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Pair production via crossed lasers

H.M. FRIED
Physics Department, Brown University, Providence, RI 02912, USA
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Abstract

The intrinsically nonperturbative "vacuum persistence probability" P for $e+e-$ production in the overlap region of a pair of high-intensity lasers is estimated in the context of three models, each of which adapt and simplify the exact Fradkin representation for the logarithm of the fermion determinant in the fields of the crossed lasers. In each case, one finds for P an expression resembling Schwinger's 1951 result for the probability of pair production in a constant electric field, proportional to an exponential factor which contains an essential singularity and hence does not admit a perturbative expansion about zero coupling. Qualitative estimates of these models suggest that realistic yields for this form of $e+e-$ production must await lasers of intensity 10^{29} W/m^2. The possibility of producing a quark–antiquark pair in this way is noted, in particular, with temporary, but large separations of the $q\bar{q}$.

Keywords: Crossed lasers; Lepton pair production

1. INTRODUCTION

Consider Schwinger's (1951) exact result for the vacuum persistence probability P when a constant electric field can act as the source of energy for the production of a lepton pair,

$$P = \sum_{n=1}^{\infty} C_n e^{-n\pi m^2/e\mathcal{E}},$$

each term of which has an essential singularity at zero coupling. Here, any perturbative approximation of the exact (operator or functional) representation of P must always give an exact result of zero; this quantity is intrinsically nonperturbative.

This presentation deals with a new, intrinsically, nonperturbative process of some experimental interest: an estimation of the order of magnitude for the production of lepton pairs in the overlap region of two crossed or intersecting high-intensity lasers (Fried *et al.*, 2001). One finds generalizations of the Schwinger essential singularity, which is the feature that controls the order of magnitude of the results; only the latter are of interest here.

It is well known that a single laser of arbitrary intensity cannot extract an $e+e-$ pair from the vacuum, since $nk_\mu \neq p_\mu + p'_\mu$, if all three momenta are on their mass shells $(k^2 = 0, p = p'^2 = -m^2)$. But this objection is removed in the overlapping region of two intersecting lasers, where $n_1 k_\mu^{(1)} + n_2 k_\mu^{(2)} = p_\mu + p'_\mu$ can be satisfied for a variety of integers $n_{1,2}$. The relevant experimental question here is the probability, the rate of this process; and for this we consider the simplest geometry, where the polarization or electric-field vectors of both laser beams are the same, but the propagation directions of the two beams are perpendicular. For simplicity, we assume that both beams have the same frequency, and that the cross-sectional area of each beam is $\sim D^2$, so that the overlap region of the beams is $\sim D^3$. If $\langle 0|S|0\rangle = \exp[-(\Gamma/2)t + i\phi]$ represents the vacuum-to-vacuum amplitude, the vacuum persistence probability is $P = \exp[-\Gamma t]$, where the lasers are turned on at $t = 0$. The probability of producing one or more pairs is then $P_1 = 1 - e^{-\Gamma t} \sim \Gamma t$, for $\Gamma t \ll 1$. We assume an "ideal" laser, with pulse duration $\tau_0 \simeq 10^{-13}$ s, a flux intensity $F_0 \simeq 10^{22}$ W/m^2, and of frequency such that $\hbar\omega \simeq 2$ eV; and ask the question: Can P_1 be significantly different from zero for such lasers?

It is interesting to examine this process from a Feynman-graph point of view. Here, n photons, each of (CM) energy $\hbar\omega$ are absorbed to produce a pair of energy $2mc^2$; that is, $n\hbar\omega \geq 2mc^2$, or $n \geq 2mc^2/\hbar\omega = 10^6$. Such a production amplitude will then have n factors of e, and the $|\text{amplitude}|^2$ will have n factors of $\propto \simeq (137)^{-1}$ and generates a probability proportional to n factors of $\propto$, which is absurdly small. What can possibly compensate this?

In the D^3 overlap region of the two lasers, there can be N "available" photons, where $N \gg n$. Then, the probability

Address correspondence and reprint requests to: Herber M. Fried, Physics Department, Brown University, Providence, RI 02912, USA. E-mail: fried@het.brown.edu

must include a "counting factor" similar to $N!/n!(N-n)!$, the number of ways of selecting n out of N available photons. But if $N/n = f \gg 1$, then by Sterling's approximation, one can see that this counting factor is proportional to f^n; and if $f > 137$, the factors of $\propto^n$ are effectively neutralized. By using a functional representation for Γ and $L[A]$, all such counting factors are automatically included.

