

A Graphic Synthesis for Space Manoeuvring

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The complex and far-from-intuitive issue of establishing how and when to act when manoeuvres are to be performed in space is discussed here in a simple graphical way. An analysis was made of the effects on the parameters of an orbit of low thrusts in three directions: tangential to the orbit; normal to it in the orbit plane; and normal to the orbit plane. An outcome is that, in order to obtain a desired effect, and at the same time minimize the undesired ones, it is better to use only a tangential thrust or only a normal one rather than a combination of simultaneous tangential and normal thrusts. Thus it is not necessary to investigate any alternative orientation of the thrusters. A manoeuvre can be accomplished by thrusts given either only once along the orbit path, or in more than one orbital point. Diagrams are produced which give a quick insight into the manoeuvre philosophy for both kinds of action.

1. introduction. In space operations, a rendezvous must be accomplished, a position must be reached or a spacecraft must be transferred to another orbit. In addition, the performances of a constellation of artificial satellites deteriorate because of motion perturbations or satellite failure, so that it becomes necessary to relocate the satellites with respect to one another. This involves 'space manoeuvring'.

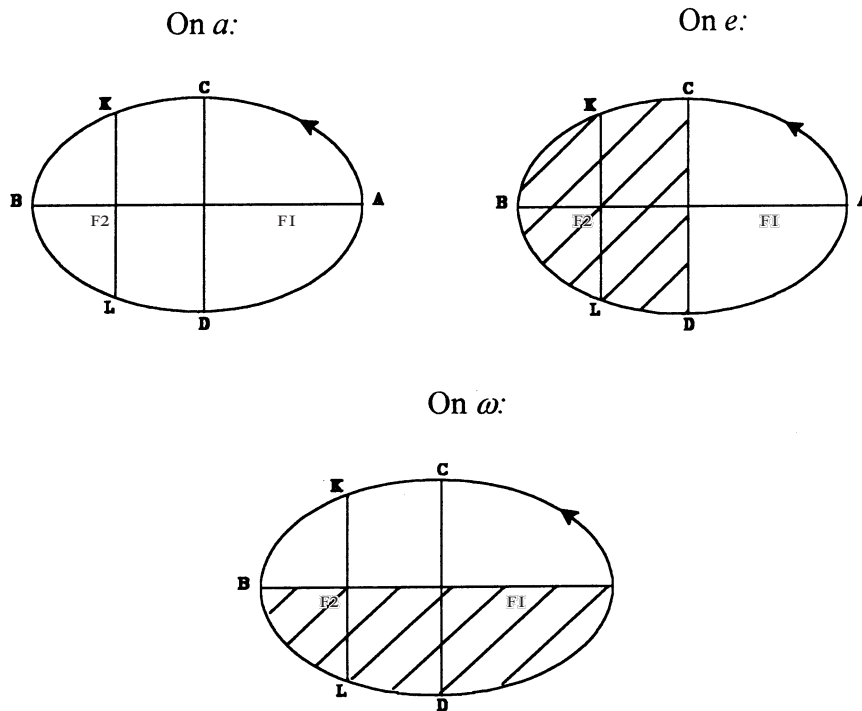
Since a space manoeuvre may be considered a perturbation specifically designed for the satellite motion, it can be accomplished by the action of 'disturbing forces' (thrusts) producing suitable variations of the values of orbital elements. Standard orbital elements will be used: a , semi-major axis; e , eccentricity; i orbit inclination; Ω , longitude of the ascending node; ω , argument of the periapsis; t_0 , time of a periapsis passage or M_0 , mean anomaly at epoch = $M - n(t - t_0)$.

The 'disturbing forces' can be expressed in the form of the following rectangular components: T , tangential, positive in the direction of motion; N , perpendicular to T in the orbital plane, positive towards the Earth; W , perpendicular to the orbital plane, positive towards the north.

2. qualitative analysis. The qualitative effects of these forces on the above-mentioned elements are well known.¹⁻³ They can be obtained by means of simple relations between v (satellite velocity) and other parameters as well as by certain geometrical properties of the orbital ellipse.

The most interesting results can be transferred directly onto the ellipse (Figure 1), which represents an orbit relative to an attracting mass at F1, using the following convention: the white areas indicate a positive variation of the elements in question and the shaded ones a negative variation due to positive force, i.e. when a positive tangential thrust is applied, the major axis always increases, while the eccentricity increases in the sector DAC and decreases in the sector CBD. For negative values of components, the results are opposite in sign.

a)



b)

