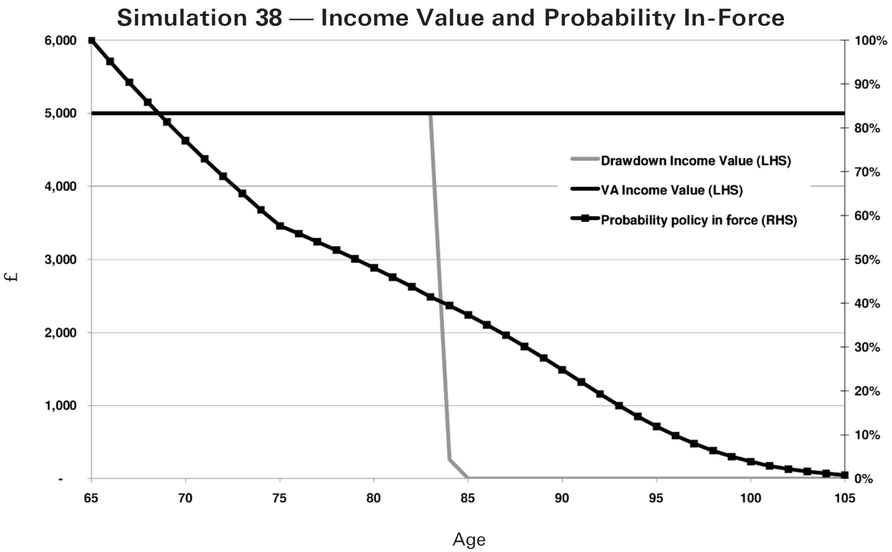

Figure 6.2.5a. Income through time, Scenario A

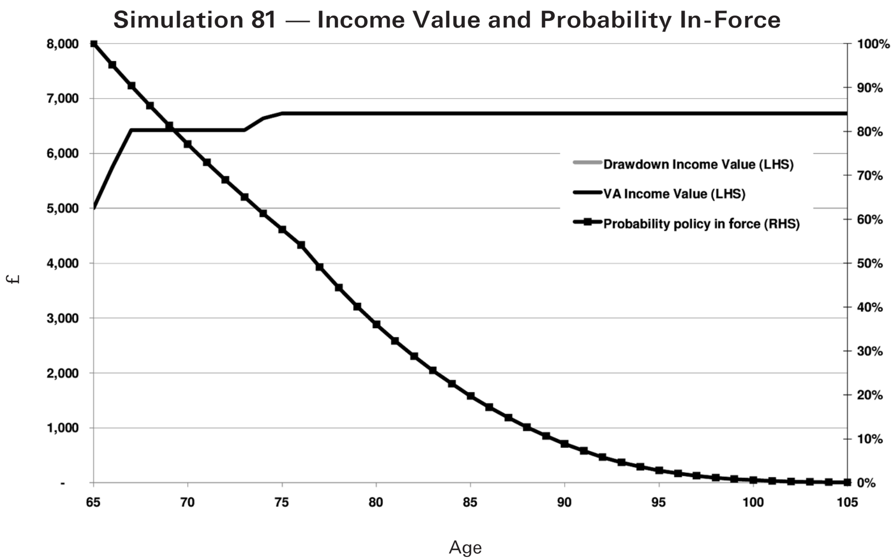

Figure 6.2.5b. Income through time, Scenario B

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6.3.2 It can be seen that, in both cases, the median fund value gradually falls through time, reaching zero by age 98 (i.e. after the product has been in force for 33 years) in the variable annuity case, and by age 103 in the drawdown case. However, in 10% of the variable annuity cases the fund has already fallen to zero, so that the guarantee starts to bite, by age 86.

6.3.3 Figure 6.3.3 shows the simulated probability distributions for the income taken from the products, again assuming that they remain in force. Remember that this is assumed always to be 5% of the guarantee base and to be the same for the two products, unless no more funds are available in the case of the drawdown product (except to the extent that the lower charges on the drawdown product mean that, if the guarantee base increases, it will be fractionally larger than for the variable annuity).

6.3.4 Note that, in both cases, the median income value rises to age 75 (when the ratchet ceases to operate), even though the median fund value falls, due to the effect of volatility in the underlying fund combined with the operation of the ratchet. Again, we emphasise that, due to the comparison performed, the income from the drawdown product is essentially the same as that from the variable annuity product. In practice, drawdown income is likely to be increased more readily in good scenarios, creating a different set of income distributions through time. Figure 6.3.3b shows clearly the increasing chances of income below that from the variable annuity contract

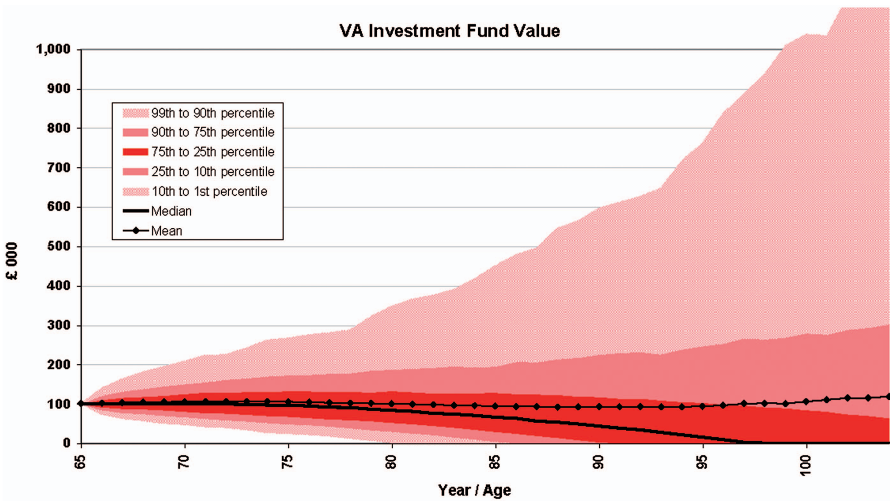

Figure 6.3.1a. Percentiles of the investment fund value through time, variable annuity

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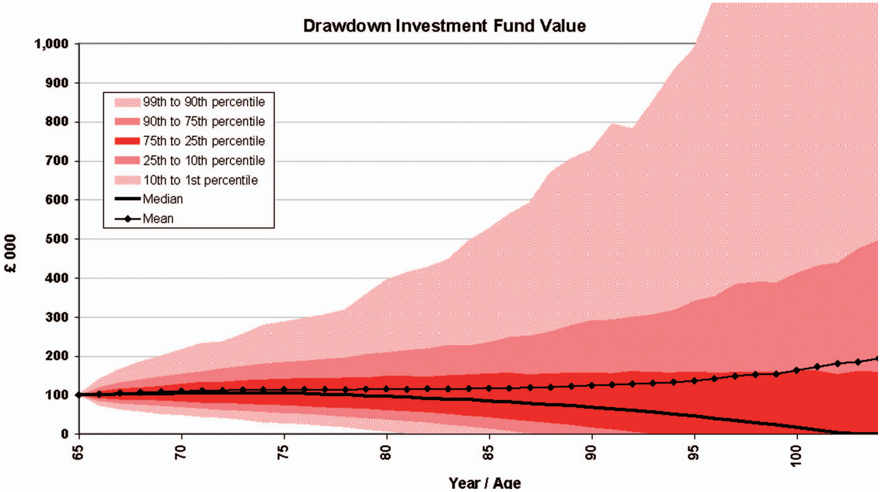

Figure 6.3.1b. Percentiles of the investment fund value through time, drawdown

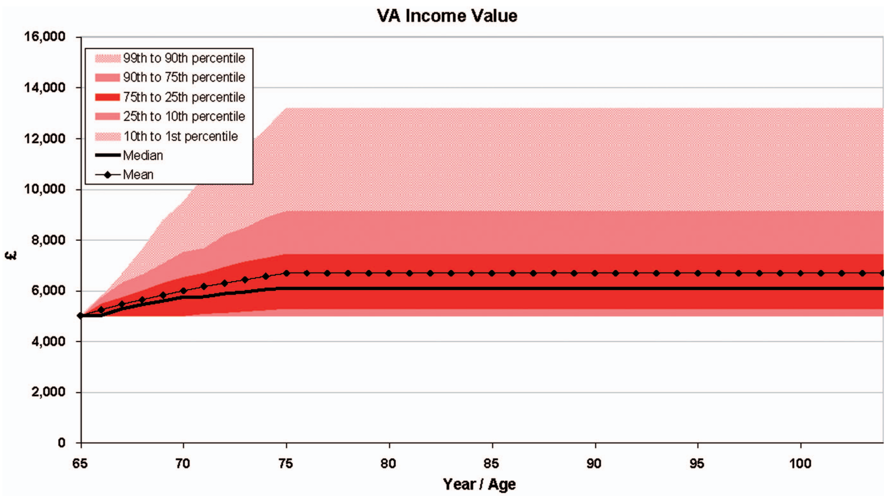

Figure 6.3.3a. Percentiles of income through time, variable annuity

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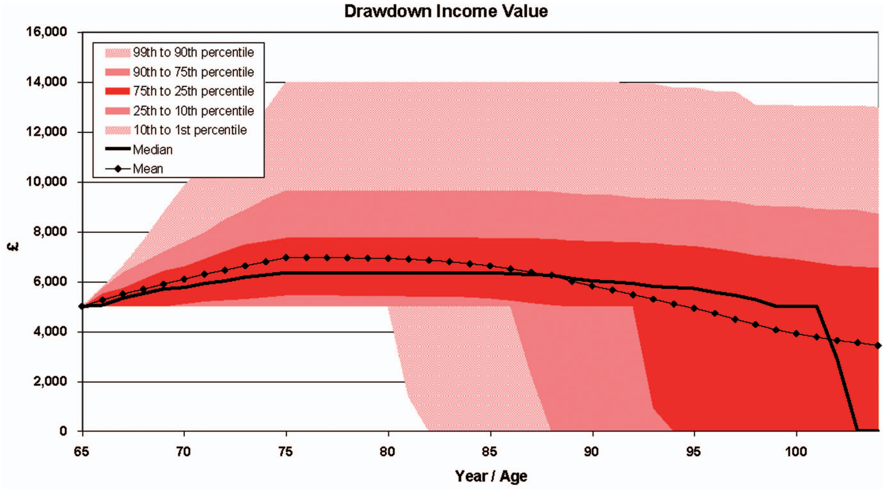

Figure 6.3.3b. Percentiles of income through time, drawdown

at older ages, or, in the absence of GAD limits (as modelled here), a significant chance of the fund running out. Conversely, the lower charges mean that the average fund size is greater for the drawdown product, resulting in increased upside chances and higher potential death benefits.

