J. Plasma Physics (2004), *vol.* 70, *part* 3, *pp.* 317–322. © 2004 Cambridge University Press 317
DOI: 10.1017/S0022377803002678 Printed in the United Kingdom

Ion–dust two-stream instability in a collisional magnetized dusty plasma

M. ROSENBERG[1] and P. K. SHUKLA[2]

[1]Department of Electrical and Computer Engineering, University of California San Diego, La Jolla, CA 92093, USA*

[2]Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 1 September 2003)

Abstract. The effect of an external magnetic field on an ion–dust two-stream instability in a collisional dusty plasma is investigated. We consider the parameter regime in which the ions and electrons are strongly magnetized, while the charged dust is unmagnetized. The presence of a magnetic field tends to hinder the growth of the instability. The implications for laboratory dusty plasmas such as 'plasma crystals' are discussed.

Ion–dust two-stream instabilities in collisional dusty plasmas have been investigated by a number of authors [1–6], with applications to laboratory dusty plasmas. For example, clouds of negatively charged dust can be trapped in the plasma–sheath interface regions where ions flow toward the electrode with a speed much larger than the ion thermal speed. Examples include dusty regions in processing plasmas and dust 'plasma crystal' experiments, where the streaming of ions relative to the dust may excite dust–acoustic type waves [1, 7]. Recently, it has been suggested that a dissipative ion–dust two-stream instability may be responsible for dust heating in dusty plasma crystallization experiments [7]. Further, it was suggested that suppression of the instability may be the trigger for a fluid-to-solid phase transition of the dust cloud (i.e. condensation to the 'plasma crystal') [7, 8].

Since future dusty plasma experiments, in particular 'plasma crystal' experiments, may involve magnetic fields [9], it is of interest to consider the effects of a magnetic field on this dissipative ion–dust two-stream instability. In this brief communication, we study this issue in the parameter regime in which the ions and electrons are strongly magnetized, while the charged dust is unmagnetized.

We consider a weakly ionized dusty plasma composed of electrons, singly charged ions, negatively charged dust grains and neutrals. The equilibrium charge neutrality condition is given by

$$n_{\mathrm{i}0} = n_{\mathrm{e}0} + Z_{\mathrm{d}} n_{\mathrm{d}0}, \tag{1}$$

where n_{j0} is the unperturbed number density of the particle species j (here the subscript $j = \mathrm{e}, \mathrm{i}$ and d refers to electrons, ions and dust, respectively), and Z_{d} is the dust charge state. There is an electric field $\mathbf{E}_0 = E_0\hat{\mathbf{z}}$ and a magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, in the $\hat{\mathbf{z}}$ direction.

* Current address: Dept. of Physics, University of San Diego, San Diego, CA 92110.

We consider electrostatic waves, with perturbed $\tilde{\mathbf{E}}(\mathbf{r},t) = -\nabla\phi(\mathbf{r},t)$, where ϕ is the wave potential. Perturbed quantities $\alpha(\mathbf{r},t)$ are assumed to vary as $\sim \alpha(x)$ $\exp(-i\omega t + ik_z z + ik_y y)$. The behavior of each of the three charged species is described by fluid equations, including the equation of continuity

$$\partial_t n_j + \nabla\cdot(n_j\mathbf{v}_j) = 0, \tag{2}$$

and the equation of motion

$$n_j m_j(\partial_t \mathbf{v}_j + \mathbf{v}_j\cdot\nabla\mathbf{v}_j) = n_j q_j\left(\mathbf{E} + \frac{\mathbf{v}_j}{c}\times\mathbf{B}_0\right) - \nabla P_j - n_j m_j \nu_j \mathbf{v}_j. \tag{3}$$

Here n_j, $\mathbf{v}_j$, m_j, q_j, and ν_j refer to the number density, velocity, mass, charge and neutral-charged particle collision frequency of species j. In the presence of an external electric field, the equilibrium flow velocity of the charged species is given by

$$\mathbf{V}_{0j} = \frac{q_j E_0}{m_j \nu_j}\mathbf{z}. \tag{4}$$

It is assumed that the plasma parameters are such that $V_{0\mathrm{e}} \ll v_{\mathrm{te}}$ and $V_{0\mathrm{d}} \ll v_{\mathrm{td}}$, while $V_{0\mathrm{i}} \gg v_{\mathrm{ti}}$, where $v_{\mathrm{t}j} = (T_j/m_j)^{1/2}$ is the thermal speed of species j with temperature T_j. In the following, we consider the frame in which the dust is stationary, namely $V_{0\mathrm{d}} = 0$.

