

The Cross Section of Recovery Rates and Default Probabilities Implied by Credit Default Swap Spreads

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Abstract

Rather than assuming a fixed recovery rate in estimation, we estimate recovery rates from credit default swap spreads, using 3 years of daily data on 152 corporations. We use a quadratic pricing model, which ensures nonnegative default probabilities and recovery rates. The estimated cross section of recovery rates is plausible, with an average recovery rate of 54% and substantial cross-sectional variation. Estimated 5-year default probabilities are on average 67% higher than default probabilities obtained using the standard 40% recovery assumption. This finding critically impacts the valuation of structured credit products. Larger firms and firms with more tangible assets have higher recovery rates.

I. Introduction

The valuation of credit default swap (CDS) contracts and other defaultable securities consists of two main components. The first is the default probability; the second is the recovery rate in the event of default. Significant attention has been devoted to understanding the likelihood of default and, more specifically, to the modeling of default intensities. Much less is known about recovery rates, and especially about risk-neutral recovery rates implied by default-risky securities, which are critical for valuation exercises.

This paper reports on the cross section of recovery rates by inferring risk-neutral recovery rates from CDS spreads. We allow for stochastic default intensity and stochastic interest rates, and we assume a constant recovery of face value. We model the default intensity using a quadratic specification, which ensures that the default intensity is always positive, and we obtain a closed-form solution for the CDS premia for different maturities. We estimate the model for each firm

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using the unscented Kalman filter, which is well equipped to deal with the substantial nonlinearities in the pricing formulas.

Part of the reason for the relative scarcity of existing results on risk-neutral recovery rates is the existence of identification problems. For instance, under the recovery of market value assumption, it is impossible to separate the recovery rate from the intensity process, as argued, for instance, by Duffie and Singleton (1999). The identification problem is less stringent when assuming recovery of face value, which is appropriate for CDS valuation, although econometric problems may still occur, depending on the data, the problem at hand, and the statistical loss function. Pan and Singleton (2008) argue convincingly that using CDS spreads for multiple maturities facilitates identification of the default intensity and the recovery rate. We therefore exploit the term structure of corporate CDS premia and restrict our analysis to 152 firms that were part of the CDX index between Oct. 7, 2004, and June 29, 2007, and for which we have daily observations for 1-, 3-, and 5-year maturity CDS spreads. Our sample spans 13 two-digit North American Industry Classification System (NAICS) industries.

We estimate the model for each firm. The resulting fit is good, similar to Chen, Cheng, Fabozzi, and Liu (2008), even though our estimation setup is more demanding, because we fit our model to three different maturities simultaneously. We also find that the root mean squared error (RMSE) of our model is substantially lower than the RMSE of a model with a 40% recovery rate. The average recovery rate across 152 firms is 53.79%. This is substantially higher than the 40% often assumed in existing studies and industry practice when estimating default probabilities. The cross-sectional standard deviation of our recovery rate estimates is similar to standard deviations computed using historical recovery rates over long time periods.

We subsequently investigate the impact of conventional recovery rate assumptions on the CDS-implied term structure of default probabilities. We find that the 5-year default probabilities implied by our model are on average 67% higher than the default probabilities implied by models with a 40% recovery rate. Relying on long-run historical averages of recovery rates can therefore lead to misspecified default probabilities. Credit risk pricing models use default probabilities as inputs and typically use the 40% recovery assumption when calibrating these probabilities. This is likely to lead to substantial valuation biases, especially for complex credit products such as collateralized debt obligations (CDOs).

Finally, we examine the impact of industry and firm characteristics on the cross-sectional differences in risk-neutral implied recovery rates, complementing the results of Acharya, Bharath, and Srinivasan (2007), who examine the determinants of historical recovery rates. We find that industry characteristics and industry distress are important determinants of recovery rates, as first suggested by Shleifer and Vishny (1992). Furthermore, we find that firm characteristics such as leverage, asset specificity, and tangibility of firm assets significantly affect recovery rates.

This paper is part of a growing literature on the estimation of recovery rates. Most papers provide estimates of historical recovery rates.¹ Historical estimates

¹See, for example, Altman, Brady, Resti, and Sironi (2005).

serve as a good benchmark for recovery rates, but they require long time series of realized defaults and are best thought of as unconditional, whereas our approach can provide conditional estimates using short samples. Moreover, historical estimates are obtained under the physical measure and cannot directly be used for pricing, while our estimate can be used directly for valuation applications jointly with risk-neutral probabilities.

Other studies estimate risk-neutral recovery rates from credit-risky securities. Bakshi, Madan, and Zhang (2006b) use bond data to analyze alternative recovery rate assumptions. Christensen (2007) estimates a stochastic recovery model using CDS data but limits himself to one firm. To mitigate identification problems, several studies estimate recovery rates by combining the valuation of credit-risky instruments with other securities. See, for instance, Le (2008), Das and Hanouna (2009), and Carr and Wu (2010), (2011). Other studies rely on securities with different seniorities to overcome identification problems; see Unal, Madan, and Guntay (2003) and Madan and Unal (1998).

Our results are most closely related to the estimates of the cross section of recovery rates in Jarrow, Li, and Ye (2009) and Schneider, Sogner, and Veza (2010). Jarrow et al. use one factor to capture the dynamics of credit risk and then estimate an affine credit risk model for each firm. They report an average recovery rate that is similar to ours, but they find very limited cross-sectional variation in this rate. Schneider et al. estimate an affine jump model with a constant recovery rate for a large cross section of firms, and they find an average recovery rate of 79%, which seems very large given their sample period. It is difficult to ascertain what drives these differences besides sample composition and sample period, but it is possible that the quadratic nature of our model allows us to more reliably estimate recoveries.

The paper proceeds as follows: Section II introduces the latent factor quadratic model used for CDS valuation. Section III discusses the data and the estimation technique. Section IV discusses the estimates and model fit. Section V presents the empirical results on the cross section of recovery rates, and Section VI concludes.

II. The Model

We first consider the pricing of credit default swaps.² Subsequently, we discuss the specification of the state variables and the resulting closed-form solution for survival probabilities and CDS spreads.

A. CDS Pricing

We use a discrete-time setup in which the uncertainty is captured by a set of latent factors denoted by $X_t = (X_t^r, X_t^j)$, where X_t^r governs the risk-free term

²For recent studies of CDS markets, see, for instance, Blanco, Brennan, and Marsh (2005), Bongaerts, De Jong, and Driessen (2011), Chen et al. (2008), Ericsson, Jacobs, and Oviedo (2009), Longstaff, Mithal, and Neis (2005), and Zhang, Zhou, and Zhu (2009).

structure, whereas X_t^j is a firm-specific state variable affecting firm j . We proceed under the risk-neutral measure. The price of a risk-free bond with maturity n is

$$(1) \quad B(t, n) = E_t \left[\prod_{i=1}^n B_{t+i-1, t+i} (X_{t+i-1}^r) \right] = E_t [B_{t, t+n} (X^r)],$$

where $\{B_{t+i-1, t+i}(X_{t+i-1}^r), i = 1, \dots, n\}$ is the discrete-time discount factor between two consecutive trading dates. For the purpose of pricing a defaultable security, we assume a constant recovery rate R , and we model the default event in a discrete-time doubly stochastic framework; see, for instance, Gouriou, Monfort, and Polimenis (2006). Denoting the potential default time of firm j by τ_j , and assuming no previous default, the survival probability is

$$(2) \quad \Pr[\tau_j > t + z] = E_t \left[\exp \left(- \sum_{i=1}^z \Lambda_{t+i-1, t+i} (X_{t+i-1}^r, X_{t+i-1}^j) \right) \right].$$

The discrete-time process $\{\Lambda_{t+i-1, t+i}(X_{t+i-1}^r, X_{t+i-1}^j), i = 1, \dots, n\}$ also defines the default intensity, which is the probability of default for a certain period conditional on no earlier default. The default probability of firm j depends on both systematic and firm-specific factors, which induces a nonzero correlation between the risk-free term structure and the default risk of the underlying firm j . The price of a zero-coupon defaultable bond with maturity n is

$$(3) \quad DB_j(t, n) = E_t \left[B_{t, t+n} (X^r) \exp \left(- \sum_{i=1}^n \Lambda_{t+i-1, t+i} (X_{t+i-1}^r, X_{t+i-1}^j) \right) \right].$$

For the purpose of CDS valuation, we assume that trading follows the discrete-time sequence $\{t, t + 1, \dots, t + n\}$ and that the coupon payments are made every p periods. The coupon dates follow the sequence $T_c = \{t, t + p, \dots, t + (n/p)p\}$, where n denotes the maturity of the CDS contract. We assume default can only occur right before a coupon date, and for notational convenience we suppress the firm-specific j . Consider two consecutive coupon dates $t + (k - 1)p$ and $t + kp$, where $1 \leq k \leq n/p$. Assuming no credit event prior to $t + (k - 1)p + 1$, the buyer of the CDS will pay a premium S on each coupon date. The present value of this premium payment at time t is $B_{t, t+kp} 1_{(\tau > t+kp)} S$. If a credit event is documented between two consecutive trading dates, the buyer receives compensation equal to the loss given default, $LGD = 1 - R$, and pays the accrued premium

$$(4) \quad \sum_{i=(k-1)p+1}^{kp} B_{t, t+kp} 1_{(t+(i-1)p < \tau < t+i)} \left(LGD - \frac{i - (k-1)p}{p} S \right).$$

Applying the same reasoning for all coupon dates and using the double stochasticity of the default time, the net present value of the CDS contract at time t is

$$(5) \quad CDS(t, n) = \sum_{k=1}^{n/p} -E_t \left[B_{t, t+kp} \exp \left(- \sum_{i=1}^{kp} \Lambda_{t+i-1, t+i} \right) \right] \times S + \sum_{k=1}^{n/p} \sum_{i=(k-1)p+1}^{kp} E_t \left[\xi_i \left(LGD - \frac{i - (k-1)p}{p} S \right) \right],$$

where

$$(6) \quad \xi_i = B_{t,t+i} \left(\exp \left(- \sum_{z=1}^{i-1} \Lambda_{t+z-1,t+z} \right) - \exp \left(- \sum_{z=1}^i \Lambda_{t+z-1,t+z} \right) \right).$$

Setting the value of the CDS contract in equation (5) equal to 0 yields the spread

$$(7) \quad S = \frac{\sum_{k=1}^{n/p} \sum_{i=(k-1)p+1}^{kp} E_t [\xi_i \text{LGD}]}{\sum_{k=1}^{n/p} \left[E_t \left[B_{t,t+kp} \exp \left(- \sum_{i=1}^{kp} \Lambda_{t+i-1,t+i} \right) \right] + \sum_{i=(k-1)p+1}^{kp} E_t \left[\xi_i \frac{(i - (k - 1)p)}{p} \right] \right]}.$$

The CDS spread depends on discounted LGD and the probability of credit events. It can be computed analytically for any maturity.