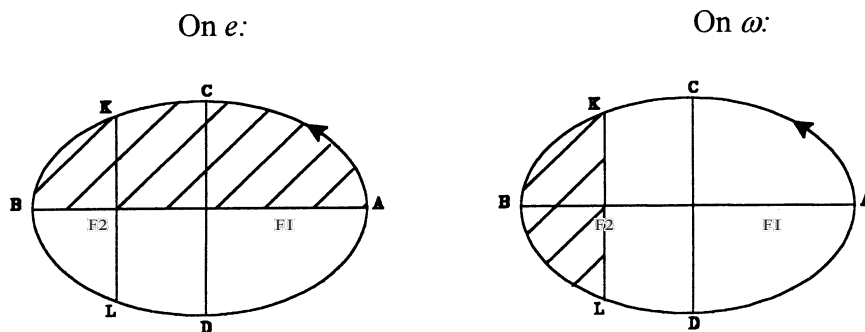
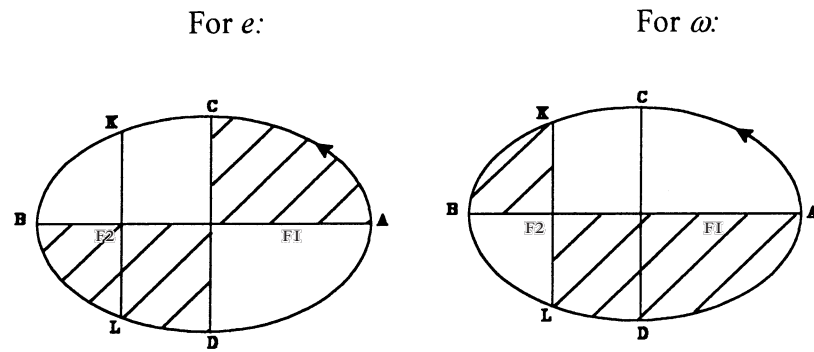


Figure 1. (a) Effects caused by T on eccentricity, major axis and line of apsides. (b) Effects caused by N on eccentricity and line of apsides. The effect of N on the major axis is zero.

Interesting results concerning the contemporary effects of thrusts T and N (synergy or contrast) may be obtained by super-impositions of the preceding figures, and these are shown in Figure 2: 'synergy' means that when positive tangential and normal forces are applied, both forces cause an increase or decrease of the elements considered; 'contrast' means that one of the forces causes an increase and the other a decrease of the elements.

a)



b)

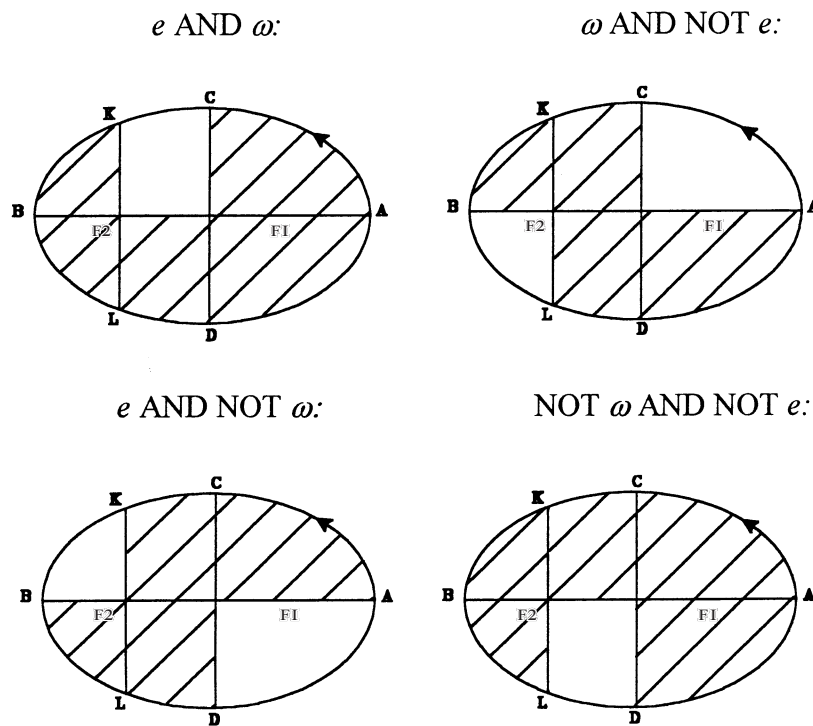


Figure 2. (a) Synergy and contrast of N and T on e or on ω ; in the white areas there is a synergy while in the shaded ones there is a contrast. (b) Synergy of N and T on e and on ω . NOT means that there is no synergy, i.e. there is a contrast. In the white areas the assertion is true; for example from L to D 'NOT ω AND NOT e ' is true; in the shaded ones the assertion is false.

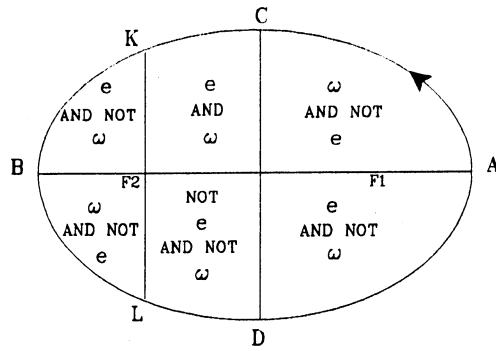


Figure 3. Areas where the inscribed assertions for forces N and T with equal sign are true.

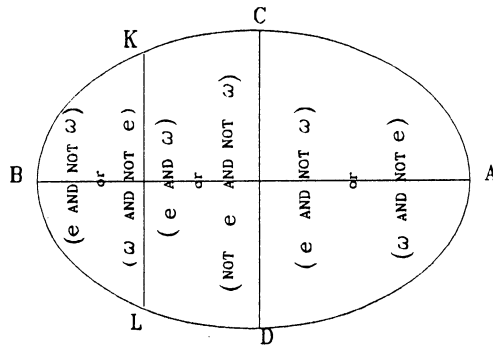


Figure 4. Synergy graph of N and T on e and ω , without distinguishing between the sign of the forces.

A synthesis of all of the above cases is given in Figure 3. As the force can be either positive or negative, a final synthesis is given in Figure 4.