We consider the case where the ions and electrons are strongly magnetized, while the dust grains are unmagnetized. The ions would be magnetized when $\Omega_{\mathrm{i}}/\nu_{\mathrm{i}} \gg 1$ and $v_{\mathrm{ti}}/\Omega_{\mathrm{i}} \ll R$, where $\Omega_{\mathrm{i}} = eB/m_{\mathrm{i}}c$ is the ion gyrofrequency and R is the radial extent of the dust cloud. There is a similar condition for the electrons. For example, these conditions could be satisfied in a reasonably sized dust cloud (e.g. of the order of $R \gtrsim 1$ cm), composed of a weakly ionized dusty Ar plasma with $T_{\mathrm{e}} \sim 3$ eV, $T_{\mathrm{i}} \sim 0.1$ eV and neutral density $n_{\mathrm{n}} \sim 10^{15}$ cm^{-3} (pressure ~ 0.03 torr at room temperature), immersed in a magnetic field of strength $B \sim 2$ T. On the other hand, dust of radius $a \sim 5$ μm would be unmagnetized, since ν_{d} would be much greater than the dust gyrofrequency, using $Z_{\mathrm{d}} \sim 10^4$ and $m_{\mathrm{d}} \sim 5\times 10^{14}$ times the proton mass.

We linearize (2) and (3), denoting the perturbed density by $\tilde{n}_j$ and the perturbed velocity by $\tilde{v}_j$. For the ions, we have

$$\bar{\omega}_{\mathrm{i}}\tilde{n}_{\mathrm{i}} - n_{\mathrm{i}0}[k_z\tilde{v}_{\mathrm{i}z} + k_y\tilde{v}_{\mathrm{i}y}] = 0, \tag{5}$$

$$\left[(\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}}) - \frac{\Omega_{\mathrm{i}}^2}{(\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}})}\right]\tilde{v}_{\mathrm{i}y} = \frac{k_y e\phi}{m_{\mathrm{i}}} + \frac{k_y T_{\mathrm{i}}\tilde{n}_{\mathrm{i}}}{m_{\mathrm{i}} n_{\mathrm{i}0}}, \tag{6a}$$

and

$$(\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}})\tilde{v}_{\mathrm{i}z} = \frac{k_z T_{\mathrm{i}}\tilde{n}_{\mathrm{i}}}{m_{\mathrm{i}} n_{\mathrm{i}0}} + \frac{k_z e\phi}{m_{\mathrm{i}}}. \tag{6b}$$

Here $\bar{\omega}_{\mathrm{i}} = \omega - k_z V_{0\mathrm{i}}$. In the limit of strongly magnetized ions with $|\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}}| \ll \Omega_{\mathrm{i}}$ and $(\omega_{\mathrm{pi}}/\Omega_{\mathrm{i}})^2 \ll 1$, and in the limit of cold ions with $|\bar{\omega}_{\mathrm{i}}(\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}})| \gg k^2 v_{\mathrm{ti}}^2$ (and $k_z \gtrsim k_y$), the perturbed ion number density from (5)–(6) becomes

$$\frac{\tilde{n}_{\mathrm{i}}}{n_{\mathrm{i}0}} \approx \frac{e}{m_{\mathrm{i}}}\frac{k_z^2\phi}{\bar{\omega}_{\mathrm{i}}(\bar{\omega}_{\mathrm{i}} + i\nu_{\mathrm{i}})}. \tag{7}$$

