

Les fonctions combinatoires et les isols, by J.C.E. Dekker. Paris, 1966. Collection de logique mathématique, série A No. 22. Gauthier-Villars. 74 pages.

This booklet is a report on the work by Myhill and Nerode on combinatorial functions (see below) and on the work by Dekker on isols (see below). It is well written and easy to read for a non-expert; only the second part requires some rudimentary knowledge of recursive functions, at least the definition.

Let  $f: N \rightarrow N$  be a function from the set of natural numbers ( $\geq 0$ ) into itself; then there exists a unique sequence of integers  $c_0, c_1 \dots$  such that

$$f(n) = \sum_{i=0}^n c_i \binom{n}{i} .$$

This is proved in a fairly complicated way on page 18, using a construction of the  $c_i$  by recurrence. The reviewer wishes to point out that, as a simple exercise in the umbral notation of Lucas, one may show that for each  $f: N \rightarrow Z$  there exists a unique  $g: N \rightarrow Z$  such that

$$f(n) = \sum_{i=1}^n (-1)^i g(i) \binom{n}{i} ,$$

to wit

$$g(i) = \sum_{n=0}^i (-1)^n f(n) \binom{i}{n} .$$

The function  $f: N \rightarrow N$  is called combinatorial if and only if all  $c_i \geq 0$ .

In the present treatment this a theorem, not the definition. The latter is rather sophisticated and depends on the possibility of extending  $f$  to a function from the set of all subsets of  $N$  into itself which satisfies certain conditions.

Two sets  $A$  and  $B$  of natural numbers are said to be recursively equivalent if there is a partial recursive function whose domain contains  $A$  and which establishes a one-one correspondence of  $A$  onto  $B$ . Assume that  $A$  has no recursively enumerable subset, then the class of all  $B$  recursively equivalent to  $A$  is called an isol. In particular, a finite number  $n$ , identified with the set  $\{0, 1, \dots, n-1\}$ , is an isol. Addition and multiplication of naturals may be extended to isols. The resulting system satisfies the usual laws and may be embedded into a ring, the so-called ring of isolic integers.

Every combinatorial function may be extended canonically to a function of the set of isols into itself. Aside from this result, it is not clear to the reviewer why the two topics are treated under one cover.

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Truth functions and the problem of their realization by two-terminal graphs, by A. Ádám. Akadémiai Kiadó, Budapest, 1968. 206 pages. U.S. \$7.80.

This book consists of two parts, the first being a survey of the mathematical