We can obtain some interesting considerations from these graphs concerning the areas where it may be possible to act in order to bring about a change in one element without changing the others. For example, we can investigate the area from L to D to see if it is possible to vary a , without varying e and ω ; in the area from C to K, we can see if the mentioned effect may be obtained by N and T which are opposite in sign. We have to investigate the remaining sectors to establish if it is possible to produce a change of e leaving ω unchanged or vice versa by acting with forces T and N .

3. quantitative analysis. One way to carry out a quantitative analysis of the problem in question is to start out from the analytic relations which express the time rate-of-change of the six parameters in terms of perturbations. The well-known Gaussian form of Lagrange's Planetary Equations will be used, where the force components are in the direction R of the radius vector, S perpendicular to the radius in the orbital plane and W orthogonal to both.^{4,5}

These equations can be also expressed in the form of the perturbing force components T , N and W by rotating the axes. A re-formulation of these equations has been obtained by Battin.⁶ Expressions equivalent to Battin's, where T , N and W are forces per unit mass, are:

$$\frac{da}{dt} = (A \sin \Phi + B \cos \Phi)T + (B \sin \Phi - A \cos \Phi)N \tag{1}$$

$$\frac{de}{dt} = (A_1 \sin \Phi + B_1 \cos \Phi)T + (B_1 \sin \Phi - A_1 \cos \Phi)N \quad (2)$$

$$\frac{di}{dt} = \frac{r \cos u}{na^2(1-e^2)^{1/2}}W \quad (3)$$

$$\frac{d\Omega}{dt} = \frac{r \sin u}{na^2(1-e^2)^{1/2} \sin i}W \quad (4)$$

$$\frac{d\omega}{dt} = (A_2 \sin \Phi + B_2 \cos \Phi)T + (B_2 \sin \Phi - A_2 \cos \Phi)N + C_2W \quad (5)$$

$$\frac{dMo}{dt} = (A_3 \sin \Phi + B_3 \cos \Phi)T + (B_3 \sin \Phi - A_3 \cos \Phi)N + C_3 \quad (6)$$

with:

$$\nu = \text{true anomaly};$$

$$u = (\omega + \nu);$$

$$n = (\mu/\alpha^3)^{1/2};$$

$$p = a(1 - e^2);$$

$$r = \frac{p}{1 + e \cos \nu};$$

$$\Phi(\text{'flight path angle'}) = \frac{e \sin \nu}{1 + e \cos \nu};$$

$$h = rv \cos \Phi; \quad h = \sqrt{p\mu};$$

$$A = \frac{2e \sin \nu}{n(1-e^2)^{1/2}}, \quad B = \frac{2a(1-e^2)^{1/2}}{nr},$$

$$A_1 = \frac{(1-e^2)^{1/2} \sin \nu}{na}; \quad B_1 = \frac{\sqrt{1-e^2}}{na^2e} \left[\frac{a^2(1-e^2)}{r} - r \right];$$

$$A_2 = -\frac{(1-e^2)^{1/2}}{nae} \sin \nu; \quad B_2 = \frac{p}{eh} \left[\sin \nu \left(1 + \frac{1}{1+e \cos \nu} \right) \right]; \quad C_2 = \frac{r \cot i \sin u}{na^2 \sqrt{1-e^2}};$$

$$A_3 = -\frac{1}{na} \left(\frac{2r}{a} - \frac{1-e^2}{e} \cos \nu \right); \quad B_3 = -\frac{(1-e^2)}{nae} \left(1 + \frac{r}{a(1-e^2)} \sin \nu \right); \quad C_3 = -t \frac{dn}{dt}$$

We may use these relations to express small, yet finite, rather than infinitesimal, variations of the elements caused by finite duration burns producing small velocity variation Δv in the direction of T , N and W^g ; and so the above equations may be modified to give the change in any element of an elliptic orbit due to a small impulse Δv .

Writing $\Delta v_N = N\Delta t$, $\Delta v_T = T\Delta t$, $\Delta v_W = W \Delta t$ the equations become:

$$\Delta a = (A \sin \Phi + B \cos \Phi) \Delta v_T + (B \sin \Phi - A \cos \Phi) \Delta v_N \quad (7)$$

$$\Delta e = (A_1 \sin \Phi + B_1 \cos \Phi) \Delta v_T + (B_1 \sin \Phi - A_1 \cos \Phi) \Delta v_N \quad (8)$$

$$\Delta i = \frac{r \cos u}{na^2(1-e^2)^{1/2}} \Delta v_W \quad (9)$$

$$\Delta\Omega = \frac{r \sin u}{na^2(1-e^2)^{1/2} \sin i} \Delta v_W \quad (10)$$

$$\Delta\omega = (A_2 \sin \Phi + B_2 \cos \Phi) \Delta v_T + (B_2 \sin \Phi - A_2 \cos \Phi) \Delta v_N + C_2 \Delta v_W \quad (11)$$

$$\Delta Mo = (A_3 \sin \Phi + B_3 \cos \Phi) \Delta v_T + (B_3 \sin \Phi - A_3 \cos \Phi) \Delta v_N + C_3 \quad (12)$$

From here on T for Δv_T , N for Δv_N and W for Δv_W will be used. Through the above equations the variations Δa , Δe , $\Delta\omega$ and ΔMo caused by a $\Delta v = 0.01$ DU/TU (in the

