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Review Article

Nonlinear acoustic–gravity waves

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Abstract. Previous results on nonlinear acoustic–gravity waves are reconsidered. It turns out that the mathematical techniques used are somewhat similar to those already adopted by the plasma physics community. Consequently, a future interaction between physicists in different fields, e.g. in meteorology and plasma physics, can be very fruitful.

1. Introduction and linear theory

Atmospheric waves have been studied by many authors during half a century (Hines 1960). They are also of increasing experimental interest (e.g. Bakhmeteva et al. 2002; Koshevaya et al. 2004). Several books and numerous review articles have been treating the linear properties of these waves, whereas comparatively much less effort has been devoted to their very complex nonlinear behaviour. In the present paper, we are therefore going to reconsider some main results in this field.

Neglecting for simplicity dissipative effects, one generally starts analytical investigations from the continuity and momentum equations together with an equation of state, i.e.

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1a}$$

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} + \mathbf{g}, \tag{1b}$$

$$(\partial_t + \mathbf{v} \cdot \nabla)(\rho^{-\gamma} p) = 0, \tag{1c}$$

where ρ is the mass density, $\mathbf{v}$ the fluid velocity, p the pressure, $\mathbf{g} = -g\hat{\mathbf{z}}$ the gravitational acceleration and γ the ratio of specific heats. In an equilibrium atmosphere with no drift velocities we have thus $\nabla p_0/\rho_0 = \mathbf{g}$.

Linearizing (1), i.e. writing $\rho = \rho_0 + \rho_1$ and $p = p_0 + p_1$, with $\rho_1 \ll \rho_0$ and $p_1 \ll p_0$, considering for simplicity the equilibrium temperature to be constant and making the ansatz $\rho_1 \sim p_1 \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r} - z/2H)$ and $\mathbf{v} \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r} + z/2H)$, with $H = p_0/g\rho_0$, we obtain the dispersion relation

$$\omega^2(\omega^2 - \omega_{\mathrm{a}}^2) - (\omega^2 - \omega_{\mathrm{g}}^2)k^2c_{\mathrm{s}}^2 - \omega_{\mathrm{g}}^2 k_z^2 c_{\mathrm{s}}^2 = 0, \tag{2}$$

where ω and $\mathbf{k}$ are the frequency and the wave vector, respectively, $c_{\mathrm{s}} = (\gamma p_0/\rho_0)^{1/2}$ is the sound speed, $\omega_{\mathrm{a}} = (\gamma g/4H)^{1/2}$ and the squared Brunt–Väisälä frequency is

$$\omega_{\mathrm{g}}^2 = (1 - 1/\gamma)(g/H). \tag{3}$$

The constant γ is typically 1.4 in the Earth's atmosphere, and as $(\gamma - 1)/\gamma \approx 0.3$ is fairly small one can clearly see that (3) has a low-frequency branch that is rather well separated from the high-frequency acoustic branch. However, in order to shed more light on the behaviour of atmospheric waves it may be instructive to consider a model atmosphere where $(\gamma - 1)/\gamma$ is much smaller than unity. In such a model Valhalla atmosphere (Stenflo 1986, 1996a) the dispersion relation (3) simplifies to

$$\omega^2 \approx \omega_{\mathrm{a}}^2 + k^2 c_{\mathrm{s}}^2 \tag{4a}$$

and

$$\omega^2 \approx \frac{k_\perp^2 \omega_{\mathrm{g}}^2}{k^2 + 1/4H^2}, \tag{4b}$$

where $k_\perp^2 = k_x^2 + k_y^2$ and $k^2 = k_\perp^2 + k_z^2$.

2. Wave–wave interactions

In an atmosphere where the equilibrium density decreases exponentially with height ($\rho_0 \sim \exp(-z/H)$) we note that the velocity perturbation instead increases exponentially ($\mathbf{v} \sim \exp(z/2H)$), whereas the energy $\rho_0 \mathbf{v}^2$ is constant as long as the atmosphere can be described by (1), where the dissipative terms have been neglected. It is then obvious that nonlinear effects will become increasingly important when a wave propagates upwards through the atmosphere. Considering for simplicity first a Valhalla atmosphere with $(\gamma - 1)/\gamma \ll 1$, we then analyse the propagation of the high-frequency wave (4a) in a background that is slowly varying due to the presence of waves with frequencies determined by (4b). We then analyse how this modified high-frequency wave nonlinearly excites the low-frequency modes (4b). Our description of this coupling is thus somewhat similar to that of an electron–ion plasma where the small electron to ion mass ratio plays the role of separating the high-frequency (the electron plasma wave) and the low-frequency (the ion acoustic wave) branches. As a result, one obtains a system of two nonlinearly coupled equations for the high-frequency and low-frequency waves. It should be stressed, however, that the Zakharov-like equations (Stenflo 1986) thus derived for a neutral atmosphere are significantly more complex than those of the well-known plasma equations (Zakharov 1972).