B. Model Specification

We assume a 3-factor affine Gaussian model for the risk-free term structure, as in Ang and Piazzesi (2003). This model is essentially the discrete-time counterpart to the 3-factor Gaussian model of Duffee (2002). The 3 factors determining risk-free rates follow a Gaussian VAR(1) (vector autoregression model of order 1)

$$(8) \quad X_t^r = \mu^r + \phi^r X_{t-1}^r + \Sigma^r \varepsilon_t^r,$$

where X^r is a (3×1) vector, ϕ^r is lower triangular, Σ^r is diagonal, and $\varepsilon_t^r \sim$ i.i.d. $N(0, I)$. The price of a 1-period zero-coupon risk-free bond is $B_{t,t+1} = \exp(-r_t)$, where $r_t = \delta_0 + \delta_1 X_t^r$ and δ_1 is a (1×3) vector. See Ang and Piazzesi for the pricing formula.

We further assume that for each firm j , there exists a specific factor that affects the firm’s default risk. This factor is assumed to be independent of the systematic factors X_t^r . Its dynamic is given by

$$(9) \quad X_t^j = \mu^j + \phi^j X_{t-1}^j + \Sigma^j \varepsilon_t^j.$$

Combining the above two equations, the dynamics of the state vector of firm j are

$$(10) \quad \begin{bmatrix} X_t^r \\ X_t^j \end{bmatrix} = \begin{bmatrix} \mu^r \\ \mu^j \end{bmatrix} + \begin{bmatrix} \phi^r & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & \phi^j \end{bmatrix} \begin{bmatrix} X_{t-1}^r \\ X_{t-1}^j \end{bmatrix} + \begin{bmatrix} \Sigma^r & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & \Sigma^j \end{bmatrix} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^j \end{bmatrix},$$

which we write as

$$(11) \quad X_t = \mu + \phi X_{t-1} + \Sigma \varepsilon_t,$$

where ϕ and Σ are (4×4) matrices, Σ is diagonal, and $\varepsilon_t \sim$ i.i.d. $N(0, I)$.

We now turn to the dynamics of the hazard rate process $\{\Lambda_{t+i-1,t+i}(X_{t+i-1}), i = 1, \dots, n\}$. To ensure positivity, we parameterize it as a quadratic function of the state variables

$$(12) \quad \Lambda_{t+i-1,t+i}(X_{t+i-1}) = (\alpha + \beta X_{t+i-1})^T (\alpha + \beta X_{t+i-1}),$$

where the superscript T denotes the transpose, α is a scalar, and β is a (1×4) vector that captures the correlation between default risk and the risk-free term structure, as well as the correlation between the default risk and the firm-specific factor. Note that at any time t , $\Lambda_{t,t+1}(X_t) = (\alpha + \beta X_t)^T(\alpha + \beta X_t)$ is known to investors. We constrain the recovery rate R between 0 and 1 by modeling it as the exponential of a negative square.

We can then use the results in Ang, Boivin, Dong, and Loo-Kung (2011) to obtain closed-form expressions involving recursions for survival probabilities and the price of the CDS contract. The 1-period survival probability for a firm at time t is given by

$$(13) \quad \Pr(\tau > t + 1) = \exp\left(-(\alpha + \beta X_t)^T(\alpha + \beta X_t)\right) = \exp\left(-\alpha^2 - 2\alpha\beta X_t + X_t^T(-\beta^T\beta)X_t\right),$$

and the survival probability after any $z \geq 2$ days is

$$(14) \quad \Pr[\tau_j > t + z] = E_t \left[\exp\left(-\sum_{i=1}^z \Lambda_{t+i-1,t+i}(X)\right) \right] = \exp\left(A_z + B_z^T X_t + X_t^T C_z X_t\right),$$

where

$$(15) \quad \begin{aligned} A_z &= -\alpha^2 + A_{z-1} + B_{z-1}^T \mu + \mu^T C_{z-1} \mu - \frac{1}{2} \ln \det(I - 2\Sigma^T C_{z-1} \Sigma) \\ &\quad + \frac{1}{2} (\Sigma^T B_{z-1} + 2\Sigma^T C_{z-1} \mu)^T (I - 2\Sigma^T C_{z-1} \Sigma)^{-1} \\ &\quad \times (\Sigma^T B_{z-1} + 2\Sigma^T C_{z-1} \mu), \\ B_z &= -2\alpha\beta + B_{z-1}^T \phi + 2\mu^T C_{z-1} \phi + 2(\Sigma^T B_{z-1} + 2\Sigma^T C_{z-1} \mu)^T \\ &\quad \times (I - 2\Sigma^T C_{z-1} \Sigma)^{-1} \Sigma^T C_{z-1} \phi, \\ C_z &= -\beta^T \beta + \phi^T C_{z-1} \phi + 2(\Sigma^T C_{z-1} \phi)^T (I - 2\Sigma^T C_{z-1} \Sigma)^{-1} \\ &\quad \times (\Sigma^T C_{z-1} \phi). \end{aligned}$$

Letting

$$\begin{aligned} G_{t,i}^0 &= E_t \left[B_{t,t+i} \exp\left(-\sum_{z=1}^i \Lambda_{t+z-1,t+z}\right) \right] \quad \text{and} \\ G_{t,i}^1 &= E_t \left[B_{t,t+i} \exp\left(-\sum_{z=1}^{i-1} \Lambda_{t+z-1,t+z}\right) \right], \end{aligned}$$

the premium paid on each coupon date on a CDS contract with maturity n is

$$(16) \quad S = \frac{(1 - R) \sum_{k=1}^{n/p} \sum_{i=(k-1)p+1}^{kp} (G_{t,i}^1 - G_{t,i}^0)}{\sum_{k=1}^{n/p} \left[G_{t,kp}^0 + \sum_{i=(k-1)p+1}^{kp} \left[\frac{(i - (k-1)p)}{p} (G_{t,i}^1 - G_{t,i}^0) \right] \right]},$$

where $G_{t,n}^m = \exp\left(A_n^m + (B_n^m)^T X_t + X_t^T C_n^m X_t\right)$ and (A_n^m, B_n^m, C_n^m) are recursively computed as

$$(17) \quad \begin{aligned} A_1^0 &= -(\delta_0 + \alpha^2), & B_1^0 &= -(\delta_1 + 2\alpha\beta), & \text{and } C_1^0 &= -\beta^T \beta, \\ A_1^1 &= -\delta_0, & B_1^1 &= -\delta_1, & \text{and } C_1^1 &= 0, \end{aligned}$$

$$(18) \quad \begin{aligned} A_n^m &= -(\delta_0 + \alpha^2) + A_{n-1}^m + (B_{n-1}^m)^T \mu + \mu^T C_{n-1}^m \mu \\ &\quad - \frac{1}{2} \ln \det(I - 2\Sigma^T C_{n-1}^m \Sigma) \\ &\quad + \frac{1}{2} (\Sigma^T B_{n-1}^m + 2\Sigma^T C_{n-1}^m \mu)^T (I - 2\Sigma^T C_{n-1}^m \Sigma)^{-1} \\ &\quad \times (\Sigma^T B_{n-1}^m + 2\Sigma^T C_{n-1}^m \mu), \\ B_n^m &= -(\delta_1 + 2\alpha\beta) + (B_{n-1}^m)^T \phi + 2\mu^T C_{n-1}^m \phi \\ &\quad + 2 (\Sigma^T B_{n-1}^m + 2\Sigma^T C_{n-1}^m \mu)^T (I - 2\Sigma^T C_{n-1}^m \Sigma)^{-1} \Sigma^T C_{n-1}^m \phi, \\ C_n^m &= -\beta^T \beta + \phi^T C_{n-1}^m \phi + 2 (\Sigma^T C_{n-1}^m \phi)^T \\ &\quad \times (I - 2\Sigma^T C_{n-1}^m \Sigma)^{-1} (\Sigma^T C_{n-1}^m \phi), \end{aligned}$$

for $n \geq 2$ and $m = 0, 1$.

In summary, we assume a 3-factor model for the risk-free term structure and a 4-factor model for the risky term structure. We assume a quadratic process for the hazard rate to ensure positivity. The resulting expression for CDS spreads can be computed recursively.

III. Data and Estimation

We first discuss the CDS sample, as well as the source of the data used in the analysis of the determinants of latent factors and recovery rates. Subsequently, we discuss our econometric setup.

A. Data

Our sample period is from Oct. 7, 2004, to June 29, 2007. We collect data for all single-name senior unsecured CDS contracts that were part of the CDX index at any time between these two dates.³ The CDS data are obtained from Markit. We obtain CDS spreads for 1-, 3-, and 5-year maturities for all firms. To have sufficiently long time series, we only retain firms that have more than 275 daily observations for each maturity. This yields a sample of 152 firms. By limiting ourselves to firms that were part of the CDX index, we reduce the sample size, but the CDS contracts on these companies are relatively more liquid. Moreover, our main

³CDS data are available starting in 2001, but market liquidity is low at the beginning of the sample, and including these observations may bias results if liquidity is not accounted for. While 5-year CDS contracts were traded more frequently starting from mid-2003, prices for 1- and 3-year contracts were often stale until mid-2004.

objective is to investigate the assumption of a 40% fixed recovery rate, and not to characterize the general population of recovery rates. From this perspective, the homogeneity of the sample biases our findings against cross-sectional variation in recovery rates, and so our sample selection provides a conservative assessment of the null hypothesis.

Figure 1 depicts the average spread across all 152 firms for each of the three tenors. The average spread varies considerably over the sample period, but the average term structure is always upward sloping. Table 1 provides descriptive statistics on the CDS spreads by rating and industry.⁴ We have few observations for some industries and rating categories. However, it seems a safe conclusion that spreads are higher for firms with lower ratings, as expected. There is considerable variation in credit spreads across industries. In our sample, spreads are rather low for finance and insurance companies, as well as for utilities and transportation firms. Spreads are highest for retail and manufacturing firms.

FIGURE 1
Average CDS Spreads across Firms

Figure 1 shows the time series of the average CDS spreads (in bps) across all 152 firms for the contracts with 1-year (light grey), 3-year (dark gray), and 5-year (black) maturity. The sample period is Oct. 7, 2004–June 29, 2007.



To estimate the risk-free term structure model, we use the daily London Interbank Offered Rate (LIBOR) with 6-month maturity and interest swap rates with maturities of 1, 2, 3, 4, and 5 years. The LIBOR and interest swap rates are obtained from Bloomberg.

For the regression analysis on the determinants of recovery rates and latent factors, we need a number of firm-specific and industry variables. Firm-specific variables for the fiscal years 2004–2007 are obtained from the Center for Research in Security Prices (CRSP)-Compustat merged database. We match the Compustat

⁴We report the firms' average ratings over the sample period. At each point in time, we assign a numerical code to the firm's rating. Subsequently, we average over time and map the average back to a rating category.