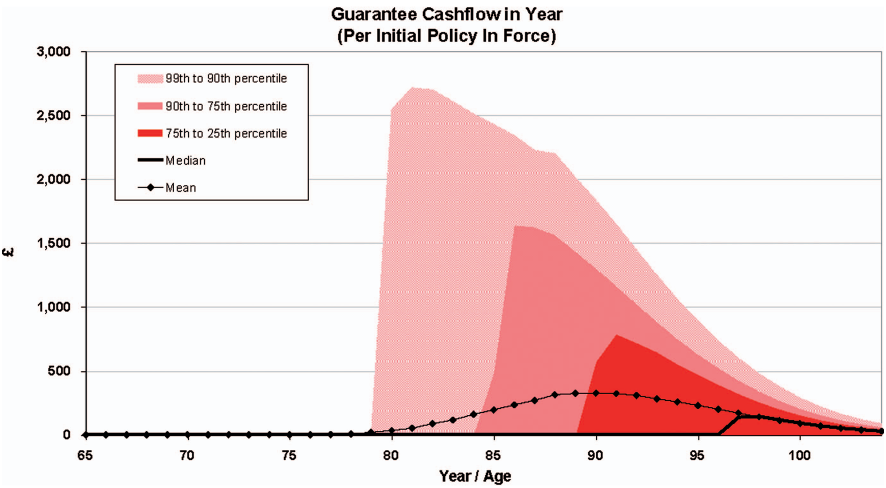

Figure 6.3.5. Percentiles of guarantee claim payments through time

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6.3.5 Figure 6.3.5 shows the probability distribution of the guarantee amounts which the insurer must pay out for the variable annuity product per initial policy in force, after allowing for lapses and deaths (assuming that the rider remains in force).

6.4 *Guarantee Outcome and Capital*

6.4.1 Finally, Figure 6.4.1 shows the probability distribution of the present value of the net guarantee profit per initial policy, with discounting at the cash roll-up rate, if the guarantee has not been hedged. The net profit is defined as the present value of the guarantee charges less the present value of the guarantee payouts. Note that the distributions are real-world/best-estimate distributions, which might be used when setting economic capital for the guarantee — it is not appropriate for determining the market-consistent cost of the guarantee, for which a risk-neutral distribution is required (see Section 7). However, the figure also shows clearly the effect of sample error in the individual values, demonstrating why it is necessary to perform more than 1,000 simulations if accurate percentiles are required, for example in order to set capital. The 95th percentile of the distribution is –£1,556, while the 99.5th percentile is –£8,699. The figure also shows the conditional tail expectations representing the average of the worst 35% of scenarios (CTE65) and the worst 10% of scenarios (CTE90), £1,873 and –£2,662 respectively, which are discussed in Section 8 on reserving.

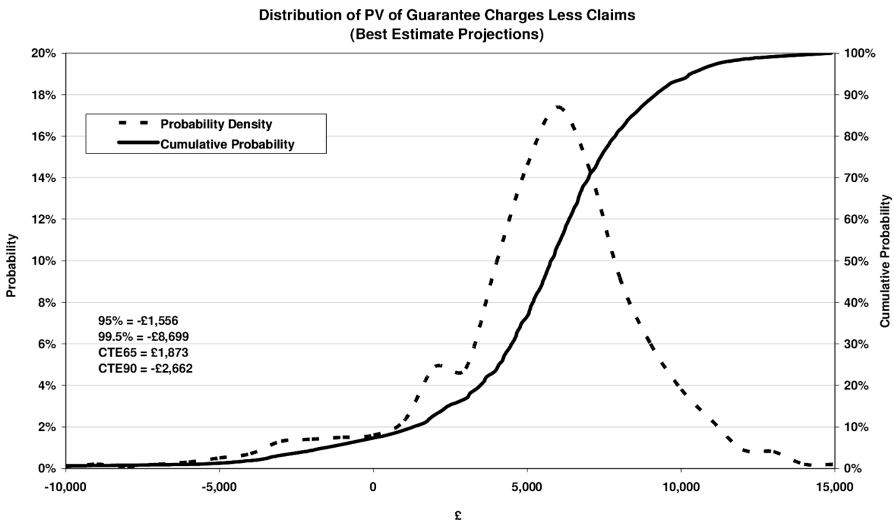

Figure 6.4.1. Probability distribution of the present value of guarantee charges less claims

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6.4.2 Figure 6.4.1 shows a high degree of variation in the possible outcome from the insurance company's perspective. As explained, the figure shows the outcome distribution if the guarantee has not been hedged. Of course, it is precisely this variation in possible outcomes which means that insurers are likely to carry out some form of hedging in practice. Possible types of hedging and the impact on the distribution of outcomes are discussed in Section 10.

6.5 *Comparison and Treating Customers Fairly*

6.5.1 A further point of comparison for the variable annuity product would be a conventional annuity with a fixed income for life, but no withdrawal or death benefits. We have chosen a single life annuity for the same male aged 65 and a guarantee period of ten years, meaning that, on death before age 75, the remaining income which would have been paid up to age 75 is paid out as a lump sum. At the end of October 2007, the approximate median internet rate for a level annuity was £7,000 p.a. and for an index-linked annuity £4,600 (source: www.fsa.gov.uk/tables/).

6.5.2 However, direct comparison between a conventional annuity, the variable annuity product modelled here, and an income drawdown product is not generally possible. In different market conditions the products will all provide different levels of income and of residual fund value or death benefit. The relative attractiveness of each product will depend on an individual's objectives, for example, how important a guaranteed level of income is, what about protecting income against future inflation, is there a need to provide for other family members on death, etc.?

6.5.3 Under the Financial Services Authority (FSA)'s Principles Based Regulation regime, all regulated firms are required to follow a series of principles in their treatment of customers. The FSA has produced, in an Annex to its Handbook, a Regulatory Guide called 'The Responsibilities of Providers and Distributors for the Fair Treatment of Customers' (the Guide). The Guide states that providers and distributors should consider the impact of their action (or inaction) on the customer at the following stages of the product lifecycle:

— design and governance;
— identifying target markets;
— marketing and promotion;
— sales and advice processes;
— after-sales information and service; and
— complaints handling.

Most of the requirements in the Guide do not have any special implications for variable-annuity-style guarantees. However, there are some specific issues which need to be considered.

6.5.4 When designing a product, the provider should perform a stress

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test to identify how it might perform in a range of market environments and how the customer might be affected. Since the value of the guarantees depends on market scenarios, it is important to consider a range of different outcomes and what the implications for customers might be.

6.5.5 The provider should ensure that the information provided to distributors is sufficient, appropriate and comprehensible in substance and in form. It should consider whether the information will enable distributors to understand it enough to give suitable advice and to extract any relevant information and to communicate to the end customer. The stochastic nature of returns on variable-annuity-style guarantees means that it can be difficult for distributors to assess their value. This places a high responsibility on the provider to communicate effectively. It is also very important that the nature of the guarantee is clearly explained to the distributor/customer, so that he or she is able to determine the charges which apply and when guaranteed payments will occur. The HMRC/GAD limits make this a more complex task in practice, and these are discussed in detail in Section 11.2.

6.5.6 Post-sale, the provider should review periodically products whose performance may vary materially, to check whether the products are continuing to meet the general needs of the target audience. This could have implications where the option becomes substantially out-of-the-money. To treat customers fairly, the company may need to consider notifying the policyholder if this happens, and, therefore, that it might be sensible to surrender or to cancel the guarantee rider, where this is possible. However, product designs which include ratchet features should help to avoid this issue by increasing the guarantee where it would otherwise become substantially out-of-the-money.

6.5.7 The principles on the fair treatment of customers will be particularly important if the provider retains the facility to amend any of the terms of the contract, such as changes to the guarantee charge, the fund mix or limits on available funds. These features can be useful risk management features for the provider. However, they can only be used where they are fully explained to customers up-front and where the provider can justify that their use is consistent with the fair treatment of customers.

6.5.8 Overall it can be seen that good quality advice will be essential for consumers. However, it is possible that further training of advisers and the development of advice tools will be required before the advice system can handle the full complexity of product features confidently.

6.5.9 In conclusion, this variable annuity product offers attractive benefits for some consumers. When compared with traditional annuities, it adds significant potential upside when markets perform strongly (see Figure 6.3.3a), and enhances the benefits on death. When compared with income drawdown, it removes the most dangerous outcomes for customers in extreme market scenarios (see Figure 6.3.3b), and may also increase the potential upside if the security offered by the guarantee enables the customer

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to invest in a riskier asset mix. Therefore, the combination of benefits delivered by variable annuities should be a welcome addition to the options available to meet customer needs.

7. Pricing

7.1 *Introduction*

7.1.1 In this section we consider methods which can be used to price variable annuity guarantees. We base the analysis on the sample product described in Section 5. The methods and techniques described are applicable to a wide range of other guarantee types.

7.1.2 The guaranteed minimum benefits of our sample product have asymmetrical risks with respect to the future investment scenarios. In general, if the unit fund performance is good and the yield curve does not move adversely against the insurer, there will be no guarantee claim payout from the insurer. In investment scenarios where the fund does not perform, the potential claim payout to policyholders from the minimum benefit guarantees can be significant. Given the asymmetric relationship of the guarantees to investment scenarios, a stochastic valuation methodology is recommended.

7.1.3 Two stochastic valuation approaches are available to value such guarantees — the traditional actuarial approach, which uses a 'real world' projection, and the market consistent approach, which typically uses a 'risk neutral' projection. Pricing practice varies across different countries and different companies. We believe that the use of the market consistent approach for pricing variable annuity guarantees is the most appropriate method for actuaries and for companies in the U.K. today. This approach uses stochastic valuation techniques consistent with the pricing of options. This flexible methodology enables most product benefit and charging structures to be accommodated, and facilitates the calculation of risk exposures which can be used to construct and to manage a dynamic hedge portfolio. For further details on the choice of valuation approaches, please refer to Hardy (2003) and Møller & Steffenson (2007).

7.1.4 In the following, we set out some of the key considerations in valuing the GMWB for life, using a market consistent approach.

7.2 *A Market Consistent Valuation Model*

7.2.1 A market consistent valuation model for financial guarantees consists of two key components — the algorithm for guarantees' cash flows and the economic scenario generator.

7.2.2 The algorithm for guarantees' cash flows needs to be defined carefully, in order to capture asymmetrical risks in the guarantee contract. This includes valuing both the guarantee charges and the claims stochastically.

7.2.3 With respect to an economic scenarios generator, a range of asset

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models are potentially available for use in pricing guarantees. A detailed discussion on the choice of the asset model is beyond the scope of this paper. For the purpose of pricing, however, the calibration of the model can arguably be more important than the model chosen. Guidance for calibration is available in professional guidance note GN47.

7.3 *Mortality Assumption*

7.3.1 It is necessary to consider the target business markets and the product design in setting the mortality assumptions. Unlike traditional annuities, where the customer has no option to surrender the contract and to receive the remaining capital value of an annuity, a GMWB for life rider on a unit-linked contract can normally be terminated at any time. Policyholders can surrender the contract and receive the remaining account value. This introduces a risk of anti-selection for the provider of the GMWB from the policyholder's behaviour. Policyholders with a GMWB for life rider, who have poor health, may be tempted to turn off the rider in order to preserve their fund from further charges. The consequence is that the remaining policyholders have better than normal longevity when compared to a pool of traditional annuity policyholders. A suitable margin for the potential risk of mortality selection should be loaded in pricing.

7.3.2 Product design can be used to reduce the sensitivity of a product to mortality exposure. For example, the death benefits offered on a GMWB product can mean that the product has less sensitivity to mortality. It is important to consider the impact of both lighter and heavier mortality in pricing.

