GRAVITATIONAL CHANNELS AND THE COSMOLOGICAL CONSTANT Λ

T. Ishizawa
Department of Astronomy, University of Kyoto

A new model of filamentary matter fields and voids is proposed. This is a gravitational version of the MIT bag model of hadrons (see a review of DeTar and Donoghue 1983). Bekenstein and Milgrom(1984) have first proposed a gravitational bag model. Their bag is closed but our bag, called a channel, is open.

We start from the action in the weak limt of gravity

$$I=\int_{-\infty}^{\infty} dt \int_{V} dx \left\{ \sum_n m_n \left[\tfrac{1}{2}\dot{\mathbf{x}}_n^2 - \phi(\mathbf{x}_n)\right] \delta(\mathbf{x}-\mathbf{x}_n) - \frac{1}{8\pi G}(\nabla\phi)^2 + B \right\}, \qquad (1)$$

where ϕ is the gravitational potential and $B=-c^4\Lambda/(8\pi G)$. Varying $\mathbf{x}_n(t)$, $\phi(\mathbf{x})$ and V under the conditions that $\delta\mathbf{x}_n(t)\to 0$ as $|t|\to\infty$ and that $\delta\phi(\mathbf{x})\to 0$ as $|\mathbf{x}^i|\to\infty$, we obtain the equations of motion for masses, Poisson equation for ϕ

$$\nabla^2\phi = 4\pi G \sum_n m_n \delta(\mathbf{x}-\mathbf{x}_n), \qquad (2)$$

and the natural boundary conditions at the free surface (S_1 in Figure 1)

$$\frac{\partial\phi}{\partial n}=0 \quad \text{and} \quad \frac{\partial\phi}{\partial \tau}=U, \qquad (3)$$

where $U=(8\pi GB)^{1/2}=c^2(-\Lambda)^{1/2}$.

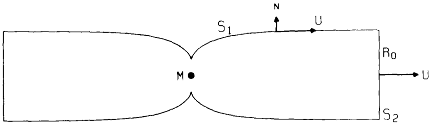

Figure 1. An axisymmetric channel of two opposite equal fluxes from a point mass.

The condition $\frac{\partial\phi}{\partial n}=0$ means that the lines of force are confined within the channels, i.e. filamentary structures. The potential ϕ is equivalent to the velocity potential in the incompressible irrotational flow which is surrounded by a gas of constant pressure and has the sources of total flux $4\pi Gm_n$ at $\mathbf{x}=\mathbf{x}_n$. The radius r_0 of the distant cross-section (S_2 in Figure 1) is obtained from Gauss's Theorem $2\pi r_0{}^2U=4\pi Gm$, so that $r_0=(Gm^2/2\pi B)^{1/4}$.

The exact solution for the two-dimensional channel from a point mass has been obtained using the standard method of conformal mapping (Milne-Thompson 1938). The force F on a test particle at the center line changes from $F\propto 1/r$ (two-dimensionally Newtonian) to F=const. at a distance of the half-width of the channel. This would be the case with the three-dimensional channel from a disk galaxy.

Assuming that the law of force is $-Gm_1m_2(1+r_{12}{}^2/A^2)^{1/2}/r_{12}{}^2$ between two masses in the disks of nearby spirals (M31, M33, NGC2403, NGC6946), we have found, from fitting the computed and observed rotation curves, that the scale length A satisfies a close relation $A=2.7\text{kpc}*(m/10^{10}M_\odot)^{1/2}$ where m is the total mass of a spiral. The acceleration $Gm/A^2=1.9\times10^{-8}$ cms^{-2} is coincident with Milgrom's(1983) limiting acceleration if $H_0=50$ $\text{kms}^{-1}\text{Mpc}^{-1}$. Equating $r_0=A$, we have $\Lambda=-1.8\times10^{-57}\text{cm}^{-2}$.

References

Bekenstein, J. and Milgrom, M.:1984, Astrophys. J. 286, 7.
Milgrom, M.: 1983, Astrophys. J. 270, 365, 371, and 384.

J. Kormendy and G. R. Knapp (eds.), Dark Matter in the Universe, 334.