Macroeconomic Dynamics, **25**, 2021, 1993–2036. Printed in the United States of America.
doi:10.1017/S1365100519000944

MACROECONOMIC CONSEQUENCES OF THE DEMOGRAPHIC AND EDUCATIONAL CHANGES IN POLAND AFTER 1990

Aleksandra Kolasa
University of Warsaw

Soon after the start of transition to a market economy in the early 1990s, Poland has experienced both a dramatic decline in the fertility rate and an increase in the share of students among young high-school graduates. These two processes significantly changed the age structure of population and average income characteristics of households. Using a general equilibrium model with heterogeneous households and uninsured income shocks, I try to assess the impact of these changes on the Polish economy as a whole and inequalities within it. I find that, in the long term, the positive effects of the educational change on output per capita will more than offset the negative impact of lower fertility. I also show that the educational change increases income and consumption inequalities, while the fertility change decreases inequality in assets.

Keywords: Low Fertility, Educational Boom, Inequalities, Heterogeneous Agents

1. INTRODUCTION

Many countries, including most of developed economies, have been facing for quite some time decreasing mortality, low fertility, and shifts in the population structure toward older ages. To arrive at this stage, however, population dynamics had to undergo a remarkable transition from a premodern regime of high mortality and high fertility.[1]

Yet, this is not the end of the story. Starting from the 1960s, the fertility rates have continued to drop even further in a number of countries, eventually falling below their replacement levels. Some researchers have dubbed this new phenomenon as the second demographic transition, which was triggered by the availability of cheap and effective contraception accompanied by ideological changes in lifestyle, views on families, self-fulfillment, or the perception of

I would like to thank two anonymous referees for useful suggestions to the previous draft. This paper also benefited from comments by Ryszard Kokoszczyński, Marcin Kolasa, Anna Nicińska, Małgorzata Rószkiewicz, Michał Rubaszek, Małgorzata Skibińska, Joanna Wolszczak-Derlacz and participants of the 5th National Bank of Poland Summer Workshop in Warsaw. All remaining errors are my own. Address correspondence to: Aleksandra Kolasa, Faculty of Economic Sciences, University of Warsaw, Długa 44/50, 00-241 Warsaw, Poland. e-mail: aleksandra.w.kolasa@gmail.com. Phone: +48 22 55 49 111, 126. Fax: +48 22 831 28 46.

 1469-8056/19

women's rights (Van de Kaa (1987), Lesthaeghe (2007)).[2] In some cases, mainly in Southern and Eastern Europe, the reduction in childbirth was so profound that the total fertility rate (TFR) eventually approached or even fell below the threshold of 1.3, called the lowest-low fertility.[3] In most countries with the lowest-low fertility, there is evidence of both a delay of childbearing and a low progression to higher-order births. Apart from the components that are associated with the second demographic transition, the underlying factors of such a substantial drop in fertility are the following: an increase in women's educational attainment, high economic uncertainty among young adults, insufficient state support of young parents, and a combination of low level of gender equality at home and high level of gender equality at the labor market (see i.a. McDonald (2000a,b), Kohler et al. (2002), Billari (2008), Goldstein et al. (2009)). All of these factors result in relatively high economic costs of having children.

The decline in fertility has a strong impact on the age structure of population and, next to rising longevity, is a primary determinant of population aging. Aging is a pressing issue in many countries, and its economic consequences are considered to be significant. First of all, an increase in the old-age dependency ratio (the ratio of dependents to the working-age population) negatively affects the public finance, raising concerns about fiscal sustainability, and stability of pension systems. Second, as individual household characteristics, such as productivity or propensity to save, vary over the life cycle, changes in the age structure of population affect many macroeconomic variables, including aggregate output, consumption, and assets. As population aging also influences the level of public debt and the real interest rate, it is important for monetary policy. Finally, aging has redistributive consequences, that is, it affects income, consumption, and assets inequalities.

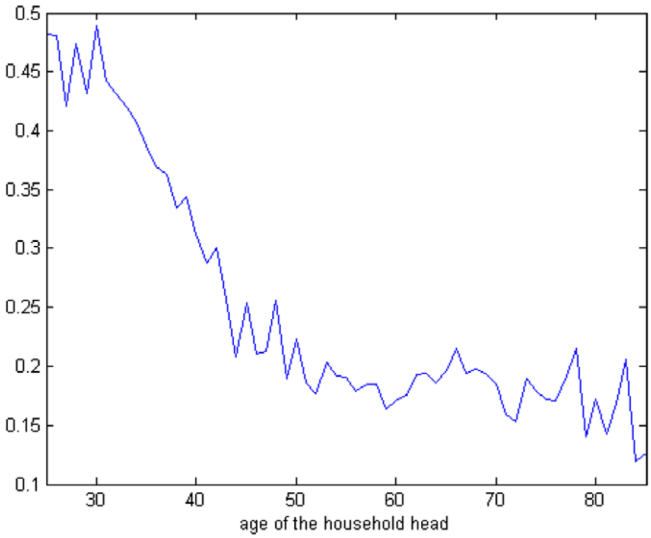
Poland is now among the countries with lowest-low fertility. Its TFR started to deteriorate dramatically in the mid-1980s, and since 2010, it has stabilized slightly above 1.3 (Figure 1). According to the CA World Factbook, Poland has currently one of the lowest fertility rates in the world and the third lowest in Europe. If this trend is continued and no other processes (such as intensified immigration) start offsetting it, population in Poland is expected to be shrinking at a high pace.