TABLE 1
Descriptive Statistics of CDS Spreads

Panel A of Table 1 reports the average 1-, 3-, and 5-year CDS spreads for 152 firms across credit ratings. We use daily data from Oct. 7, 2004, to June 29, 2007. Ratings are obtained from Compustat. Panel B reports the average 1-, 3-, and 5-year CDS spreads across industries. Firms are matched using the North American Industry Classification System (NAICS), and industry is identified by the first 2 digits of the NAICS number.

Panel A. Average CDS Spreads across Ratings

Category	Spreads (bps)			No. of Firms
	1 Year	3 Year	5 Year	
All firms	21.58	41.91	63.55	152
AAA	4.34	9.23	15.25	2
AA	4.21	8.70	14.23	3
A	6.82	15.80	27.27	50
BBB	12.62	30.39	52.43	75
BB	36.75	71.26	110.72	16
B	230.40	352.71	419.84	6

Panel B. Average CDS Spreads across Industries

Industry		Spreads (bps)			No. of Firms
		1 Year	3 Year	5 Year	
Health care and social assistance	Hlth	13.68	32.80	53.78	2
Utilities	Util	9.89	21.42	34.64	7
Finance and insurance	Fin	8.79	18.87	30.56	22
Manufacturing	Manu	35.30	58.13	78.67	54
Professional, scientific, and technical services	Pro	14.96	35.53	63.78	5
Information	Info	13.48	32.92	58.10	19
Construction	Cons	16.82	42.44	73.20	4
Transportation and warehousing	Tran	6.55	17.00	31.30	5
Wholesale trade	Whol	12.27	28.08	48.81	3
Retail trade	Retl	24.44	57.85	91.39	17
Mining	Min	10.19	22.06	36.77	5
Real estate and rental and leasing	Este	14.89	37.17	63.26	2
Accommodation and food services	Acco	17.07	41.39	72.32	7

variables with the CDS data using the Compustat identifier GVKEY. To compute industry variables, we define the firm's industry according to the first 3 digits of the NAICS code. We discuss the firm-specific and industry variables in more detail in the online Appendix (www.jfqa.org).

B. Estimation

1. Loss Function

We conduct several estimation exercises. For maturity τ , denote the model spread from equation (7) at time t as $S_{M,t}^\tau$. Our main measure of model fit is the RMSE. Using the insight of Granger (1969) that the choice of loss function affects model estimates and that identical loss functions in the estimation and evaluation stage are preferred, we therefore first use a nonlinear least squares procedure, minimizing the RMSE based on the differences between the model spreads and the data. We conduct one optimization exercise per firm, using the data from the three maturities jointly by summing the squared errors across maturities. We thus minimize

$$(19) \quad \sqrt{\frac{1}{3T} \sum_{t=1}^T \left[(S_{M,t}^{1Y} - S_{D,t}^{1Y})^2 + (S_{M,t}^{3Y} - S_{D,t}^{3Y})^2 + (S_{M,t}^{5Y} - S_{D,t}^{5Y})^2 \right]},$$

where $S_{D,t}^\tau$ is the observed spread for maturity τ at time t , and T is the sample size. To construct this loss function, the latent state variables have to be filtered from the data. We address the nonlinearity in the measurement equation by using the square-root unscented Kalman filter proposed by Van der Merwe and Wan (2001), which we found to be numerically stable and computationally feasible.

To investigate robustness, we also estimate model parameters by minimizing the mean absolute error, and we conduct a likelihood-based time-series estimation based on the prediction errors from the Kalman filter (see Bakshi, Madan, and Zhang (2006a) and Li and Zhao (2006) for more details on this loss function).

2. A Two-Step Procedure

There are 4 factors: 3 risk-free factors that are common to all firms, and 1 firm-specific factor. To ensure that the risk-free factors are the same for all firms, we estimate their dynamics in a first step. This exercise is therefore only performed once, and the 3 filtered factors are subsequently taken as given in the nonlinear least squares procedure performed on the firm's CDS spreads.⁵ The CDS data are subsequently used to filter the firm-specific latent factor and the parameters of the default risk process. This two-step procedure closely follows the approach used by Duffee (1999) to estimate latent factor models for risky bonds.

For each firm, we estimate two recovery models. In the first model, we set the recovery rate equal to 40% and let the model fit the CDS data. In the second model, we let the recovery rate be a free parameter. For this case, there are 12 parameters in total to be estimated: three parameters of the firm-specific factor in equation (9), five parameters of the hazard rate in equation (12), three measurement error parameters, and the recovery rate.

3. Numerical Optimization

Due to the rich parameterization of these models and the extensive cross section of firms, a feasible framework to conduct the numerical optimization is needed.⁶ In the case of the RMSE-based optimization, we proceed as follows:⁷ We start with the model with 40% recovery rate and use a grid to identify a parameter combination that produces a good initial fit for a diverse sample of CDS spreads. Then we generate 50 random arrays from the normal distribution, with mean equal to this parameter set and standard deviation equal to the absolute value of this parameter set. We calculate the RMSEs for these parameters and use the three parameter combinations with the best fit as starting points for the numerical optimization. We select the parameter vector with the lowest optimized RMSE.

For the second model, where the recovery rate is a free parameter, we start from the firm's optimal values for the first model and a 40% recovery rate. We generate 100 random arrays from the normal distribution, with mean equal to this

⁵The estimation of the risk-free term structure is also conducted using nonlinear least squares in conjunction with the unscented Kalman filter.

⁶See, for instance, Duffee (1999) for a good discussion of this issue in the context of Gaussian term structure models.

⁷A similar approach is followed for the other loss functions.

parameter vector and standard deviation equal to the absolute value of this parameter vector. We calculate the RMSE for the 100 sets of parameters and use the 10 parameter vectors with the best fit as starting values in numerical optimization. We then select the parameter vector with the lowest optimized RMSE.

This procedure is satisfactory for the majority of firms. In some cases however, the resulting RMSE is fairly large, or the resulting default probabilities and recovery rates are not plausible. This may indicate a local optimum. For these firms, we repeat the numerical optimization procedure using the optima for five different firms in similar industries as starting values. If this results in a lower RMSE, we select this as the optimum.

Our approach could be problematic if it turns out that the numerical optimization does not yield optima for the recovery rate that are substantially different from 40%, or if the results for different firms are very similar. Our results below indicate the opposite. There is a large amount of cross-sectional variation in estimated recovery rates, parameter vectors, and model properties in general.

IV. Estimates and Model Fit

We first discuss the in-sample and out-of-sample fit of our model. Subsequently, we conduct a Monte Carlo analysis to assess how identification problems impact the estimation exercise.

A. The Risk-Free Term Structure

Panel A of Table 2 presents the estimation results for the risk-free term structure. Two of the factors are very persistent. Consistent with existing evidence, the first factor is highly correlated with the level of the term structure, the second factor is highly correlated with the slope, and the third factor is highly correlated with curvature. We do not report the RMSEs to save space. They are consistent with existing studies, such as Jagannathan, Kaplin, and Sun (2003), Li and Zhao (2006), Duffie and Singleton (1997), and Chen et al. (2008).

TABLE 2
Risk-Free and Risky Term Structure Estimates

Panel A of Table 2 reports parameter estimates for the risk-free term structure. X_1 , X_2 , and X_3 are latent factors. The factor dynamics and loadings for the risk-free term structure are estimated using the 6-month LIBOR and the 1-, 2-, 3-, 4-, and 5-year swap rates. Panel B reports the estimated parameter distribution across all firms. Here, α , β_1 , β_2 , β_3 , and β_4 capture the dynamics of the hazard rate process; μ_j , ϕ_j , and Σ_j capture the dynamics of firm j 's firm-specific latent factor; and u_1 , u_2 , and u_3 are the standard deviations of measurement errors of 1-, 3-, and 5-year CDS contracts estimated using the unscented Kalman filter.

Panel A. Factor Loadings and Dynamics of Risk-Free Term Structure

Parameters	X_1	X_2	X_3
δ_0		9.16E-05	
δ_1	3.79E-06		
δ_2		1.14E-05	
δ_3			2.84E-06
μ	0.0232122	0.0009766	5.12E-05
ϕ_1	0.9988641	0	0
ϕ_2	0.0057737	0.9529954	0
ϕ_3	0.0041623	-0.0022862	0.77535574

(continued on next page)

TABLE 2 (continued)
Risk-Free and Risky Term Structure Estimates

Panel B. Parameter Distribution across All Firms

Percentile	Hazard Rate Process Parameters					Latent-Factor Parameters			Std. Dev. of Measurement Error		
	α	β_1	β_2	β_3	β_4	μ_j	ϕ_j	Σ_j	u_1	u_2	u_3
Average	0.00057	-0.00006	0.00011	0.00017	-0.00239	-0.00236	0.99991	0.00773	0.00000	0.00083	0.00001
2.5th	-0.00274	-0.00033	-0.00042	-0.00158	-0.00770	-0.00521	0.99936	0.00000	0.00000	0.00000	0.00000
25th	-0.00114	-0.00016	-0.00012	-0.00044	-0.00333	-0.00322	0.99991	0.00009	0.00000	0.00000	0.00000
50th	0.00041	-0.00006	0.00012	0.00010	-0.00257	-0.00257	0.99999	0.00059	0.00000	0.00000	0.00000
75th	0.00138	0.00004	0.00034	0.00079	-0.00167	-0.00190	1.00000	0.00283	0.00000	0.00000	0.00000
97.5th	0.00583	0.00022	0.00072	0.00181	0.00396	0.00349	1.00000	0.08796	0.00001	0.00004	0.00000
Std. dev.	0.00239	0.00015	0.00033	0.00090	0.00271	0.00206	0.00019	0.02512	0.00001	0.01024	0.00006

B. The Risky Term Structure: In-Sample Model Fit

Panel B of Table 2 presents the cross-sectional distribution of the parameter estimates for 152 CDS contracts, obtained using the RMSE-based loss function (19). The distributions of the parameter estimates for the two other estimation exercises are qualitatively similar. The most important conclusion from Table 2 is that the firm-specific latent factor is very persistent for all firms. The online Appendix contains an analysis of the economic and financial determinants of the firm-specific latent factor.

Table 3 contains results for all three estimation exercises as described in Section III.B.1: RMSE-based, absolute error-based, and time-series based, for each of the 152 firms. For each firm, we report results for the model with a fixed 40% recovery rate and for the model with estimated recovery rate. Column 5 reports the RMSE for the model with estimated recovery rate. On average across firms and maturities, the RMSE of the model with estimated recovery rate is 4.16 basis points (bps), whereas for the model with 40% recovery rate, reported in column 8, the RMSE is on average 7.24 bps. The improvement in fit is over 40%, which is very substantial. We also statistically compare the two models using the generalized method of moments (GMM) test in Bakshi et al. (2006a). The null hypothesis that the average RMSE for the model with estimated recovery rate is higher than for the model with 40% recovery rate is rejected at the 5% level for 69.74% of the firms.

