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Stimulated scattering of electromagnetic waves in a two-electron plasma

P. K. SHUKLA[1] and M. A. HELLBERG[2]

[1]Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany

[2]School of Pure and Applied Physics, University of Natal, Durban 4041, South Africa

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Abstract. The nonlinear interaction between large-amplitude electromagnetic waves and electron-acoustic (EA) waves in a two-electron-temperature plasma is considered, taking into account the combined effects of the radiation pressure and the thermal force involving the differential Joule heating of the electrons caused by the electromagnetic waves. By employing a fluid approach, we derive a system of coupled equations for the electromagnetic waves and the EA waves; the latter are nonlinearly driven by the radiation and thermal forces. We have carried out a normal mode analysis of our nonlinearly coupled equations, and have derived a general dispersion relation that is useful for studying different types of parametric instabilities. A new class of modulational instability in the collision-dominated regime is identified. The implications for space and laboratory plasmas are pointed out.

1. Introduction

It is well known (Jones et al. 1975; Bezzerides et al. 1978; Barkat et al. 1995; Denton et al. 1995; Choe et al. 1995) that laser-irradiated laboratory and space plasmas usually contain two distinct groups of electrons. The latter are characterized by their different energy distributions: the low-energy electrons are much colder in comparison with the high-energy hotter electrons. The so-called two-electron-temperature plasmas (with fixed ion background) exhibit a new type of electron-acoustic (EA) wave (Jones et al. 1975; Watanabe and Taniuti 1977) in which the restoring force comes from the pressure of the hot-electron component, while the inertia comes from the mass of the cold electrons. Various aspects of the linear mode have been studied (Gary and Tokar 1985; Mace and Hellberg 1990), and drift-driven instabilities invoked in space plasmas (Tokar and Gary 1984; Mace and Hellberg 1993a,b). Laboratory experiments have also been carried out (Hellberg et al. 2000). Nonlinear wave interactions in two-electron-temperature plasmas have previously been examined by, for instance, Skaeraasen et al. (1996). It has been shown that EA waves can give rise to non-envelope solitons and double layers (Bharuthram and Shukla 1986; Dubouloz et al. 1991, 1993; Mace et al. 1991, 1992; Berthomier et al. 2000; Mace and Hellberg 2001), as well as envelope solitons (Khirseli and Tsintsadze 1980; Hansen et al. 1994; Rao and Shukla 1997; Pottelette et al. 1999), although weak double layers do not exist (Mace and Hellberg 1993). The importance of EA waves in the magnetosphere has recently been emphasized by Singh and Lakhina (2001).

In this paper, we consider the nonlinear coupling between large-amplitude electromagnetic (EM) and EA waves, taking into account the EM ponderomotive force and the differential Joule heating of electrons in the EM wave's electric field. The paper is organized as follows. In Sec. 2, we present the EM wave equation and derive equations for the EA wave potential that is reinforced by the radiation pressure and the thermal nonlinear force. The latter produces electron-temperature perturbations that are governed by the energy equation. The mode-coupling equations are Fourier-analyzed to obtain nonlinear dispersion relations. The latter are analyzed and explicit results for the growth rates of decay and modulational instabilities are obtained. Furthermore, assuming that the electron collision frequency is much larger than the modulation frequency, we obtain a new class of thermal modulational instability. Possible applications of our work to space and laboratory plasmas are pointed out in Sec. 6.