For the electrons, we assume $|\bar{\omega}_{\mathrm{e}}| = |\omega + k_z V_{0\mathrm{e}}| \ll \nu_{\mathrm{e}}$, so (2) and (3) for the electrons become

$$\bar{\omega}_{\mathrm{e}}\tilde{n}_{\mathrm{e}} - n_{\mathrm{e}0}[k_z\tilde{v}_{\mathrm{e}z} + k_y\tilde{v}_{\mathrm{e}y}] = 0, \tag{8}$$

$$\left[1+\frac{\Omega_{\mathrm{e}}^2}{\nu_{\mathrm{e}}^2}\right]\tilde{v}_{\mathrm{e}y}\approx\frac{ik_ye\phi}{m_{\mathrm{e}}\nu_{\mathrm{e}}}-\frac{ik_yT_{\mathrm{e}}}{m_{\mathrm{e}}\nu_{\mathrm{e}}}\frac{\tilde{n}_{\mathrm{e}}}{n_{\mathrm{e}0}}, \tag{9a}$$

and

$$\tilde{v}_{\mathrm{e}z}\approx\frac{ik_ze\phi}{m_{\mathrm{e}}\nu_{\mathrm{e}}}-\frac{ik_zT_{\mathrm{e}}}{m_{\mathrm{e}}\nu_{\mathrm{e}}}\frac{\tilde{n}_{\mathrm{e}}}{n_{\mathrm{e}0}}, \tag{9b}$$

where $\Omega_{\mathrm{e}}=eB_0/m_{\mathrm{e}}c$ is the electron gyrofrequency. Since $\nu_{\mathrm{e}}\ll\Omega_{\mathrm{e}}$ and in the limit of warm electrons with $k_z^2v_{\mathrm{te}}^2\gg|\bar{\omega}_{\mathrm{e}}\nu_{\mathrm{e}}|$, the perturbed electron number density from (8)–(9) becomes

$$\frac{\tilde{n}_{\mathrm{e}}}{n_{\mathrm{e}0}}\approx\frac{e\phi}{T_{\mathrm{e}}}. \tag{10}$$

The dynamics of the unmagnetized dust (also assuming cold dust with thermal energy $T_{\mathrm{d}}=0$) is described by

$$\omega(\omega+i\nu_{\mathrm{d}})\frac{\tilde{n}_{\mathrm{d}}}{n_{\mathrm{d}0}}-\frac{Z_{\mathrm{d}}e}{m_{\mathrm{d}}}\nabla^2\phi\approx 0. \tag{11}$$

Inserting (7) and (10)–(11) into Poisson's equation, we obtain the dispersion relation

$$1-\frac{k_z^2}{k^2}\frac{\omega_{\mathrm{pi}}^2}{A_{\mathrm{e}}\bar{\omega}_{\mathrm{i}}(\bar{\omega}_{\mathrm{i}}+i\nu_{\mathrm{i}})}-\frac{\omega_{\mathrm{pd}}^2}{A_{\mathrm{e}}\omega(\omega+i\nu_{\mathrm{d}})}\approx 0, \tag{12}$$

where A_{e} is given by

$$A_{\mathrm{e}}=1+\frac{1}{k^2\lambda_{\mathrm{De}}^2}. \tag{13}$$

Note that (12) has the same form as the dispersion relation for an ion–dust two-stream instability in a collisional, non-magnetized dusty plasma given in [1, 7], apart from the factor $(k_z/k)^2$ multiplying ω_{pi}^2. However, it is this factor arising from strongly magnetized ions that hinders the growth of the instability. (A similar effect occurs for the Buneman instability in an electron–ion plasma when the electrons are strongly magnetized [10].)

In the limit $\omega\ll k_zV_{0\mathrm{i}}$, $\nu_{\mathrm{i}}\ll k_zV_{0\mathrm{i}}$ and $\nu_{\mathrm{d}}\ll\omega$, the dispersion relation (12) becomes approximately

$$1-\frac{\omega_{\mathrm{pi}}^2}{A_{\mathrm{e}}k^2V_{0\mathrm{i}}^2}\left[1+\frac{2\omega}{k_zV_{0\mathrm{i}}}+\frac{i\nu_{\mathrm{i}}}{k_zV_{0\mathrm{i}}}\right]-\frac{\omega_{\mathrm{pd}}^2}{A_{\mathrm{e}}\omega^2}\left[1-\frac{i\nu_{\mathrm{d}}}{\omega}\right]\approx 0. \tag{14}$$