The online Appendix presents the ratio of the RMSEs for both models across firms for all three tenors. The most important observation is that for many firms, the RMSE for the model with estimated recovery rate is only a fraction of the RMSE for the model with 40% recovery rate. Note that even though the model with estimated recovery rate nests the model with 40% recovery rate, for some firms the ratio is larger than 1 for one of the maturities because we use all three maturities jointly in estimation.

As a percentage of the spread, the RMSE is 5.63% for the 5-year tenor. Chen et al. (2008) report an average RMSE of 5.78% using a sample of 30 financial firms and 4.04% using a sample of 30 industrial firms, but they only use the

TABLE 3
Firm-by-Firm Recovery Rate Estimates

For each firm, we report rating, industry, estimated recovery rate, model-implied average 5-year survival probability, RMSE, and absolute errors (Abs. Err.). We list estimation results based on the following loss functions: RMSE^a, absolute error^b, and maximum-likelihood estimation (MLE)^c. Errors are averaged across three maturities and are reported in basis points.

Company Name	Rating	Industry	Recovery Rate ^a	5-Year Survival Prob. ^a	Estimated Recovery			40% Recovery			
					RMSE ^a	Abs. Err. ^a	Abs. Err. ^b	RMSE ^a	Abs. Err. ^a	Recovery Rate ^b	Recovery Rate ^c
	1	2	3	4	5	6	7	8	9	10	11
Sun Microsystems Inc	BB	Manu	0.826	0.771	3.492	2.384	2.365	3.881	2.730	0.824	0.867
Honeywell International Inc	A	Manu	0.887	0.926	0.636	0.457	0.449	3.294	1.830	0.899	0.905
Fortune Brands Inc	BBB	Manu	0.861	0.856	2.294	1.645	1.620	3.098	1.815	0.858	0.847
AT&T Corp	BB	Info	0.335	0.967	5.770	4.015	2.744	7.015	4.969	0.524	0.205
Du Pont (E I) De Nemours	A	Manu	0.915	0.904	0.647	0.486	0.481	5.779	5.006	0.918	0.873
Eastman Kodak Co	B	Manu	0.209	0.872	8.767	6.710	6.522	10.238	7.787	0.322	0.256
General Electric Co	AAA	Manu	0.734	0.964	0.583	0.447	0.443	0.761	0.604	0.755	0.775
General Motors Corp	B	Manu	0.368	0.737	58.912	44.019	41.364	60.428	47.380	0.492	0.360
Goodrich Corp	BBB	Manu	0.094	0.978	2.144	1.406	1.386	2.772	1.819	0.055	0.181
Ingersoll-Rand Co Ltd	A	Manu	0.119	0.986	1.135	0.776	0.760	2.563	1.509	0.147	0.075
Intl Business Machines Corp	A	Pro	0.540	0.982	0.761	0.506	0.492	1.399	0.922	0.606	0.590
Maytag Corp	BB	Manu	0.135	0.929	12.873	6.552	6.376	43.725	27.734	0.362	0.177
Olin Corp	BBB	Manu	0.486	0.927	2.969	2.204	2.090	4.843	3.361	0.552	0.522
Altria Group Inc	BBB	Manu	0.925	0.621	4.870	3.750	2.946	11.467	8.530	0.902	0.663
Conocophillips	A	Manu	0.662	0.970	0.881	0.652	0.606	0.950	0.692	0.684	0.656
Amgen Inc	A	Manu	0.148	0.990	0.689	0.526	0.507	0.994	0.720	0.218	0.132
Sears Roebuck & Co	BB	Retl	0.280	0.923	3.573	2.610	2.474	10.876	7.310	0.356	0.188
RadioShack Corp	BB	Retl	0.483	0.898	4.501	3.547	3.485	5.099	3.683	0.549	0.467
Wyeth	A	Manu	0.698	0.963	1.396	0.824	0.748	1.937	1.274	0.750	0.710
Kroger Co	BBB	Retl	0.410	0.958	1.957	1.423	1.393	5.592	3.957	0.424	0.339
CVS Caremark Corp	BBB	Retl	0.959	0.964	1.417	1.033	0.997	2.736	1.679	0.644	0.699
General Mills Inc	BBB	Manu	0.112	0.985	1.109	0.790	0.771	1.475	0.953	0.146	0.182
Penney (J C) Co	BBB	Retl	0.825	0.795	9.615	4.152	3.768	10.413	4.130	0.830	0.836
Caterpillar Inc	A	Manu	0.780	0.958	0.819	0.554	0.539	1.198	0.792	0.820	0.840
Deere & Co	A	Manu	0.790	0.950	0.959	0.659	0.634	1.240	0.891	0.769	0.679
Bristol-Myers Squibb Co	A	Manu	0.863	0.938	0.701	0.532	0.513	0.978	0.678	0.860	0.871
Boeing Co	A	Manu	0.825	0.949	0.727	0.519	0.503	2.122	1.377	0.830	0.905
Dow Chemical	A	Manu	0.414	0.977	1.561	0.963	0.905	1.852	0.954	0.459	0.320
Lockheed Martin Corp	BBB	Manu	0.649	0.965	1.145	0.711	0.700	9.576	5.174	0.642	0.586
MeadWestvaco Corp	BBB	Manu	0.789	0.872	2.005	1.413	1.402	12.490	9.370	0.779	0.564
Cardinal Health Inc	BBB	Whol	0.395	0.963	3.950	1.455	1.326	10.153	7.424	0.427	0.209
Intl Paper Co	BBB	Manu	0.763	0.887	2.341	1.659	1.608	6.699	4.454	0.749	0.750
Motorola Inc	BBB	Manu	0.700	0.945	1.394	0.997	0.982	2.423	1.851	0.706	0.765
Sara Lee Corp	BBB	Manu	0.471	0.954	2.902	1.841	1.776	3.343	1.995	0.486	0.475
FirstEnergy Corp	BBB	Util	0.856	0.862	1.308	0.972	0.957	4.129	2.597	0.856	0.845
Progress Energy Inc	BBB	Util	0.629	0.951	2.359	1.614	1.489	5.358	4.107	0.747	0.599
Hilton Hotels Corp	BB	Acco	0.285	0.938	4.009	2.701	2.678	6.135	3.377	0.240	0.242
Textron Inc	A	Manu	0.213	0.986	1.009	0.777	0.701	1.105	0.828	0.252	0.308
Halliburton Co	BBB	Min	0.388	0.976	0.985	0.669	0.647	1.650	1.238	0.377	0.679
Rohm and Haas Co	A	Manu	0.855	0.913	1.254	0.896	0.887	1.660	1.074	0.857	0.863
Clear Channel Communications	BB	Pro	0.236	0.913	9.925	7.351	6.980	33.202	27.610	0.072	0.151
American Electric Power	BBB	Util	0.950	0.698	1.395	1.109	1.068	15.817	6.474	0.954	0.964
Constellation Energy Grp Inc	BBB	Util	0.249	0.978	1.756	1.242	1.192	2.038	1.283	0.282	0.353
Alcan Inc	BBB	Manu	0.334	0.976	1.451	0.964	0.926	2.938	1.753	0.322	0.336
Alcoa Inc	A	Manu	0.448	0.977	1.946	1.243	0.983	3.135	1.853	0.352	0.355
Northrop Grumman Corp	BBB	Manu	0.729	0.955	1.168	0.712	0.696	9.238	4.079	0.698	0.711
Raytheon Co	BBB	Manu	0.281	0.980	1.668	1.131	1.107	4.454	3.434	0.278	0.270
Campbell Soup Co	A	Manu	0.742	0.958	1.301	0.937	0.880	3.591	2.550	0.805	0.574
Whirlpool Corp	BBB	Manu	0.431	0.955	2.140	1.513	1.491	2.676	1.656	0.480	0.424
Avis Budget Group Inc	BB	Este	0.505	0.933	3.787	2.384	2.276	5.997	4.094	0.531	0.467
Kerr-McGee Corp	BBB	Min	0.552	0.930	5.396	3.213	3.147	7.092	3.819	0.523	0.323
CA Inc	BB	Info	0.875	0.742	3.309	1.999	1.849	4.070	2.660	0.887	0.863
Disney (Walt) Co	A	Info	0.901	0.876	1.244	0.879	0.855	4.823	2.791	0.895	0.880
Loews Corp	A	Fin	0.819	0.933	1.021	0.713	0.698	1.111	0.791	0.817	0.859
Hewlett-Packard Co	A	Manu	0.494	0.978	1.368	0.928	0.880	2.268	1.583	0.501	0.599
Baxter International Inc	A	Manu	0.128	0.987	0.983	0.717	0.707	2.326	1.442	0.137	0.080
Duke Energy Corp	BBB	Util	0.613	0.957	0.842	0.595	0.588	2.431	1.690	0.616	0.626
Hess Corp	BB	Manu	0.594	0.929	3.061	2.182	2.137	3.210	2.316	0.598	0.697

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TABLE 3 (continued)
Firm-by-Firm Recovery Rate Estimates