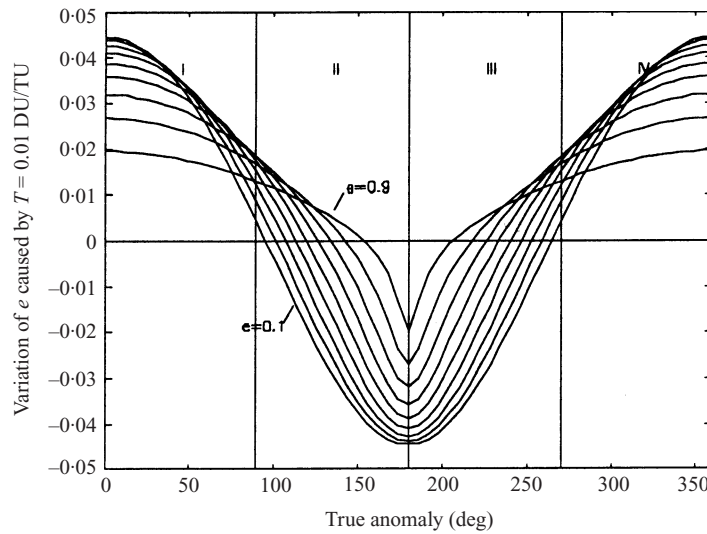


Figure 5. Variation Δe caused by a $T = 0.01$ DU/TU versus true anomaly, for $a = 5$ DU and nine values of e , from $e = 0.1$ to $e = 0.9$ with step of 0.1 .

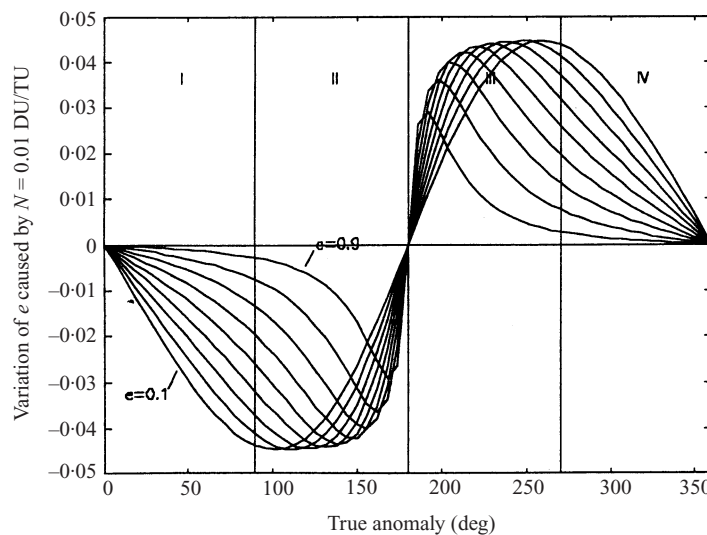


Figure 6. Variation Δe caused by a N thrust = 0.01 DU/TU versus ν , for $a = 5$ DU and nine values of e , from $e = 0.1$ to $e = 0.9$ with step of 0.1 .

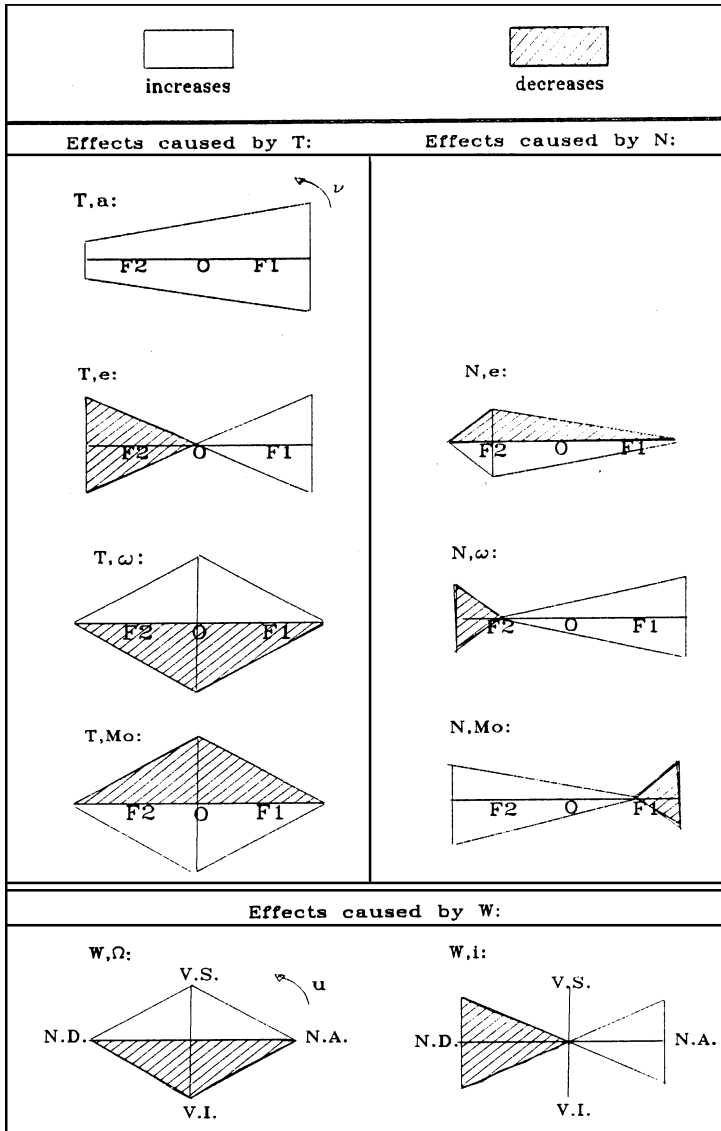


Figure 7. Schematic representation of the effects caused by $T, N, W > 0$.