7.4 *Policyholder Behaviour Assumptions*

7.4.1 The lack of credible data on policyholder lapse behaviour presents a challenge when pricing variable annuity guarantee riders in the U.K. Insurers may have data on the base product lapse rates, but the GMWB for life may be sold as a rider on the base contract, and the impact on the overall aggregate lapse rate is less clear. In general a low lapse rate increases the cost of guarantees, as more policies will benefit from the potential future claims. For companies hedging their market risks, if the actual lapse rate is lower than the original hedging assumption, then the company could run the risk of under-hedging. Indeed, equally, there can be difficulties making assumptions which are too conservative leading to over-hedging and, consequently, to losses when lapses actually emerge.

7.4.2 Policyholder lapse behaviour may depend on the investment scenarios. From the policyholder's perspective, the optimal behaviour is usually to keep the rider when the guarantee is in-the-money and lapse the rider when the guarantee is out-of-the-money. U.S. experience, to date, indicates noticeable anti-selective behaviour, varying by product and by market segment, but not necessarily 'perfectly rational on a market value basis'. Consequently, U.S. insurers generally use dynamic lapse rates when valuing guaranteed minimum benefits, although the allowances may only be semi-optimal, i.e. they do not assume that every policy would lapse if the guarantee were out of money. A survey carried out by the Society of Actuaries (2005) showed that 15 out of 18 companies assumed dynamic lapse behaviour in their statutory valuation. In pricing this product, we have used the dynamic lapse formula shown in Section 5.3. Further details can be found in Mo (2007) and Drinkwater (2007).

7.4.3 Apart from lapse experience, other policyholder behaviours are likely to impact the insurer. For example, policyholders have options regarding when to turn on the guarantees, when to take income, and how much income should be taken. Modelling the potential policyholder behaviour presents further challenges. Effective product designs are necessary to mitigate any unfavourable policyholder behaviour. For example, if a ratchet or surrender penalty is not included in the rider design, in times when fund performance is good, logical policyholders should lapse and re-enter the contract in order to lock in the fund growth. Hence, careful product design and wording are necessary to act as the first line of defence against any policyholder anti-selection.

7.5 *Sensitivity Analysis of the Market Consistent Cost of the Guarantee*

7.5.1 Table 7.5.1 illustrates the market consistent present value of guarantee claims and rider charges for the working example shown in Section 5. The final column in the table is the market consistent cost of the guarantee, expressed as a constant annual charge for the period whilst the rider is selected. Sensitivity analysis is provided for key modelling parameters.

7.5.2 The cost of the guarantee is very sensitive to capital market parameters. In general, higher volatility increases the cost of the guarantee as the probability of the policyholder's fund being depleted to zero increases. A lower interest rate yield curve also gives a higher cost of the guarantee. This is because, under a risk neutral valuation, a lower yield curve implies lower expected fund returns and discount rates — hence the probability of the insurer paying guarantee claims is higher, and the present value of the expected cost of the guarantee is higher.

7.5.3 The cost of the guarantee is also very sensitive to mortality rates and policyholder's withdrawal and surrender behaviours. As expected, using a lighter mortality table, lower lapse rates or introducing a dynamic lapse function increases the cost of the guarantee. From our analysis, encouraging policyholders to defer taking withdrawals can also reduce the cost of the guarantee. For further product design discussions and pricing analysis of GMWB for life, please refer to Su (2007).

7.6 *Setting the Price*

7.6.1 In considering the actual pricing level for the guaranteed minimum

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Table 7.5.1. Sensitivity analysis of the market consistent (MC) cost of the guarantee

Sensitivity analysis	MC cost of guarantee claims (£)	MC cost of rider charges (£)	Cost of the guarantee in bps p.a. charge
1. Base case	3,385	5,201	49
2. Capital market assumptions			
(a) +1% shift in yield curve	1,835	5,021	27
(b) −1% shift in yield curve	5,849	5,366	82
(c) 25% relative increase in equity implied volatility	4,644	5,164	67
(d) 25% relative decrease in equity implied volatility	2,256	5,238	32
(e) 25% relative increase in swaption implied volatility	3,494	5,201	50
(f) 25% relative decrease in swaption implied volatility	3,304	5,201	48
3. Longevity risk			
(a) 80% of the base case mortality table	4,355	5,467	60
(b) 120% of the base case mortality table	2,692	4,974	41
4. Policyholder behaviour risk			
(a) Defer taking withdrawals for 3 years	2,974	5,890	38
(b) Defer taking withdrawals for 5 years and increased 5.5% income at age 70	3,993	6,515	46
(c) 6% base lapse rate with dynamic lapse	2,609	4,635	42
(d) 2% base lapse rate with dynamic lapse factor	4,404	5,895	56
(e) 4% fixed lapse rate with no dynamic lapse factor	2,576	5,193	37

benefits, the insurer will need to take account of: the current market offering; the level of competition; the perceived value of the guarantees from customers; the cost of providing or reinsuring guarantees; the asset liability management strategy, and the targeted return on capital. The market consistent cost of guarantees provides a baseline for the charges required from the policyholders.

7.6.2 One challenging aspect of setting the guarantee charges is that, unlike a traditional annuity contract where insurers typically vary their prices for new business on a frequent, often weekly basis, the guarantee charges are generally changed infrequently. The guarantee charges are expected to be fixed for a period of time, but the market consistent value of the guarantees changes constantly, and so the market consistent profit margin will be very

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sensitive to market conditions at the point of sale. A policy sold this week will have a different profit margin to a policy sold next week. This will result in volatile new business profit. A risk premium can be loaded to compensate for this pricing risk. Alternatively, the insurer could choose to pre-hedge the market risk exposures according to some expected business volume and mix to lock in the expected profit margin, or to set trigger points for re-pricing new business.

7.6.3 A profit testing or emerging cost analysis is required to assess the shareholder's return on both statutory and economic capital for different pricing and hedging strategies. The rider guarantees and the base unit-linked contract ideally should be profit tested together, in order to understand the total value of the combined product propositions. For example, if lapse rates decrease, this makes the base product more profitable, but is likely to lead to reduced profits on the rider guarantee.

7.7 *Profit Testing*

7.7.1 Different levels of sophistication can be used for profit testing a product with financial guarantees. One simple approximation is to express the market consistent cost of guarantees in terms of the annual management charge and to calculate the return on capital using a simple deterministic model. For example, if the guarantee charge is 0.75% p.a. and the cost of guarantee plus expenses is 0.50% p.a., then the net profit margin is 0.25% p.a. If we assume that the capital requirement is 4.0% of the unit-linked funds, and we assume that the capital earns a cash yield of 5.50% p.a., then the rate of return on the capital can be approximated as $(0.25\% + 4.0\% \times 5.50\%)/4.0\% = 11.75\%$ p.a. The implicit assumption of this simple approximation is that market risk hedging is used, and that the cost of hedging is the market consistent cost of guarantees.

7.7.2 The simple approximation above serves as a useful ballpark estimate of the level of return on the capital, but it fails to capture the projected cash flows for a company under different investment scenarios. Understanding the projected cash flows is important for insurers adopting hedging strategies to minimise the market risks. It is worth pointing out that, because of the exotic nature of the guarantees and the presence of demographic risks, no hedging strategy or hedge assets will be perfect. For companies targeting to hedge 100% of market risk, there will always be an element of noise in the profit and loss accounts, due to the imperfect nature of any hedge. To understand the hedge effectiveness and to project cash flows, a more sophisticated financial projection algorithm is required. A nested stochastic financial projection tool is recommended to project the values of future hedge assets and liabilities under a range of investment scenarios.

7.7.3 Under a nested stochastic projection, the outer scenarios are typically projected on a real world basis. At each time step of an outer

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scenario, the cost of guarantees is recalculated with inner risk neutral scenarios, and hedge assets are also projected forward with any rebalancing algorithm. A nested stochastic projection allows the balance sheet position at different points in time to be determined, and allows the investigation of the effectiveness of hedging under different investment scenarios. Transaction costs and the liquidity requirements of maintaining a hedge strategy can also be studied using a nested stochastic projection. We illustrate the use of nested stochastic projections in Section 10.9. The nested stochastic projection technique can be used to provide in-depth management information, but, as for all actuarial modelling, care should be taken to understand the limitations of the results produced by the model.

8. Reserving and Accounting

8.1 In this section we consider the reserving requirements in two markets which are well known to the authors, the U.K. and Ireland. The Irish section may be of interest to many U.K. actuaries, as a significant proportion of products offered in the U.K. and the European markets today are sold from Ireland. The approaches and the methodologies commonly used in Ireland have been adapted from the U.S.A. and they provide a useful benchmark approach for U.K. actuaries to consider. We also consider asset admissibility requirements in the U.K. and accounting issues.

8.1.1 The amount of reserve and the methodology used will be influenced by whether or not an active hedging programme is in place. This section assumes that hedging is in place. Section 10 calculates different levels of capital which may be required, depending on whether hedging is in place or not.

8.1.2 The interaction between capital and reserves will vary from company to company, depending on the particular capital measure which is appropriate for the company. We do not attempt to cover all of these possibilities.

8.2 *U.K. Reserving*

8.2.1 We believe that this contract should be considered as a unit-linked assurance with guarantees. Under this classification, the gross reserves for the product will be calculated, in accordance with generally accepted actuarial practices (INSPRU 1.2.10 (7) R), as:

- a unit reserve; plus
- an option reserve for the investment guarantees (if the guarantee rider option was purchased); plus
- an additional sterling reserve.

8.2.2 As a non-profit unit-linked product, only the Peak 1 rules apply.

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Peak 2 rules would only apply if the with-profits fund was used amongst the list of funds on which the guarantees are written. Unit, additional sterling and aggregate expense reserves will be determined in the normal way for unit-linked policies without investment guarantees.

8.2.3 The value of the option reserve will be determined using stochastic methods, reflecting the wide range of scenario outcomes which need to be valued. The choice of model would need to take into account the professional and regulatory guidance in force at the time. A risk neutral, arbitrage free, market consistent model is recommended. This would be consistent with the hedging approach which will be used to support the guarantees (and regulatory guidance applicable to similar Peak 2 embedded option liabilities). Margins for adverse deviations would need to be added to the otherwise risk neutral valuation basis. This suggests valuing the rider using risk neutral market implied parameters (reflective of the hedge assets backing the rider), but with the volatility and other assumptions adjusted to provide the margin. The minimum requirement is for the margins for adverse deviation to be greater than, or equal to, the relevant market price for the risks being valued.

8.2.4 The option reserve will be calculated net of the value of future guarantee charges. Thus, within the stochastic projections, guarantee claims are treated as a liability and guarantee charges as a negative cash flow, both of which are discounted by the valuation interest rate. The important principles are the avoidance of a future valuation strain and that the collection of future guarantee charges are consistent with TCF.