Company Name	Rating	Industry	Recovery Rate ^a	5-Year Survival Prob. ^a	Estimated Recovery			40% Recovery		Recovery Rate ^b	Recovery Rate ^c
					RMSE ^a	Abs. Err. ^a	Abs. Err. ^b	RMSE ^a	Abs. Err. ^a		
	1	2	3	4	5	6	7	8	9	10	11
Arrow Electronics Inc	BBB	Whol	0.902	0.702	1.827	1.379	1.360	2.559	1.545	0.898	0.903
Omnicom Group	A	Pro	0.921	0.843	0.935	0.662	0.661	1.145	0.743	0.921	0.908
Sherwin-Williams Co	A	Manu	0.843	0.882	2.470	1.584	1.532	6.107	3.699	0.821	0.771
Donnelley (R R) & Sons Co	BBB	Manu	0.630	0.910	6.437	3.913	3.758	13.255	8.355	0.627	0.358
Wells Fargo & Co	AA	Fin	0.675	0.980	0.481	0.361	0.358	6.516	4.824	0.679	0.674
Weyerhaeuser Co	BBB	Manu	0.427	0.956	2.550	1.620	1.501	3.478	1.870	0.448	0.423
Computer Sciences Corp	A	Pro	0.521	0.940	5.963	3.548	3.252	13.312	9.003	0.523	0.505
Alltel Corp	A	Info	0.450	0.947	6.934	4.606	3.846	14.149	7.033	0.462	0.372
McDonalds Corp	A	Acco	0.755	0.959	1.187	0.926	0.895	4.345	3.085	0.785	0.769
SuperValu Inc	BB	Retl	0.367	0.914	6.719	4.914	4.710	17.797	14.059	0.402	0.634
Marsh & McLennan Cos	BBB	Fin	0.418	0.946	6.571	3.215	3.081	7.965	3.372	0.457	0.743
Gannett Co	A	Info	0.608	0.950	1.787	1.252	1.239	1.967	1.428	0.609	0.324
Union Pacific Corp	BBB	Tran	0.321	0.976	1.696	1.115	1.082	2.361	1.515	0.340	0.295
Knight-Ridder Inc	BBB	Info	0.257	0.937	8.552	6.622	6.285	11.662	6.896	0.237	0.173
Target Corp	A	Retl	0.202	0.992	0.835	0.545	0.465	3.155	2.003	0.482	0.390
Liz Claiborne Inc	BBB	Manu	0.251	0.966	3.127	1.961	1.895	4.569	3.112	0.238	0.181
Albertsons Inc	BBB	Retl	0.099	0.918	12.365	9.172	8.751	36.667	26.815	0.123	0.194
Fannie Mae	AAA	Fin	0.410	0.989	0.452	0.354	0.342	1.237	0.837	0.386	0.528
Lennar Corp	BBB	Cons	0.739	0.870	3.033	2.228	2.127	4.499	2.622	0.774	0.770
Centex Corp	BBB	Cons	0.231	0.959	3.412	2.288	2.066	14.916	11.500	0.315	0.437
Pulte Homes Inc	BBB	Cons	0.613	0.906	2.705	1.864	1.805	8.404	6.433	0.620	0.628
Wal-Mart Stores Inc	AA	Retl	0.447	0.990	0.480	0.362	0.358	1.014	0.721	0.460	0.342
Conagra Foods Inc	BBB	Manu	0.600	0.955	1.848	1.431	1.413	4.113	3.110	0.574	0.472
Nordstrom Inc	A	Retl	0.626	0.961	1.357	0.850	0.835	4.399	3.590	0.659	0.832
Southwest Airlines	A	Tran	0.354	0.971	1.223	0.951	0.891	2.561	1.825	0.355	0.345
Gap Inc	BB	Retl	0.333	0.931	4.297	3.103	2.970	8.100	5.276	0.357	0.298
American Express Co	A	Fin	0.788	0.954	0.766	0.594	0.588	0.821	0.634	0.795	0.754
Chubb Corp	A	Fin	0.920	0.880	0.925	0.617	0.610	3.115	2.303	0.922	0.950
Centurytel Inc	BBB	Info	0.291	0.953	2.655	1.966	1.928	5.956	4.139	0.265	0.227
Newell Rubbermaid Inc	BBB	Manu	0.391	0.967	1.861	1.319	1.292	2.574	1.705	0.430	0.489
Toys R Us Inc	B	Retl	0.466	0.580	24.661	18.731	18.348	34.245	24.950	0.499	0.313
CSX Corp	BBB	Tran	0.247	0.977	1.957	1.400	1.302	2.801	1.815	0.379	0.220
Wendys International Inc	BB	Acco	0.575	0.885	7.388	4.753	4.558	29.483	22.388	0.523	0.804
Cigna Corp	BBB	Fin	0.695	0.943	2.073	1.040	0.990	2.628	1.359	0.681	0.620
Limited Brands Inc	BBB	Retl	0.338	0.951	3.142	2.086	2.058	3.610	2.234	0.344	0.349
Norfolk Southern Corp	BBB	Tran	0.888	0.880	1.803	1.151	1.143	2.614	1.570	0.894	0.969
Countrywide Financial Corp	A	Fin	0.522	0.950	2.569	1.852	1.793	2.821	1.939	0.517	0.599
Dominion Resources Inc	BBB	Util	0.432	0.969	1.343	1.014	0.977	3.150	2.072	0.441	0.520
Belo Corp -Ser A Com	BBB	Info	0.833	0.757	4.267	3.149	3.114	5.079	3.596	0.834	0.859
Tribune Co	BB	Info	0.110	0.930	15.887	11.268	9.753	31.537	16.270	0.093	0.196
Verizon Communications Inc	A	Info	0.456	0.968	3.437	2.414	1.997	5.713	3.197	0.504	0.244
BellSouth Corp	A	Info	0.618	0.969	0.871	0.649	0.631	1.328	1.015	0.639	0.415
AT&T Inc	A	Info	0.875	0.894	0.991	0.752	0.724	1.389	1.069	0.869	0.882
Temple-Inland Inc	BBB	Manu	0.419	0.941	2.966	2.074	2.024	3.376	2.328	0.441	0.233
Home Depot Inc	A	Retl	0.497	0.986	0.786	0.540	0.522	2.079	0.785	0.520	0.341
American International Group	AA	Fin	0.859	0.931	1.232	0.791	0.750	3.520	2.574	0.868	0.777
Toll Brothers Inc	BBB	Cons	0.319	0.935	7.173	4.792	4.401	12.832	8.982	0.249	0.275
Anadarko Petroleum Corp	BBB	Min	0.404	0.972	1.945	1.334	1.272	2.799	1.863	0.389	0.405
Carnival Corp	A	Tran	0.537	0.972	0.900	0.649	0.621	1.583	1.159	0.506	0.567
Istar Financial Inc	BBB	Este	0.517	0.939	3.260	2.247	2.160	4.187	2.637	0.498	0.469
Harrahs Entertainment Inc	BB	Acco	0.386	0.914	9.397	6.921	6.792	29.182	18.546	0.304	0.216
Safeway Inc	BBB	Retl	0.572	0.934	2.355	1.715	1.656	2.659	1.885	0.572	0.446
CBS Corp	BBB	Info	0.425	0.950	2.275	1.628	1.585	6.554	4.482	0.432	0.238
MBNA Corp	BBB	Fin	0.690	0.942	2.696	1.803	1.590	2.941	1.840	0.774	0.926
Autozone Inc	BBB	Retl	0.347	0.948	2.575	1.920	1.897	18.596	15.748	0.330	0.608
Jones Apparel Group Inc	BBB	Manu	0.283	0.934	5.660	3.841	3.598	6.269	3.889	0.314	0.132
Time Warner Inc	BBB	Info	0.536	0.952	1.616	1.228	1.153	3.006	2.192	0.523	0.457
Macys Inc	BBB	Retl	0.366	0.967	1.682	1.196	1.135	2.076	1.527	0.236	0.343
First Data Corp	A	Info	0.316	0.954	10.746	8.206	6.284	22.578	9.271	0.241	0.240
Boston Scientific Corp	BBB	Manu	0.444	0.959	2.335	1.757	1.603	11.238	8.261	0.511	0.558
Tyson Foods Inc -CL A	BBB	Manu	0.599	0.903	2.711	2.008	1.894	12.873	7.227	0.536	0.576

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TABLE 3 (continued)
Firm-by-Firm Recovery Rate Estimates

Company Name	Rating	Industry	Recovery Rate ^a	5-Year Survival Prob. ^a	Estimated Recovery			40% Recovery			
					RMSE ^a	Abs. Err. ^a	Abs. Err. ^b	RMSE ^a	Abs. Err. ^a	Recovery Rate ^b	Recovery Rate ^c
	1	2	3	4	5	6	7	8	9	10	11
Radian Group Inc	A	Fin	0.750	0.909	2.318	1.590	1.548	2.739	1.733	0.766	0.739
Ace Ltd	A	Fin	0.771	0.917	2.396	1.319	1.295	2.499	1.371	0.760	0.704
Transocean Offshore Inc	A	Min	0.815	0.924	1.244	0.845	0.821	1.549	1.083	0.820	0.918
Allstate Corp	A	Fin	0.590	0.976	0.938	0.662	0.649	1.165	0.783	0.589	0.596
Universal Health Svcs—CL B	BBB	Hlth	0.522	0.932	3.192	2.318	2.283	3.618	2.421	0.504	0.550
Eastman Chemical Co	BBB	Manu	0.913	0.744	2.473	1.771	1.525	3.330	2.223	0.878	0.884
Simon Property Group Inc	A	Fin	0.868	0.886	1.449	0.999	0.931	1.690	1.128	0.854	0.863
Lear Corp	B	Manu	0.736	0.464	37.184	26.579	24.736	44.140	35.028	0.841	0.834
Capital One Financial Corp	BBB	Fin	0.519	0.968	1.568	1.022	0.982	9.865	4.931	0.584	0.484
McKesson Corp	BBB	Whol	0.386	0.966	1.916	1.187	1.145	2.165	1.362	0.444	0.530
Cox Communications Inc	BBB	Info	0.370	0.954	2.121	1.590	1.564	2.510	1.875	0.416	0.362
Washington Mutual Inc	A	Fin	0.520	0.962	1.890	1.403	1.319	1.982	1.473	0.557	0.541
Darden Restaurants Inc	BBB	Acco	0.602	0.946	2.446	1.865	1.813	6.342	4.417	0.556	0.602
Hartford Financial Services	A	Fin	0.373	0.980	1.292	0.758	0.709	1.449	0.841	0.378	0.379
Electronic Data Systems Corp	BBB	Pro	0.476	0.917	3.862	2.419	2.290	4.219	2.638	0.482	0.585
Sabre Holdings Corp—CL A	BBB	Info	0.079	0.928	11.363	8.779	8.572	14.338	9.956	0.060	0.319
Quest Diagnostics Inc	BBB	Hlth	0.861	0.851	3.370	2.507	2.397	4.197	2.895	0.870	0.698
Equity Office Properties Tr	BBB	Fin	0.590	0.943	3.318	2.235	1.970	5.856	4.345	0.715	0.531
Valero Energy Corp	BBB	Manu	0.796	0.912	1.653	1.226	1.168	4.314	3.204	0.794	0.798
Marriott Intl Inc	BBB	Acco	0.689	0.943	1.071	0.792	0.782	2.074	1.581	0.697	0.546
Sempra Energy	BBB	Util	0.605	0.958	1.662	1.188	1.140	2.023	1.476	0.608	0.681
XL Capital Ltd	A	Fin	0.255	0.974	1.692	1.219	1.140	2.810	1.882	0.256	0.416
American Axle & Mfg Holdings	BB	Manu	0.385	0.740	25.078	18.324	17.375	42.422	31.527	0.341	0.290
Devon Energy Corp	BBB	Min	0.390	0.973	1.857	1.236	1.181	2.658	1.663	0.397	0.393
MetLife Inc	A	Fin	0.256	0.985	0.876	0.613	0.592	1.145	0.784	0.264	0.088
Visteon Corp	B	Manu	0.348	0.625	48.731	30.058	29.712	61.921	42.712	0.364	0.269
Ford Motor Co	B	Manu	0.545	0.647	30.077	22.181	20.159	34.702	27.795	0.759	0.818
Aetna Inc	A	Fin	0.465	0.976	1.768	0.943	0.892	2.658	1.375	0.468	0.442
Kraft Foods Inc	A	Manu	0.565	0.972	0.980	0.712	0.691	5.287	4.249	0.607	0.519
Cit Group Inc	A	Fin	0.644	0.953	1.620	1.045	0.989	2.118	1.361	0.659	0.537
Comcast Corp	BBB	Info	0.443	0.966	1.377	1.028	1.009	1.531	1.066	0.463	0.210
News Corp	BBB	Info	0.950	0.627	1.503	1.088	1.067	2.371	1.270	0.944	0.941
Starwood Hotels & Resorts Wrld	BBB	Acco	0.523	0.887	6.306	4.772	4.704	6.600	4.771	0.532	0.654

5-year tenor in estimation. We explain three tenors using one set of parameters, which is a considerably more challenging task. Our model therefore seems to provide a good fit. This confirms the findings of many papers in the literature that latent factor models generally provide a very good fit for credit risk modeling. In light of this finding, the improvement in fit provided by the single extra recovery parameter is particularly impressive.

