

## ARTICLES

# NONLINEARITIES IN THE DYNAMICS OF THE EURO AREA DEMAND FOR M1

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This paper finds evidence of nonlinearities in the dynamics of the euro area demand for the narrow aggregate M1. A long-run money demand relationship is first estimated over a sample period covering the past three decades. Although the parameters of the relationship are jointly stable, there are indications of nonlinearity in the residuals of the error-correction model. This nonlinearity is explicitly modeled using a fairly general Markov switching error-correction model with satisfactory results. The empirical findings of the paper are consistent with theoretical predictions of nonlinearities in the dynamics of adjustment to equilibrium stemming from “buffer stock” and “target-threshold” models and with analogous empirical evidence for European countries and the United States.

**Keywords:** Euro Area, Cointegration, Nonlinear Error-Correction, Money Demand

## 1. INTRODUCTION

Linear models embodying error-correction mechanisms have become the standard macroeconometric tool in the empirical literature on money demand [see Sriram (2001); Duca and van Hoose (2004)]. They combine a theoretically grounded description of the behavior of money demand at equilibrium with a data-driven specification of the (linear) dynamics of disequilibrium correction in the short run. One of the main reasons for their popularity is that these models have been able to provide a statistically meaningful representation of the observed sluggishness in the portfolio allocation behavior of economic agents. Yet such sluggishness derives from the existence of market rigidities, such as portfolio adjustment costs, that may also translate into nonlinearities in the dynamics of adjustment to equilibrium.

Nonlinearities in the dynamics of money demand are typically rationalized on the basis of “target-threshold” and “buffer stock” theoretical models.<sup>1</sup> Miller and

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Orr's (1966) inventory-theoretic model of the demand for transaction balances by firms is a representative example of a target-threshold model, whereas the models of Cuthbertson and Taylor (1987) and Gandolfi and Lothian (1983) fall under the category of buffer stock models. These models start from the observation that, due to shocks of various nature, the monetary holdings of individual agents may depart from their desired or "target" levels. However, in the presence of adjustment costs, it may not be optimal for agents to readjust their asset portfolios immediately to bring their balances back to the target straight away. The optimal response may be to let monetary balances fluctuate as a temporary buffer stock until the other assets can be adjusted. Only when the deviations of monetary holdings from the desired levels become relatively large or exceed some specified threshold do agents engage in those transactions needed to bring their balances back to the target.

These theoretical frameworks have been developed to explain nonlinearities in the money demand behavior of individual agents. However, the microeconomic frictions arising from portfolio adjustment costs may also result—under certain conditions—in persistent deviations of the aggregate long-run money demand from the equilibrium level and in nonlinearities in the short-run monetary dynamics. Bertola and Caballero (1990) argue that, in the presence of kinked adjustment costs, such conditions are related to the degree of coordination and synchronization across individual agents, which is—in turn—likely to depend on the relative importance of aggregate and idiosyncratic uncertainty. When the former predominates, the aggregate variables also display, at least to some extent, the type of sluggish dynamic adjustment associated with microeconomic money demand frictions.

Consistent with these theoretical predictions, in recent years some authors have found empirical evidence of nonlinearities in the short-run dynamics of monetary aggregates. Sarno (1999), Lütkepohl et al. (1999), Teräsvirta and Eliasson (2001), Ordóñez (2003), Sarno et al. (2003), and Chen and Wu (2005) model such nonlinearities for various European countries and the United States using regime-dependent models, usually smooth-transition regressions. Hendry and Ericsson (1991) and Escribano (2004) instead model the nonlinearities in the money demand in the United Kingdom using polynomial error-correction models (e.g., cubic polynomials), with the latter author also considering semiparametric methods.

Although these papers differ in terms of sample period, country coverage, and econometric methodology, a common finding is that the strength of the short-run dynamics of money demand is state-dependent. In particular, these papers typically find that the loading factor of the money demand error-correction term varies over time, depending on the regime prevailing (with such regimes governed by the error-correction term itself or by movements in specific real or nominal variables). A regime-switching loading factor implies that the size of the adjustment relative to the previous disequilibrium is not constant over the full sample period, but varies across regimes.<sup>2</sup> In addition, some studies find that the coefficients of the explanatory variables, notably the autoregressive terms, may also be regime-dependent.

The purpose of this paper is to investigate whether there is similar evidence of nonlinearity in the (short-run) dynamics of the demand for euro area M1. In general terms, the focus on a narrow aggregate such as M1 can be explained by the fact that it is a close empirical counterpart of the notional monetary balances featuring in the relevant theoretical models [e.g., Miller and Orr's (1966) target-threshold model]. Besides, like other monetary aggregates, M1 can effectively summarize the information available in key macroeconomic fundamentals, such as output, prices, and interest rates. Nelson (2003) has recently noted that, by proxying a spectrum of yields that matter for aggregate demand but are not always directly observable, monetary aggregates such as M1 may provide incremental information about aggregate demand.

There are also factors specific to the euro area that render the analysis of the dynamics of M1 of significant interest for monetary policy purposes. Indeed, in the euro area M1 exhibits a number of empirical properties that make it an important component of the information set available to policy makers.<sup>3</sup> In particular, changes in real M1 seem to contain useful information about developments in area-wide output up to three years ahead. In addition, over the past two decades, turning points in M1 growth have often reliably predicted those in the general euro area business cycle with a lead of around three to four quarters. Against this background, an in-depth understanding of the dynamics of M1, with a focus on its potential nonlinearities, would enhance the information content of this monetary aggregate for future output activity and, ultimately, prices.