2. Governing equations

We consider the nonlinear propagation of a large-amplitude EM wave in a two-electron temperature plasma. The nonlinear interaction between the coherent EM wave and EA waves is governed by (Forslund et al. 1975; Shukla et al. 1986)

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_{pe}^2\right)\mathbf{A} = -\omega_{pe}^2\frac{n_{es}}{n_0}\mathbf{A}, \tag{2.1}$$

which is obtained from

$$\boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{2.2}$$

by using $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$, $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$, and $\mathbf{J} = -e(n_0 + n_{es})\mathbf{V}_e$, where $\mathbf{V}_e = e\mathbf{A}/m_e c$ is the electron quiver velocity in the EM wave vector potential $\mathbf{A}$. The EM wave frequency is taken to be much larger than the effective electron–ion collision frequency. Here, c is the speed of light in vacuum, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the unperturbed electron plasma frequency, $n_0 = n_{0h} + n_{0c}$ is the sum of the hot- and cold-electron number densities, e is the magnitude of the electron charge, m_e is the electron mass, and

$$n_{es} = n_{1h} + n_{1c} \equiv \frac{1}{4\pi e}\nabla^2\phi \tag{2.3}$$

is the electron density perturbation associated with the EA waves. In (2.3), ϕ is the electric potential of the EA waves. The right-hand side of (2.1) is associated with the nonlinear current arising from the coupling of the EM pump-induced electron quiver velocity and the electron density perturbation of the EA waves.

We assume that the phase velocity of the EA waves is much smaller (larger) than the thermal speed of the hot (cold) electron component and that their wavelength is smaller than V_{Te}/ν_{eh}, where $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal speed, T_e is the temperature of the hot electrons, and ν_{eh} is the collision frequency of hot electrons with stationary ions. Thus, the equation of motion for the hot-electron component is

$$n_{1h} = n_{0h}\frac{e\phi}{T_e} - n_{0h}\frac{T_{e1}}{T_e} - \frac{n_{0h}e^2|\mathbf{A}|^2}{2m_e T_e c^2}, \tag{2.4}$$

where T_{e1} is a small temperature fluctuation ($\ll T_e$), and the third term on the right-hand side of (2.4) represents the ponderomotive potential of the EM waves.

The electron temperature perturbation is determined from (Stenflo 1985)

$$\frac{3}{2}\frac{\partial T_{e1}}{\partial t} + \nu_r T_{e1} - \frac{\chi_e}{n_{0h}}\nabla^2 T_{e1} - \frac{T_e}{n_{0h}}\frac{\partial n_{1h}}{\partial t} = 2m_e\nu_{eh}|\mathbf{V}_e|^2, \tag{2.5}$$

where $\chi_e = 3.2 n_{0h}V_{Te}^2/\nu_{eh}$ is the electron thermal conductivity, $\nu_r = 3m_e\nu_{eh}/m_i$ is the electron energy relaxation rate due to electron–ion collisions, and m_i is the ion mass. The right-hand side of (2.5) represents the differential Joule heating of the hot electrons due to collisions in the EM wave fields.

The dynamics of cold electrons is governed by

$$\frac{\partial n_{1c}}{\partial t} + n_{0c}\boldsymbol{\nabla}\cdot\mathbf{v}_c = 0, \tag{2.6}$$

and

$$\frac{\partial \mathbf{v}_c}{\partial t} = \frac{e}{m_e}\boldsymbol{\nabla}\phi - \frac{e^2}{2m_e^2c^2}\boldsymbol{\nabla}|\mathbf{A}|^2, \tag{2.7}$$

where n_{1c} ($\ll n_{0c}$) is the small cold-electron number-density perturbation and $\mathbf{v}_c$ is the cold-electron fluid velocity. Equations (2.3)–(2.7) form a closed system for the EA waves.

Eliminating $\mathbf{v}_c$ from (2.6) and (2.7), we obtain

$$\frac{\partial^2 n_{1c}}{\partial t^2} + \frac{n_{0c}e}{m_e}\nabla^2\phi - \frac{n_{0c}e^2}{2m_e^2c^2}\nabla^2|\mathbf{A}|^2 = 0. \tag{2.8}$$