8.2.5 Although new regulations allow negative mathematical reserves, the Consolidated Life Directive still requires that no contract should be valued at less than its surrender value. It is therefore likely that any negative option reserve would be floored to zero on a per policy basis. In particular, this will be necessary if the rider option can be lapsed in isolation without a surrender penalty. By zeroising the reserve, expected profits inherent in the rider charge premium basis will not be capitalised on the day when the policy is sold, but will fall into surplus over time, as the charges are received and the actual cost of providing the guarantee unfolds.

8.2.6 Selection of the mortality table would need to reflect the fact that there is a longevity guarantee, as well as an early (non-guaranteed) death benefit prior to age 75.

8.2.7 Traditionally, a zero lapse assumption was used for similar statutory valuations, although a case can be made to use an assumption involving dynamic policyholder behaviour (higher lapses in rising markets when options are out of the money, and lower lapse rates in falling markets when options are in the money).

8.2.8 For any contracts with a GMDB, there is capital at risk on death. For unit-linked business, U.K. regulation stipulates that 0.3% of this capital at risk (technically netted down for reinsurance) be held as an insurance death risk capital component of the solvency margin (INSPRU 1.1.81-84R).

8.2.9 U.K. regulation also requires that an additional solvency margin component of 4% of reserves (unit, sterling, aggregate expense and option) is held for products with guarantees. This is likely to apply regardless of whether the guarantee is written as a rider policy or not (INSPRU 1.1.92G). The requirement to hold solvency capital of 4% of reserves applies regardless of how onerous (or non-onerous) the investment guarantee is. It is therefore recommended, that, for prudent and sound management of the business, the amount of capital actually required to support the risk be assessed.

8.3 *Irish Reserving*

8.3.1 Insurance companies in Ireland are regulated by the Irish Financial Regulator (FR) under the European Communities (Life Assurance) Framework Regulations 1994 (1994 Regulations), which transpose the European Union's Third Life Directive. The regulatory structure is generally principles based, and is quite similar to the structure which applied in the U.K. prior to the introduction of realistic balance sheets.

8.3.2 As for the U.K., the product will be classified as investment linked with guarantees. Where only the guarantee rider is carried on an Irish balance sheet, it would be treated as Class I business rather than as linked business. However, the same reserving methodology would normally apply to the guarantee rider. A unit reserve, option reserve for the investment guarantees, plus an additional sterling reserve would be held. In some situations the Irish company underwrites only the guaranteed benefits and, therefore, will only hold (for this line of business) the option reserve, which will incorporate any additional expense reserve.

8.3.3 As with the U.K., the value of the option reserve will be determined using stochastic methods projecting both guarantee cost claims and guarantee charge premiums.

8.3.4 Where the company operates a dynamic hedging regime, it has become normal practice (in line with the method being used by the U.S. regulators) to make an appropriate allowance for future movements in hedge assets in the calculation of reserves and of the total capital required. Before taking credit for a hedging strategy, it will be important to demonstrate that the hedge strategy is actually in use and that a 'clearly defined hedging strategy' is in place.

8.3.5 The principles-based method of regulation in Ireland allows Irish companies to adapt reserving methods in use in other countries. It has become established to use the U.S. conditional tail expectation (CTE) approach. For example, the reserve would be set at a CTE 65 level, which means that the reserve represents the average of the worst 35% of scenarios. This means that a full set of stochastic projections of claims, guarantee charges, expenses and investment returns on backing assets is required. Note that a series of nested stochastic calculations is required to project hedge

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asset transactions, typically using real world scenarios with risk neutral and reserving bases nested within.

8.3.6 The Appointed Actuary must he satisfied that the approach adopted and the assumptions used are prudent.

8.3.7 The Appointed Actuary should ensure that appropriate account is taken of the requirement for a resilience reserve, though this can be covered by the basic methodology above. In calculating the resilience reserve, allowance is made for hedge asset movements.

8.3.8 Companies are subject to the normal E.U. solvency rules. In reality it is important to demonstrate that the total level of capital is appropriate and provides a sufficient level of prudence.

8.3.9 The general E.U. solvency margin appropriate for this type of business is 4% of reserves. However the FR will normally expect companies to hold capital equal to at least 150% of the required minimum solvency margin (a higher level would be expected of new companies). To test capital adequacy, the Appointed Actuary will normally check that the total capital is at least equal to the level calculated on a CTE90 basis. With the arrival of Solvency II, it is likely that the total capital measure will change to a 99.5% VAR approach.

8.4 *Admissibility of Derivatives as an Asset*

Assuming that a company selling variable annuities uses hedging to manage its risks, there is the requirement to conform with the FSA rules on derivatives. INSPRU 3.2 sets out the main criteria which apply. While all hedge assets are normally admissible, it is important to take account of the appropriate rules when implementing a hedging strategy. Similar rules apply in Ireland.

8.5 *Accounting*

8.5.1 This paper does not claim to cover accounting issues in any great detail. In the U.S.A., the financial reporting of variable annuity products is subject to both U.S. GAAP and local regulatory standards. Many companies design their hedging strategies to reduce the volatility of U.S. GAAP earnings, and the classification of the product as an insurance or an investment vehicle is an important issue, as this will determine how the benefit liability is to be calculated. As with most products, if the charges are insufficient, this can lead to profitability and DAC recoverability issues.

8.5.2 Outside of the U.S.A., accounting for these products will generally follow IFRS principles. Given the level of mortality or longevity risk involved, these products are generally considered to be insurance business for accounting purposes. However, each product needs to be considered separately to ensure that a suitable level of risk is included.

8.5.3 Any hedge assets normally have a readily available market value, and therefore can be valued in a quite straightforward manner.

8.5.4 The valuation of liabilities is more complex. The most obvious approach is to value the guarantee liability as the present value of future guarantee claims less the present value of future guarantee charges. This valuation would be performed on a market consistent basis, with best estimate assumptions for each of the variables required. However, this can lead to a negative reserve at the policy inception, since the present value of future guarantee premiums is likely to be greater than the present value of claims. Should the company hold a negative reserve in this case? If so, a large profit would flow through the profit and loss in the year of sale. This may not be consistent with the measurement of profit for other products.

8.5.5 An alternative is to calculate the expected cost of the guarantee at inception on best estimate assumptions. The liability at time zero would be the present value of claims less the present value of the best estimate cost of guarantee. This would create an initial reserve of zero, and hence there would be no initial profit or loss. The margin in the guarantee premium would then flow to profit in subsequent years, plus or minus any experience differences or hedge profits/losses. The company must ensure that the method used is consistent with the basis used for other products.

9. Risk Management

9.1 *Overview*

9.1.1 When variable annuities were first developed in the U.S.A., companies were prepared to take risk exposures directly on their balance sheets, with limited or no hedging of the inherent market risk. The main risk management applied was to limit the extent of the guarantees to levels which were seen as unlikely to bite. The very severe bear market of 2000 to 2002 brought the risks of such a strategy clearly into view, with very significant financial impact for a number of market participants. As a result of this experience, U.S. companies have developed comprehensive financial risk management strategies to allow them to offer guarantees which customers value, without themselves being exposed to the full impact of adverse market movements.

9.1.2 There are a number of parallels to the experiences in the U.K. with-profits market, where a relatively unsophisticated approach to managing risks was applied prior to 2001 but subsequently hedging strategies have become quite common, if not the norm.

9.1.3 In 2008 we find ourselves in an environment where risk management has become a much more central and recognised discipline within financial institutions. Increasingly, the external expectation is that companies will have in place enterprise-wide risk management (ERM)

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processes. The objective behind these processes will be to give senior management a clear, consistent and comprehensive view of all the risk exposures in the corporate entity, including all subsidiary businesses.

9.1.4 Within the context of a company's ERM process, the variable annuity product provides a very interesting challenge. The product introduces a wide range of different risk exposures which need to be analysed, and which can impact the risk profile of the organisation.

9.1.5 Companies with effective risk management programmes will have a well defined risk appetite in relation to each risk category. The approach and strategy in managing risks inherent in the variable annuity product should be influenced by the company's risk strategy and appetite statements. A company with a significant appetite for market risk may be happy to put in place no, or limited, hedging, whereas a company with a limited appetite for market risk will need to manage its exposure by sophisticated hedging, or, may, indeed, prefer to reinsure all or part of the product. The company will also need to ensure that its approach to non-market risks is consistent with its attitude to market risks.

9.1.6 In the rest of this section we cover, in more detail, the range of risk exposures associated with the product.

9.2 *Market Risk*

9.2.1 A key feature of the product design is an inherent exposure to market risk — the objective of the product, after all, is to offer customers protection from the full volatility of the market. The management of this market exposure is such a key part of the proposition that it is covered in detail in Section 10.

9.2.2 Market risk can be decomposed into so-called systematic market risk and non-systematic risk, or so-called basis risk. Basis risk relates to the extent to which the underlying managed funds do not perfectly track market indices, while systematic risk covers the risk of the indices and other market variables moving adversely.

9.3 *Credit Risk*

The implementation of a hedging programme will involve a range of derivative and structured instruments being employed. This will introduce an exposure to a range of counterparties which will need to be managed. This is in addition to the credit risk component of exposures to defaultable bonds in the underlying product investments.

9.4 *Longevity/Mortality Risk*

9.4.1 The various types of variable annuity product which can be offered will generally include some element of mortality or longevity risk exposure. Exposures to mortality risk, typically, will be relatively straightforward to manage, although this is somewhat complicated by the fact that the level of

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exposure varies with market levels. A significant fall in market levels could lead to a material increase in the mortality exposure, and, indeed, this was an issue faced by many U.S. insurers and reinsurers in 2000 to 2002. There is also a risk of selective lapse behaviour in such circumstances, with less healthy customers retaining their contracts to benefit from the increasingly valuable life benefits. A variety of reinsurance strategies will be available to mitigate these risks, and consideration should be given to establishing triggers and thresholds to prompt a review of the risk management strategy.

9.4.2 Products offering a guaranteed minimum income or withdrawal benefit for life present a more interesting challenge. This is due partly to uncertainty over longevity improvements, but also because risk management solutions for longevity exposures are less well developed. Also, many companies in the U.K. will have substantial existing exposures to longevity, and may be writing other products, such as pension scheme buyouts or annuities, which further increase this exposure. The nature of longevity risk within these products is complex, and will depend on the investment experience. Very often there is a geared exposure to longevity risk, where a small increase in mortality improvements can lead to a substantial increase in the value of the guarantee. The reinsurance market for longevity risk is relatively immature, although capacity has been increasing markedly in recent years. The ideal risk management solution will be a form of contingent insurance which reflects the fact that an adverse outcome only arises for the company where both the investment performance has been poor and the expected longevity experience has worsened. This type of insurance may be difficult to put in place, and many insurers are likely to accept on balance sheet the additional longevity risk exposure which the product brings. The conservatism in the pricing basis may reflect overall corporate appetite for this risk, and is often the first line of defence for this risk source. Another risk management solution is to add a death benefit to the product, so that there is a reduced exposure to longevity risk. This is common in the U.S.A., but may be less possible in the U.K., due to HMRC restrictions on death benefits on pension products.