A strong hydrodynamic instability can occur roughly when $\omega_{\mathrm{pi}}/\sqrt{A_{\mathrm{e}}}\geqslant kV_{0\mathrm{i}}$, which can be written as

$$\frac{v_{\mathrm{ti}}^2}{V_{0\mathrm{i}}^2}-\left(\frac{T_{\mathrm{i}}}{\delta T_{\mathrm{e}}}\right)\geqslant k^2\lambda_{\mathrm{Di}}^2, \tag{15}$$

where $\delta=n_{\mathrm{i}0}/n_{\mathrm{e}0}$. The maximum growth rate of this ion–dust dissipative instability, in the limit $\nu_{\mathrm{i}}\gg\omega$, occurs when the equality in (15) approximately holds and the spectrum is given by

$$\frac{\omega}{\omega_{\mathrm{pd}}}\sim\frac{(1+i)}{\sqrt{2}}\left(\frac{k_z}{k}\frac{\omega_{\mathrm{pi}}}{\nu_{\mathrm{i}}}\right)^{1/2}\frac{1}{A_{\mathrm{e}}^{3/4}}-\frac{i\nu_{\mathrm{d}}}{2\omega_{\mathrm{pd}}}. \tag{16}$$

Condition (15) is more restrictive than the condition for the dissipative instability in the non-magnetized case, which is roughly [1]

$$\frac{v_{\mathrm{ti}}^2}{V_{0\mathrm{i}}^2}-\frac{k_z^2}{k^2}\frac{T_{\mathrm{i}}}{\delta T_{\mathrm{e}}}\geqslant k_z^2\lambda_{\mathrm{Di}}^2. \tag{17}$$

https://doi.org/10.1017/S0022377803002678 Published online by Cambridge University Press

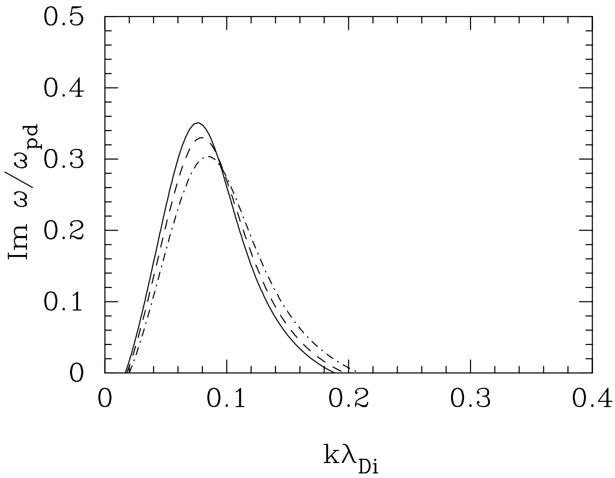


Figure 1. Imaginary part of ω (normalized to ω_{pd}) obtained by solving (12). The parameters are $m_{\mathrm{i}}/m_{\mathrm{p}} = 40$, $m_{\mathrm{d}}/m_{\mathrm{p}} = 5 \times 10^{14}$, $T_{\mathrm{e}}/T_{\mathrm{i}} = T_{\mathrm{e}}/T_{\mathrm{d}} = 30$, $\omega_{\mathrm{pi}}/\Omega_{\mathrm{i}} = 0.4$, $Z_{\mathrm{d}} = 10^4$, $n_{\mathrm{d}}/n_{\mathrm{i}} = 10^{-5}$, $\nu_{\mathrm{i}}/\omega_{\mathrm{pi}} = 0.1$, $\nu_{\mathrm{d}}/\omega_{\mathrm{pd}} = 0.1$ and $V_{0\mathrm{i}}/v_{\mathrm{ti}} = 5.5$. Angle between $\mathbf{k}$ and $\mathbf{V}_{0\mathrm{i}}$ is $\theta = 0°$ (solid line), $\theta = 30°$ (dash line) and $\theta = 45°$ (dash-dot line).