Substituting (2.4) and (2.8) into (2.3), we have

$$\left[(\nabla^2 - k_D^2)\frac{\partial^2}{\partial t^2} + \omega_{pc}^2\nabla^2\right]\phi + \frac{k_D^2}{e}\frac{\partial^2 T_{e1}}{\partial t^2} = -\frac{k_D^2 e}{2m_e c^2}\left(\frac{\partial^2}{\partial t^2} - C_e^2\nabla^2\right)|\mathbf{A}|^2, \tag{2.9}$$

where $\omega_{pc} = (4\pi n_{0c}^2/m_e)^{1/2}$ is the plasma frequency of the cold-electron component, $k_D = (4\pi n_{0h}e^2/T_e)^{1/2} \equiv \lambda_D^{-1}$ is the Debye wavenumber of the hot electrons, and $C_e = \omega_{pc}/k_D$ is the EA wave speed. In the absence of electron-temperature fluctuations, (2.9) reduces to

$$\left[(1 - \lambda_D^2\nabla^2)\frac{\partial^2}{\partial t^2} - C_e^2\nabla^2\right]\phi = \frac{e}{2m_ec^2}\left(\frac{\partial^2}{\partial t^2} - C_e^2\nabla^2\right)|\mathbf{A}|^2, \tag{2.10}$$

which for $\lambda_D^2\nabla^2\phi \ll \phi$ gives

$$\phi = \frac{e}{2m_ec^2}|\mathbf{A}|^2. \tag{2.11}$$

Correspondingly, we have from (2.4)

$$n_{es} = \frac{1}{8\pi m_e c^2}\nabla^2|\mathbf{A}|^2, \tag{2.12}$$

which exhibits the radiation-pressure-driven non-resonant density response in an electron plasma.

On the other hand, from (2.4) and (2.5), we obtain

$$\left(\frac{5}{2}\frac{\partial}{\partial t} + \nu_r - 3.2\frac{V_{Te}^2}{\nu_{eh}}\nabla^2\right)T_{e1} - e\frac{\partial\phi}{\partial t} = \frac{2\nu_{eh}e^2}{m_ec^2}\left(1 - \frac{1}{4\nu_{eh}}\frac{\partial}{\partial t}\right)|\mathbf{A}|^2. \tag{2.13}$$

Combining (2.9) and (2.13), we readily obtain

$$\left(\frac{5}{2}\frac{\partial}{\partial t}+\nu_r-\frac{3.2V_{Te}^2}{\nu_{eh}}\nabla^2\right)\left[(1-\lambda_D^2\nabla^2)\frac{\partial^2}{\partial t^2}-C_e^2\nabla^2\right]\phi-\frac{\partial^3\phi}{\partial t^3}$$
$$=\frac{e}{2m_ec^2}\left(\frac{5}{2}\frac{\partial}{\partial t}+\nu_r-\frac{3.2V_{Te}^2}{\nu_{eh}}\nabla^2\right)\left(\frac{\partial^2}{\partial t^2}-C_e^2\nabla^2\right)|\mathbf{A}|^2$$
$$+\frac{2e\nu_{eh}}{m_ec^2}\left(1-\frac{1}{4\nu_{eh}}\frac{\partial}{\partial t}\right)\frac{\partial^2|\mathbf{A}|^2}{\partial t^2}, \tag{2.14}$$

which is the EA wave equation in the presence of radiation pressure and the thermal nonlinearity produced by the differential Joule heating of electrons in the presence of EM waves.

Equations (2.1), (2.2), (2.9), and (2.14) are the desired set for nonlinearly coupled EM and EA waves in a plasma containing two types of electrons. In the following, we consider the parametric excitation of the EA waves by EM waves.