9.5 *Persistency Risk*

9.5.1 Like most life and pensions products the profitability of the product will depend on the levels of persistency. Most variable annuity products will offer guarantees which become more or less valuable in changing market conditions. Higher lapses increase profits for the company when the guarantee is in the money, while lower lapses increase profits when the guarantee is out of the money. However, persistency may increase when valuable guarantees are biting, and may markedly reduce if guarantees are seen to have become less valuable following strong market performance.

9.5.2 When modelling the product, some attempt to model policyholder behaviour will be required, but will inevitably be difficult, due to lack of

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credible data. Most companies will model an element of dynamic lapse, though it is unlikely that they will allow fully for rational behaviour. When profit testing the product, it will be helpful to profit test the base product and the guarantee together, as lower persistency usually means greater profitability from the base product, which can partly offset losses made on the guarantee component.

9.6 *Election Rate Risk*

Most GMIB/GMWB products give the customer the option to elect to start taking income over a period of time. Also, generally, they allow flexibility over the amount of income. Higher levels of income and/or earlier withdrawal generally lead to greater costs for the insurer. The most common method to manage this risk is through offering improved terms for those who defer commencement of their income withdrawal. Ratchets can also lead to a reduction in income election, especially when the option is out of the money.

9.7 *Asset Allocation Risk*

Many variable annuity products allow the customer the option to select among a range of different funds, which may have different asset allocation mixes. Economic costs will normally increase with increased exposure to equities or to equity markets with higher volatilities. Generally, pricing will be based on an assumed average allocation. There is a risk to the company that the actual mix of selected funds will differ from the assumptions. The risk can be reduced by limiting the range of funds, varying price with asset mix, or introducing policy clauses which allow the company to change the asset mix if volatility increases. Obviously, any ability to change the asset mix or the charge must be consistent with the TCF principles.

9.8 *Operational Risk*

The variable annuity product, particularly if it is a new development for a company, introduces a wide range of potential operational risk issues. These risks will include pricing and modelling risks, and, perhaps most significantly, a range of risks associated with any new market risk hedging programme which is put in place by the company. The product is likely to introduce a range of complex exposures to market risk, and interactions between market and behaviour risks which do not exist in other products. Therefore, it will be important that the company introduces additional reporting systems to track these risks on an ongoing basis. Similarly, a range of additional skills will be required to implement a hedging programme, especially if dynamic hedging is employed.

9.9 *Strategic Risk*

9.9.1 When reviewing their business plans, companies will need to

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consider the strategic risks associated with the variable annuity market. Will the variable annuity market become a significant part of the U.K. market? Is the company adequately positioned to respond to this development? Will this development provide an opportunity for new overseas market entrants? Will there be any disruption to established market positions in the annuity or drawdown markets, for example? The impacts which market activity has on business volumes, and consequential unit costs, and expense risks also need to be considered.

9.9.2 Risks of this type will need to be assessed by companies (whether they decide to enter the market or not) as part of their business and strategic planning processes, and appropriate risk mitigants considered.

9.10 *Pricing Risk*

The long-term profitability of the product will depend on a wide range of assumptions within pricing models. In practice, the experience will differ from these assumptions, and the profitability may turn out to be less than hoped for. The complexity of the product increases the risk of model errors or the results proving more sensitive to pricing assumptions than expected.

9.11 *Extreme Events*

Through a combination of an effective hedging strategy and an appropriate product design, it should be possible to maintain risk exposures within agreed risk appetites. It will, however, be important to think carefully through the impact of extreme market and financial conditions, where the normal assumptions underlying the hedging programme could become invalid. Market volatilities could increase to much higher levels than those assumed, or it could become impossible to trade in the securities which are required to maintain a well hedged position. It will be important to work through these scenarios on a regular basis to quantify the maximum exposures should such unusual events arise. Rating agencies will also want to understand the impact of these extreme scenarios.

9.12 In the next section we consider, in some detail, the management of market risk. We quantify the impact of these market risks, and also lapse and longevity risks.

10. Hedging and Market Risk Management

10.1 *Introduction to Market Risks*

10.1.1 As mentioned in previous sections, the variable annuity product introduces significant exposures to market risks. We start this section with a brief introduction to the terminology commonly used by those actively managing complex market risk exposures — the so called 'Greeks' — delta, gamma, rho and vega.

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Delta

10.1.2 The first risk to consider is the risk that the unit-linked fund assets fall in value. As the asset values fall, the guaranteed benefits become more valuable, with the guarantee becoming more likely to bite. This is measured through delta, which captures how the value of the embedded option liability changes relative to changes in the underlying assets.

10.1.3 Symbolically:

$$\text{Delta} = \partial V_L / \partial V_A$$

where V_L denotes the value of the guarantee option liability, and V_A denotes the value of the underlying managed fund assets (e.g. the unit-linked fund).

10.1.4 Matching the delta of the liability options will be at the heart of the dynamic hedging process, which we describe below. However, as the value of delta, itself, varies as the value of the underlying assets change, this will only provide a temporary immunisation against small immediate changes in the underlying. Therefore, the convexity of the liabilities may also be important. This is analogous to Redington's (1952) theory of immunisation of traditional insurance portfolios with bond investments.

Gamma

10.1.5 The convexity of the value of the liability options with respect to the underlying assets is gamma. Equivalently, gamma measures the rate of change of delta with respect to changes in the underlying asset values, as:

$$\text{Gamma} = \partial \Delta / \partial V_A = \partial^2 V_L / \partial V_A^2.$$

Rho

10.1.6 In addition to the risk of the underlying assets falling in value, there is the risk that market interest rates can change. A fall in interest rates would lead to an increase in the capital value of future guarantee claims, and, hence, the value of the guarantee liability would normally increase. This risk is captured through rho, which measures how the value of the embedded option liability changes with changing interest rates.

10.1.7 Symbolically:

$$\text{Rho} = \partial V_L / \partial r.$$

10.1.8 Here r denotes the value of the interest rates used in the valuation of the embedded option. As there is a term structure to the interest rates, there will also be a term structure to rho. For this reason, it is quite common to group the yield curve into so-called buckets, and to analyse the rho for each of these buckets, which is referred to as key rate rho analysis.

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Vega

10.1.9 Another key risk measure is the volatility of the underlying assets; the more volatile the underlying assets, the more uncertainty about whether future asset values will be high or low when guarantee benefits fall due. The value of the liability option with respect to changes in volatility is vega:

$$\text{Vega} = \partial V_L / \partial \sigma_A.$$

Other Greeks

10.1.10 The above are usually the most important sources of market risk. There are clearly other sensitivities which can be measured through further 'Greeks'. In most circumstances the above four are sufficient for any practical hedging programme. Some of the other Greeks, which determine how the value of the liability options change and influence the design of more sophisticated hedge programmes, include:

- *volga*, which measures the volatility of volatility; and
- *vanna*, which is the sensitivity of the option value with respect to the underlying price and volatility. This can be considered as either:
  - the sensitivity of delta with respect to a unit change of volatility; and
  - the sensitivity of vega with respect to a unit change of delta. This is related to the volatility skew/smile, which measures the variation in the implied volatility with respect to the strike (i.e. the depth in or out of the money).

10.1.11 In addition, there are also Greeks which measure how the liability option value changes over time. As the passage of time is predictable, these are not sensitivities with respect to risk variables, but they can still be important when hedging positions over time intervals, such as a weekend or a holiday period. Examples include:

- *theta*, which is the sensitivity of the option value to changes in the unexpired option term, which is usually measured as the change in the option price to the calendar day passing; and
- *charm*, which is also referred to as delta decay, the sensitivity of delta to changes in the unexpired option term, usually measured as the change in delta to a calendar day passing.

10.2 *Dynamic Hedging*

10.2.1 There is a large body of research (Black & Scholes (1973), Harrison & Kreps (1979), Harrison & Pliska (1981), Hull (2006)) which has established that, under certain conditions, by investing in a portfolio consisting of risk-free and other assets, which together match the current market value of the liability options, as well as their Greeks, then, no matter what market movements take place (asset value changes, interest rate changes, volatility fluctuations, etc.), the portfolio of assets will always equal the liability option value. In particular, at the point(s) where the guarantee claims fall due, the portfolio of assets will match the value of the guarantee claims, and we say that the portfolio of assets replicates and hedges the liability options.

10.2.2 Since the values of the Greeks are continuously changing as market conditions change, it turns out that the component hedge assets used in the replicating portfolio need to be adjusted dynamically on a frequent basis — hence the term dynamic hedging.

10.2.3 Included within the theoretical conditions, for the dynamic hedge to replicate the liability options, are that the market is frictionless, trading can take place continuously, in arbitrary quantities, and that there are no riskless arbitrage opportunities.[2] Thus, it follows that the dynamic hedging strategies employed by banks or insurance companies, who manufacture embedded options in this way, will gear towards using instruments which are highly liquid, and for which trading costs are as low as possible. For this reason, futures contracts are popular for hedging delta risk, while swaps are popular for hedging interest rate risk. Equity options or variance swaps might be considered to hedge equity vega, while swaptions might be considered if interest rate volatility risk is significant.

10.2.4 The process involves valuing the liability options several times on a frequent basis. Most variable annuity writers carry out dozens of stochastic valuations on an individual policy basis, with each valuation using several thousand scenarios, on a nightly basis. These valuation results are used to compare regularly (almost continuously) the risk measures (i.e. the Greeks) of the liability options with the Greeks of the hedge assets, using the valuation results derived the previous night.

10.2.5 The process also consists of setting risk thresholds, which, if breached, would trigger a rebalancing trade recommendation. The exposures to the various hedge assets are effectively determined by solving simultaneous equations, which equate each of the Greeks of the liability options being hedged with the Greeks of the hedge assets. This imposes constraints on the assets which can be used within the hedge programme — they must be linearly independent. Thus, for example, the same option cannot be used simultaneously for matching vega and gamma — two independent options would be needed.

10.2.6 In view of the limitations of the market (i.e. it is not frictionless, continuous trading is not quite possible, and, in some cases, there is limited market capacity for long-term volatility protection), most practical hedges do

[2] Note that the list of conditions for dynamic hedging is slightly different for the list of conditions needed for certain closed form formulae, such as the Black-Scholes one.

not eliminate all the risk perfectly, but rather reduce the exposures significantly. The parameters of the hedge programme (risk thresholds, valuation frequencies, preferences towards futures or options, etc.), would be selected by investigating the financial impacts of alternative parameter choices and hedge strategies. These analyses could also bring in basis risk (also referred to as tracking error) and non-market risks, so that the limitations of the hedge with respect to all the main risk sources present can be studied.

10.2.7 The process also consists of monitoring the performance of the hedge, once it is implemented. By studying how the movements in the hedge assets in force match the liability option values, adjustments can be made to the parameters of the hedge and to the choice of instruments. The analyses and adjustments again would take into account the impact of basis risk, as well as non-market risks, such as the evolving dynamic lapse behaviour experience.

10.2.8 Ultimately, risk management is about reducing the risks to acceptable levels, ensuring that the amount of capital at risk is sufficiently low and that the product is priced with satisfactory returns on the residual capital which is at risk. Total elimination of the risk is not possible.