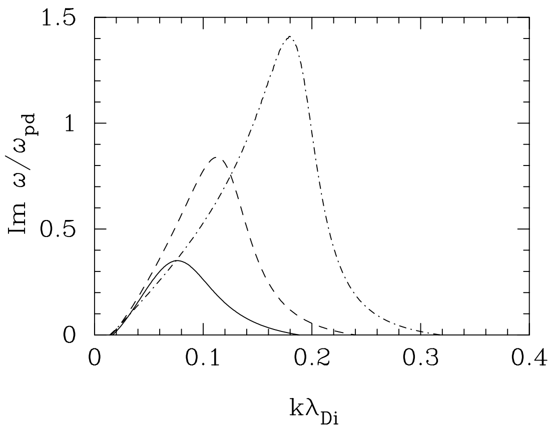
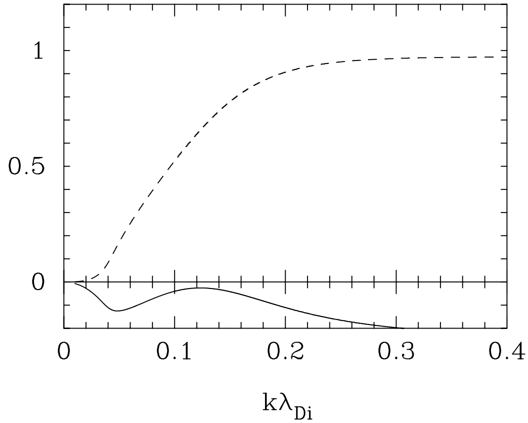
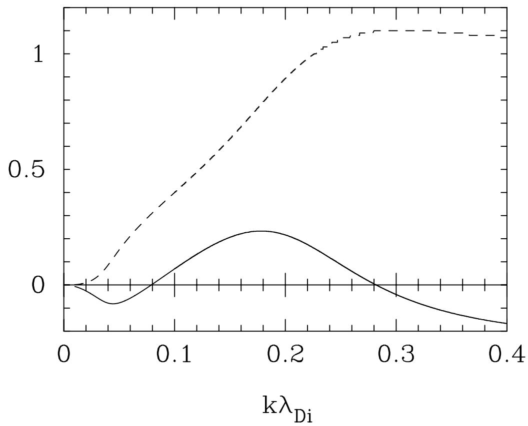


Figure 2. Imaginary part of ω (normalized to ω_{pd}) for the non-magnetized case. Other parameters are the same as in Fig. 1.

The range of k where the instability occurs given by (15) is narrower than the range given by (17) for waves that propagate obliquely to the ion flow direction (see also [10]). Consider for example a dusty 'plasma crystal' radiofrequency experiment where dust is localized in the plasma–sheath boundary region and ions flow into the sheath with speed near the ion sound speed $c_{\mathrm{s}} \approx (T_{\mathrm{e}}/m_{\mathrm{i}})^{1/2} \gg v_{\mathrm{ti}}$ (since typically $T_{\mathrm{e}} \gg T_{\mathrm{i}}$). If $\delta \approx 1$ in the dust cloud, it would be difficult to satisfy condition (15) and thus the instability could be hindered if the ions were strongly magnetized. This is in contrast to the non-magnetized case where the instability could still occur for waves that propagate obliquely to the ion flow direction and where damping of the instability requires high ion and dust collision frequencies and thus high neutral density [7]. Thus we suggest that, if suppression of an ion–dust streaming



(a)



(b)

Figure 3. Real (dash line) and imaginary (solid line) parts of $\omega/\omega_{\rm pd}$. (a) Solution of (12), for the same parameters as in Fig. 1, but with $\nu_{\rm i}/\omega_{\rm pi} = 0.5$, $\nu_{\rm d}/\omega_{\rm pd} = 0.5$ and $\theta = 0°$. (b) Solution for non-magnetized case, for the same parameters as in (a), but with $\theta = 45°$.

instability is the trigger for dusty plasma condensation, this could more easily be accomplished in a plasma with strongly magnetized ions, even at lower neutral pressures.