The improvement in fit provided by the model with estimated recovery rate is confirmed when using other loss functions. Column 7 of Table 3 reports the fit based on the mean absolute error for the model with estimated recovery rate. Column 6 reports the mean absolute error using the parameter values estimated using the RMSE-based loss function. Consistent with the insights of Granger (1969), the errors in column 6 are larger than the ones in column 7, where the estimated parameters are based on the mean absolute errors. Column 9 reports the mean absolute error with 40% recovery rate; the deterioration in fit compared to the model with estimated recovery in column 6 is again substantial.

C. The Risky Term Structure: Out-of-Sample Model Fit

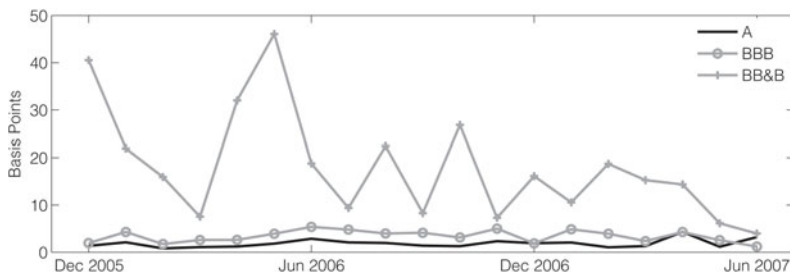
A model's out-of-sample performance is as important as its in-sample fit. For our model, it is important to assess out-of-sample performance using a recursive exercise, but reliably reestimating the model very frequently for all firms in the sample is prohibitively expensive from a computational perspective. We therefore conduct a more limited recursive analysis. We aggregate sample spreads by rating categories. We use the rating categories with the largest number of firms: A, BBB, and a combination of BB and B-rated firms. Estimation is based on minimizing expression (19). We always forecast the state variables and the CDS spreads up to 1 month out of sample, using the prior 12 months to estimate the model parameters. For example, our first out-of-sample month is Dec. 2005. We compute RMSEs for this month for all days for which data are available. The parameters and latent state variables used for this forecasting exercise are estimated using daily data for the period Dec. 1, 2004–Nov. 30, 2005. Subsequently, we forecast daily data for Jan. 2006 using estimates obtained using the sample Jan. 1, 2005–Dec. 31, 2005, and so on.

Figure 2 reports out-of-sample RMSEs for the 5-year maturity. We aggregate the RMSEs by month to avoid reporting different forecasting horizons in the same figure. Out-of-sample RMSEs in Graph A strongly depend on ratings, which confirms the in-sample results in Table 3. The relative RMSEs in Graph B indicate that as a percentage of the spreads, the errors are roughly similar across rating

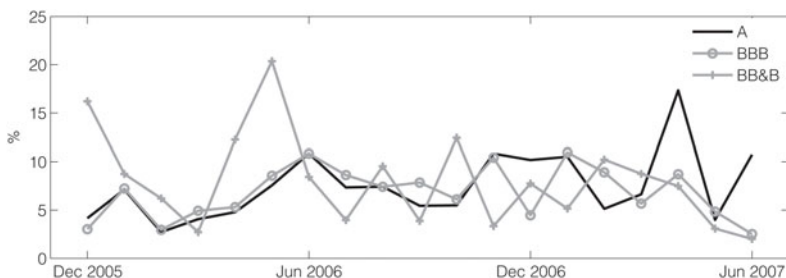
FIGURE 2
Out-of-Sample RMSE and Relative RMSE

Graph A of Figure 2 shows the time series of the out-of-sample RMSE for the 5-year CDS spread in basis points for firms in three different ratings categories. Graph B shows the time series of the relative RMSE (i.e., RMSE divided by the CDS spread). Estimation is based on minimizing the RMSE for all three maturities.

Graph A. RMSE



Graph B. Relative RMSE



categories. RMSEs and relative RMSEs fluctuate considerably over the sample period for the lower-rated companies. However, keeping in mind that we forecast up to 1 month out of sample, the out-of-sample performance of the model is generally good: On average the relative RMSE is 7.49%, 6.81%, and 8.03% for the firms rated A, BBB, and BB/B, respectively.

D. Econometric Identification

Econometric identification is an important consideration when estimating recovery rates. See, for instance, Pan and Singleton (2008) and Schneider et al. (2010) for discussions. Pan and Singleton (2008) argue that the use of multiple tenors in estimation helps identification, and for this reason we simultaneously use three tenors in estimation.

We perform a Monte Carlo analysis to assess the robustness of our estimation methodology and to detect potential identification problems. We simulate time series of 3 years worth of daily CDS spreads by Monte Carlo for a typical firm in our sample, for all three tenors. We subsequently perturb the parameters of the data generating process by adding a random noise, drawing from a normal distribution with zero mean and standard deviation equal to 2 standard deviations of the empirical distribution of parameters. We use the resulting parameter values as starting values for the numerical search that fits the simulated CDS spreads. We repeat this experiment 100 times.

The online Appendix presents the results of the Monte Carlo exercise and discusses the results in more detail. The distribution of the parameter estimates is tight around the parameters of the data generating process. The averages of the estimated recovery rates are very close to the true recovery rate, and the same applies to the parameters governing default probabilities. The *t*-statistics for the differences between the estimated parameters and the true parameters suggest that the differences are not statistically significant. This Monte Carlo experiment confirms that our econometric methodology is able to reliably estimate model parameters, and recovery rates in particular.

V. The Cross Section of Recovery Rates

We first discuss estimates of recovery rates in the existing literature. Subsequently, we discuss our recovery estimates, and how differences between these estimates and the standard 40% recovery rate assumption affect estimates of default probabilities. Finally, we document the economic determinants of the estimated recovery rates.

A. Existing Estimates of Recovery Rates

This paper is part of a growing literature on the estimation of recovery rates. Most papers provide estimates of long-run (unconditional) historical average recovery rates.⁸ Based on these estimates, a fixed 40% recovery rate is often

⁸Altman and Karlin (2008) report an average of 36.95% for senior unsecured bonds for the 1978–2007 period, Altman et al. (2005) report an average of 41.8% for all bonds for 1982–2001,

assumed in industry practice and many existing studies when estimating credit risk models.⁹ To illustrate the importance of estimating the recovery rate more reliably, the online Appendix contains a figure indicating that an incorrect estimate of the recovery rate affects the CDS premium even more than an incorrect estimate of the survival probability, especially for large deviations. It also shows that the resulting changes in CDS premiums are large compared to the RMSEs in our empirical exercise.

Historical estimates require long time series of realized defaults and serve as a good benchmark for recovery rates. However, these estimates are different from ours in several ways. First, reliable estimation requires long sample periods, and thus these estimates are best thought of as unconditional, whereas our approach can provide conditional estimates using short samples. Second, realized recovery rates in historical studies are computed up to 30 days after the trigger event. This introduces substantial heterogeneity between firms in the sample, as argued, for example, by Guo et al. (2008). Our estimates avoid this heterogeneity problem, because we rely on market information at the same time for all firms in the sample. Third, historical estimates are obtained under the physical measure and cannot be used directly for pricing, while our estimate can be used directly and jointly with risk-neutral probabilities for different valuation applications. Fourth, as noted by many prior researchers, papers reporting historical estimates do not report their estimation methodology and are therefore uninformative about the statistical significance of the estimates. See, for instance, Emery, Ou, and Tennant (2008).

Other studies estimate risk-neutral recovery rates from credit-risky securities. Christensen (2007) estimates a stochastic recovery model using CDS data but limits himself to one firm. Bakshi et al. (2006b) also estimate recovery rates, but their study is different from ours along several dimensions. The most important difference between both studies is the data: Bakshi et al. (2006b) estimate BBB-rated bonds, while we use a large cross section of CDS contracts on firms with different ratings. Bakshi et al. (2006b) assume that the dynamic of the default intensity is governed solely by the factor that drives the risk-free rate, while we let the default intensity depend on latent firm-specific factors as well as the risk-free term structure. Moreover, our default intensity is a quadratic function of the state variables, as opposed to an affine function in their approach. Bakshi et al. (2006b) compare the fit of alternative recovery rate assumptions and model stochastic recovery as opposed to constant recovery, by inversely relating the recovery rate to the default probability. They also relate risk premia to moments of the physical distribution.¹⁰

and Altman and Kishore (1996) report an average of 47.65% for senior unsecured bonds for the 1978–1995 period. Emery, Ou, Tennant, Matos, and Cantor (2009) report an average of 36.4% for senior unsecured bonds for 1982–2008. These studies also report recovery rates on a year-by-year basis.

⁹See, for instance, the discussion in Pan and Singleton (2006) on industry practice. The 40% assumption is based on long-term historical averages (see, e.g., the discussion in Duffie and Lando (2001) and Guo, Jarrow, and Lin (2008)). Chen et al. (2008), Almeida and Philippon (2007), and Thorburn (1997) use a 40% recovery assumption; Duffee (1999) assumes 44%; Ju, Parrino, Potoshman, and Weisbach (2005) assume 45%; and Longstaff et al. (2005) use a 50% fixed recovery rate.

¹⁰We do not attempt to analyze risk premia because the richer parameterization might compound identification problems, but this is definitely an interesting avenue for further research.

To remedy identification problems, several studies estimate recovery rates by combining the valuation of credit-risky instruments with other securities. Jarrow (2001) proposes a methodology using both debt and equity prices. Le (2008) uses information from option prices to estimate the dynamics of the risk-neutral default intensity. The implied default intensity is then used along with 5-year CDS spreads to compute the implied recovery rates. Das and Hanouna (2009) use the term structure of CDS spreads together with stock prices and volatility to extract the term structure of default probabilities and recovery rates. Their model requires calibration at each point in time, while we estimate our model using the time-series information for the term structure of CDS spreads. Using an arbitrage argument between out-of-the-money put options and credit markets, Carr and Wu (2010) estimate a constant recovery rate model for eight firms for which the \$1 in default security is traded. Alternatively, one can rely on securities with different seniorities to overcome the identifiability problem. Unal et al. (2003) and Madan and Unal (1998) use data on junior and senior debt to identify risk-neutral recovery rates.