Fortunately for Poland, the observed population aging, and in particular declining fertility, has been accompanied by an increase in educational attainment. Indeed, another important process, which started in this country in the early 1990s, is a rising share of students among young high-school graduates. According to the Polish Households Budget Survey (HBS) from 2013, only less than 20% of all households' heads aged 50 years or older have an academic degree, while for those aged between 25 and 30, the corresponding statistics is slightly less than 50% (Figure 2).[4] Since worker's individual productivity is strongly linked to his/her educational attainment level, one can expect positive effects of this process on the Polish economic performance.

Naturally, for such a large expansion of higher education, the previously observed relationship between education and productivity might be disrupted.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

FIGURE 1. Total fertility rate in Poland.



Note: Based on the data from the Polish Household Budget Survey 2013. Post secondary education is included.

FIGURE 2. The share of households whose household head has an academic degree.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

First, there is a threat of educational inflation, that is, a constant rise of minimum credentials required for a given position and the devaluation of degrees and grades (for more, see Johnson (2003), Hadjar and Becker (2009)). Second, the rising supply of academy graduates in the labor market might not coincide with the demand for them and thus amplify the educational mismatch, that is, the share of overqualified workers.[5] All these aspects might negatively impact the education premium, but whether this risk will materialize is far from certain. Indeed, Walker and Zhu (2008), Valletta (2016), and Parro (2017) present examples with the education premium remaining unchanged or even increasing in the presence of a strong acceleration of the share of academy graduates.

A boom in tertiary education (educational change) and a rapid fertility decline (fertility change) emerged in Poland almost simultaneously and with exceptional intensity. The main objective of this paper is to quantify their macroeconomic consequences and implications for inequalities. To this end, I develop a general equilibrium model with heterogeneous agents and incomplete financial markets. In the model, there is a large number of households in different age cohorts. As in Gourinchas and Parker (2002), their deterministic productivity profiles differ between educational groups. I also introduce the education-dependent individual income processes. The life span is stochastic, and similar to Hubbard and Judd (1987), the survival probabilities depend on age. During their working life, households experience uninsured, idiosyncratic shocks, as in Aiyagari (1994). The model is cast in a small open economy setup, with the domestic interest rate reacting to changes in foreign debt. The model is calibrated to the Polish economy and replicates the key specific features of the life cycle characteristics of Polish households, which are obtained from the household level data from the Polish HBS. The model-based simulations allow me to assess both the long-term effects of the fertility and educational changes, and their transitional dynamics.

There is considerable literature that analyzes the macroeconomic and welfare effects of population aging in the US economy (De Nardi et al. (1999), Storesletten (2000), Krueger and Ludwig (2007), Ludwig et al. (2012)). Nevertheless, the US has not recently experienced such dramatic changes in the fertility rate as Poland, and hence the main focus of these studies is rather on the effects of rising longevity. There are also several papers that deal with pension system stability in Poland using classical overlapping generaltions models, i.a., with no intracohort heterogeneity (see, among others, Hagemejer et al. (2015)). Only very few studies look at the relationship between population aging and inequalities (one of the exceptions is Krueger and Ludwig (2007)), and I am not aware of any for Poland.

An important feature of this study is that it focuses not only on the development of fertility, but also on changes in educational attainment. Historically, these two processes often occurred simultaneously. Therefore, a possible causal relationship between them has been a subject of several empirical studies, the results of which are nevertheless not conclusive. For example, while Snopkowski et al. (2016) and Hansen et al. (2018) confirm that, at least to some extent, an increase in educational attainment is responsible for changes in fertility, Diebolt et al. (2016) find the causal effect of education on fertility to be insignificant. From a theoretical perspective, a particularly interesting contribution linking fertility and education is the Unified Growth Theory (Galor and Weil (1999, 2000)).[6] It tries to explain in one conceptual framework the process of demographic transition from the Malthusian regime to the recent times. At the heart are the endogenous decisions concerning both the number of children and their educational attainment. Parents choose how much to invest in their children's human capital and thus face the quality–quantity tradeoffs, with the returns on investment in human capital depending, i.a., on the level of technological progress.[7]