Considering next an atmosphere with arbitrary γ (e.g. the Earth's atmosphere where $\gamma \approx 1.4$) we note that the resonance conditions for three-wave interactions can be approximately satisfied (e.g. Yeh and Liu 1970; Juren and Stenflo 1973) and that three-wave interactions thus can play a major role in the amplitude development of atmospheric waves. Due to algebraic difficulties the coupling coefficients appeared first as rather complex expressions (Dysthe et al. 1974; Ostrovskii and

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Petrukin 1981), which limited the interest in applying them. After two decades of efforts, it was however fortunately possible to present them in their final explicit form (Axelsson et al. 1996a, 1996b) consistent with the wave action conservation laws.

3. Acoustic–gravity modons

In 1987 it was shown that the equations governing low-frequency acoustic–gravity waves can have localized (Stenflo 1987; Wu and Yao 1990) dipole-vortex solutions (modons). They propagate in the horizontal direction with a speed that is larger than that of the linear internal waves. In their two-dimensional version ($\partial_y = 0$) these equations appear in the form (Stenflo 1987, 1994; Stenflo and Stepanyants 1995; Horton et al. 2008)

$$d_t\left(\nabla^2\psi - \frac{\psi}{4H^2}\right) = -\partial_x\chi \tag{5a}$$

and

$$d_t\chi = \omega_{\mathrm{g}}^2\partial_x\chi, \tag{5b}$$

where $\psi(x,z,t)$ is the stream function, $\chi(x,z,t)$ is the normalized density perturbation, $d_t = \partial_t - (\partial_z\psi)\partial_x + (\partial_x\psi)\partial_z$ and $\nabla^2 = \partial_x^2 + \partial_z^2$.

In this case we allow the equilibrium temperature (p_0/ρ_0) to be an arbitrary function of z, and thus use the squared Brunt–Väisälä frequency expressions

$$\omega_{\mathrm{g}}^2 = \left(\rho_0\frac{d\rho_0^{-1}}{dz} - \frac{p_0}{\gamma}\frac{dp_0^{-1}}{dz}\right)\left(p_0\frac{dp_0^{-1}}{dz}\right)\frac{p_0}{\rho_0}. \tag{6}$$

We note that ω_{g}^2 reduces to the positive quantity (3) for an atmosphere with constant equilibrium temperature. In the Earth's atmosphere there are regions where the expression (6) can be positive, whereas in other regions it is negative.

Equations (5) indicate that a nonlinear energy transfer from small- to large-scale fluctuations (inverse cascade) is possible. Thus, using (5), it has been shown that low-frequency large-scale zonal flows can be generated by higher-frequency small-scale internal gravity waves (Horton et al. 2008).

Let us now consider perturbations moving with a constant velocity V in the x-direction. Thus, we replace ∂_t by $-V\partial_x$. The simplest solution of (5b) is then

$$\psi = -(V/\omega_{\mathrm{g}}^2)\chi. \tag{7}$$

Inserting (7) into (5a), one then obtains an equation which we here study in a polar coordinate system so that $x - Vt = r\sin\theta$ and $z = r\cos\theta$, where $r^2 = (x-Vt)^2 + z^2$, to look for a solution of (5a) in the form $\psi = R(r)\cos\theta$.

Outside the vortex the solution is then

$$R_{\mathrm{e}}(r) = \alpha_{\mathrm{e}}K_1(r/\lambda_{\mathrm{e}}), \tag{8}$$

where K_1 is the modified Bessel function of order one, $\lambda_{\mathrm{e}}^{-2} = 1/4H^2 - \omega_{\mathrm{g}}^2/V^2$ and α_{e} is a constant.