10.3 *Static Hedging*

10.3.1 An alternative to dynamic hedging is static hedging. Static hedging involves purchasing structured products 'over the counter' (OTC) to match the guarantee liabilities. This is easier for the simpler guaranteed minimum accumulation benefits (GMABs), which can be matched with a basket of vanilla European options, than for the more complicated living benefit guarantees with path dependent ratchets, such as the example considered in Section 5. For such products, the embedded option is highly exotic, and the matching OTC asset would need to be highly structured, which gives rise to liquidity and pricing issues.

10.3.2 Where an insurer invests in OTC options to hedge its liabilities, then it is likely that the counterparty (typically an investment bank) will use dynamic hedging techniques to enable it to manage the risks which it takes on. The insurance company is effectively purchasing structured products wholesale and selling retail.

10.3.3 Static hedges are so called because, if an exotic structured product can be found to match perfectly the guarantee option sold by the insurance company, then all Greeks of the embedded option are also matched, and, in a world of perfect knowledge about demography, at least, it would not be necessary to rebalance the hedge as a result of market movements. In reality, most static hedges also only approximately match the liability options sold, and some residual capital will be at risk, as well as some rebalancing being necessary.

10.3.4 Examples of static hedges that have been used include:

— vanilla European put options for GMABs or simpler GMDBs, where the notional amount of put options would depend on the demographic (mortality/lapse) assumptions on the block of business; and
— vanilla European swaptions for the interest rate risk on guaranteed annuity options (GAOs) (a special case of GMIBs). See, for example, Pelsser (2002) or Wilkie *et al.* (2004).

10.4 *Dynamic vs. Static*

10.4.1 One of the key advantages of dynamic hedging is that it enables the insurance company effectively to manufacture its own guarantees. This would be a valuable source of competitive advantage, as purchasing structured products or risk protection from a third party would inevitably lead to some profit opportunity being ceded to the third party. It is likely that the infrastructure required to run a dynamic hedging programme and to manufacture the guarantee options internally would be greater than for a static hedge. Therefore, it could be argued that a business case needs to be made to compare the profit advantages from internal manufacturing with the expense costs of establishing and maintaining this capability.

10.4.2 The perceived advantage of static hedging, if customised appropriately, is that the effectiveness of the hedge, typically, is not affected by large market movements, as compared with most dynamic hedges, which require more frequent rebalancing and which can be vulnerable to sudden market movements (unless gamma protected). However, dynamic hedges aim to use liquid assets as much as possible, with very low transaction and rebalancing costs. By contrast, when static hedges need to be rebalanced, the transaction costs can be very significant. Even if a static hedge matches market risk perfectly, it will need to be rebalanced to take account of new business and other non-market risks.

10.4.3 In practice, it is possible to combine static and dynamic hedges. For example, a company could use exchange traded futures and swaps to mitigate delta and rho risks, and could use OTC assets to mitigate vega and/or gamma. By decomposing the main sources of risk, the insurer can seek third party protection for only those sources of risk which it is unwilling or uncomfortable in carrying on its own balance sheet. The operational risk controls in place for dynamic hedges generally extend, naturally, to cover also the static or semi-static components of the hedge programme.

10.4.4 Static and dynamic hedges can also be combined if, for example, a static hedge is put in place to cover a proportion of the liability with relatively simple options, and then to use dynamic hedging to reduce further the residual risk after the limitations of the static hedge have been reached. This is sometimes referred to as a 'static core with dynamic overlay'.

10.4.5 Systems and expertise which facilitate both dynamic and static hedge decisions are likely to lead to the most optimal hedge decisions, as circumstances and product designs evolve.

10.5 *Other Hedge Design Considerations*

10.5.1 Another key hedge design question is whether the hedge aims to reduce the volatility of earnings on some accounting basis, such as IFRS, GAAP or on a statutory solvency reporting basis. Alternatively, companies may seek to set up their hedge programmes to match their best estimates of the values of the liability — reflecting their internal assessment of the economic value of the business being hedged.

10.5.2 Companies generally seek to employ economic hedges, as hedging against a more prudent basis would likely lead to over-hedging and higher hedge costs.

10.6 *Industry Approaches to Hedging*

10.6.1 Moody's (2006) surveyed the practices of 20 major variable annuity writers, and found that differing risk management philosophies and risk exposures, as well as multiple accounting regimes, have resulted in life insurance companies employing a variety of risk management approaches to managing variable annuity guarantees. At the time of the survey, over 85% of the variable annuity industry had some form of hedging in place, and this proportion is believed to have increased since the time of the survey. The summary opinion states: "Whether or not a company hedges its variable annuity risk, a key credit concern in Moody's view is whether the insurer has appropriately quantified possible returns in extreme down scenarios.''

10.6.2 Moody's (2006) found that dynamic hedging involving the so-called Greeks was the approach taken by the majority (60%) of companies. While 15% of companies described their hedge as static, Moody's noted that very few hedges are truly static, and, invariably, they need to be adjusted at some stage in the product life — particularly when demographic experience is dynamic in behaviour. Generally, the portfolios described as static were limited to the simpler GMAB/GMDB designs. Static hedging is very rare for modern GMWB or ratchet type features, although OTC structures may be used as part of the overall hedge programme.

10.6.3 In the Moody's' survey, economic hedges were the most popular type (approximately two-thirds of respondents described their hedges as economic). 20% of companies described their hedges as GAAP based and perhaps unsurprisingly, Moody's notes that all of these were publicly traded companies. Consistent with this, publicly traded companies selling GMAB and GMWB risks were more likely to hedge multiple Greeks, partly because these products are accounted for under FAS-133 rather than SOP 03-1 — the former being a market consistent accounting standard, which results in less income volatility for hedged liabilities.

10.7 *Numerical Examples*

There are two main (complementary) ways to demonstrate the effectiveness of a hedge programme and to quantify the economic capital at

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risk under different hedging strategies. The first approach involves looking at the impact of immediate stress scenarios on an economic (or some other) balance sheet, whilst the second approach involves studying the effectiveness of the hedge over time, using cash flow projections. There are, of course, variations under each of these extremes; most U.K. companies use variations of immediate balance sheet stresses to evaluate their individual capital assessments (ICAs), whilst U.S. companies generally use cash flow projections and either a VAR or CTE measure of the residual profit and loss (P&L) or net claims liability under some time horizon.

10.8 *Immediate Balance Sheet Stresses*

10.8.1 Table 10.8.1 shows a simplified economic balance sheet for the model point, described in Section 5, with a dynamic lapse function. The unit reserve is £100,000 and the value of the liability option is –£1,816 (the value of future charges being higher than the value of futures claims).

10.8.2 Table 10.8.2 shows the values of the unit reserves and the liability option values immediately after a 5% fall in equity values. The unit reserve has fallen to £97,000 (it was only 60% invested in equities), and the liability option value has risen to –£1,563. If there were no hedge assets such as futures, the net asset position would reflect a strain of £253, being the change in the liability option value. The table also shows what would happen had a delta (only) hedge been in place. The unit reserve and liability option values are still £97,000 and –£1,563, but the net asset position benefits from the

Table 10.8.1. Economic balance sheet, current balance condition

Liabilities		Assets			
		Equities	Fixed income	Risk free	Futures
Unit reserve	100,000	60,000	40,000		
Liability option	–1,816			–1,816	0
Balance					
Total	98,184	60,000	40,000	–1,816	

Table 10.8.2. Economic balance sheet, after a 5% fall in equities

Liabilities		Assets			
		Equities	Fixed income	Risk free	Futures
Unit reserve	97,000	57,000	40,000		
Liability option	–1,563			–1,816	239
Balance	–14				
Total	95,423	57,000	40,000	–1,816	239

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increase in the value of the hedge assets, and the capital strain is now only £14.

10.8.3 In Table 10.8.4a to Table 10.8.4d, we repeat this analysis for various other market and ICA type stresses, illustrating the capital strain which results under (1) an unhedged strategy, (2) a delta-rho strategy, and (3) a delta-rho-vega strategy. Particularly, we illustrate the difference between marking the liability value against market consistent economic value — Table 10.8.4a — and some statutory pricing basis — Table 10.8.4b. In the case of 10.8.4a, we assume an economic valuation based on a swap rate curve. In the case of 10.8.4b we assume that the statutory valuation is based on a risk-free yield curve based on gilts + 10 bps, using the same hedge assets as in 10.8.4a. The stresses on the statutory balance sheet are typically used for ICA purposes, while economic balance sheets would also be internally monitored.

10.8.4 For the delta-rho strategy, we are using equity futures for delta. As for the rho strategy, we use a bucket approach (referred to as key-rate rho

Table 10.8.4a

	Market movement				Capital strain (economic value)		
#	Equities	Interest rates	Equity volatility	Swaption volatility	Unhedged	Delta rho	Delta rho vega
(1)	−5%				−253	−14	−2
(2)	−25%				−1,959	−762	−286
(3)		−1%			−1,939	−266	−189
(4)			+6%		−1,542	−1,542	54
(5)				+3%	−200	−200	−200
(6)	−25%	−1%	+6%	+3%	−5,767	−2,900	4

Table 10.8.4b

	Market movement				Capital strain (marked to statutory value)		
#	Equities	Interest rates	Equity volatility	Swaption volatility	Unhedged	Delta rho	Delta rho vega
(1)	−5%				−281	−42	−30
(2)	−25%				−2,170	−974	−498
(3)		−1%			−2,235	−562	−485
(4)			+6%		−1,674	−1,674	−78
(5)				+3%	−211	−211	−211
(6)	−25%	−1%	+6%	+3%	−6,300	−3,433	−529

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Table 10.8.6

	Demographic stress	Capital strain	
		Economic	Statutory
(7)	Lapse stress (50% base lapse + larger sensitivity)	−325	−480
(8)	Mortality (25% reduction in mortality)	−934	−1,662

analysis) to provide a more efficient response to changes in different points of the term structure of interest rates, using swaps with maturities in two, five, ten and 25 years. For simplicity in the vega hedge, we use a ten-year vanilla European put. In practice, a bucket approach is suggested for both rho and vega hedges, where the choice of the buckets must be tailored to take into account the characteristics of the liability, the liquidity in the market and cash requirements of the possible hedge assets.

10.8.5 We note that the combined stress impact is similar to, but not identical to, the sum of the individual impacts. This is a consequence of the second order dependencies of the liability *and* the asset risk factors, i.e. the cross Greeks. The combined stress leads to a capital requirement of about 6% of the unit reserve when unhedged, but then reduces to about 3% with a delta-rho hedge, and to almost 0% with a delta-rho-vega hedge. This is before allowing for capital requirements from others sources of risk, which we explore below.

10.8.6 We also consider some non-market risk stresses. Firstly, we consider mortality and lapse risks. These stresses do not change the value of hedge assets directly, and we show, in Table 10.8.6, the impact of the stresses on an economic and statutory basis.[3]

10.8.7 It can be seen that, for small or modest market movements, the dynamic hedge is very effective at limiting the impact of the market risk exposure (examples (1) - (4)). However, one of the limitations of dynamic hedging arises when the company is unable to rebalance during severe market falls. Tables 10.8.4a and 10.8.4b illustrate the capital impacts of such falls. This is because the Greek hedging considered here is operating at a local level; that is, the hedge considers the sensitivities of the liability around prevailing market conditions. However, it is possible to tailor solutions that help to mitigate the effect of large movements in market conditions. Clearly, different results will arise for alternative hedge strategies involving different hedge instruments.

