Over-tension in a Condenser Battery during a sudden discharge.

By P. Kapitza, Ph.D. Fellow of Trinity College, Assistant Director of Magnetic Research at the Cavendish Laboratory, Clerk Maxwell Student of Cambridge University.

[*Read* 18 January 1926.]

(Plate II.)

In the recent experiments on the Zeeman effect in strong magnetic fields* made by Mr H. W. B. Skinner and myself for obtaining an intense source of light for a small fraction of a second, we have been discharging a condenser battery consisting of 32 Leyden jars connected in parallel with a capacity of one-tenth of a microfarad. The discharge was made through a small spark gap by means of an oil immersed switch (the details of this arrangement are to be found in the above-mentioned paper). When the discharge was produced a curious phenomenon was sometimes observed. After the battery had been charged to its maximum tension, and the switch had been put into operation, only a small spark occurred in the spark gap, and the main part of the discharge went over the top of the insulators of one of the Leyden jars.

By taking the photograph of the spark on a film moving rapidly before a slit which was illuminated by the spark, it was possible to see the details of the phenomenon.

In Figs. 1, 2 and 3 are shown three photographs taken with different self-inductions in series with the spark gap. On Photograph 1 a normal discharge is seen in which no flash over the insulation occurs in the condenser battery. About 20 full oscillations can be seen, after which the spark is extinguished. Photographs 2 and 3 represent the spark in the case when the discharge flashes over one of the condensers. From these photographs it can be seen that in the case of Photograph 2 the spark is extinguished after the first half of the oscillation, as only one line is seen, and that in the case of Photograph 3 it is extinguished after the full oscillation, as two lines are seen. This shows the very peculiar character of the over-tension occurring during a sudden discharge in a condenser battery. In this paper a short account of these phenomena will be given in which it will be shown how it is possible to account for these over-tensions, and how they can be avoided.

From general considerations it can be seen that during the discharge of a condenser battery, when the self-induction of the

* *Proc. Roy. Soc.* A, vol. 109, p. 225 (1925).

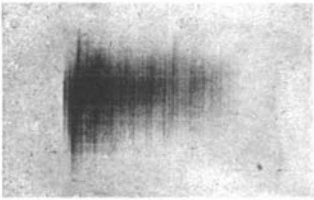

Fig. 1

Fig. 2

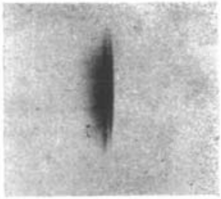

Fig. 3

connecting leads together with the self-induction of the Leyden jars is of the same order as the self-induction of the spark circuit, electrical oscillations can be set up between the Leyden jars which, being superimposed on the tension of the general discharge, may produce over-tension in separate Leyden jars.

Let us consider a large number of Leyden jars connected in series as shown on Fig. 4.

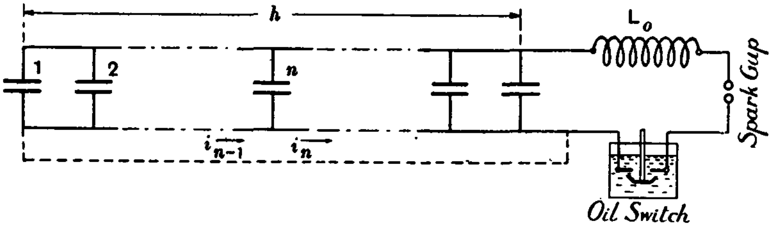

Fig. 4

Let the capacity of each of the Leyden jars be c; the self-induction of the connecting leads l; the current in the connecting leads of the Leyden jars i_n, i_{n-1}, and the tension of the Leyden jar e_n. We then get a series of equations of the following type:

$$i_n - i_{n-1} = -c\frac{de_n}{dt} \qquad \text{......(1)},$$

$$e_n - e_{n-1} = -l\frac{di_{n-1}}{dt} \qquad \text{......(2)}.$$

The number of these equations is twice the number of the jars, and it is rather difficult to treat the system of these differential equations for a large number of Leyden jars, but with a quite sufficient approximation we can replace them with two partial differential equations. Taking the capacity and the self-induction as equally distributed along the condenser battery we get

$$\left.\begin{aligned} c &= C\,dx \\ l &= L\,dx \end{aligned}\right\} \qquad \text{......(3)},$$

$$i_n - i_{n-1} = di,$$

$$e_n - e_{n-1} = de.$$

Taking the axis of coordinates along the battery as shown in Fig. 4, we obtain the equations