In line with much of the quoted empirical literature, this paper characterises nonlinearity in terms of regime dependence in the dynamic behavior of money, that is, allowing for the possibility that the short-run dynamics of money demand varies across different states of the economy. However, an innovation of this study is the choice—based on an extensive specification search—of a Markov-switching error-correction model to characterize such regime dependence.<sup>4</sup> In particular, the study applies Hamilton's (1989) Markov-switching model, as extended to cointegrated vector autoregression models by Krolzig (1997).<sup>5</sup>

One attractive feature of the Markov-switching modeling approach relative to other nonlinear approaches (e.g., smooth transition models) is that it does not impose specific assumptions about the observability of the underlying stochastic process. In contrast, it allows the regimes to be characterized by an unobservable process that is endogenously determined by the evolution of the system over time. This greater flexibility of the Markov-switching approach comes, though, at the cost of a larger reliance on judgment in interpreting the regimes and of inability to establish with certainty the occurrence of a particular regime at one point in time (an event to which one can only assign an estimated probability).

Our empirical model is estimated over a sample period covering the past three decades. To our knowledge, this is the first money demand study for the euro area estimated over such an extended sample. Consistent with theoretical predictions by buffer stock and target-threshold models and with previous empirical results for the United States and some European countries, we find that the error-correction model

of real euro area M1 is characterized by nonlinear dynamics of disequilibrium adjustment. In particular, when the deviations of aggregate demand for monetary balances from equilibrium are large, the speed of adjustment to the desired level of monetary balances is greater.

## 2. THE LONG-RUN MONEY DEMAND RELATIONSHIP

Our empirical investigation relies on Krolzig's (1997) two-stage approach to the cointegration analysis of vector autoregression (VAR) models with Markovian regime-shifts.<sup>6</sup> In the first stage (which is the object of this section), Johansen's (1995) multivariate cointegration procedure is applied to a system of variables in order to determine the cointegrating rank and estimate the identified long-run money demand relationship.<sup>7</sup> In the second stage, a Markov-switching model of the dynamics of monetary balances is selected and estimated, conditional on the previously obtained cointegrating matrix.

The analysis is based on quarterly data for the euro area—defined according to the principle of changing composition (the 11 original countries up to 2000Q4; these plus Greece, thereafter)—over the period 1971Q4 to 2003Q3. The variables modeled consist of the monetary aggregate M1 ( $M_t$ ) deflated by the GDP deflator ( $P_t$ ), the real GDP ( $Y_t$ ), and the short-term market interest rate ( $R_t$ ). Nominal M1 is the period average of the end-of-month seasonally adjusted (s.a.) notional stock compiled by the ECB. The GDP data are based on the aggregation of s.a. national accounts data (ESA95 whenever available) up to 1998Q4; thereafter, on area-wide Eurostat statistics. The national data on M1 and GDP prior to the introduction of the euro have been aggregated using the irrevocable conversion rates announced on 31 December 1998 (19 June 2000 for Greece). The interest rate is a weighted average (based on GDP weights at 2002 purchasing power parities) of national three-month interbank interest rates up to 1998Q4; thereafter, it corresponds to the three-month EURIBOR.

The long-run money demand function is specified in the log-log form

$$(m - p)_t = \beta_1 y_t - \beta_2 r_t + k, \quad (1)$$

where all variables are in natural logarithms and  $k$  denotes an intercept unrestricted to the cointegrating space. As noted by Lucas (2000) for the United States, this functional form presents significant advantages over alternative specifications in terms of sounder microfoundations and a more accurate calculation of the welfare costs of inflation at low interest rates. In addition, in the framework of the shopping-time model of money demand determination by McCallum and Goodfriend (1987), Lucas (2000) observes that—for reasonable estimates of the interest rate elasticity—the log-log specification is more in line with theoretical models, such as Miller and Orr (1966).<sup>8</sup> For the euro area, Stracca (2003) investigates the issue of the choice of the functional form for the long-run demand for M1, providing empirical evidence in support of the log-log specification.

As a preliminary step, the statistical properties of the variables forming the system  $z = [(m - p), y, r]$  are examined using standard unit root tests (augmented Dickey–Fuller and Phillips–Perron) as well as the KPSS stationarity test. The results—not reported for the sake of brevity—suggest that over the sample period considered all the variables should be modeled as  $I(1)$  in levels.

The cointegrating properties of the system  $z_t$  are subsequently tested by means of the multivariate cointegration procedure of Johansen (1995),

$$\Delta z_t = v + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + \Psi D_t + u_t, \quad (2)$$

where the parameters of the model are represented by the vector  $v$  of deterministic components, the matrices  $\Gamma$  and  $\Psi$  of short-run coefficients, and the matrix  $\Pi = \alpha\beta'$ , with  $\alpha$  the matrix of loading factors and  $\beta$  the matrix of long-run coefficients. In particular,  $\beta'z_{t-1}$  includes the one-period lagged money-demand error-correction term implied by the cointegrating vector,  $D_t$  is a vector of  $I(0)$  exogenous variables, and  $u_t$  is the errors vector (assumed to be serially noncorrelated with zero mean and constant covariance matrix). Consistent with Stracca (2003),  $D_t$  includes two impulse dummies (ID99Q1 and ID00Q1) taking the value 1 in the first quarter of 1999 and 2000, respectively, and zero elsewhere, as exogenous variables.<sup>9</sup>

The application of the Johansen (1995) procedure enables us to determine the number of cointegrating vectors and, subject to appropriate specification testing, makes it possible to identify and estimate such vectors. On the basis of the Akaike, Hannan–Quinn, and Schwartz information criteria, the lag order  $p$  of the testing VAR (including linear trends in the data and an unrestricted intercept in the cointegrating vector) is set at 2 in levels. Panel A of Table 1 reports the Johansen's trace ( $\lambda_{\text{trace}}$ ) and maximum eigenvalue ( $\lambda_{\text{max}}$ ) cointegration tests. Both tests reject the hypothesis of no cointegration at the conventional significance levels, accepting that of at most one cointegrating relationship. The evidence of cointegration is robust to the use of test statistics adjusted for degrees of freedom (as suggested in Reimers [1992]) in order to control for potential small-sample bias.

