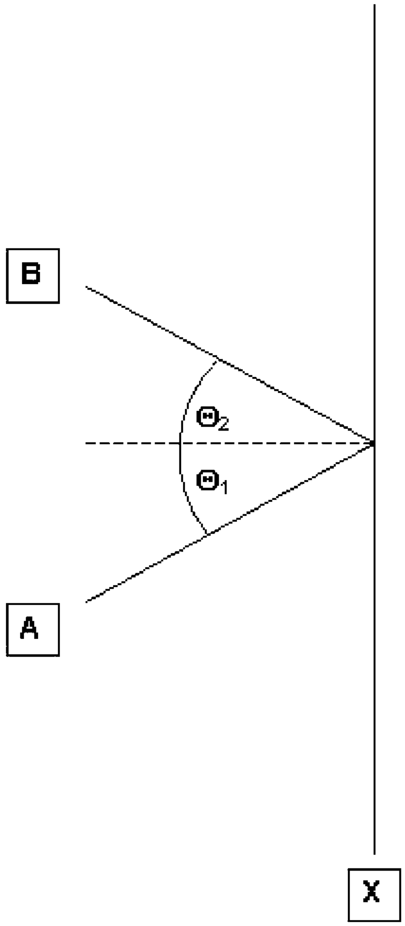

Figure 3 (Gaertner). Reflection of (a single beam of) light emanating from A at surface X.

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Author's Response

Huygens versus Fermat: No clear winner

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Abstract: How should we assess the appeal of multiple scientific theories when they can all explain a particular empirical phenomenon of interest? We contrast Huygens' and Fermat's explanations of the law of refraction of light and find that neither dominates the other when considering multiple criteria for assessing the overall appeal of a scientific theory. The absence of teleology in Huygens' account is a strong plus compared to Fermat's. But Huygens' wave theory scores less well with respect to other desiderata for a scientific theory. In this case, there does not appear to be a clear winner, nor need there be one.

Gaertner provides valuable insights into my original discussion of Fermat's principle of least time (Schoemaker 1991). He reminds us – as well as demonstrates – that Huygens' wave theory can also be used to derive Snell's law of refraction but without resort to the metaphysical notion that light travels the shortest distance in time. He considers this "a case against optimality."

If we desire a purely causal or descriptive account of nature's laws, Huygens will be much more appealing than Fermat. Huygens clearly offers an alternative account of Snell's law, but whether this constitutes a case against optimality is debatable. For example, if parsimony or simplicity is important, Fermat's account may be preferred. It is far simpler to explain – to, say, high school students – than Huygens' interference pattern. The latter entails the cumulative effect of waves that are slightly or greatly out of phase.

Further, suppose we judge a scientific theory not only on its ability to explain observed empirical phenomena ex post facto, but also on its ability to predict new phenomena ex ante. In that case, Fermat may have the upper hand. The principle of least time is deceptively simple, both to apply and to remember. And the deception lies in its appeal to teleological explanations. But as long as the teleological approach results in valid new predictions – as Fermat's principle did about the behavior of light in converging lenses – its metaphysical nature may well be a price worth paying. The key is to view the search for optimality as a powerful heuristic. This may sound paradoxical since heuristics are by definition not optimal. But we should not take this perceived optimality too literally, as though it were a deep scientific truth. At this stage, we simply don't know.

R1. Multiple criteria

It would have been interesting if Gaertner could have scored Fermat versus Huygens on these multiple criteria, rather than place all his weight on one (the absence of teleology). This perhaps underscores the subjectivity of science. What constitutes an adequate scientific explanation? Clearly, explaining the empirical phenomenon at hand ranks very high. But I think that simplicity and elegance, as well as the propensity to spawn new predictions, constitute important criteria, too. And consistency with other prevailing scientific explanations, or the breadth of the domain of application, should perhaps matter as well. Table R1 summarizes some of the criteria I would personally use to evaluate competing theories.

Table R1. Evaluating a Scientific Theory

Possible Criteria to Use:
a. Does it explain the phenomenon well?
b. Is the explanation easy to follow or apply?
c. Is it a parsimoniuous explanation (fewest assumptions)?
d. Does it generate interesting new predictions?
e. Can the theory be falsified in principle?
f. How consistent is it with other theories?
g. How broad is the potential domain of application?
h. How widely is it used by practicing scientists?
i. Does it have strong competitors (i.e., alternative theories)?
j. Has it been widely tested, with positive results?

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Because science is a cumulative process, where better theories replace inferior ones in the Darwinian struggle for intellectual survival, it is interesting that Fermat's principle is still taught in physics courses at both the high school and university levels. And the twin existence of wave and corpuscular theories of light, more generally, underscores that neither one dominates the other on all accounts. In some applications, such as simple optics, we may prefer Fermat. In others, involving interference patterns or the phenomenon of reflection, we would presumably favor Huygens.