direction of T and N respectively) for $a = 5$ DU and different values of e have been calculated and then plotted versus true anomaly ν . As to the variations Δi and $\Delta \Omega$ caused by a thrust in the direction of W , we fixed a value of variations Δi and $\Delta \Omega$ caused by a thrust in the direction of W , we fixed a value of $i = 30$ deg, $a = 5$ DU, $\omega = 90$ and 180 deg respectively and then plotted the variations versus u . Figures 5 and 6 show examples of these graphs, where Δe versus ν is represented.

By carefully analysing the aforesaid graphs it was possible to obtain the areas where a given effect is positive or negative and the points of the orbit where it is null or maximum, depending on the value of e .

These results were used to obtain the schematic graphics of Figure 7. In each of them, the x -axis represents the hypothetical orbit's major axis with the centre in O and the foci in $F1$ and $F2$. In each of the ordinate axes, the approximate effects of the forces on the parameters are represented when they are applied in the points of the elliptic contour represented by the abscissas on the x -axis; the upper part of the contour is represented above the x -axis and the lower one under the x -axis. In the white areas, the parameter considered increases by the action of the positive force and in the shaded areas it decreases. Only the points where a certain effect is null or maximum are shown and then joined up by straight lines. The null or maximum points are precisely indicated in the diagrams concerning the effects caused by T (on e , a and on ω) for every e , as are the points in the diagrams concerning the effects caused by N and by W for a slightly eccentric orbit; in the latter cases the maximum points are moderately displaced at higher eccentricity.

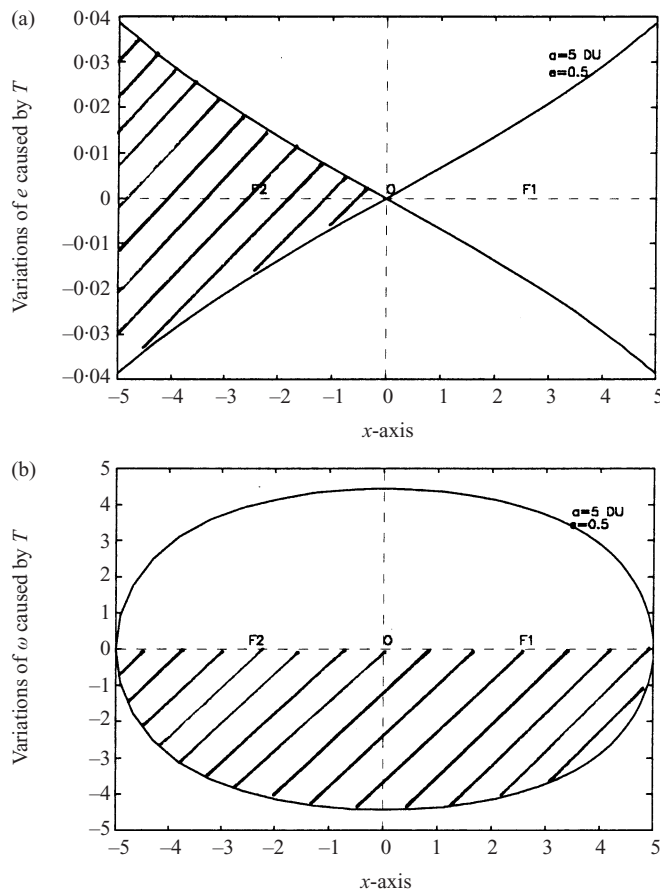


Figure 8. Examples of the real variations of e and ω respectively according to the convention described above, caused by a $\Delta v = 0.01$ DU/TU in the direction of T , for $a = 5$ DU and $e = 0.5$. The abscissas were calculated by means of the relation $x = r \cos \nu + c$ with $r =$ radius vector and $c = ea$ which is the abscissa of the first focus; thus the x -axis actually represents the hypothetical orbit's major axis. As before, in the white areas there is an increase of the elements and a decrease in the shaded ones.

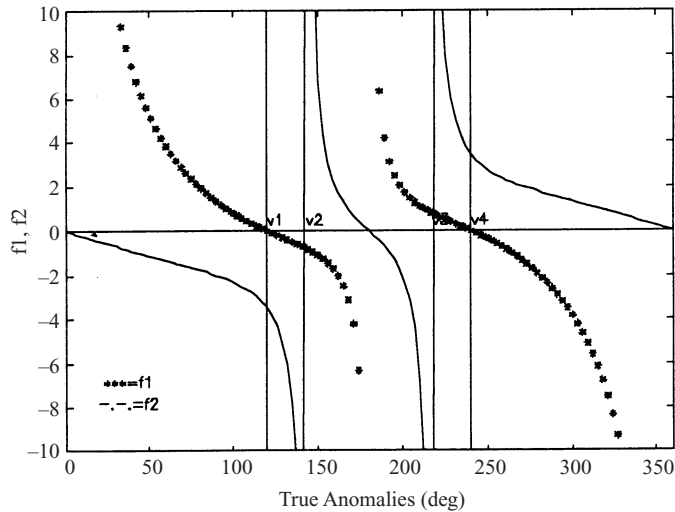


Figure 9. Ratios f_1 and f_2 versus ν , for $a = 5$ DU and $e = 0.5$.