[3] The mortality stress was performed by lightening the mortality rates uniformly by 25%. A more sophisticated stress would stress the mortality rates in a non-uniform way. For the lapse stress, we assume a 50% base lapse rate (2% instead of 4%), and higher sensitivity to the moneyness of the product (double).

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10.9 *Cash Flow Projections*

10.9.1 An alternative method of assessing hedge effectiveness is to investigate the net profit and loss over time through cash flow projections.

10.9.2 Cash flow projection under realistic scenarios is an intensive computing task. In addition to projecting future guarantee charges collected and guarantee claims, one must also project future liability option movements, together with their Greeks, in order to be able to project the hedge transactions and the asset values that would be held. As the valuation of the liability options, their Greeks, and, in some cases, the hedge assets, require the use of a stochastic model, the overall analysis will require nested stochastic functionality. In general, the outer scenarios would either be realistic or selected stress conditions, while the inner scenarios would be market consistent, conditional on the path of the outer scenario.

10.9.3 A suitable model should adequately address aspects such as transaction costs, movements in the main market and demographic risks, collateral, margins and other cash requirements amongst other items. Given the complexity of the task, it is helpful to begin the analysis with simplifying assumptions, such as deterministic volatility and lapses, and then refine the model and the analysis incrementally, as the behaviour of the liability and the hedge is understood. Relatively short time steps, such as weekly, are also required, given the nature and potential of hedge transaction activity.

10.9.4 In the analysis which follows, we have carried illustrative nested stochastic projections, using the same Milliman scenario generator as in Section 7, with stochastic volatility included. For this, we have used a relatively simple model for equity volatility, whereby the volatility surface shifts according to a mean-reverting process, and the spread between realised and implied volatility is kept constant. The outer scenarios have been calibrated to target a long-term volatility level of 20%. The scenario generator also reflects two basic types of movement in the term structure (so called 'shifts' and 'twists'). The inner scenarios reflect the market consistent or statutory parameters, as appropriate, to compute the relevant value of the liability options and the market value of hedge assets with weekly time steps. No explicit transaction costs are assumed in this exercise.

10.9.5 Figures 10.9.5a to 10.9.5c illustrate the dispersion of the economic P&L results for the example product in this paper, with an unhedged, delta-rho, and delta-rho-vega hedge strategy. One hundred outer scenarios with weekly projection time steps (and 1,000 nested scenarios at each time step) have been used. The delta-rho strategy involves the use of index futures and interest rate swaps, selected to match the delta and rho of the liabilities. The delta-rho-vega strategy utilises the use of index options to obtain temporary partial protection against the risk of equity volatility being higher than expected. As the option selected is not the same exotic option as the liability sold, its delta and rho will not match those of the liability, and therefore different quantities of futures and swaps are used to

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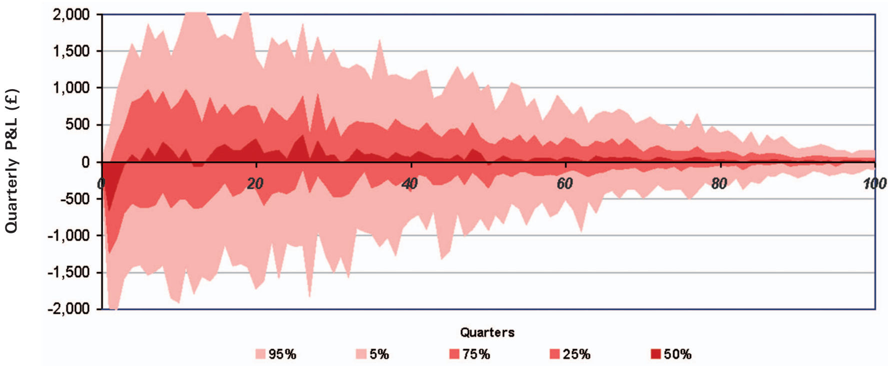

Figure 10.9.5a. Unhedged quarterly P&L distribution WB product (£)

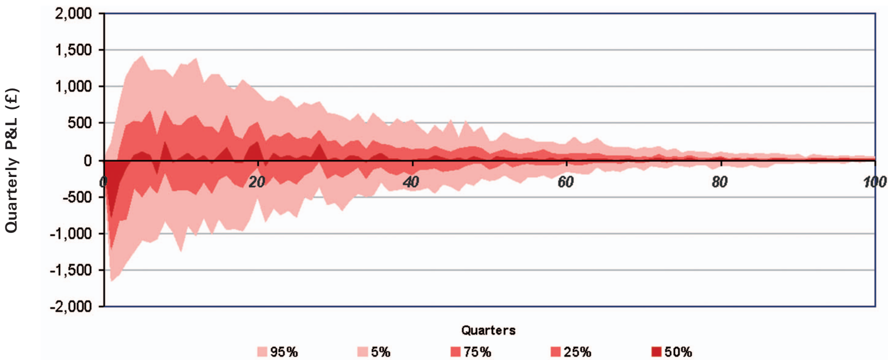

Figure 10.9.5b. Delta-rho hedged quarterly P&L distribution WB product (£)

neutralise these risk measures. A more detailed analysis would also involve investigating the use of alternative instruments to hedge the liability.

10.9.6 Each P&L is distributed around zero as the economic liability which has been projected is the value of the guarantee claims net of the expected cost of hedging, without any pricing margin included. The very significant reduction in P&L volatility, as a consequence of hedging, is clearly apparent.

10.9.7 The impact of changing certain risk factors (e.g. including stochastic volatility, or changing a hedge parameter), can be difficult to

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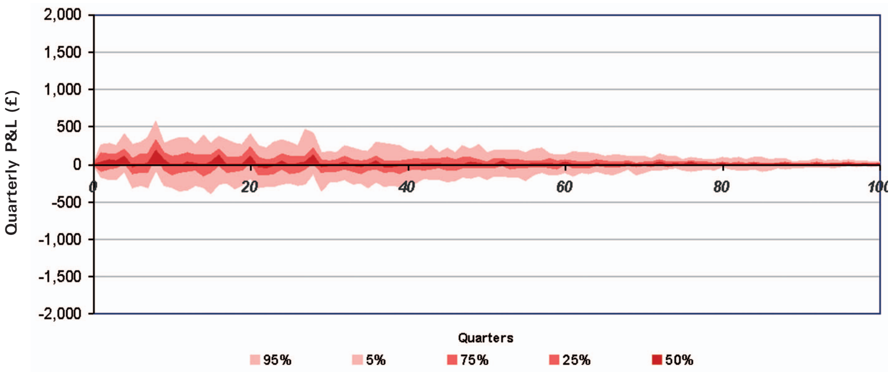

Figure 10.9.5c. Delta-rho-vega hedged quarterly P&L distribution WB product (£)

discern from graphical output alone, and a more detailed numerical analysis is needed. A useful measure which captures the combined financial effect of all P&L outcomes, over the life of the product, into a single variable is the present value of future profits (PVFP). This measure is popular, as it is consistent with the embedded value techniques which many insurance companies use as part of their financial control process. In Figure 10.9.7 we show the distribution of PVFP as a percentage of the starting account value for the different hedging strategies. We note the significant tail of a substantial loss of value under the unhedged strategy.

10.9.8 We also investigated alternative projections, focusing on volatility

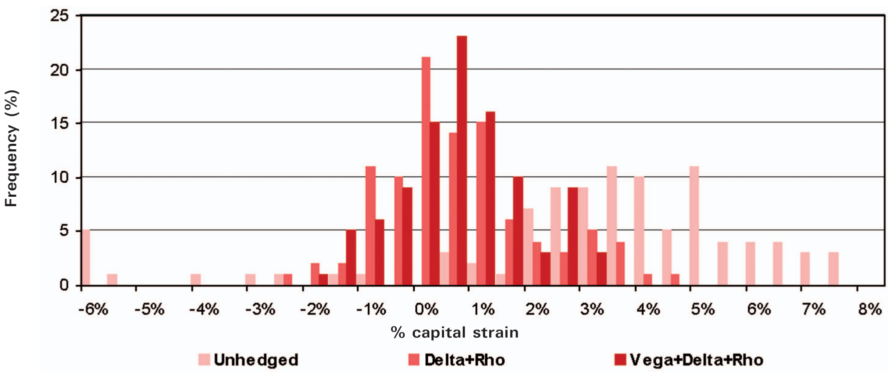

Figure 10.9.7. PVFP distribution WB product with dynamic lapses and stochastic volatility (%AV)

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and lapses. We found that the impact of changing from deterministic lapses and volatility to stochastic lapses and volatility was to increase the standard deviation of the PVFP variable from 2.30% to 3.82%, in the case of an unhedged strategy, as a consequence of the additional risk factors. The standard deviation of the delta-rho result also increased from 1.03% to 1.38%, and for the delta-rho-vega from 0.83% to 1.13%. Importantly, the benefit of hedging, measured in terms of the reduction in the standard deviation of the PVFP variable, is greater when stochastic volatility and lapses are allowed for (the standard deviation result is reduced by a factor of 64% for a delta-rho hedge and 70% for a delta-rho-vega hedge, compared with 55% for a delta-rho hedge and 64% for a delta-rho-vega hedge, when these variables were deterministic).

10.9.9 A possible metric for the economic capital at risk of being lost can be defined in terms of the present capital value of the losses over a selection of the worst scenarios. In this case, we use the present value of losses incurred under the worst one in 100 scenarios as the measure of the economic capital at risk (or the first percentile of the PVFP variable).

10.9.10 In the analysis below, we have again used the pricing basis as the starting point, with stochastic volatility in the outer scenarios. Any deliberate parameterisation of experience inconsistent with the pricing basis will reveal the impact of mis-pricing the product, and will distort observations about hedge effectiveness when the hedge is priced 'perfectly'. Table 10.9.10 also shows the comparable figures with deterministic lapses and volatility to illustrate the impact of these risk variables.

10.9.11 It is evident that the tail of the distribution (the first percentile is our choice of the measure of risk for economic capital purposes in this example) widens considerably, due to the additional risks related to dynamic lapses and stochastic volatility, particularly in an unhedged portfolio. The relative impact of these risks is reduced significantly with a dynamic hedge strategy, as these can respond to dynamic market conditions.

10.9.12 With economic capital analyses, the outcome is influenced by the choice of method and the selection of scenarios, and therefore the results derived from this sort of exercise must be interpreted in this context. Table 10.9.12 compares alternative measures of economic capital at risk using either immediate balance sheet stress test or cash flow projections under the

Table 10.9.10. One in 100 present value of losses

	Dynamic lapses and stochastic volatility	Deterministic lapses and volatility
Unhedged	−12,853	−3,214
Delta-rho hedge	−2,531	−1,042
Delta-rho-vega hedge	−2,005	−1,899

Table 10.9.12. Measures of economic capital at risk

	Immediate stress (capital strain/AV)	Cash flow projection (PVFP/AV)
Unhedged	5.8%	12.9%
Delta-rho hedge	2.9%	2.5%
Delta-rho-vega hedge	0.0%	2.0%

alternative hedging strategies. For this comparison, we have used the combined stress (6), as outlined in Table 10.8.4.