Inside the vortex the solution is

$$R_i(r) = \alpha_{\mathrm{i}}J_1(r/\lambda_{\mathrm{i}}) - (\lambda_{\mathrm{i}}/\lambda)^2 Vr, \tag{9}$$

where J_1 is the Bessel function of order one, $\lambda_{\mathrm{i}} = (\lambda^{-2} - \lambda_{\mathrm{e}}^{-2})^{-1/2}$ and α_{i} and λ are constants. Using the boundary conditions at the vortex radius r_0, we can then determine the constants α_{e} and α_{i}, and also obtain the relation (Stenflo and

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Stepanyants 1995)

$$\frac{\lambda_i J_2(r_0/\lambda_i)}{J_1(r_0/\lambda_i)} + \frac{\lambda_e K_2(r_0/\lambda_e)}{K_1(r_0/\lambda_e)} = 0. \tag{10}$$

Equation (10) contains the three parameters λ, r_0 and V, and two of them, for example r_0 and V, can therefore be considered as independent. Using typical parameter values we estimate r_0 to be of the order of 100–1000 m in the equatorial atmosphere (e.g. Stepanyants 1989).

We have thus constructed a two-dimensional localized solution that decreases exponentially far away from the vortex centre ($r \gg r_0$). Our solution represents a dipole-type vortex. Our modons can propagate faster than the maximum phase velocity $V_{\max}$ in any horizontal direction. According to crude estimates (Stenflo 1994) for an atmosphere with constant equilibrium temperature we have $V_{\max} = 2c_s(\gamma - 1)^{1/2}/\gamma$, i.e. with $\gamma \approx 1.4$ we use $V_{\max} = 0.9c_s$. However, with equilibrium temperature gradients close to the instability threshold (Stenflo 1994) one can obtain much slower velocities for the stationary modons.

Observations (Ramamurthy et al. 1990; Widdel et al. 1994) of nonlinear acoustic gravity waves in the atmosphere indicate qualitative similarities with theory (Stenflo 1991, 1994). Further studies (Jovanovic et al. 2002) have resulted in a catalogue of nonlinear vortex structures associated with acoustic–gravity perturbations in the Earth's atmosphere. This includes Kelvin–Stewarts cats eyes, tripolar (Jovanovic et al. 2001) structures as well as new solutions having the form of a row of counter-rotating vortices. Generalizations to three-dimensional solitary vortex structures (Pokhotelov et al. 2001) show that monopole vortices of finite height can also be found. Further studies of this kind will probably shed much light on the interplay (Stenflo 1996b; Pokhotelov et al. 2001) between chaos and order in the atmosphere.

Using these equations, we can also find the spectral properties of the atmospheric wave turbulence. Thus, it has recently been shown that the characteristic turbulent spectrum associated with (5) has a Kolmogorov-like feature (Shaikh et al. 2008).

4. Generalized Lorenz equations

Equations (5) have been generalized in order to take into account the rotation of the Earth, as well as viscosity and thermal diffusion effects (Stenflo 1996a). Due to the complexity of these equations one then has to adopt simplifying assumptions. Following previous attempts (Lorenz 1963; Stenflo 1996a) we therefore characterize the atmospheric disturbances by four time-dependent functions $X(t)$, $Y(t)$, $Z(t)$ and $U(t)$ to obtain the so-called L-S system of equations

$$\dot{X} = \sigma(Y - X) + sU, \tag{11a}$$

$$\dot{Y} = r_1 X - XZ - Y, \tag{11b}$$

$$\dot{Z} = XY - bZ \tag{11c}$$

and

$$\dot{U} = -X - \sigma U, \tag{11d}$$

where the dot stands for the normalized time derivative, σ is the Prandtl number, r_1 is the generalized Rayleigh parameter (Stenflo 1996a), $b = 4k_z^2/k^2$ and $s = 4\Omega^2 k_z^2/\kappa^2 k^6$, where Ω is the angular frequency of the Earth's rotation and κ is the

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thermal diffusion coefficient. We note that in a non-rotating atmosphere we can put $\Omega = 0$, i.e. $s = 0$. The system of equations (11a), (11b) and (11c) is then identical to that of Lorenz (1963).

The L-S equations (11) have been studied in detail by many physicists. It has thus been shown that the physical (e.g. Yu and Yang 1996; Zhou et al. 1997a, 1997b; Yu 1999; Liu 2000; Banerjee et al. 2001; Lonngren and Bai 2001; Zhou 2002) as well as the mathematical properties of the Lorenz system (Ekola 2005) can be significantly altered even if a very small rotation frequency is included. The L-S equations (11) have in addition been generalized to a system of five equations for five time-dependent amplitudes (Stenflo 1996b). Such equations could describe both localized dipole-type vortices as well as chaotic behaviour.