$$\frac{di}{dx} = -C\frac{de}{dt} \qquad \text{......(4)},$$

$$\frac{de}{dx} = -L\frac{di}{dt} \qquad \text{......(5)}.$$

Now to obtain a solution of these equations it is necessary to satisfy the following initial conditions: that when

$$t = 0,$$
$$\left.\begin{array}{ll}\text{(A)} & i = 0 \\ \text{(B)} & e = e_0\end{array}\right\} \quad (0 \leq x \leq h),$$

where e_0 is the potential to which the battery is charged and h may be called the total length of our condenser battery. We must also satisfy the following conditions: that

$$\left.\begin{array}{lll}\text{(C)} & \text{at } x = 0, & i = 0 \\ \text{(D)} & \text{at } x = h, & e = L_0 \dfrac{di}{dt}\end{array}\right\} \quad (any)\ t,$$

where L_0 is the outside self-induction.

The most general solution for i and e in these equations can be shown to be of the following form:

$$\left.\begin{array}{l} i, e = \Sigma\,(A \sin mx + B \cos mx) \sin amt + \\ \quad + \Sigma\,(A' \sin mx + B' \cos mx) \cos amt \end{array}\right\} \quad \text{......(6)},$$

where

$$a = \sqrt{\frac{1}{CL}} \quad \text{......(7)}.$$

It is easily seen that in satisfying equations (4) and (5) and conditions (A) and (C), we obtain the following expressions for i and e:

$$i = \Sigma A \sin mx \sin amt \quad \text{......(8)},$$

$$e = La\Sigma A \cos mx \cos amt \quad \text{......(9)}.$$

The boundary condition (D) requires that

$$L \cos mh = L_0 m \sin mh \quad \text{......(10)}.$$

This gives the value for m. If, for brevity, we put

$$\mu_m = mh \quad \text{......(11)},$$

and

$$\gamma = \frac{hL}{L_0} \quad \text{......(12)},$$

we obtain from (10) the following transcendental equation

$$\gamma = \mu_m \tan \mu_m \quad \text{......(13)},$$

the roots of which determine the value of m. Introducing these roots in the general expressions for e and i we obtain

$$i = \Sigma A_m \sin \frac{\mu_m x}{h} \sin \frac{\mu_m at}{h} \quad \text{......(14)},$$

$$e = aL\Sigma A_m \cos \frac{\mu_m x}{h} \cos \frac{\mu_m at}{h} \quad \text{......(15)}.$$

Now the coefficient A_m will be determined from the initial condition (B). It has been shown by Poisson* that expansions in Fourier series with coefficients which are roots of transcendental equations such as (13) can be treated similarly to ordinary Fourier series.

We find $$A_m = \frac{e_0}{La}\frac{1}{N_m}\int_0^h \cos\frac{\mu_m x}{h}\,dx \qquad \text{......(16)},$$

where $$N_m = \int_0^h \cos^2\frac{\mu_m x}{h}\,dx \qquad \text{......(17)},$$

and we obtain $$A_m = \frac{4e_0}{La}\frac{\sin\mu_m}{2\mu_m + \sin 2\mu_m} \qquad \text{......(18)}.$$

The final solution in our case is therefore

$$e = 4e_0 \sum_0^\infty \frac{\sin\mu_m}{2\mu_m + \sin 2\mu_m}\cos\frac{\mu_m}{h}x\cos\mu_m\frac{a}{h}t \quad \text{......(19)},$$

$$i = \frac{4e_0}{La}\sum_0^\infty \frac{\sin\mu_m}{2\mu_m + \sin 2\mu_m}\sin\frac{\mu_m}{h}x\sin\mu_m\frac{a}{h}t \quad \text{......(20)}.$$

Let us first consider the case when the outside self-induction L_0 is very large. It is easily seen that γ in equation (13) represents the ratio of the total internal self-induction hL to the outside self-induction L_0. When γ is very small we find that the roots of the equation are close to $n\pi$, where n is any positive integer from 0 to ∞. In this case all the A's vanish except $A_0 = \frac{1}{4}$, and if we take into consideration that the smallest root of the equation (13) is

$$\mu_0 = \sqrt{\frac{hL}{L_0}} \qquad \text{......(21)},$$

we easily obtain from (19) and (20), taking into consideration (7),

$$e = e_0\cos\frac{t}{\sqrt{ChL_0}} \qquad \text{......(22)},$$

$$i = e_0\sqrt{\frac{Ch}{L_0}}\frac{x}{h}\sin\frac{t}{\sqrt{ChL_0}} \qquad \text{......(23)}.$$

If we remember that Ch is the total capacity of the battery, we see that with a large outside self-induction there are, in the normal way, no oscillations in the battery during the discharge, as e is independent of x, and the discharge is of the same type as would

* *Journal de la Polytechnique*, cahier 18: Poisson, "Mémoire sur la manière d'exprimer les fonctions en série de quantités périodiques". In this work Poisson treats a problem very similar to ours. He calculates the vibrations of a load attached to a bar with an equally distributed mass. The problem dealt with in my case is very like that treated by Poisson, as the self-induction of the outside circuit is equivalent to the inertia of the load, and the distributed capacity and self-induction of the condensers correspond to the distributed mass and to the elasticity of the load.

be obtained with a condenser of a capacity Ch, with an outside self-induction L. From (23) we see that the current i increases linearly along the battery.