For example, consider a dusty plasma with the following parameters: Argon plasma, $B \sim 2$ T, $n_{\rm n} \sim 10^{15}$ cm^{-3}, $n_{\rm i0} \sim 10^8$ cm^{-3}, $T_{\rm e} \sim 3$ eV, $T_{\rm i} \sim 0.1$ eV; dust with $a \sim 5$ μm, $m_{\rm d}/m_{\rm p} \sim 5 \times 10^{14}$, $n_{\rm d0} \sim 10^3$ cm^{-3} (intergrain spacing $d \sim 0.06$ cm) and $Z_{\rm d} \sim 10^4$. The dust could be strongly coupled and may be crystallized, with the dust Coulomb coupling parameter $\Gamma = Z_{\rm d}^2 e^2/dk_{\rm B}T_{\rm d} \sim 2400$, if $T_{\rm d} \sim T_{\rm i}$. We estimate the collision frequencies using $\nu_{\rm i} \sim \sigma_{\rm in} n_{\rm n} v_{\rm ti}$ (with $\sigma_{\rm in} \sim 5 \times 10^{-15}$ cm^2) and $\nu_{\rm d} \sim 4\pi a^2 n_{\rm n} v_{\rm tn} m_{\rm n}/3m_{\rm d}$. For these parameters, $\nu_{\rm i}/\Omega_{\rm i} \sim 0.05$, $\omega_{\rm pi}/\Omega_{\rm i} \sim 0.4$, $\nu_{\rm d}/\Omega_{\rm d} \sim 400$ and $\omega_{\rm pd}/\Omega_{\rm d} \sim 3700$, so the ions are magnetized but the dust is non-magnetized. Figure 1 shows solutions of (12) for a set of corresponding dimensionless parameters, with $V_{0\rm i} \sim (T_{\rm e}/T_{\rm i})^{1/2} v_{\rm ti}$, for various values of the angle θ between $\mathbf{k}$ and $\mathbf{V}_{0\rm i}$ (note that $k_z = k\cos\theta$). For comparison, in Fig. 2 we show solutions of (12) with the factor k_z^2/k^2 in the second term of (12) replaced by 1, which would correspond to the case where the ions are not magnetized (see [1]). As can be seen, strong hydrodynamic growth can occur in the non-magnetized case over a wider range of k for oblique propagation, whereas the growth rate and range of unstable k remain smaller for the strongly magnetized case. For the parameters of Fig. 1, modes with wavelengths $\lambda < 10\pi\lambda_{\rm Di} \sim 0.75$ cm would be stable.

Figure 3(a) shows the real and imaginary parts of ω (normalized to $\omega_{\rm pd}$) obtained from (12) for the same parameters as in Fig. 1 and for $\theta = 0°$, but with the collision frequencies increased by a factor of five corresponding to increasing $n_{\rm n}$. (Note that B would also have to increase in order that $\nu_{\rm i}/\Omega_{\rm i} < 1$). As can be seen, there is no growth in the magnetized case. On the other hand, Fig. 3(b) shows solutions of (12) with the factor k_z^2/k^2 in the second term replaced by 1 (corresponding to the non-magnetized case), for the same parameters as in Fig. 3(a) but with $\theta = 45°$. Thus for these parameters, the dissipative ion–dust streaming instability can occur in the non-magnetized case, but is suppressed in the case that the ions are strongly magnetized.

In summary, we have presented linear dispersion relations for an ion–dust dissipative instability in a collisional, magnetized dusty plasma, in the case where the electrons and ions are strongly magnetized while the dust is unmagnetized. The condition for growth of the instability is more restrictive than in the non-magnetized case, which may have implications for laboratory dusty plasmas such as 'plasma crystals'. However, we point out that we have only considered simple wave effects and that further work should consider the complications introduced by a magnetic field such as ion $\mathbf{E} \times \mathbf{B}_0$ drifts due to the electric field of the dust grains.

Acknowledgements

The work of MR was partly supported by DOE grant no. DE-FG03-97ER54444 and DE-FG02-03ER54730 and by LANL/IGPP grant no. 02-1217R.

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