As the aim of my paper is to quantify the macroeconomic effects of decreased fertility and increased education, and not their interdependencies, both of these processes are treated as exogenous. This approach is common in papers that rely on a similar modeling framework (Gourinchas and Parker (2002), Börsch-Supan et al. (2006), Díaz-Giménez and Díaz-Saavedra (2009)). What is new in my study is that the same model is used to examine the impact of changes in education, a decrease in the fertility rate, and the total impact of both processes, which ensures comparability of the results for each of these scenarios. Research that analyzes simultaneously the macroeconomic effects of these two processes within a life cycle framework is very scarce. The exceptions are Díaz-Giménez and Díaz-Saavedra (2009) and Catalano and Pezzolla (2016), but none of them discuss changes in inequality.

The main findings of this paper are the following. First, in the long term, the positive effect of the educational change on output per capita will more than offset the negative impact of lower fertility. Second, a drop in fertility has stronger negative consequences for Polish output per capita compared to rising longevity. In contrast, the latter is relatively more important for the long-term response of the real interest rate and domestic assets. Due to population aging, the Polish real interest rate should decline, and a significant rise in the contribution rate would be required in order to keep the pension system balanced. Finally, the educational change positively impacts income and consumption inequalities, while the fertility change decreases the inequality of assets.

The rest of the paper is organized as follows. In Section 2, I shortly discuss the Polish economic developments after the end of the communist regime, with special emphasis on fertility and education attainment. Section 3 presents the model and definitions of equilibrium. Section 4 discusses the model assumptions and calibration. In Section 5, I present the baseline results, that is, the new steady states and transitional dynamics. In Section 6, I relax or modify some of the modeling assumptions and calculate the effects of rising longevity, imperfect substitution between workers, different interest rate rules, and pension system design. Section 7 summarizes the main findings and discusses the resulting policy challenges. The algorithms used to find the model solution are described in Appendix A.

2. POLAND AFTER 1990

In 1989, after more than four decades of being a communist state, Poland started its transition from central planning to a market economy. That was unquestionably an enormous challenge. The Polish economy was relying on heavy industry and small-farm, subsidized agriculture, while the service sector was largely neglected. Production and foreign trade (mainly within the Soviet bloc) was planned by the state. As a result, the (state-owned) enterprises were often inefficient, uncompetitive, and operated outdated technologies. On the other hand, full employment, low inequality, and a variety of state-provided welfare rights and benefits (including childcare services, subsidies, etc.) were guaranteed, at least officially.

The task of a deep restructuring of the Polish economy was even more challenging and imperative due to the macroeconomic crisis in 1989 with hyperinflation, shortages, soaring external debt, and nearly depleted foreign exchange reserves. In 1990, a set of radical reforms, called the “Balcerowicz Plan” (after the name of the minister of finance) or “Shock Therapy”, was introduced. It involved eliminating price controls and most of subsidies, putting limits on an extensive wage rise, large devaluation of the Polish exchange rate, cutting back or eliminating trade barriers, and reforming the National Bank of Poland (NBP), that stopped granting subsidized loans to state-owned enterprises and raised the interest rates. Fiscal and monetary tightening relatively soon stabilized inflation, eliminated fiscal deficit, and ensured currency convertibility. Yet, all of this did not come without costs, from which the major one was a substantial rise in unemployment.[8] In the following years, Poland continued structural reforms that modernized its economy and in 2004 the country became a member of the European Union (EU).

In the rest of this section, I give a brief overview of the Polish economy after 1990. Special attention is put on the two processes that are the main focus of this paper, namely, an educational boom and fertility decline. All cited statistics come from the official sources, including the Polish Central Statistical Office (CSO), Eurostat, and World Bank.

Macroeconomic developments. Since 1992, Poland experienced stable and relatively fast growth of output and consumption. Real GDP (gross domestic product) was growing on average at 3.6% annually between 2000 and 2015. Poland was also the only country in the EU that did not report a fall in GDP during the Great Recession in 2009. The Polish economy was supported by the inflow of foreign capital, including structural funds from the EU, and free access to the EU market. Due to fast convergence that was accompanied by complex structural and social changes, Polish transition is widely perceived as a success story (World Bank Group, 2017).