Some studies have suggested that the Johansen procedure may not be robust to nonlinearities in the short-run dynamics [e.g., Barnett et al. (2000)]. Thus, as a further robustness test, we run the nonparametric cointegration test of Bierens (1997). Because of its nonparametric nature, the results of this test are independent of potential nonlinearities in the data generating process. The results of applying Bierens' (1997) test to the system  $z_t$  confirm the existence of a single cointegration vector at the 95% significance level.

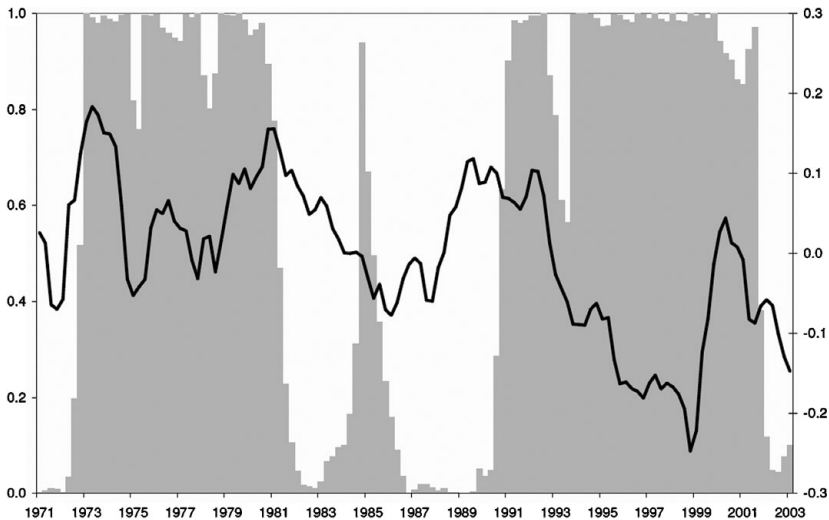
The results of the long-run exclusion tests in Panel B show that none of the variables can be excluded from the cointegrating vector at the conventional significance levels. Furthermore, the tests for weak exogeneity reveal that  $y$  and  $r$  can be treated as weakly exogenous to the system, both individually and jointly.

**TABLE 1.** Johansen procedure

A. Cointegration tests					
Eigenvalues	Rank	$\lambda_{\text{trace}}$	$\lambda_{\text{trace}}^{\dagger}$	$\lambda_{\text{max}}$	$\lambda_{\text{max}}^{\dagger}$
0.22109	= 0	45.46**	43.33**	31.98**	30.48**
0.09891	≤ 1	13.47	12.84	13.33	12.71
0.00112	≤ 2	0.14	0.14	0.14	0.14
B. $\chi^2$ restriction tests (conditional on unitary rank)					
	$(m - p)$	$y$		$r$	
Exclusion	$\chi_1^2 = 17.54$ [0.00]	$\chi_1^2 = 13.00$ [0.00]	$\chi_1^2 = 18.00$ [0.00]		
Weak exogeneity	$\chi_1^2 = 13.44$ [0.00]	$\chi_1^2 = 2.43$ [0.12]	$\chi_1^2 = 1.80$ [0.18]		
Joint weak exogeneity ( $y$ and $r$ )		$\chi_2^2 = 2.99$ [0.22]			
C. Estimated cointegrating vector (conditional on weak exogeneity of $y$ and $r$ )					
$(m - p) = 0.744y - 0.392r$					
(0.07) (0.04)					
D. Dynamic money demand equation					
$\Delta(m - p)_t = 0.005 - 0.051\text{ECT}_{t-1} + 0.178\Delta(m - p)_{t-1} - 0.084\Delta y_t$ $+ 0.013\Delta y_{t-1} - 0.024\Delta r_t - 0.022\Delta r_{t-1} + 0.027\text{ID99Q1}_t$ $+ 0.031\text{ID00Q1}_t + \varepsilon_t$					
$T = 128$ ; $\text{Adj}R^2 = 0.65$ ; $\text{s.e.}(\varepsilon_t) = 0.68\%$ ; $\text{LM}(1) : F(1, 118) = 1.52$ [0.22]; $\text{LM}(1 - 5) : F(5, 114) = 0.79$ [0.56]; $\text{ARCH}(1 - 4) : F(4, 111) = 0.63$ [0.64]; $\text{NORM} : \chi_2^2 = 1.58$ [0.45]; $\text{HET} : F(14, 104) = 1.21$ [0.28]; $\text{RESET} : F(1, 118) = 6.01$ [0.02]					

Note:  $\dagger$  denotes adjustment for degrees of freedom as in Reimers (1992); \*\* (\*) rejection of the null hypothesis at 1% (5%) critical level;  $p$ -values in square brackets; standard errors in parentheses.

The estimated cointegrating vector, normalized with respect to real M1 and to zero mean, is also presented in Table 1. From Panel C it is possible to see that the estimated income elasticity is 0.744. This value is consistent with theoretical predictions, as it falls between the value of 0.5 anticipated by the Baumol–Tobin inventory-theoretic model of transaction demand for money and the unitary elasticity implied by the quantity theory.<sup>10</sup> The interest rate elasticity of the demand for real M1 is estimated at  $-0.392$ . Because of the relatively low and sluggish average remuneration of the deposits included in M1 (which also includes zero-remunerated currency in circulation), this interest rate can be interpreted as approximating the opportunity cost of holding the monetary aggregate. Given the functional log-log form, the interest rate elasticity is constant across interest rates and measures the percentage change in the demand for money in response to a one percent change in the short-term interest rate. On the basis of the magnitude and



**FIGURE 1.** Money demand error-correction term and regime probabilities. *Note:* Error-correction term (line, right-hand-side axis) rescaled to zero mean; regime probabilities (bars, left-hand-side axis) are the smoothed probabilities of Regime 1.

sign of the coefficients, this cointegrating vector can be interpreted as representing a long-run demand function for real M1.