5. Related plasma physics phenomena

The nonlinear theory for acoustic–gravity waves has previously mainly been developed by plasma physicists who noted that the theoretical methods used in plasma physics were also applicable to atmospheric waves (e.g. Dysthe et al. 1974; Petviashvili and Pokhotelov 1992; Onishchenko et al. 2008). Now, however, it is possible to reverse the exchange of ideas, i.e. the recent results found in studies of acoustic–gravity waves could also be used for more insight into plasma physics phenomena (e.g. Hultqvist and Stenflo 1975). Below we shall point out a few examples.

(a) Dusty plasmas are of much interest nowadays (e.g. Shukla and Eliasson 2009). Considering a non-uniform dusty plasma, Shukla and Shaikh (1998) found dust–acoustic vortices which are described by (5) if ω_{g}^2 in (5b) is replaced by

$$\omega_{\mathrm{d}}^2 = \left(\rho_{\mathrm{d}0}\frac{d\rho_{\mathrm{d}0}^{-1}}{dz} - \frac{p_{\mathrm{i}0}}{\gamma_{\mathrm{i}}}\frac{dp_{\mathrm{i}0}^{-1}}{dz}\right)\left(p_{\mathrm{i}0}\frac{dp_{\mathrm{i}0}^{-1}}{dz}\right)\frac{p_{\mathrm{i}0}}{\rho_{\mathrm{d}0}}, \tag{12}$$

where $\rho_{\mathrm{d}0}$ is the equilibrium dust mass density, $p_{\mathrm{i}0}$ is the equilibrium ion pressure and γ_{i} is the adiabatic exponent of the ion fluid. These investigations were later extended to a collisional dusty plasma (Shaikh and Bhatt 2003).

(b) The equations (Nycander et al. 1987; Shaikh and Shukla 2009) describing magnetic-electron-drift turbulence are similar to (5) if ψ and χ are replaced by the magnetic field and electron temperature perturbations. We then also have to replace H by the ratio between the velocity of light and the electron plasma frequency, whereas ω_{g}^2 in (5b) (which, due to (6), is proportional to dp_0/dz) in this case is replaced by a squared frequency that is proportional to the electron density gradient.

(c) The interaction of acoustic–gravity waves with the ionosphere is also of much interest (e.g. Nekrasov et al. 1995; Aburdzhaniya 1996; Kotsarenko et al. 1999; Sorokin and Chmyrev 1999; Koshevaya et al. 2005; Kaladze et al. 2008a, 2008b). Kaladze et al. (2008a) thus generalized (5a) to

$$d_t\left(\nabla^2\psi - \frac{\psi}{4H^2}\right) = -\partial_x\chi - \frac{\sigma_{\mathrm{p}}B_0^2}{\rho_0}\left(\partial_z^2\psi - \frac{\psi}{4H^2}\right), \tag{13}$$

where σ_{p} is the Pedersen conductivity and B_0 is the geomagnetic field magnitude. Equation (13) is supposed to be valid at high latitudes where the geomagnetic field is essentially vertical. Together with (5b) it describes two-dimensional acoustic–gravity waves in the Earth's ionosphere.

(d) Considering electromagnetic modes in non-uniform electron–ion plasmas, and adopting mathematical techniques analogous to those used in deriving (11), it turns out that equations similar to (11), where the number of time-dependent functions can be three, four, five or six (Mirza and Shukla 1997; Murtaza et al. 1999; Azeem and Mirza 2006), can appear.

Due to the examples (a), (b), (c) and (d) above it seems as if a thorough investigation of acoustic–gravity-plasma waves ought to be considered as a prerequisite to most further studies of non-uniform plasmas.

6. Summary and conclusion

In this short review paper, we have presented main theories for nonlinear acoustic–gravity waves in non-uniform media. Nonlinear acoustic–gravity waves may appear in the form of solitary dipole vortices (Stenflo 1987), and possess dual turbulence cascades that are responsible for the formation of structures (Shaikh et al. 2008). The nonlinear equations for the dynamics of acoustic–gravity waves can also be represented by the Lorenz–Stenflo equations (Stenflo 1996a). The latter admit chaotic trajectories. Furthermore, equations similar to those of the nonlinear acoustic–gravity waves also appear in the theories for dusty plasmas (Shukla and Shaikh 1998) and for the Earth's ionosphere (Kaladze et al. 2008a, 2008b). In conclusion, we stress that the results of the present paper describe basic nonlinear features of middle (Fritts and Alexander 2003) and solar (McKenzie and Axford 2000) atmospheric turbulence.

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