The attracting mass is at F1, so above the x -axis the effects for true anomalies ν from 0° to 180° are represented and below it, those for ν from 180° to 360° . In the case of W , the x -axis represents the line of the nodes and the y -axis represents the line of the vertices.

Obviously these representations do not exactly express the variations in the y -axis, which are \sin/\cos type oscillations and depend on a and e . However, the symmetries of the diagrams in Figure 7 for the T thrust are also present in the actual diagrams (Figure 8).

4. discussion. The qualitative and quantitative analysis provide the following conclusions concerning certain combined effects between the variations Δa , $\Delta\omega$, Δe and ΔMo in a single time action:

4.1. Δa and not (Δe and $\Delta\omega$). It is not possible to vary a and completely avoid both of the undesired effects by balancing the N and T thrusts in the suitable areas of Figure 3 or Figure 4.

Let us consider Figure 9, where f_1 is the ratio N/T for which the variation Δe caused by T cancels the one due to N ; f_2 is the same ratio for $\Delta\omega$. Lines f_1 and f_2 do not intersect and therefore we cannot obtain any ν value in which both of the undesired effects can be cancelled by a balanced action of T and N . A reduction of the undesired effects by the action of both T and N can nevertheless be obtained, but only from $\nu_1 = \cos^{-1}(-e)$ to $\nu_2 = \tan^{-1}(e^2 - 1)/2e$ and from $\nu_3 = \tan^{-1}(1 - e^2)/2e$ to $\nu_4 = \cos^{-1}(-e)$, corresponding to the arcs CK and LD of Figure 4 as we expected.

Figure 7 provides us with some immediate operational insight: for instance, a desired Δa can be obtained by the smallest T value at the perigee; here, no undesired $\Delta\omega$ is produced by T and furthermore the undesired Δe is minimal due to the low value of T .

4.2. Δa and not $\Delta\omega$ – Δa and not Δe . Figure 7 clearly indicates that by applying a T at the perigee, a high Δa can be obtained with no $\Delta\omega$, and by applying a T at the ends of the minor axis, a Δa can be obtained with no Δe .

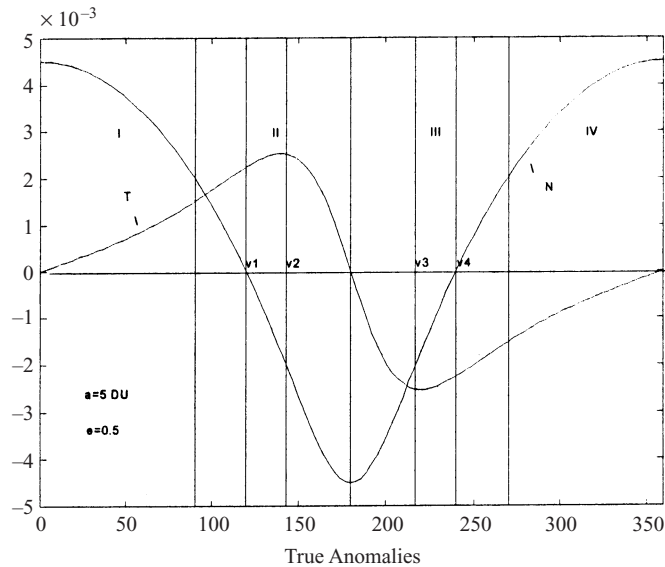


Figure 10. Thrusts T and N , in terms of ν , to obtain $\Delta\omega = 1^\circ$ and $\Delta e = 0$ ($a = 5$ DU and $e = 0.5$).

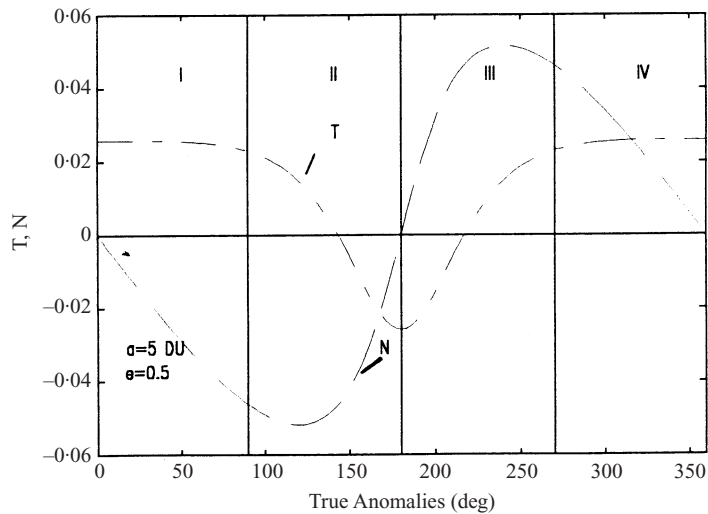


Figure 11. Thrusts T and N , in terms of ν , to obtain $\Delta e = 0.1$ and $\Delta\omega = 0^\circ$, with $a = 5$ DU and $e = 0.5$.