3. Dispersion relations

Let us suppose that an EM pump, $\mathbf{A}_0\exp(i\mathbf{k}_0\cdot\mathbf{r}-i\omega_0 t)$ + complex conjugate, interacting with low-frequency $[\omega(\ll\omega_0),\ \mathbf{k}]$ EA waves, generates EM sidebands $\mathbf{A}_\pm\exp(i\mathbf{k}_\pm\cdot\mathbf{r}-i\omega_\pm t)$, where $\omega_\pm=\omega\pm\omega_0$ and $\mathbf{k}_\pm=\mathbf{k}\pm\mathbf{k}_0$ are the frequencies and wavenumbers for the EM sidebands. Hence, we obtain from (2.1), (2.2), (2.9), and (2.14), after Fourier transformation,

$$\epsilon_\pm\mathbf{A}_\pm=-\frac{k^2e}{m_e}\phi_s\mathbf{A}_{0\pm}, \tag{3.1}$$

$$\epsilon_l\phi_s=\frac{e^2}{2m_e^2c^2}(\omega^2-\omega_e^2)(\mathbf{A}_{0-}\cdot\mathbf{A}_++\mathbf{A}_{0+}\cdot\mathbf{A}_-), \tag{3.2}$$

and

$$(\Omega\epsilon_l+\omega^3)\phi_s=\frac{e}{2m_ec^2}\left[\Omega(\omega^2-\omega_e^2)+i2\nu_{eh}\omega^2\left(1+i\frac{\omega}{\nu_{eh}}\right)\right]$$
$$\times(\mathbf{A}_{0-}\cdot\mathbf{A}_++\mathbf{A}_{0+}\cdot\mathbf{A}_-), \tag{3.3}$$

where $\epsilon_\pm=\omega_\pm^2-\omega_{pe}^2-k_\pm^2c^2$, $\Omega=\frac{5}{2}\omega+i\nu_r+i\omega_\chi$, $\omega_\chi=3.2k^2V_{te}^2/\nu_{eh}$, $\epsilon_l=(1+k^2\lambda_D^2)\omega^2-\omega_e^2$, $\omega_e=kC_e$, $\phi=\phi_s\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, $\mathbf{A}_{0+}=\mathbf{A}_0$, $\mathbf{A}_{0-}=\mathbf{A}_0^*$, and the asterisk denotes the complex conjugate.

From (3.1)–(3.3) we readily obtain the nonlinear dispersion relation

$$\epsilon_l=-k^2c^2(\omega^2-\omega_e^2)W_0\sum_\pm\epsilon_\pm^{-1}, \tag{3.4}$$

and

$$\Omega\epsilon_l+\omega^3=-k^2c^2[\Omega(\omega^2-\omega_e^2)+i\omega^2(2\nu_{eh}+\tfrac{1}{4}i\omega)]W_0\sum_\pm\epsilon_\pm^{-1}, \tag{3.5}$$

where $W_0=e^2|\mathbf{A}_0|^2/2m_e^2c^4$. We note that $\epsilon_\pm\approx\pm2\omega_0(\omega-\mathbf{k}\cdot\mathbf{v}_g\mp\delta)$, where $\mathbf{v}_g\approx\mathbf{k}_0c^2/\omega_0$ is the group velocity of the pump, $\omega_0=(k_0^2c^2+\omega_{pe}^2)^{1/2}$ is the pump wave frequency, and $\delta=k^2c^2/2\omega_0$ is the frequency shift caused by the nonlinear interaction of EM waves with the EA waves. The pump and sidebands are assumed

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to be coplanar. In the following sections, we analyze (3.4) and (3.5) to show the existence of decay and modulational instabilities.

4. Instabilities without Joule heating

For the three-wave decay interaction, a large-amplitude EM pump decays into a daughter EM wave and a EA wave. Hence, $\epsilon_- = 0$ and $\epsilon_l = 0$. Ignoring the upper sideband in (3.4), we then obtain

$$\epsilon_l(\omega - \mathbf{k}\cdot\mathbf{v}_g + \delta) = \frac{k^2c^2}{2\omega_0}(\omega^2 - \omega_e^2)W_0. \tag{4.1}$$