Jointly modeling equity or options together with CDS contracts presumes, of course, that these markets are fully integrated. This may bias results, as pointed out by Carr and Wu (2011). For example, the portfolios of insurance companies, which are important investors in credit markets, are restricted to investment-grade securities. Using different seniorities is limited to firms with traded senior and junior securities, which amounts to less than 3% of available trades. Such institutional constraints may imply different premia on restricted versus unrestricted markets. We therefore pursue an alternative approach and attempt to improve identification by using credit default swaps with different maturities in estimation.

Our results are most closely related to Jarrow et al. (2009) and Schneider et al. (2010). Jarrow et al. focus on potential statistical arbitrage opportunities in the term structure of CDS spreads, but they also provide estimates of implied recovery rates. They use one factor to capture the dynamics of credit risk and then estimate an affine credit risk model for each firm. Schneider et al. estimate an affine jump model with a constant recovery rate for a large cross section of firms. We obtain very different results from these two studies. Jarrow et al. report an average recovery rate of approximately 50% but find very limited cross-sectional variation in this rate. Schneider et al. find a much higher average recovery rate of 79% and a median of 90%.

B. The Cross Section of Risk-Neutral Recovery Rates

Column 3 of Table 3 indicates that the average recovery rate across 152 firms is 53.79%, much higher than the 40% estimate used in most of the existing literature.¹¹ As discussed above, there are several possible explanations for this

¹¹The last two columns of Table 3 report recovery rates estimated based on absolute errors and the time-series estimation. Recovery rates are in most cases similar to the ones in column 3, and the pairwise correlations between the three sets of recovery rates are all over 0.95. We therefore focus on the recovery rate estimates in column 3.

difference. Perhaps most importantly, our sample covers the years 2004–2007, which is a relatively expansionary part of the business cycle. For senior unsecured bonds, Altman and Karlin (2008) report a recovery rate of 56.77% for 2004, 45.88% for 2005, 60.90% for 2006, and 47.70% for 2007, all substantially above the long-run average. Emery et al. (2009) report the following recovery rates for senior unsecured bonds: 52.09% for 2004, 54.88% for 2005, 55.02% for 2006, and 53.25% for 2007. These numbers are very close to our estimates.

The CDS data embed expectations of ultimate recoveries. Emery et al. (2009) report that ultimate recoveries may substantially exceed conventional historical estimates based on data available up to 30 days after the trigger event. For 2007, the ultimate recovery rate for their sample is estimated at 60.0%, as opposed to 53.3% using the conventional method. For 2008, the difference is much greater, 74.0% compared to 33.8%. Moreover, our estimated recovery rate is forward looking, and the implied recovery rate therefore ought to be the market's expectation of recovery over the life of the contract. In our case, the effective maturity is a weighted average of 1, 3, and 5 years, as all three maturities are used in estimation. Finally, we estimate risk-neutral recovery rates, which may differ from historical ones due to a risk adjustment. Not much is known about the difference between historical and risk-neutral recovery rates, and the implied price of recovery risk. Unal et al. (2003), using corporate bond data, find that, on average, implied risk-neutral recovery rates lie systematically below the physical recovery rates reported in Altman and Kishore (1996).

Graph A of Figure 3 depicts the histogram of estimated recovery rates. Our results reveal substantial cross-sectional variation in estimated recovery rates. Table 3 indicates that while the average recovery rate is 53.79%, the smallest recovery rate is 7.85%, for Sabre Holdings, a technology company, and the largest recovery rate is 95.04%, for News Corporation. The cross-sectional standard deviation of the estimated recovery rates is 22.96%. This is very close to the historical standard deviation of 25% reported by Altman and Karlin (2008). However, our findings contrast with Jarrow et al. (2009), who report very limited variation in recovery rates across firms. Schneider et al. (2010) report a standard deviation of 21.93%, similar to our finding. The minimum recovery rate in their sample is 3.36% and the maximum is 99.84%.

It is also interesting to compare the histogram in Figure 3 with reported distributions for historical recovery rates, which are typically obtained by taking a grand time-series cross-sectional average. Schuermann (2004) reports that the distribution of recovery rates is not unimodal, and this can also be seen from the evidence in Altman and Karlin (2008). Our findings in Figure 3 seem to indicate relatively more cases of high recovery rates than the distributions reported in these papers, but once again we must keep in mind that the sample period is very different.

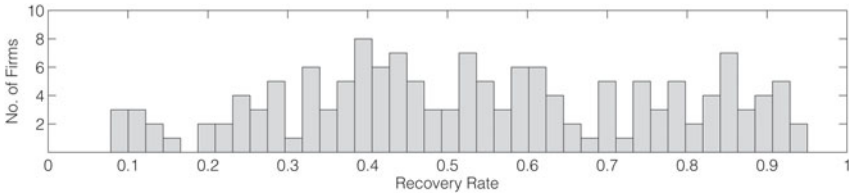
In summary, we believe our estimated recovery rates are plausible. Compared to historical estimates, they are relatively similar, in view of the sample period, and the estimated cross-sectional standard deviation is very close to the long-run historical average. Our average recovery estimate is almost identical to Jarrow et al. (2009), but the cross-sectional variation in our estimates is higher. Compared to Schneider et al. (2010), on average our estimates are much lower,

FIGURE 3

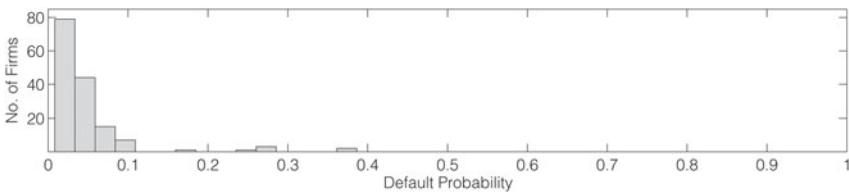
Distribution of Estimated Recovery Rates and 5-Year Default Probabilities

Figure 3 shows the histogram of estimated recovery rates and estimated 5-year default probabilities using all 152 firms. In Graph A, the y-axis indicates the number of firms with a particular recovery rate as indicated by the x-axis. In Graphs B and C, the y-axis indicates the number of firms with a particular 5-year default probability as indicated by the x-axis. Graph B contains the case of 40% recovery, and Graph C contains the case of estimated recovery.

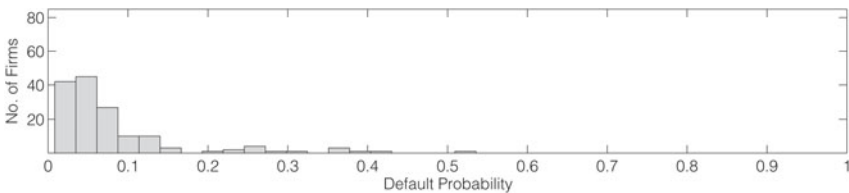
Graph A. Distribution of Recovery Rates



Graph B. Distribution of Default Probabilities with 40% Recovery Rates



Graph C. Distribution of Default Probabilities with Estimated Recovery Rates



and in our opinion much more plausible, especially because historical recovery rates are on average smaller in their sample period. It is difficult to ascertain what drives these differences besides sample composition and sample period, but it is possible that the quadratic nature of our model allows us to more reliably estimate recoveries.

C. Recovery Rates and Default Probabilities

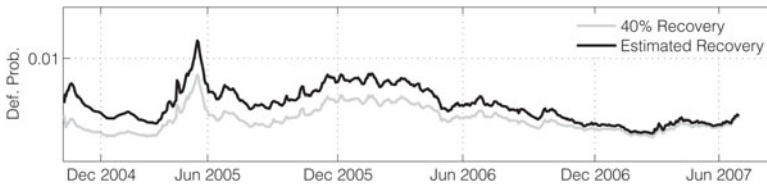
We now investigate the relation between estimated recovery rates and estimated default probabilities. We first investigate how estimating the recovery rate, as opposed to keeping it fixed at 40%, affects the estimates of default probabilities. We then compute simple cross-sectional correlations between the CDS-implied recovery rates and default probabilities. This relationship is of substantial interest, especially because little is known about it from historical data. Indeed, by definition the study of this relationship at the firm level is problematic using historical data, and one has to resort to computing correlations between ex post recovery and ex post default probabilities for groups of firms.

Graphs B and C of Figure 3 present the histogram of estimated 5-year default probabilities obtained when estimating the recovery rate, and contrast it with the histogram of estimated 5-year default probabilities obtained when fixing the recovery rate at 40%. Figure 4 presents the time path of average default probabilities under both assumptions, for all three tenors. Estimating the recovery rate clearly has a substantial impact on estimated default probabilities, making them larger on average. This is not surprising, because the average estimated recovery rate is 53.79%, substantially higher than 40%. To obtain the same price for a given credit-risky security with a higher recovery rate, the default probability needs to be higher. What is more surprising is how much higher the estimated default probability paths are. For the 1-year tenor in Graph A of Figure 4, the average of the path obtained with an estimated recovery rate is 34% higher than in the case of the fixed recovery rate. For the 3-year tenor, it is 53% higher. For the 5-year tenor, it is 67% higher.

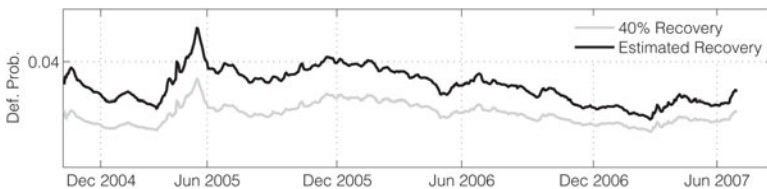
FIGURE 4
Default Probabilities with 40% Recovery and Estimated Recovery

Graph A of Figure 4 shows the time series of 1-year default probabilities averaged across all firms. Graph B shows the time series of 3-year default probabilities averaged across all firms. Graph C shows the time series of 5-year default probabilities averaged across all firms. In each case we show probabilities obtained when recovery rates are set equal to 40% (in grey), as well as probabilities obtained when recovery rates are estimated (in black).

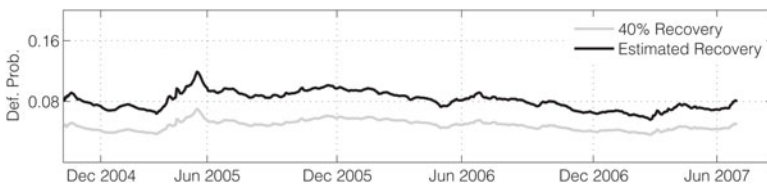
Graph A. 1-Year Default Probabilities



Graph B. 3-Year Default Probabilities



Graph C. 5-Year Default Probabilities



The average estimated recovery rate of 53.79% is 34% higher than the standard 40% assumption. The difference between the 1-year default probabilities is of approximately the same order of magnitude, but the difference is substantially greater for 5-year default probabilities. It is necessary to understand why this is

the case, because 5-year default probabilities are very important for credit markets. When pricing credit-sensitive securities such as CDOs and CDO tranches, default probabilities estimated from CDS contracts are one of the most critical inputs into the model, and 5-year default probabilities are most often used. Our results therefore indicate that one of the most important inputs in these models is subject to a 67% misspecification if a 40% recovery rate is used, which is a standard assumption in industry. This would cause dramatic mispricing of these securities. Our findings are consistent with recent Securities and Exchange Commission (2010), (2011) proposals on the regulation of structured products, which emphasize the importance of heterogeneity in the pool of CDO assets and the importance of a thorough assessment of the model inputs.