In QED, the exact functional expression for the vacuum-to-vacuum amplitude, in the presence of an external field (of vector potential A_{ext}) is given by

$$\langle 0|S[A_{ext}]|0\rangle = e^{\mathcal{D}_A} e^{L[A+A_{ext}]}|_{A\to 0},$$

where

$$\mathcal{D}_A = -\frac{i}{2}\int \frac{\delta}{\delta A_\mu} D_{c,\mu\nu} \frac{\delta}{\delta A_\nu},$$

and $D_{c,\mu\nu}$ is the (free) photon propagator. An alternate form of the linkage operator $e^{\mathcal{D}_A}$ is given by the functional integral

$$\langle 0|S[A_{ext}]|0\rangle = N\int d[A] e^{(i/2)\int A_\mu (D_c^{-1})_{\mu\nu} A\nu} \cdot e^{L[A+A_{ext}]},$$

where the normalization integral N is defined by

$$N^{-1} = \int d[A] e^{(i/2)\int A_\mu (D_c^{-1})_{\mu\nu} A_\nu}.$$

(It should be noted that $L[A]$ is really $L[F]$, since $L[A]$ is rigorously gauge invariant.) For simplicity, we neglect all radiative corrections, so that $\langle 0|S|0\rangle \simeq e^{L[A_{ext}]}$, where $A_\mu(x)$ is the vector potential of the two laser beams, and henceforth we suppress the designation "ext":

$$A_\mu^{ext} \to A_\mu(x) = \epsilon_\mu^{(1)} \sin(k^{(1)}\cdot x) + \epsilon_\mu^{(2)} \sin(k^{(2)}\cdot x + \delta).$$

We do this here because the produced $e+e-$ appear in the midst of intense laser beams, and their subsequent motion would be essentially classical. However, neglecting the radiative corrections of gluons in QCD is not tenable; we return to this point below.

To begin the calculation, we first write down an exact, Fradkin-functional representation for $L[A]$ (see, e.g., Fried, 1992)

$$L[A] = -\frac{1}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4} \int_0^\infty \frac{ds}{s} e^{-is(m^2+p^2)}$$
$$\cdot e^{i\int_0^s ds' \sum_\mu [\delta^2/\delta v_\mu^2(s')]}$$
$$\cdot \left\{ e^{-ie\int_0^s ds' [v_\mu(s') - 2p_\mu] A_\mu \left(x + 2s'p - \int_0^{s'} v\right)} \right.$$
$$\left. \cdot \mathrm{tr}\left(e^{e\int_0^s ds' \sigma_{\mu\nu} F_{\mu\nu}\left(x+2s'p - \int_0^{s'} v\right)} \right)_+ - (e\to 0) \right\}\Big|_{v_\mu \to 0},$$

and then perform the following, simplifying approximations.

1. Treat the absorbed photons as "soft," since (using "natural" variables, with $c = \hbar = 1$) $\omega/m = 10^{-6}$. We then introduce a "no-recoil" approximation, of which many are available; but the simplest in this problem is to drop the $\int_0^{s'} v$ dependence in the arguments of A and F: $A_\mu \to A_\mu(x + 2s'p), F_{\mu\nu} \to F_{\mu\nu}(x + 2s'p)$.
2. Keep only the dependence that produces the essential singularities. In the Schwinger constant electric-field calculation, the essential singularities come from the A-dependence, while the F-dependence just contributes a multiplicative, normalization factor, so that $ReL < 0$, or $\Gamma > 0$. Here, the same division applies . . . and since we are only interested in the order of magnitude of Γ—which depends primarily on the largest essential singularity—we here neglect the $\sigma\cdot F$ dependence. Thus the trace factor becomes: $+4$, and we assume the physically obvious result: $\Gamma > 0$.

With these approximations,

$$L[A] \to -2\int_0^\infty \frac{ds}{s} e^{-ism^2} \int d^4x \int \frac{d^4p}{(2\pi)^4} e^{-isp^2} \cdot e^{i\int_0^s ds' (\delta^2/\delta v^2)}$$
$$\cdot \left\{ e^{-ie\int_0^s ds' [v_\mu(s') - 2P_\mu] A_\mu(s+2s'p)} - 1 \right\}\Big|_{v_\mu \to 0},$$

and, holding $\int_0^\infty ds$ for the last operation, we must next decide in which sequence to perform $\int d^4x, \int d^4p$, and $\exp[i\int_0^s \delta^2/\delta v^2]$. Unfortunately, these operations cannot be calculated analytically, and further approximations are needed. In the order of increasing complexity, we can display three models, as follows.