Given the relatively broad time span covered by the sample period, which comprises periods of both high and low interest rates, it is important to test for the stability of the coefficients of the equilibrium money demand relationship. For this purpose, we apply two types of Nyblom tests for parameter constancy of the cointegrating vector as extended to cointegrated VARs by Hansen and Johansen (1999). The null hypothesis of the tests—which are respectively based on the maximum (Sup) and the mean (Mean) of a weighted LM-type statistics over the sample period—is the joint stability of the parameters of the cointegrating vector. The supremum and mean test statistics yield 1.60 ( $p$ -value = 0.53) and 0.98 ( $p$ -value = 0.20), respectively.<sup>11</sup> The high level of the  $p$ -values indicates that the null hypothesis cannot be rejected at the conventional significance levels, suggesting that the long-run parameters are jointly stable over a sample period covering the last three decades.

Conditional on the finding of joint weak exogeneity for  $y$  and  $r$ , the dynamic model is specified as a single-equation error-correction model. The estimated equation is reported in Panel D. In particular, the coefficient of the error-correction term is negative and statistically significant, supporting the interpretation of the cointegrating vector as a long-run money demand function. Yet the relatively small size of the coefficient ( $-0.051$ ) reveals a rather sluggish adjustment to equilibrium in case of deviations. This is illustrated in Figure 1, which plots the developments of the money demand error-correction term, by the slow rate at

which monetary disequilibria are corrected. The mean-reverting behavior of the error-correction term shows that disequilibria in the money market are eventually corrected. However, the process of return to equilibrium can be very sluggish, reflecting the presence of market frictions, such as portfolio adjustment costs. In particular, the figure shows that over the last three decades there have been occasions on which the deviations from equilibrium have become rather large before the adjustment process prevailed.

Finally, the statistical properties of the residuals of the model are evaluated by means of several standard misspecification tests for autocorrelation, nonnormality, and heteroscedasticity. The results are satisfactory and suggest that the model is adequately specified. However, we fail to reject the null hypothesis of no misspecification of the RESET test. Originally developed to test for omitted regressors, a significant value of the RESET statistic is often indicative of nonlinearity in the residuals [see Granger and Teräsvirta (1993)]. The evidence of misspecification provided by this test suggests that the specification of the equation may be improved by modeling such nonlinearity explicitly. The next section formally investigates this issue.

### 3. MODELING THE NONLINEAR DYNAMICS OF M1

The analysis of the residuals of the linear error-correction model suggests that a standard model with time-invariant parameters may not provide an appropriate representation of the short-run dynamics of M1. Such dynamics may be better captured by a model allowing for some form of regime-dependent behavior. In particular, if the nonlinear process is time-invariant conditional on an (unobservable) regime variable  $s_t$ , a Markov-switching model may be considered as an appropriate framework. The idea behind this class of models is that the parameters of the underlying data generating process of the observed time series vector  $z_t$  depend upon an unobservable regime variable  $s_t$ , representing the probability of being in a certain state of the world.

Letting  $s_t \in \{1, \dots, M\}$  indicate the regime prevailing at time  $t$ , the properties of the MS( $M$ )–ECM( $p$ ) model for the euro area real money demand can be analyzed depending on the realization of the regime,

$$\Delta(m - p)_t = v(s_t) + \gamma_0(s_t) \Delta x_t + \sum_{i=1}^{p-1} \gamma_i(s_t) \Delta z_{t-i} + \alpha(s_t) \beta' z_{t-1} + \psi(s_t) D_t + u_t, \tag{3}$$

where  $v(s_t)$  is the regime-dependent intercept term,  $\gamma_0(s_t)$ ,  $\gamma_i(s_t)$ , and  $\psi(s_t)$  are the vectors of state-dependent short-run parameters,  $\alpha(s_t)$  is the regime-dependent adjustment coefficient,  $x_t$  is the vector of contemporaneous values of  $y_t$  and  $r_t$ ,  $\beta' z_{t-1}$  is the cointegrating vector, and  $D_t$  is the vector including the two dummies. The hypothesis underlying the model is that the equilibrium relationship does not vary across regimes; only the strength of disequilibrium adjustment and



**TABLE 2.** Identification procedure

	LR-linearity	LR-restrictions	RCM	Adj $R^2$	LR-regimes
MSI	0.143	0.000	36.6	0.64	0.182
MSA	0.367	0.179	63.2	0.65	0.013
MSH	0.999	0.000	99.8	0.62	0.999
MSAH	0.294	0.408	18.5	0.65	0.951
MSIA	0.048	0.981	31.3	0.63	0.132
MSIH	0.047	0.002	25.7	0.64	0.728
MSIAH	0.034	—	21.7	0.66	0.093

*Note:* For the LR tests [see Hansen (1992, 1996)], only  $p$ -values are reported. LR-linearity is a test of the null hypothesis of linearity against each possible Markov-switching specification. LR-restrictions tests each Markov-switching specification against the more general MSIAH model. RCM is the regime classification measure (RCM) of Ang and Bekaert (2002). LR-regimes is a test of the null hypothesis of 2-versus 3-regimes.

the short-run dynamics of the model are allowed to vary. Finally, note that the distribution of the error term  $u_t$  also depends on the realization of the regime, because  $u_t \sim NID[0, \Sigma(s_t)]$ .