Why are estimated 5-year default probabilities so much higher when estimating the recovery rate? The reasons for this are multiple, and can be traced back to several model parameters. However, we found that the main reason is the estimated persistence of the firm-specific factor (9). In order to demonstrate this, some back-of-the-envelope calculations are very instructive. Consider the 1-day survival probability (13). We can compute this for every day of our sample, which yields a 1-day default probability of $1.56e^{-5}$ when the recovery rate is estimated, on average across time and across firms. When the recovery rate is fixed at 40%, this average is equal to $1.32e^{-5}$. Therefore, for our sample the 1-day default probability is 17% higher when the recovery rate is estimated. Now consider the multiperiod default probability implied by expression (14). If the hazard rate process is characterized by an AR(1) (autoregressive specification of order 1) process with unit root without drift, future expected hazard rates are equal to the current hazard rate. We can therefore compute 1-, 3-, and 5-year default probabilities with estimated or 40% recovery rates, and also the ratio between these default probabilities. Under the unit root assumption, the default probabilities are approximately 17% higher when recovery is estimated, regardless of the maturity.

However, Table 2 indicates that while the persistence of the state vector governing the hazard rate is very high, it is of course not always exactly 1. In fact, on average, the persistence of the firm-specific factor (9) is 0.9999. When assuming a 40% recovery rate, the average persistence is slightly lower, at 0.9994. This small difference in persistence can have substantial implications, because the persistence is daily, and the maturity of the CDS contracts is up to 5 years. To see this, consider deterministic hazard rate processes with persistence of 0.9999 and 0.9994 and no intercept, and compute default probabilities using initial 1-day default probabilities of $1.56e^{-5}$ and $1.32e^{-5}$. For the 1-year maturity, resulting default probabilities are 28% higher in the case of estimated recovery rates. For 3- and 5-year maturities, default probabilities are 51% and 75% higher. Small differences in the daily persistence of the state vectors, and by implication of the hazard rate, can therefore lead to large differences in default probabilities. While this simple back-of-the-envelope calculation does not take all model features into account, the resulting differences in default probabilities are similar to those documented in Figure 4 and discussed previously.

Moreover, the higher persistence of the firm-specific factor (9) when recovery rates are estimated is a robust stylized fact, and it obtains for 126 out of 152 companies in the sample. We therefore conclude that for a given sample,

assuming a 40% recovery rate leads to important biases in estimated 5-year default probabilities for at least two reasons. First, the actual or expected recovery rate in the sample period may be different from 40%. Second, the 40% assumption leads to a bias in the estimated daily persistence of the hazard process, and this substantially impacts the pricing of CDS contracts with long maturities.

Given prices on a credit-risky security, a higher estimated recovery rate implies a higher default rate. However, this relationship is different from the cross-sectional relation between the estimated recovery rate and average default probability for different firms. The online Appendix contains a scatterplot of estimated recoveries and average 5-year default probabilities for all 152 firms. The correlation between average default probabilities and recoveries is positive, at 38.79%. Correlations for 1- and 3-year default probabilities are also positive, but somewhat lower. Note that different results may obtain if one estimates both the recovery rate and the default probability as time varying.

This estimated cross-sectional correlation is also very different in nature from the existing evidence on the time-series correlation between aggregate ex post default and recovery rates. The existing literature for the most part finds a negative time-series correlation between default and recovery rates (see, e.g., Altman et al. (2005), Carey (1998), Schuermann (2004), and Emery et al. (2009)). Carey and Gordy (2003) find that the sign of the correlation depends on the sample period. By construction, our estimates are uninformative about this, because our estimated recovery rate is constant over time.

D. Determinants of Recovery Rates

We now analyze the determinants of recovery rates. We first analyze how recovery rates differ between industries and rating categories. Subsequently, we closely follow the regression approach in Acharya et al. (2007) to analyze the determinants of cross-sectional variation in recovery rates. Whereas Acharya et al. use historical recovery rates in their analysis, we use the risk-neutral recovery rates from Table 3.

Table 1 contains descriptive statistics on spreads by industry and rating, and it indicates that spreads are higher for lower-rated firms and that there is substantial variation in average spreads across industries. To what extent are these differences due to differences in expected recovery? While, on average, higher ratings are associated with higher recovery rates for our estimates, the relationship is not monotonic. This is consistent with the literature on historical recovery rates, which emphasizes that seniority and industry are more important determinants of recovery than ratings. Indeed, we find substantial variation of estimated recovery rates across industries.

We therefore attempt to explain recovery rates using firm characteristics and industry conditions. We compute firm-specific and industry variables following Acharya et al. (2007). The data are discussed in detail in the online Appendix. We have 4 years of data, but only one recovery rate per firm.¹² We run panel

¹²Using a long time series of CDS spreads facilitates the reliable estimation of model parameters, including recovery rates. While it may prove difficult to reliably estimate recovery rates using very

regressions where we keep the firm's recovery rate constant over the 4 years, but we allow the firm-specific variables to change from year to year. The panel regressions therefore contain $4 \times 152 = 608$ observations. Table 4 presents the results from the panel data regressions, using White (1980) standard errors. Results using two industry distress measures, as described in the online Appendix, are very similar.

TABLE 4
Determinants of Recovery Rates

Table 4 reports ordinary least squares (OLS) estimates for regressions of recovery rates on firm and industry variables. The *t*-statistics based on White (1980) standard errors are reported in parentheses. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	1	2	3	4	5	6	7
Firm size	0.0361*** (3.98)	0.0304*** (3.91)	0.0356*** (3.91)	0.0378*** (4.29)	0.0369*** (4.09)	0.0360*** (3.90)	0.0363*** (3.98)
Leverage	-0.1570** (-2.02)	-0.1360* (-1.86)	-0.1480* (-1.86)	-0.1570* (-1.94)	-0.1440* (-1.77)	-0.1760** (-2.19)	-0.1530* (-1.93)
Tangibility	0.0811** (2.10)	0.0077 (0.26)	0.0787** (2.01)	0.0446 (1.18)	0.0622* (1.68)	0.0524 (1.40)	0.0802** (2.06)
Industry Q	-0.0283 (-0.98)	0.0095 (0.42)	-0.0312 (-1.06)	-0.0331 (-1.10)	-0.0378 (-1.25)	-0.0276 (-0.91)	-0.0294 (-0.99)
Industry asset specificity	-0.1230* (-1.79)		-0.1210* (-1.76)				-0.1230* (-1.79)
Industry distress		-0.0197 (-0.54)	-0.0205 (-0.55)	-0.0483 (-0.96)	-0.0277 (-0.74)	-0.0230 (-0.62)	
Distress \times Specificity				0.1110 (0.70)			
Leverage \times Specificity					-0.2200 (-1.12)		
Illiquidity \times Specificity						-0.0636 (-0.88)	
Peer \times Distress							-0.0000261 (-0.31)
Intercept	0.2230** (2.08)	0.2600*** (2.73)	0.2310** (2.13)	0.2100** (1.97)	0.2180** (2.01)	0.2320** (2.12)	0.2210** (2.06)
No. of obs.	399	546	399	399	394	386	399
R^2 (%)	6.4	3.3	6.5	5.8	6.1	6.1	6.4

Most coefficients are estimated with the a priori expected sign and are consistent with intuition, similar to the findings of Acharya et al. (2007). Large firms have higher recovery rates. High tangibility of assets implies high recovery rates. All of the estimated tangibility coefficients are positive, and many are statistically significant. Acharya et al. also find a positive effect, but their estimates are largely statistically insignificant. Similar to Acharya et al., we find that the direct effect of an industry's asset specificity on recovery rates is negative and significant. We also find that the firm's leverage negatively affects the recovery rate.

We investigate a number of interactions of these variables. The interaction of industry-level leverage and industry asset specificity is negative but not statistically significant. An industry's asset specificity causes lower recovery rates if the

short time series, a compromise where one reestimates the model every year should yield good results, and for the purpose of the exercise in this section it would be preferable. However, the computational cost of this exercise is very high, because we are working with a very large cross section of firms.

leverage of this industry is generally high. Also, similar to Acharya et al. (2007), we find that the interaction of industry illiquidity and industry asset specificity is negative.

Our estimates yield the expected sign for the direct effect of industry distress on recovery rates: When a firm is in a distressed industry, it tends to recover less. Note that for both distress measures used, only approximately 5% of the firms are in distress. Coupled with the limited sample size, this may explain why estimates are not statistically significant. We limit ourselves to firms that were part of the CDX index during our sample period. For estimation, this is beneficial because the population of firms is more homogeneous. For the analysis of the determinants of recovery rates, it is a disadvantage because it reduces the sample size, and sample heterogeneity may actually be an advantage when identifying these determinants.

In summary, Table 4 indicates that variables such as asset specificity, tangibility, and leverage are important determinants of recovery rates. This conclusion is consistent with the available evidence from historical recovery rates.

VI. Conclusion

We estimate recovery rates implied by CDS spreads, using more than 3 years of daily CDS spreads for three maturities on 152 North American corporations. We use a quadratic pricing model, which ensures nonnegative default probabilities and recovery rates. The estimated cross section of recovery rates is plausible, with an average recovery rate of 53.79% and substantial cross-sectional variation. Estimated 5-year default probabilities are on average 67% higher than default probabilities obtained using the standard 40% recovery assumption, suggesting potentially large biases in the valuation of CDOs when relying on estimates obtained with fixed 40% recovery rates. This is due to the fact that in our sample, the estimated recovery rate is higher than 40% on average, but also due to biases in the estimated persistence of the state vector, which critically impacts long-horizon default probabilities. Consistent with the evidence on historical recovery rates, larger firms and firms with more tangible assets have statistically significantly higher recovery rates, and the opposite is true for firms with higher leverage and more assets that are industry specific.

Our results suggest a number of extensions. First, the high persistence of the firm-specific factor, and the importance of this persistence for default probabilities at long horizons, suggest a detailed evaluation of alternative specifications of the state vector. Second, it might be interesting to subdivide the sample to study changes in the recovery rate over time. Alternatively, analyzing stochastic recovery could yield new insights. Finally, a detailed exploration of the implications of differences in estimated 5-year default probabilities for the valuation of different CDO tranches could prove interesting.

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