If γ be large, the tension along the condenser will not be equally distributed during the discharge, and from expression (19) we can easily see that the maximum value of e will be at $x = 0$, the end of the condenser opposite to the end from which the current is taken. e in this place will equal

$$e = e_0 \Sigma \frac{4 \sin \mu_m}{2\mu_m + \sin 2\mu_m} \cos \frac{\mu_m at}{h} \qquad \text{......(24).}$$

From the time $t = 0$ to the time $t = \frac{h}{a}$ the tension at this place is constant, as can at once be seen from the way in which we determine the coefficient A_m, (16) and (17). Further on it decreases to zero and then increases again with opposite sign. In order to see what really happens we have to calculate the value of this sum for a longer time than $t = \frac{a}{h}$ and for different γ's. This is not difficult as it is quite sufficient to calculate four terms of this expression which approximately converges as $\frac{1}{m^2}$. If $\gamma = 1$ we find that after the first half of the oscillation the tension at $x = 0$ will be 1·28 of the initial one. If $\gamma = 2$ we find that after $1\frac{1}{2}$ full oscillations the tension at the same place will be equal to 1·35 of the initial one. If $\gamma = 4$ after ten full oscillations the tension at $x = 0$ will be 1·56 of the initial one. This shows that with increasing γ the over-tension will gradually increase, but the maximum will occur later. This explains the fact that sometimes the breaking down of the condenser happens only after several half-oscillations. Calculating the terms in this series it is easily seen that the first two terms are the most important ones. At the beginning they have opposite signs, but after some time of oscillation they may have the same sign and maximum amplitudes together. This moment will give the over-tension. If the first root of equation (13) is μ_0, and the second μ_1, and the ratio $\frac{\mu_1}{\mu_0} = k$, then the over-tension will happen after n half-oscillations where n is defined by

$$kn\pi - n\pi = m\pi \qquad \text{......(25),}$$

and m is any odd number. From this formula (25) we find that the ratio of the first and second roots of this equation has to be 4 in order to get the over-tension after the first half-oscillation, $3\frac{1}{2}$ to get it after the second half-oscillation, $3\frac{1}{3}$ to get it after the third oscillation and so on.

In these calculations we have not taken into consideration the damping of the oscillations due to the resistance in the circuit. This fact, indeed, makes it impossible for the over-tension to occur very late during the discharge, but for the first two or three cycles it is probably permissible to ignore the resistance, and this very much simplifies the analysis.

From the analysis it is also seen that in general the over-tension during the discharge of a condenser battery could easily be avoided if the sparking circuit was connected to different ends of the condenser battery, as is shown by a dotted line on Fig. 4. In this case each Leyden jar when discharging has the same self-induction in each circuit, and when all the Leyden jars discharge with the same period, no oscillations between the jars can occur, as there is no difference of potential between them. This also applies to a separate Leyden jar which, in order to avoid electrical over-tension, must be connected in such a way that the leads are taken from opposite ends. It may also be mentioned that, as can be seen from (20), some irregularity must be observed in the current, as it is expressed by an equation similar to (19) for the tension. On Photograph 1 it can be observed that the intensity of the spark varies in an irregular way from one oscillation to another, and this is probably due to variations in the current.

In certain cases in practice it is possible to use oscillations in Leyden jars in order to obtain very short and sharp impulses of the electrical current. For this purpose it is only necessary to place a spark gap at the opposite end of the condenser battery, connecting this spark gap to the condenser battery without any self-induction, and adjusting it so as to make it slightly larger in size than the spark gap in the circuit with the oil switch. By adjusting the self-induction in the series with the main spark gap, it will be possible to produce over-tension in the subsidiary spark gap after one, two or more oscillations, and as the discharge will at this moment pass in the subsidiary spark gap, we shall only obtain one or two oscillations in the main spark gap.

In conclusion I would like to express my thanks to Prof. A. N. Kryloff, with whom I have discussed the mathematical part of this paper.