1.1. Model A: First cumulant approximation

Here, we first calculate the Fradkin linkages:

$$e^{i\int_0^s ds'(\delta^2/\delta v^2)} \cdot e^{i\int_0^s v_\mu(s') Q_\mu(s')}\Big|_{v\to 0} = e^{-i\int_0^s ds' Q^2(s')},$$

and then perform

$$\int d_p^4 e^{-isp^2} \mathcal{F}(\epsilon\cdot p, p\cdot k^{(1)}, p\cdot k^{(2)}) \to -i\frac{\pi^2}{s^2}\left(\frac{2s}{\omega^2}\right)$$
$$\cdot \frac{1}{2\pi} \iint_{-\infty}^{+\infty} du_1\, du_2\, e^{2isu_1u_2/\omega^2} \cdot e^{\mathcal{R}(x|u_1,u_2)},$$

with

$$\mathcal{R}(x|u_1, u_2) = -ie^2\epsilon^2 s \int_0^{\prime} d\lambda [S^2 - \langle S\rangle^2],$$

$$S = \sin(k^{(1)}\cdot x + 2s\lambda u_1) + \sin(k^{(2)}\cdot x + 2s\lambda u_2 + \delta),$$

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and $\langle S\rangle = \int_0' d\lambda S$. This leaves the third operation: $\int d^4x \to (D^3ct)\cdot(1/D^3)\int d^3x$, but $(1/D^3)\int d^3x \exp[\mathcal{R}(x)]$ is too complicated to evaluate explicitly.

We therefore consider the simplest "first cumulant" approximation, replacing the integral over an exponential by the exponential of the integral,

$$\exp\left[\frac{1}{D^3}\int d^3x\mathcal{R}(x)\right].$$

In statistical mechanics, this is perhaps the simplest way of approximating a full, cluster expansion.

This integration can be done explicitly, and generates for our Model L_A a function of two independent variables, of form

$$L_A = \iint_{-\lambda\max}^{+\lambda\max} d\lambda_1\, d\lambda_2\, \mathcal{J}\left(\frac{e\epsilon\omega}{m^2}\,|\,\lambda_1,\lambda_2\right),\ \lambda_{max} \sim \frac{m}{\omega} = 10^6$$

We make the "natural approximation" of replacing this by

$$L_A \simeq \iint_{-\infty}^{+\infty} d\lambda_1\, d\lambda_2\, \mathcal{J},$$

and then find the function of a single variable, expressed in terms of a single (proper-time) integral,

$$L_A \to \frac{i}{2}\,\frac{(D^3ct)}{(2\pi)^2}\cdot m^4 \int_0^\infty \frac{dt}{t^3}\, e^{-it}\left[\frac{1}{\sqrt{1-(\gamma t)^4}} - 1\right]$$

with

$$t \equiv m^2 s,\ \gamma \equiv \left(\frac{e\epsilon\omega}{\sqrt{6}\cdot m^2}\right).$$

It is important to note that the (improper) perturbative expansion of the square root, in powers of γ^4, gives a sequence of purely imaginary terms; that is, ReL_A has no perturbative expansion. In fact,

$$ReL_A \simeq -\frac{(D^3ct)}{2(2\pi)^2}\cdot m^4 \int_{\tau_0}^\infty \frac{d\tau}{\tau^3}\,\frac{e^{-\tau}}{\sqrt{(\tau/\tau_0)^4 - 1}},\ \tau_0 = \gamma^{-1} \gg 1;$$

and

$$\Gamma_A = -\frac{2}{t}\,ReL_A \simeq \frac{(D^3c)m^4}{2(2\pi)^2}\cdot\gamma^3 e^{-1/\gamma}.$$

Our exponential factor $e^{-1/\gamma}$ is reminiscent of Schwinger's (for $n=1$), and one may ask: Why? To understand this, consider that special case of "crossed" lasers which corresponds to beams moving in exactly opposite directions. One can carry through the same analysis as above, and then note that this new Γ_A will be the same as our Γ_A if one introduces an extra averaging over the time dependence (which was trivially extracted above). Surely this averaging is physically reasonable, which means that the Γ of the two situations are physically equivalent.

But in the new, head-on beam collision geometry, if the electric fields are in the same direction, as assumed above, then the magnetic fields cancel; and in the limit $\omega \to 0$ we are dealing with Schwinger's problem, and we should expect (for $n=1$) the factor $\sim e^{-\pi m^2/e\mathcal{E}}$ where the Schwinger $\mathcal{E}$ corresponds in our problem to $\epsilon\omega$. Thus we should here expect a similar form, $e^{-\kappa m^2/e\epsilon\omega}$. In the Schwinger calculation, $\Gamma = \mathcal{F}(m^2/e\mathcal{E}\omega)$ but here, $\Gamma_A \Rightarrow \mathcal{F}((m^2/e\epsilon\omega),(\omega/m)) \to \mathcal{F}((m^2/e\epsilon\omega),0)$ as we let $\lambda_{max} \to \infty$. It is therefore not surprising that we produce an exponential factor similar to that of Schwinger's exact solution.