Since the parameters depend on a regime which is assumed to be stochastic and unobservable, a generating process for the states  $s_t$  needs to be formulated. In particular, the stochastic process generating the unobservable regimes is assumed to be an ergodic Markov chain defined by the transition probabilities

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}. \quad (4)$$

By inferring the probabilities of the unobservable regimes conditional on the available information set, it is possible to reconstruct the evolution of the regimes.

In order to select the best specification of the model for the euro area data, we run a battery of tests. We start with linearity tests against the various types of Markov-switching models with two regimes and subsequently use different statistics to select the most appropriate among the possible Markov-switching specifications. The first column of Table 2 reports the  $p$ -value of the (upper bound of the) likelihood-ratio (LR) statistic testing the null hypothesis of linearity against the alternative of a specific type of Markov-switching nonlinearity.<sup>12</sup>

On the basis of the LR test, the data fail to reject the null of linearity for the models specifying the regime switching behavior for either the intercept term (MSI), the short-run parameters of the error-correction model (MSA), or the variance-covariance matrix (MSH). By contrast, the null of linearity is easily rejected at the conventional significance levels for the specifications combining different types of regime-dependence behavior: in the intercept and short-run coefficients (MSIA), in the intercept and variance-covariance matrix (MSIH), and in the intercept, short-term parameters and variance-covariance matrix (MSIAH). Only for the MSAH model, which combines regime dependence in the short-run parameters and the variance-covariance matrix, can the null of linearity not be rejected. These

results suggest that in order to identify and describe the regimes it is necessary to use models specifying general forms of regime-switching dynamics, such as the MSIA, MSIH, or MSIAH models. Interestingly, all of these models include a time-varying intercept (unlike the MSAH model), suggesting that allowing for regime switching in the intercept term is particularly important in order to adequately specify the nonlinearities in the monetary dynamics.<sup>13</sup>

The second column of Table 2 shows the  $p$ -values of LR restriction tests designed to select the most parsimonious among the candidate Markov-switching specifications. In practice, these tests assess each specification against the more general MSIAH model. Based on the results of the nonlinearity LR tests, we restrict the discussion to the last three specifications. The null hypothesis of no shifting in the short-run parameters (MSIH versus MSIAH) is strongly rejected by the data. By contrast, the null of no shifting in the variance-covariance matrix (MSIA versus MSIAH) cannot be rejected.

On the basis of this test, the MSIA specification presents some advantages over the less parsimonious MSIAH model. However, there are some indications that the MSIAH specification is to be preferred. First, the regime classification measure (RCM) proposed by Ang and Bekaert (2002) to discriminate among different types of Markov-switching models (third column of Table 2) suggests a better fit of the MSIAH. The RCM is a summary point statistic of the degree of accuracy with which a model identifies the regime-switching behavior over the sample period. The statistic ranges between 0 and 100, with 0 denoting a perfect regime classification performance and 100, indicating that the model fails to provide any information on the regime dependence. The value of the RCM statistic recorded for the MSIAH specification is fairly low, and significantly smaller than that for the MSIA. In addition, the MSIAH model seems to have an higher explanatory power as can be evinced from the larger value of the coefficient of determination (adjusted for degrees of freedom): 0.66 versus 0.63 (fourth column of Table 2). Finally, the dating cycle identified by the MSIA model is relatively volatile and hard to relate to economic developments in the euro area over the sample period.

On the basis of the above considerations, we restrict our attention to the MSIAH specification. As a final check, we test whether it may be statistically more appropriate to use a model allowing three instead of two regimes (fifth column of Table 2). The results of the test are not clear-cut. The null of a two-regime MSIAH model versus a three-regime model can be rejected only at the 10% significance level. However, given the size of the sample (128 observations), we retain the specification allowing for fewer regimes.

The results for the estimation of the MSIAH(2)-ECM(1) model for the euro area (real) M1 are presented in Table 3. The number of observations in each regime is large enough to allow for robust statistical inference. The regimes are fairly persistent, with the conditional probabilities ( $p_{11} = 0.94$ ,  $p_{22} = 0.90$ ) implying an expected duration of around  $4\frac{1}{2}$  years and  $2\frac{1}{2}$  years for the first and second regime, respectively.

**TABLE 3.** MSIAH(2)-ECT(1) estimation for  $\Delta(m - p)_t$

Transition probabilities			Regime properties		
	Reg 1	Reg 2		nObs	Duration
Reg 1	0.9429	0.0571	Reg 1	80.5	17.5
Reg 2	0.0971	0.9029	Reg 2	47.5	10.3
Regime 1			Regime 2		
	Coef	s.e.		Coef	s.e.
Const	0.004	0.001	Const	0.009	0.002
$\Delta(m - p)_{t-1}$	-0.036	0.094	$\Delta(m - p)_{t-1}$	0.267	0.130
$\Delta y_t$	-0.310	0.165	$\Delta y_t$	0.024	0.129
$\Delta y_{t-1}$	0.089	0.159	$\Delta y_{t-1}$	-0.243	0.124
$\Delta r_t$	-0.028	0.011	$\Delta r_t$	-0.005	0.009
$\Delta r_{t-1}$	-0.021	0.013	$\Delta r_{t-1}$	-0.024	0.012
$ECT_{t-1}$	-0.073	0.012	$ECT_{t-1}$	-0.053	0.014
ID00Q1	0.033	0.007	ID00Q1	0.027	0.039
ID99Q1	0.031	0.007	ID99Q1	0.022	0.029
Std. error (Reg. 1)		0.62%	Std. error (Reg. 2)		0.41%