R3. Reflection of light

Gaertner justly notes that the application of Fermat's principle may require the specification of clear boundary conditions before it can be usefully invoked. The example Gaertner gives about light reflecting off a glass plate (see his Fig. 3) illustrates the need for boundary conditions well. The shortest path from *A* to *B* is indeed a straight line. But once the boundary condition has been imposed, that the light beam must strike the mirror en route, Fermat's principle makes the right prediction. Any other path from *A* to *B* along the mirror would be longer than the symmetric triangle drawn in that figure. And indeed, if points *A* and *B* had not been equidistant from the mirror, as in my Figure R1 here, Fermat's principle would allow for an easy solution. To find the shortest path, we can set up an algebraic distance equation to prove that *C* is indeed the point of shortest distance (and hence, time, in this isotropic medium). A simpler approach, however, is geometric analysis. If we simply reflect point *B* through the mirror to the other side (called *B**) and then draw the straight line *A* to *B**, point *C* emerges as the shortest route. Any other point beside *C,* such as, for example, *C′*, will make route *A–C′–B** longer than *A–C–B**. And because *A–C–B* is equal in distance to *A–C–B**, we have proved that *C* is the reflection point of shortest distance and time.

This visual example illustrates the intuitive simplicity and ease of application of Fermat's principle. But yes, it does require that some clear boundary conditions are set beforehand. However, all theories will require boundary conditions, and it is not so evident to me that Fermat's is more restrictive or arbitrary in this regard than Huygens'. Indeed, I wish that Gaertner had also indicated the specific boundary conditions needed for wave theory's account. And I would challenge him to offer a wave theory explanation as simple as the example here, as to why point *C* will indeed be the point of reflection when a beam of light is directed from *A* to *B* via the mirror (assuming the general case where *A* and *B* are not equidistant from the mirror). To me, the beauty of Fermat's principle lies in its simplicity.

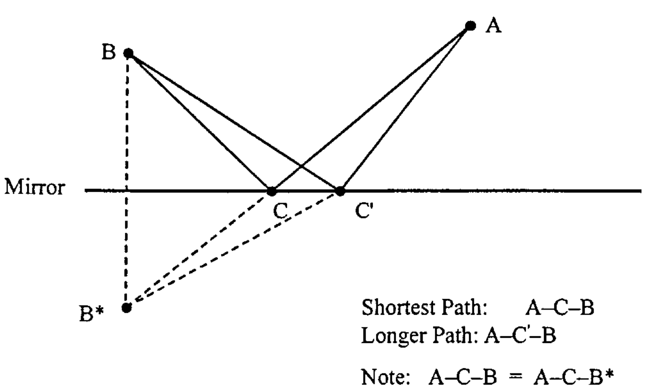

Figure R1. Reflection of light

R4. Beyond physics

Of course, Snell's law is just one example – the one I happen to focus on in my target article – to make a broader statement about the pros and cons of optimality arguments. In my rejoinder to commentators in 1991, I addressed an example from the social sciences, namely, the economist's view that humans are utility-maximizing creatures. In this case, the power of a simple optimality model is quite evident. Simple explanations can be constructed and clear predictions made, about the economic affairs of organizations as well as individuals. It would require much heavier machinery to derive these same conclusions from a deeper process model that acknowledges the heuristic nature and bounded rationality of the human actor. And even psychologists, who generally reject the optimality assumptions of economics, have used optimality arguments in their descriptive models (e.g., prospect theory in risky choice, signal detection models, linguistics, etc.).

Similarly, survival of the fittest offers biologists an easy principle via which to explain the diverse and complex behavior of species. Indeed, the principle is so easy to apply that it has been criticized as too facile in hindsight, while offering limited predictive power in going forward. But the principle's basic premises (of random mutation and natural selection) are undeniable and have been demonstrated in laboratory experiments with fruit flies and the like. The challenge comes when we are applying a broad principle, such as survival of the fittest or utility maximization in economics, without undue plasticity or excessive hindsight bias. More specific principles, based on narrow causal mechanisms, may do better in this regard but also may lack the metaphoric qualities of the broader principles in sparking human imagination. This may be the fundamental trade-off we have to face when judging competing scientific accounts of observed phenomena in nature.

R5. In conclusion

I appreciate Gaertner's view that wave theory can explain Snell's law as well as Fermat's, without having to resort to metaphysics. However, I am not persuaded that Fermat's principle (and the implicit argument of optimality) is inferior to Huygens on all other counts. The need for simplicity (of explanation and application), parsimony in assumptions (beyond the teleological one) and the ability to stimulate the mind to new hypotheses are important criteria as well. Fermat's principle scores rather well on these criteria, and this was one of the prime reasons I profiled this particular principle in my 1991 target article. So the basic question remains as to why optimality arguments – which seem so deeply flawed in their explicit assumption that nature optimizes – are so effective in explaining observed phenomena in nature and, even more puzzling, in predicting entirely new ones. We are getting closer to an answer, thanks to Gaertner as well as many other commentators, but I am not sure we have a really good account yet.

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Letter "r" appearing before author's initials refers to Continuing Commentary Response references

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