1.2. Model B: A "modified first cumulant" approximation

For our second model, we adopt the sequence:

1. Perform the exact $\int d^4x$, and then the exact linkage operation;
2. Convert the exact $\int d^4p$ into a pair of integrals, as in Model A;
3. Introduce an exact and useful representation for the Bessel function J_0 (which appears as a result of the spatial averaging), and approximate the result in a "first cumulant" manner.

The result is

$$\Gamma_B(\gamma) \sim \frac{D^3c}{2(4\pi)^2}\, m^4\cdot\gamma^3 e^{-1/\gamma}$$

which is equivalent to that of Γ_A.

1.3. Model C: Nonperturbative approximation to the full cluster expansion

Here, one tries to extract and sum the "most important" part of every perturbative order in the cluster expansion of L_B, with the result

$$\Gamma_c(\gamma) = \sqrt{\frac{2}{\pi}}\int_0^\infty du e^{-u^2/2}\Gamma_B(u\gamma),$$

or, after an approximate evaluation,

$$\Gamma_c(\gamma) \sim \frac{2}{\sqrt{3}}\cdot\frac{(D^3c)}{(4\pi)^2}\, m^4\cdot\gamma^2 e^{-(3/2\gamma 2/3)}$$

which displays a "weakened" essential singularity. Numerically, if $\gamma \ll 1$, then

$$\gamma^3 e^{-1/\gamma} < \gamma^2 e^{-(3/2\gamma 2/3)}.$$

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This change in the form of the essential singularity may be given a physical interpretation, as follows. One understands that elementary QED processes occur over distances $\sim\lambda_c$. But here, because so many low-energy photons must be coherently absorbed, we can anticipate pair production by absorption over larger distances $\sim u_0\lambda_c$, where u_0 is that value of the u-variable in the Γ_c integral where the integrand is peaked: $u_0 \sim \gamma^{-1/3} \geq 10^2$. Extracting contributions from every term of the cluster expansion can be thought of as allowing absorption to proceed without the spatial restrictions contained in the "first cumulant" approximations.

Moving, finally, to practical matters, one must now ask how large are these Γ, numerically. Can one expect reasonable production with $F_0 = 10^{22}$ W/m^2, $D \simeq 10^{-5}$ m, $\hbar\omega \sim 2$ eV, and pulse duration $\tau_0 \simeq 10^{-13}$ s? Let us suppose that $P \simeq \tau_0\Gamma \sim 0.1$, so that roughly 10 pulses are needed to produce one pair. With intensity $F_0, \gamma = 10^{-6}$; so that for an arbitrary F, one can write $\gamma = 10^{-6}(F/F_0)^{1/2}$. Then, for Model C, a little algebra shows that this rate requires $F/F_0 \sim 10^7$. (For Models A and B, $F/F_0 \sim 10^8 - 10^9$.) Since laser intensities appear to increase by one or two orders of magnitude each year, these estimates suggest that one must wait at least a few years before this form of production is measurable. However, a better evaluation of these overlapping integrals may show an even "more weakened" essential singularity, and therefore a lower F/F_0 value. This is an interesting, and eventually a practical problem, which should attract the attention of capable people, both theoretical and experimental.

2. APPLICATION TO QCD

Suppose it were possible to produce $e+e-$ and $\mu_+\mu_-$ pairs with intersecting, high-intensity lasers. Then it should be possible to produce quark–antiquark pairs in the same way, although what happens after production would be completely different from the QED case, because of the strong, gluonic interactions between q and $\bar{q}$. Here, gluonic radiative corrections cannot be neglected . . . but one can imagine two extreme, and differing, situations:

1. q and $\bar{q}$ materialize with their connecting flux tube/string "in place," so that, in effect, one has produced a π^0, which the laser fields cannot tear apart.
2. q and $\bar{q}$ materialize, each surrounded by appropriate gluonic structure, which immediately starts to form itself into a tube/string, joining q and $\bar{q}$.

It is this second possibility which is most interesting, for the formation of the tube/string is surely not an instantaneous affair; rather, it should be a process that can be characterized by a "string-formation velocity," v_f. As a physical process, $v_f \leq c$. But it is possible for the q and $\bar{q}$ to be accelerated away from each other by the intense, crossed lasers so that their relative velocity of separation, v_s, satisfies $v_s \geq v_f$. Of course, after a quarter-wavelength of the laser pulse passes over these charged particles, deceleration occurs, and the tube/string wins. But this argument suggests that the q and $\bar{q}$ might temporarily reach separations considerably larger than a few fermis.

What would be the signal of such a process? Clearly, there would be large energy deposition in a small spatial region. Perhaps a pair of hadronic jets; perhaps q–$\bar{q}$ annihilation à la positronium, with the production of a few very-high-energy gammas, or even a small "fireball" of X rays. Because one is dealing with the conversion of a virtual quantum state into a real pair, one simply does not know if the second possibility above can occur; but it would seem to be an interesting question, which deserves some critical attention.

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