10.9.13 There are three main conclusions to be made from this analysis:

— The benefit of hedging is substantial under either method of measuring risk capital. Generally, the level of hedging will be selected such that the economic capital at risk is below the Solvency I solvency margin requirement of 4% of account value.
— Delta-rho hedging reduces the economic capital at risk to around the 2% to 3% level, depending upon the measure of economic capital at risk, and the scenarios/stresses used in the exercise.
— The amount of capital at risk can be reduced further through the use of more advanced vega and gamma hedging, as well as the consideration of other higher order Greeks. A more detailed study would consider the trade-off between the savings of economic capital at risk and the trading costs associated with more exotic hedges.

11. Pensions Legislation

11.1 As discussed previously, we believe that a major area of development for variable annuities in the U.K. market will be unit-linked pensions with guaranteed minimum withdrawal benefits. Section 6 indicates that such a product offers significant customer benefits, and would seem to meet the customer demand for good investment returns coupled with the safety net of a valuable guarantee.

11.2 *HMRC Limits on Income Drawdown*

11.2.1 The sample GMWB product which we described in Section 5, would be considered as an income drawdown product under U.K. pensions legislation. This means that there are lower and upper limits that restrict the income which a member can take each year. Although the income limits are set by HMRC, they are commonly referred to as the 'GAD limits'. The limits vary, depending on whether the individual is below or above age 75:

— Below age 75, the minimum income is zero, while the maximum income is 120% of the age-related figure derived by using tables produced by

GAD. This maximum figure is broadly equivalent to the (single-life, level) annuity that could be purchased with the pension fund.

— Above age 75, the maximum and minimum income limits are 90% and 55%, respectively, of the figure derived using the GAD tables, but always based on age 75. Therefore, the maximum income allowable does not increase as the customer ages beyond age 75 (all else being equal).

11.2.2 This means that the maximum withdrawal amount is set by reference to the customer's fund size, gender, age and the interest rate on U.K. government gilts. So, for example, assuming that a male pensioner aged 65 has a fund of £100,000 and that the yield on gilts is 4.3%, then the maximum annual pension which can be drawn down is 120% $*$ £7,000 = £8,400.[4]

11.2.3 The reasons why income drawdown is subject to GAD limits can be summarised as:

— The GAD maximum is designed to prevent customers from exhausting their drawdown funds, and then having to rely on state benefits.

— The GAD minimum has been introduced for those over age 75, to ensure "that tax-relieved pension saving is used to secure an income in retirement".[5]

11.2.4 Our sample GMWB product already meets both of these aims, since it means that the customer buys a minimum level of income for the rest of his or her life. However, the current income restrictions hinder the ability for GMWB products to meet customer needs.

11.3 *GAD Maximum Income*

11.3.1 As they stand, the rules cause a problem for GMWBs, by limiting the amount of money which the customer is allowed to take in a pre-defined manner. On occasion, this would restrict the amount which the life assurance company can pay to below the amount which it is guaranteeing that the customer will receive for the rest of his or her life. This appears to be an unintended consequence of the GAD rules on the new GMWB designs, since the GMWB guarantees that the customer will never run out of money if they retain their guarantee.

11.3.2 Consider the following example. A customer at age 65 selects a GMWB for £100,000. In return for the extra charge which they pay, they are guaranteed to be able to take £5,000 a year for the rest of their life.

11.3.3 Initially, this withdrawal is well within the maximum GAD limit of £8,400 p.a.

[4] The GAD table sets the limit at £70 per £1,000 purchase price for a 65 year old male for yields between 4.25% and 4.49%. The reference yield is the yield on 15 year gilts, as published in the *Financial Times* on the previous 15th of the month.

[5] 'Tax relief for pensions: inheriting tax-relieved pension savings'. HMRC, 2007.

11.3.4 Markets then go through a very poor spell, and the fund value reduces by age 70 to £45,000. This fund value results in a maximum GAD limit of only £4,482. So, under the current rules, the company would not be able to pay the customer the full £5,000 out of their fund.

11.3.5 However, the company is guaranteeing to pay £5,000 for the rest of the customer's life, so they will never run out of income, even if their fund reduces to zero.

11.4 *GAD Minimum Income after Age 75*

11.4.1 The rules would also cause a problem for a GMWB product by requiring some customers who have purchased a GMWB to take more than they have been guaranteed — hence resulting in a lower guarantee for the remaining years.

11.4.2 Again consider an example. A customer at age 65 selects a GMWB for £100,000 in their SIPP. In return for the extra charge which they pay, they are guaranteed to be able to take £5,000 a year for the rest of their life. They immediately start taking this maximum guaranteed withdrawal each year.

11.4.3 By age 75, their remaining fund is worth £95,000. This fund value results in a minimum GAD limit of £5,225. So under the current rules the customer would have to take more than their guaranteed minimum income of £5,000.

11.4.4 The insurance company would then reduce the value of the remaining guarantee — say to £4,950.

11.4.5 As a result, the value of the customer's guarantee is reduced, even though they are using their pension to take an income and are trying to protect themselves against outliving their funds.

11.5 *Recent Developments*

11.5.1 Therefore, the industry has been lobbying the U.K. Government to remove the need for maximum and minimum income limits where a GMWB applies to the contract, on the grounds that:

— the provider is guaranteeing that the customer will never have to reduce their income from their pension, whatever happens to the value of their pension funds. This prevents customers from exhausting their drawdown funds, and then having to rely on state benefits.
— Also, the customer is paying for this product to guarantee an income, not to leave the money untouched in a tax-free environment. Indeed, product designs could include an enforced withdrawal of the guaranteed amount from age 75, to protect HMRC's tax revenues if required.

11.5.2 Initial signs from the Government were very positive. In HM Treasury's 'The Annuity Market' paper (December 2006), it was announced:

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"The Government is keen to take representations from industry where they feel that there are innovative products which meet the above characteristics but cannot be offered because of a specific tax or other reason."

11.5.3 However, HM Treasury subsequently reported in the 2007 Pre-Budget Report and Comprehensive Spending Review:

"Following a commitment in the 2006 Pre-Budget Report, the Government has consulted widely with industry on tax barriers to the further development of 'hybrid' decumulation products, which combine an element of drawdown with a guaranteed income. The Government has decided not to change the tax rules as this would add complexity to the tax system and potentially benefit only a small number of consumers with large pension savings."

11.5.4 Providers launching variable-annuity-style guarantees in the U.K. so far have derived a range of possible solutions to try to overcome this problem; for example, by offering to convert the guaranteed payment to an immediate annuity for life if the GAD limits bite. Whilst this solution fits with current legislation, it is a compromise in meeting customer needs.

11.5.5 This is unfortunate, and it means that the GAD limits, which are designed to protect the pensioner against exhausting the pension fund, can have the opposite effect, and can act against the pensioner's interest. They also increase the complexity of the product, and hence make it more difficult for a distributor to give appropriate advice to customers.

11.5.6 HM Treasury's reasoning for not adapting the rules to cope with product evolution, that a legislation change would only benefit a small number of consumers, seems potentially flawed:

— One target market, if we broaden our analysis to GMWB products in general, is those who would otherwise buy an annuity at a relatively young age (e.g. age 60 to 65). The mortality cross-subsidy element of annuities at these ages is minimal, and the customer is still likely to be investing for the long term. However, these customers may not feel comfortable with the risk associated with income drawdown. This target market is substantial. More than 16,500 people bought an annuity in 2006, with £40,000 or more after tax-free lump sums had been taken (Source: ABI data). These customers would have qualified for income drawdown. Over the next ten years, with the baby boomers approaching retirement, this population is expected to grow significantly. If growth is 5% p.a., the volume of these customers who could benefit from a change in legislation over the ten years following 2007 is about 250,000.

— Another target market is those who would buy income drawdown, but would value the ability to guarantee a minimum income level for their retirement. As mentioned in Section 6, a guarantee like this could enable them to take more risk with their investments in search of higher growth. Since 2000, about 115,000 drawdown plans have been written in the U.K., with more than 25,000 of these being in 2006. With 5% p.a. growth over the next ten years, the market of drawdown clients (before deaths and lapses) could reach nearly 500,000.

11.6 *Legislation Surrounding Guaranteed Minimum Death Benefits*

11.6.1 The pension rules also limit the death benefits which may be paid from drawdown funds. These, effectively, cap any lump sum death benefit to the fund value at death. Income payments are also allowed. Beyond age 75 no lump sum payment can be made. This legislation protects tax revenues which could otherwise be at risk if customers defer taking income to build up their pension as an inheritance vehicle.

11.6.2 In other markets, the standard GMWB for life product also offers a minimum death benefit equal to the fund or a higher GMDB benefit. This would not seem to be possible within a U.K. defined contribution pension (although some providers seem to have structured this form of guarantee using defined benefit legislation).

11.6.3 This is, perhaps, another area where the legislation could be relaxed. If a customer is committed to withdrawing a guaranteed level of income, it would seem possible to meet the customer's need for a minimum death benefit without risking HM Treasury's tax revenues. Indeed, the inheritance tax paid would be higher when the GMDB bites to top up a payout.

11.7 *Future Development of Legislation*

We believe that new variable annuity product designs offer considerable opportunities to develop propositions which meet the needs of a wide range of consumers, and have the potential to support government objectives of increased individual saving. They also offer the potential for more efficient savings in higher return investments, rather than the current dominance of fixed-interest investments post-retirement. We believe that the Actuarial Profession should continue to study ways in which legislation could be developed to meet all the objectives of government, consumer and product provider.

12. Conclusions

12.1 For a number of years, the U.K. market has been looking for product designs which might provide a suitable replacement for with-profits products. We believe that the variable annuity may well be this product.

12.2 The product offers valuable guarantees which meet real customer needs. The new hedging techniques which have been developed offer the opportunity to design a wide range of attractive guarantees. The product has been very successful in other international markets, and we believe that there is every prospect that this success will be repeated in the U.K.

12.3 Actuaries have the potential to play a key role in many aspects of the development of this product. As the product combines a wide range of risk types (for example market risk, longevity, customer behaviour), the actuary is better placed than other professionals to look holistically at the product. Pricing, reserving, hedging all provide interesting challenges, where the actuary can apply his or her skills. However, it will be essential for today's actuary to keep up to date with modern developments in the areas of investment hedging and economic capital. Just five years ago these techniques were used in only a small proportion of life business. We believe that, within the next five years, it is likely that an increasingly high proportion of new sales will involve guarantees backed by complex hedging programmes.

12.4 With the development of new products and markets, new types of risk will evolve. One area which we believe merits further research by the profession is in managing extreme events. Whilst hedging programmes work effectively in a wide range of financial markets, there remains a risk that market behaviour could change more rapidly and significantly than predicted by models. How significant are the risks in these more extreme scenarios?

12.5 Finally, as noted in Section 11, we believe that the Actuarial Profession should continue to examine ways in which legislation could be developed better to allow the use of modern techniques to deliver better outcomes for consumers.

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