Standard misspecification tests (not reported for the sake of brevity) fail to reveal signs of autocorrelation, nonnormality, or heteroscedasticity for both the standardized residuals and the one-step prediction errors, suggesting that the model is satisfactorily specified.<sup>14</sup>

Figure 2 depicts the smoothed probabilities of being in Regime 1 together with the annual growth rate of real M1.<sup>15</sup> Regime 1 includes the periods of highest volatility in real monetary growth over the last 30 years. In particular, it comprises a protracted period of relatively low but volatile growth in real M1 throughout most of the 1970s and beginning of the 1980s as well as a long time span of relatively high and volatile monetary growth throughout the 1990s. By contrast, the probabilities of Regime 2 are associated with periods of more stable monetary conditions. More formal evidence in support of this observation is provided by the analysis of real money’s regime-dependent variances using Warne’s (1998) probability weighted estimator of conditional moments.<sup>16</sup> The conditional variance for real M1 growth in Regime 1 (0.016%) is indeed more than twice that in Regime 2 (0.006%).

To provide a historical interpretation of the estimated regimes, it is useful to recall some of the main phases in monetary developments in the euro area over the past thirty years. The beginning of our sample (1971) coincides with the end of the Bretton Woods fixed-exchange rate era and the start of the “Great Inflation,” a period characterized by monetary instability and by large fluctuations in exchange rates. Indeed, over most of the 1970s high and volatile inflation and nominal interest rates (and, more generally, macroeconomic instability) led to



**FIGURE 2.** Real money growth and regime probabilities. *Note:* Real money growth (line, right-hand-side axis) is the annual percentage change in deflated M1; regime probabilities (bars, left-hand-side axis) are the smoothed probabilities of Regime 1.

significant fluctuations in monetary growth. In 1979 an international exchange rate agreement—the European Monetary System (EMS)—was established among European countries, also as a monetary stabilization tool. However, monetary conditions initially failed to stabilize, as policy stances remained accommodative in most countries, whereas exchange rates continued to fluctuate within relatively large bands around their established parities. As noted by Juselius (1999), only in the early 1980s, when the bands were narrowed and parity realignments became less frequent, did the EMS become effective in reducing flexibility in exchange rates. At the same time, the upward trend in inflation and nominal interest rates came to a halt. In particular, in Germany, monetary growth was effectively constrained by the Deutsche Bundesbank's price-stability-oriented policy of monetary targeting. The ensuing long period of relatively tighter monetary conditions and narrower exchange rate movements came to an end with the collapse of the EMS in 1992. The rest of this decade was characterized by renewed volatility in monetary growth as an acceleration in the process of disinflation led to significant decreases in the opportunity costs of holding money. By the beginning of the present decade the process of disinflation was completed and inflation and interest rates were locked at low levels, whereas the launch of the European Monetary Union was finalized with the replacement of the national currencies by the euro.

The evolution of the estimated regimes matches fairly closely the shifts in monetary and exchange rate regimes described above. According to the model, the euro area economy was initially in Regime 2 (the regime associated with relatively stable monetary conditions). The shift to the post-Bretton Woods period

of macroeconomic instability is captured by the model's switch to Regime 1 (the regime associated with more volatile monetary conditions) in the early 1970s. The economy remained in this regime throughout the rest of the 1970s and the beginning of the 1980s. It subsequently returned to Regime 2 in correspondence with the shift toward less flexible exchange rates and the end of the inflationary acceleration. Except for a short period in 1985, the euro area economy remained in this regime of relatively lower monetary volatility between 1981 and 1991. A shift to more volatile monetary conditions was recorded in correspondence with the collapse of the EMS. The economy remained in Regime 1 as the process of disinflation gained momentum throughout the rest of the 1990s. Finally, with the end of the disinflation process and the completion of the European Monetary Union toward the end of the sample period, the economy shifted back to the more stable monetary regime.

The theoretical models surveyed in the introductory section lead to the prediction that the process of adjustment to equilibrium should be more effective during the first regime—characterized by more extreme developments in monetary balances—than in the second one. Indeed, buffer stock models would suggest that in periods when the behavior of money deviates significantly from its norm, agents should adjust to the “desired” level at a higher speed than in tranquil periods. The regime-dependent coefficients of adjustment provide some support for this hypothesis. In both regimes the coefficients of adjustment have the expected negative sign and are significantly different from zero. However, in Regime 1 the estimated coefficient is larger in absolute terms than in Regime 2 (0.073 and 0.053, respectively), confirming the hypothesis that the differences in the speed of monetary disequilibria adjustment depend on the prevailing monetary conditions.<sup>17</sup> Although the value of the coefficient of adjustment in Regime 2 is fairly close to the estimate for the linear model, the estimated loading factor in Regime 1 implies a faster correction to the equilibrium. *Ceteris paribus*, the process of monetary disequilibrium adjustment should be about  $1\frac{1}{4}$  years shorter in the first regime than in the second one.<sup>18</sup>

These stylized facts find further confirmation in the behavior of the error-correction term. The probability-weighted conditional variance of the error-correction term (based on the methodology of Warne, 1998) is noticeably higher in Regime 1 (1.13%) than in Regime 2 (0.50%), reflecting the concentration of large disequilibria in the former regime. High probabilities of being in Regime 1—the regime in which the coefficient of adjustment of the error-correction term is higher—are typically associated with periods in which the deviations from equilibrium are large. By contrast, the probabilities of being in Regime 2—in which the adjustment to equilibrium is slower—are usually higher in correspondence with periods characterized by relatively small deviations from equilibrium.