4.3. Δe and not ($\Delta\omega$ and Δa) – Δe and not Δa . An impulse N does not produce variations of a . Thus, when such an impulse is applied to the points where the highest Δe and no $\Delta\omega$ are produced, i.e. at the ends of the second focus (see Figure 7), the desired manoeuvres may be performed.

4.4. ΔMo and not (Δa and Δe) – ΔMo and not Δa . As in the above case, an impulse N at the apogee should be applied.

4.5. $\Delta\omega$ and not Δe . By solving the system formed by equations (8) and (11) with $\Delta\omega$ equal to the desired value and $\Delta e = 0$, we can obtain the diagram illustrating the suitable thrusts N and T versus ν . As can be seen in Figure 10, the best solution is to apply only one of the two thrusts N or T in the points ν in the graph where the other thrust is represented as null. For optimal energy-saving purposes it is better to apply T at the points of true anomaly $\nu_1 = \nu_4 = \cos^{-1}(-e)$, i.e. at the ends of the minor axis, rather than N at the perigee or apogee.

4.6. Δe and not $\Delta\omega - \Delta Mo$ and not Δe . By the same process as before, it proves convenient to use only one impulse for Δe and not $\Delta\omega$, i.e. a tangential thrust at the apogee or perigee (see Figure 11); for ΔMo and not Δe , a thrust T may be used at the points of anomalies ν_1 and $\nu_2 = \cos^{-1}(-e)$, i.e. at the ends of the minor axis (Figure 12).

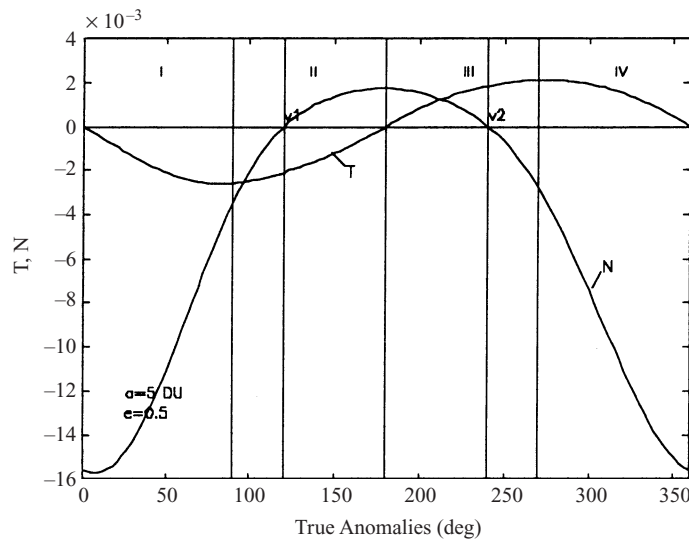


Figure 12. Thrusts T and N , versus ν , to obtain $\Delta Mo = 1^\circ$ and $\Delta e = 0$, ($a = 5$ DU and $e = 0.5$).

One interesting outcome of the above analysis (4.1–4.6) is that, in order to obtain a desired effect and at the same time minimize the undesired ones, it is better to use only a thrust T or only a thrust N rather than a combination of T and N . Therefore, the orientations of T and N are the best ones for the thrusters in the in-plane manoeuvres. Concerning the out-of-plane effects the situation is quite clear (Figure 7): in order to obtain a Δi , a W thrust should be given at a node, or at both nodes in the opposite direction, and a similar action by W should be performed at the vertices of the orbit in order to obtain a $\Delta\Omega$.

Figure 13 shows the synthesis of the suggested thrusts to obtain the combined effects between Δa , $\Delta\omega$, Δe and ΔMo .

In the vertex $\Delta\omega$ we can also read ΔMo ; it is not necessary to make any changes to the force T and, as far as force N is concerned, 'N at the apogee or perigee' must be replaced by 'N at the apogee'.

As far as the out-of-plane effects are concerned, Figure 7 may be of further help: for example, if a variation Δi without a variation $\Delta\Omega$ is required, it is obvious that the

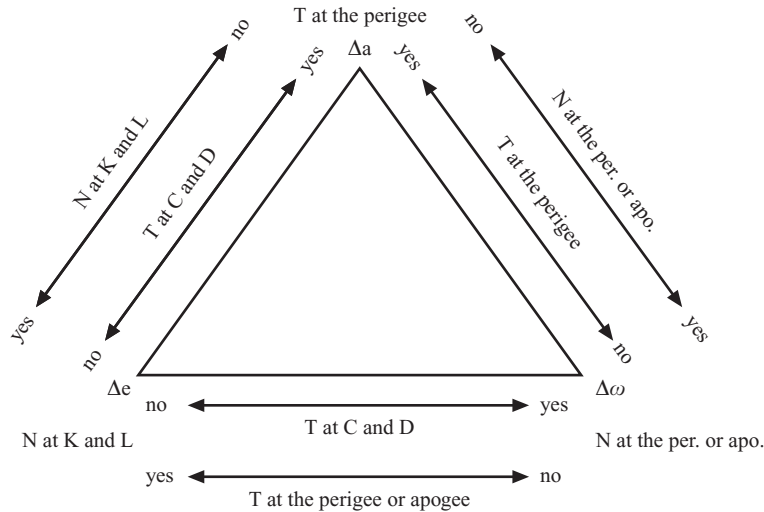


Figure 13. Diagram illustrating which thrust and orbital position is the best according to the desired manoeuvre, giving a synthesis of the suggested thrusts in order to obtain some certain combined effects between $\Delta\omega$, Δa and Δe . It indicates the best solutions and also takes into account the need to save energy, i.e. to give the lowest thrust. At each vertex the best action to obtain the indicated effect with the smallest variations of the other two parameters is indicated; at the sides of the triangle the action to obtain the effect at the 'yes' end of the arrow is shown, thus avoiding the effect at the 'no' end. Points C, D, K and L are the same as those shown in Figure 7. All of the above concerns the so-called 'single time manoeuvres'.