Letting $\omega = \omega_e(1 + k^2\lambda_D^2)^{-1/2} + i\gamma_d$ and $\omega = \mathbf{k}\cdot\mathbf{v}_g - \delta + i\gamma_d$ in (4.1), we obtain the growth rate

$$\gamma_d = \frac{kc}{2}\frac{k\lambda_D}{(1 + k^2\lambda_d^2)^{3/4}}\left(\frac{\omega_e W_0}{\omega_0}\right)^{1/2}. \tag{4.2}$$

For the modulational interactions, we have $\epsilon_\pm \approx 0$ and $\epsilon_l \neq 0$. Here, (3.4) yields

$$[(\omega - \mathbf{k}\cdot\mathbf{v}_g)^2 - \delta^2](1 + k^2\lambda_D^2)\omega^2 - \omega_e^2 = -k^2c^2(\omega^2 - \omega_e^2)\frac{\delta}{\omega_0}W_0. \tag{4.3}$$

It can be easily shown that for $|\omega - \mathbf{k}\cdot\mathbf{v}_g| \gg \delta$, ω_e and $k^2\lambda_D^2 \ll 1$, (4.3) reduces to

$$\omega - \mathbf{k}\cdot\mathbf{v}_g = \pm ikc\left(\frac{\delta W_0}{\omega_0}\right)^{1/2}, \tag{4.4}$$

which admits an oscillatory modulational instability.

5. Thermal modulational instability

Let us now focus on the thermal modulational instabilities. Here (3.5) takes the form

$$\begin{aligned}[(\omega - \mathbf{k}\cdot\mathbf{v}_g)^2 - \delta^2]\{\Omega[(1 + k^2\lambda_D^2)\omega^2 - \omega_e^2] + \omega^3\} \\ = -k^2c^2[\Omega(\omega^2 - \omega_e^2) + i\omega^2(2\nu_{eh} + \tfrac{1}{4}i\omega)]\frac{\delta}{\omega_0}W_0.\end{aligned} \tag{5.1}$$

Equation (5.1) is a fifth-order polynomial in ω. It can be analyzed numerically. However, some interesting analytical result follows for $\omega_\chi, \nu_r, \omega_e \ll |\omega| \ll \nu_{eh}$, $k^2\lambda_D^2 \ll 1$ and $\mathbf{k}\cdot\mathbf{v}_g = 0$. Here, we have from (5.1)

$$\omega^3 = -ik^2c^2\nu_{eh}\frac{4\delta}{7\omega_0}W_0 \equiv (i\Omega_0)^3, \tag{5.2}$$

which predicts an instability whose increment is

$$\gamma \approx (k^2c^2\nu_{eh})^{1/3}\left(\frac{\delta}{\omega_0}\right)^{1/3}W_0^{1/3}. \tag{5.3}$$

6. Summary

In this paper, we have considered the nonlinear coupling between large-amplitude electromagnetic and electron-acoustic waves in a two-electron-temperature plasma.

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Accounting for the radiation pressure and the differential Joule heating of the electrons in the radiation field, we have obtained general dispersion relations. The latter have been analyzed for the decay and modulational interactions. Explicit expressions for the growth rates have been obtained. Furthermore, assuming that the electron collision frequency far exceeds the modulation frequency, we have shown that our general dispersion relation admits a new class of modulational instability. The growth rate of that instability is proportional to $\nu_{eh}^{1/3}$ and $|\mathbf{A}_0|^{2/3}$. The present nonlinear instability may thus produce non-thermal fluctuations that can affect the propagation of EM waves in space and inertially confined fusion plasmas. We note that our theoretical prediction of stimulated scattering of EM waves off the EA waves may lend support to a recent observation from the Trident laser facility (Montgomery et al. 2001). Furthermore, our theoretical results may also help to design new experiments for exploring the physics of complex nonlinear interactions in the Earth's ionosphere during heating by powerful radar beams. Here, the backscattered signal will provide the signature of the EA waves, thereby helping to determine the ambient plasma parameters in situ.

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