In addition, the results in Table 3 show evidence of regime dependence also in other short-run coefficients, notably in that of the autoregressive term.<sup>19</sup> To illustrate the differences in the broader dynamic properties of the model (and not only of the error-correction term), we report the responses of real money to

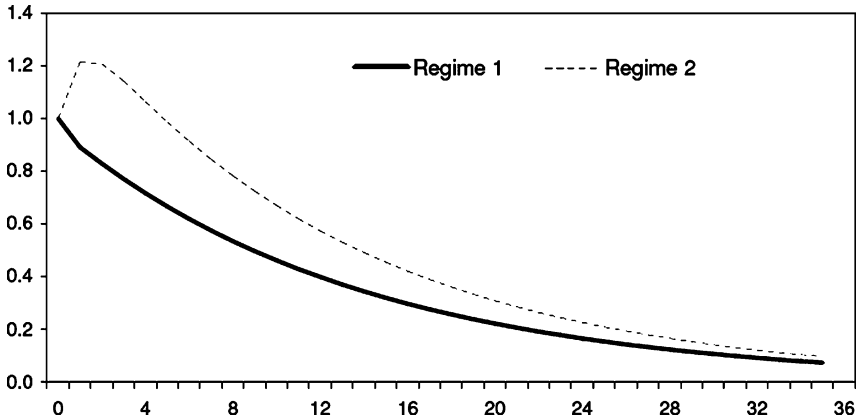


FIGURE 3. Regime-dependent impulse responses of real money to (own) unit shocks.

(own) unit innovations in the two separate regimes identified by the estimated Markov-switching equations. Following Ehrmann et al. (2003), we compute regime-dependent impulse responses, namely, responses conditional on a given regime prevailing at the time of the shock and throughout the horizon. These conditional responses provide an useful analytical tool to unveil nonlinearities in the dynamics of the model when the responses across regimes to analogous shocks reveal asymmetries in terms of size, sign and persistence.<sup>20</sup> However, by excluding regime shifts over the duration of the response, they may not be suitable to assess the ultimate macroeconomic impact of a shock.<sup>21</sup>

As Figure 3 shows, the responses to monetary innovations across Regime 1 and 2 of the estimated model differ in terms of both the direct impact of the shock and its persistence. Indeed, under Regime 1 the shock is followed by an immediate correction in real money, with the response function steadily returning towards the baseline. By contrast, under Regime 2 the response function exhibits a “hump-shaped” profile, initially rising over the first two quarters following the shock and only subsequently returning toward the baseline. The impulse responses also differ across regimes in terms of their persistence, with the effect of the shock dying out significantly faster under Regime 1 than under Regime 2. Indeed, the half-life of the impulse response function is noticeably lower under the first than under the second regime ( $2\frac{1}{4}$  years versus  $3\frac{1}{2}$  years).<sup>22</sup> Thus, the analysis of the dynamic properties of the model confirms that the process of adjustment to equilibrium works more effectively under Regime 1, which is characterized by relatively larger disequilibria.

To sum up, our empirical findings provide evidence of nonlinearities in the dynamic behavior of the demand for euro area M1. These findings are in line with the theoretical predictions of buffer stock and target-threshold models that postulate frictional adjustment in individual money demand behavior. Our findings

are also consistent with the results of empirical studies for various countries (both inside and outside the euro area) based on alternative econometric methods that show evidence of asymmetries in the estimated short-run dynamics of money demand.

A somewhat unexpected aspect of our results is that the regime associated with more volatile monetary dynamics and faster disequilibrium adjustment (Regime 1) is relatively more frequent than that associated with slower error-correction and less extraordinary monetary developments (Regime 2). One possible explanation is that our empirical analysis is based on quarterly data covering a relatively short span (three decades) compared to previous studies based on annual data spanning over more than one century [e.g., Sarno (1999); Teräsvirta and Eliasson (2001); and Sarno et al. (2003)]. The use of annual observations obtained by averaging may smooth out higher-frequency monetary fluctuations and induce slower dynamic adjustment in the data. In addition, samples spanning secular periods may reduce the impact of phases of significant monetary and macroeconomic instability such as the 1970s.

#### 4. CONCLUDING REMARKS

The empirical analysis presented in this paper supports the use of M1 as an information variable for the conduct of monetary policy in the euro area. Using a log-log functional form, we find evidence of the existence of a stable equilibrium relationship linking the demand for M1 with output, prices, and interest rates over a sample period comprising the past 30 years. To our knowledge, this is the first euro area money demand study using such an extended sample period. Given the switch to a regime of low and stable inflation within the sample period, it is interesting to note that a formal Nyblom test of parameter constancy indicates that the estimated equilibrium relationship is fairly stable. More generally, the empirical investigation in this study provides further support to Lucas's (2000) arguments in favor of the use of log-log functional forms to specify the long-run behavior of money demand.

The stability of the estimated relationship suggests that it may provide an adequate benchmark against which to assess actual movements in M1. Large and persistent deviations of monetary balances from the equilibrium level implied by the estimated relationship may reveal the emergence of potential pressures on future economic activity and, ultimately, prices. More generally, periods of excessively fast or slow monetary growth compared to that predicted by the model may signal the build-up of imbalances in asset markets and balance sheets that may lead to macroeconomic instability [Borio and Lowe (2002)].

Based on a fairly general Markov-switching error-correction model, our empirical investigation reveals evidence of nonlinearities in the behavior of the short-run demand for euro area real M1. We find that the dynamic behavior of M1—whereby deviations from equilibrium are corrected—varies depending on the prevailing regime of monetary conditions. In particular, the probabilities of being in the

regime in which the error-correction adjustment is faster are typically higher, in periods associated with large deviations from equilibrium. By contrast, the probabilities of being in the regime in which the adjustment to equilibrium is slower are usually higher in correspondence with periods of relatively small deviations from equilibrium.