best solution is to apply an orthogonal component W at nodes ($u = 0^\circ, 180^\circ$), where its effect on i is maximum and that on Ω is null.

5. conclusions. A manoeuvre may be accomplished by means of suitable variations of the orbital parameters. It may not be possible to choose the point or the points where the thrusts are to be applied, i.e. one has to manoeuvre 'here and now', generally by means of low thrusts. This may happen, for instance in the final phase of a rendezvous, at the apogee of the current orbit; it is then necessary to know which thrusts offer the best solution. In these cases a single impulse T or N may be more suitable than two simultaneous T and N impulses, as discussed before. The diagrams presented in Figures 7 and 13 can give rapid indications of the most suitable kind of thrust necessary according to the point of action when the manoeuvre must be performed at a single time.

When we consider the techniques of multiple thrusting around the orbit, Figure 7 can still give a quick insight into the manoeuvre philosophy, while Figure 13 is no longer helpful. The reason is that, as shown in Figure 7, a T thrust is more effective than an N one, therefore only the former will be used to obtain the desired effect – when it is applied for the first time – and cancel the undesired effects, when it is applied the second time. This can also lead to a strengthening of the desired effect.

The following manoeuvres can be used as examples:

(i) *To obtain a Δa without altering the other parameters.* From Figure 7, it can be seen that two T thrusts at the ends of the major axis can give rise to two Δa 's which add together; they do not produce a $\Delta\omega$ and, if they are balanced according to

equation (8), they produce a couple of counterbalancing Δe 's. This is the classical Hohman transfer.

Two T thrusts at the ends of the minor axis can produce a final Δa , no Δe , and, if they are balanced according to equation (11), they produce a couple of counterbalancing $\Delta \omega$'s.

In practice, for small impulses and variations of the parameters, two equal T thrusts applied at 180° from each other, can give rise to two Δa 's which add together as well as couples of counter-balancing Δe 's, $\Delta \omega$'s, ΔMo 's (as we said, the symmetries of the schematic diagrams in Figure 7 for the T thrust are also present in the corresponding actual Figures).

(ii) *To change everything except a in the plane parameters.* Two T burns having opposite signs, at opposite anomalies in the actual orbits, can produce these effects, as shown in Figure 7. The intensity of the T forces are to be balanced according to the equation (7).

(iii) *To let an orbit spin around its active focus, without changing its final shape and size.* The optimal way involves giving an initial T burn at one end of the minor axis and a second T burn, which is opposite in sign, at the second end of the minor axis of the intermediate transfer ellipse. The two burns should produce two Δa 's which cancel each other out, so they should be balanced in intensity according to equation (7).

(iv) *If only e must be varied*, i.e. to obtain a highly eccentric orbit for communication purposes in the area under the apogee, two opposite impulses at the perigee and apogee may be applied. Their ratio can be easily established by means of equation (7).

In the case of a circular orbit, the point of the first impulse becomes the new orbit perigee, which is elliptic. The second impulse given at the apogee will produce a new increase of eccentricity and cancel Δa , and the perigee will approach the Earth. In this way it is possible to obtain orbits which are increasingly more eccentric whose perigees are increasingly nearer the Earth.

As stated above, in order to vary only e , a number of T thrusts at the perigee alternated by an equal number of thrusts at the apogee should be used: each apogee burn must compensate the Δa produced by the preceding perigee burn.

In this way the orbital period T is restored each $T/2$ time, but the time of perigee transit will prove different at each transit. In the case of a geostationary orbit a shift in longitude of the sub-satellite point will be experienced¹¹; in order to avoid this effect, an over-compensation is necessary at the apogee, by a Δv which is twice the perigee Δv one and opposite in sign. This excess in Δv will be compensated yet again by a new perigee thrust which is equal to the preceding perigee one.

Until now we have dealt with impulsive thrusts. If the thrusters are weak it may be necessary to perform thrusts which are prolonged in time. Their effects may be worked out by integrating equations from (7) to (12). It is clear from Figure 7 that, generally speaking, there is a loss of efficiency for each given Δv global value. As to the variations of the orbit inclination i the effect was shown^{5,11} to be the same as that given by an impulsive thrust of equal Δv performed at the centre of the thrust arc Δs , except that its size is reduced by the factor $2 \sin (0.5 \Delta s) / \Delta s$.

For the reasons stated above, a higher number of impulsive thrusts performed in the best points in successive orbital periods is preferable to a single prolonged thrust, where feasible. It was nevertheless shown that in the case of circular orbits, the change in semi-major axis (by a T thrust) depends exclusively on the total Δv , regardless of the thrust length^{5,11}.

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key words

1. Satellites.
2. Space.
3. Command & Control