Our empirical findings of nonlinearity in the dynamics of euro area money demand are consistent with theoretical predictions by buffer stock and target-threshold models. In addition, they are consistent with evidence of state dependence in the estimated short-run dynamics of money demand for several European countries and the United States reported in recent empirical contributions [see Sarno (1999); Teräsvirta and Eliasson (2001); Ordóñez (2003); Sarno et al. (2003); and Chen and Wu (2005)]. Because these studies use alternative types of nonlinear error-correction models (typically, cubic polynomial or smooth-transition models), the finding that frictions in individual money demand behavior translate into rigidities at the aggregate level seems to be fairly robust to the choice of econometric methodology.

One potential implication of our findings of nonlinearities in euro area monetary dynamics is that the effects of excessively fast or slow monetary growth on the economy could also be regime-dependent. This would imply that the assessment of the implications for output of monetary imbalances should be preceded by an accurate analysis of the monetary conditions characterising the state of the economy. A failure to do so may lead to an inappropriate interpretation of the information contained in monetary developments.

The empirical analysis in the paper is based on a simple-sum monetary aggregate. Future work should aim to establish whether similar asymmetries can be identified also in the dynamics of weighted monetary aggregates, particularly the Divisia multicountry aggregates recently proposed for the euro area by Barnett (2006).

## NOTES

1. For a discussion on the notion of buffer stock in monetary economics see Laidler (1984). Mizen (1994) is a comprehensive study of buffer stock money demand models, also including target-threshold models as a specific type.

2. This empirical finding is in contrast with the standard assumption in linearly specified error-correction models of a time-invariant loading factor, which implies a proportional adjustment to disequilibrium [see Escribano (2004), for a discussion]. Indeed, although linear error-correction models allow the magnitude of the adjustment to vary with the size of the disequilibria, they impose the restriction that the adjustment must be a constant proportion of the previous disequilibrium (with the proportion measured by the estimated loading factor).

3. See Issing (2003) and the studies and references therein.

4. A number of specifications for smooth-transition models (mainly single equations) were also tested. However, in most cases we experienced severe difficulties in achieving convergence of the estimation algorithm. In those cases when it converged, it was often not possible to estimate with precision the parameters governing the regime transition.



5. Camacho (2005) has recently proposed an alternative model of Markov-switching equilibrium adjustment based on a common trends representation.

6. The empirical results have been obtained using the packages Ox and PcGive and the program MSVAR by H.-M. Krolzig (downloadable from <http://www.kent.ac.uk/economics/staff/hmk/index.htm>).

7. Note that in the first stage it is not necessary to model the Markovian regime shifts explicitly in order to derive the equilibrium relationships [Saikkonen (1992)].

8. Chadha et al. (1998) concur on the theoretical superiority of the log-log form. Based on McCallum and Goodfriend's (1987) model, they show that the choice of any well-behaved utility function and transactions technology (e.g., Cobb-Douglas, CES, and translog functions) is likely to result in a log-log specification of long-run money demand. However, using U.K. data, they find that the empirical advantages of the log-log specification may be more relevant for the short-run dynamics of money demand than for its equilibrium behavior.

9. The first dummy is introduced to control for the exceptionally large rise in the demand for M1 holdings (especially for overnight deposits) recorded after the start of Stage Three of the European Monetary Union in January 1999. This rise probably reflected institutional innovations associated with the new monetary policy regime (e.g., the introduction of a new reserve requirements system) as well as the changes in statistical reporting procedures. The second dummy controls for the temporary acceleration in demand for currency at the time of the "millennium bug" scare, when concerns about possible disruptions to retail payment systems and cash dispensing machines became widespread in several euro area countries.

10. In the conditional model, the hypothesis of unitary income elasticity is rejected by the data [ $\chi_1^2 = 8.70$  ( $p$ -value = 0.04)].

11. The distributions of the tests are bootstrapped using 1,000 replications. The computations are performed using the program Structural VAR, version 0.19, by Anders Warne (downloadable from [www.texlips.hypermart.net/svar](http://www.texlips.hypermart.net/svar)).

12. The application of LR tests in the context of Markov-switching models is discussed in Hansen (1992, 1996).

13. We are grateful to an anonymous referee for drawing our attention to this issue.

14. However, the results of these tests should be interpreted with caution, given that their asymptotic distributions may not be valid for residuals from Markov-switching models.

15. In the context of Markov-switching models, the estimated probabilities of each regime occurring at time  $t$  are called "smoothed" when they are obtained using full sample information.

16. Warne (1998) proposes to compute the conditional moments of a variable by weighting the observed data with the estimated smoothed regime probabilities.

17. Based on a Wald test, the null hypothesis that the coefficient of adjustment of the conditional model for Regime 1 equals that of the Regime 2 model could be rejected at the 10% significance level [ $\chi_1^2 = 3.13$  ( $p$ -value = 0.08)].

18. The finding that the coefficient of adjustment may vary depending on the size of the deviations from equilibrium is consistent with similar evidence for the United States and some European countries reported in the empirical studies quoted in the Introduction [e.g.; Sarno (1999); Teräsvirta and Eliasson (2001); Sarno et al. (2003); Escribano (2004); and Chen and Wu (2005)].

19. See Sarno (1999) and Sarno et al. (2003) for similar findings.

20. It should be noted that the computation of the impulse responses in a single-equation framework requires the assumption of orthogonality of the exogenous regressors.

21. For a critical view of regime-dependent impulse responses see Krolzig (2006). The author notes that, because they bind the system to remain within the regime prevailing at the time of the initial shock and ignore the Markovian regime-switching dynamics, these impulse responses cannot appropriately represent the nonlinear behavior of the economy.

22. The half-life of an impulse response function is a measure of persistence indicating the number of periods required for the response to an unit shock to the time series to dissipate by half.

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