The major struggle of the Polish economy after 1990 was high unemployment. It grew rapidly at the beginning of the transition to a market economy, peaking at above 16% in 1994. The economic slowdown in 2000–2004 was another period of high unemployment. In 2002, about more than one-fifth of all Poles willing to work were unemployed. Since 2005, unemployment has been systematically

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

decreasing (with the exception of the Great Recession), and in 2017, it stood below 5%, that is, at the lowest level since the transition. Although Polish labor market conditions are much better now compared to the 1990s and 2000s, there are still some unresolved issues. One of the major concerns is temporary employment, whose share remains one of the highest in the EU. Moreover, with relatively low labor force participation of older workers and rapid population aging, Polish labor supply is expected to shrink considerably in the future.[9]

Poland's accession to the EU in 2004, together with a difficult situation on the labor market in the early 2000s, triggered labor migration. According to the official estimates, at the end of 2017, more than two and a half million Poles were abroad for more than three months. The outflow of Polish workers and rapid population aging resulted in labor shortages. In recent years, however, they have been compensated by temporary immigration, mostly from Ukraine. For lack of comprehensive unified data sources, the scale of this inflow is difficult to quantify (Roszkowska et al. (2017)). According to the NBP report (Chmielewska et al. (2018)), in the first three quarters of 2017, there were around 900 thousand Ukrainian migrants in Poland.

Another important challenge has been the presence of the informal sector. However, the share of the Polish shadow economy in GDP has been systematically declining since 1991 and now constitutes between 6 and 24% of GDP, depending on the estimation method (Medina and Schneider (2018)). It is hence still higher than observed in the Western EU countries.

Inequalities. Between 1990 and 2005, Poland experienced a significant rise in income inequality, which is mainly attributed to two factors: the economic transition process and cyclical fluctuations (Keane and Prasad (2002), Newell and Socha (2007), Brzeziński et al. (2013)). Indeed, expansion of the private sector changed the determinants of wages. They became more volatile, better linked to individual productivity, and the education premium increased. The second factor behind rising income inequality was high unemployment during the economic slowdown in 1998–2002.

Since then, income inequality in Poland has been stable or even slightly decreasing. In 2017, the Gini coefficient for income after taxes and transfers stood at 0.3, which points to a relatively high level of income equality.[10] Similarly, according to the Polish survey of household finance, net-wealth inequality in Poland is moderate compared to euro-area countries (Grejcz and Żółkiewski (2017)).

Fertility patterns. When the process of transition to a market economy began, Poland had already undergone all of the four stages of the demographic transition. Indeed, after a period of decline, its fertility rate was generally stable between 1970 and 1985 and stood on average at 2.2, while the life expectancy increased significantly compared to the first half of the 20th century (see Figure 1 and Mackenbach (2013)).[11]

It was only later—during the 1990s—when the Polish TFR decreased further from 2.2 to the levels associated with the lowest-low fertility. A reduction in childbearing following transition to a market economy was a common feature among postcommunist countries. Frejka (2008) distinguishes three main reasons of decreasing fertility in the postcommunist countries in the 1990s: economic crisis at the early stage of transition, the diffusion of western values and lifestyles, and institutional conditions specific to a market economy (such as, among others, job insecurity, expensive housing, or new career opportunities). According to Kotowska et al. (2008), the major driver of the observed shift in Polish fertility behavior was the changing role of the state. It was no longer a provider of certain services and social benefits, and thus, some of its responsibilities had to be taken over by individual households. The new labor market conditions made jobs more insecure, unemployment risk grew, especially for women, and households economic situation became more uncertain. Adding to that missing state response in form of a proper family policy, households found it hard to reconcile work and family. Mishtal (2009) also highlights the difficult situation of Polish women on the labor market as a factor that makes them postpone or forego childbearing. However, in recent years, the labor market improved significantly, new family benefits were introduced, but fertility remained low.

Regarding the diffusion of western values, as stated by Kotowska et al. (2008), the ideological change in Poland was relatively slow, although there were some signs of an increase in childbearing age or the acceptance of cohabitation, especially for younger cohorts (see also Matysiak (2009)). Recent data show that both the mean age of women at birth of a first child and the proportion of live births outside marriage continue to rise. The former reached 27.2 in 2016 compared to 24.5 in 2000, and the latter increased from 12.2 to 25% in a similar period. It might indicate that the ideological change keeps progressing.

The postponement of childbearing results in underestimated values of the TFR. However, after 2010, the speed of growth of the childbearing age slowed down to slightly more than one month per year. According to rough calculations, it should depress the current TFR by about one-twelfth, which still points to relatively low fertility (Goldstein et al. (2009)). Other arguments that support the thesis that the decline in childbearing in Poland is not just a temporary phenomenon are the following. First, Polish women still have their children at a relatively young age compared to other European countries. Second, the age of 30 is seen as an ultimate deadline (a biological limitation) for having a first child by many Polish women (Matysiak (2009)).

Finally, more educated women (parents) in Poland tend to have fewer children, and interestingly, this pattern was also present during the communist period (Brzozowska (2014), Osiewalska (2017)).

Tertiary education. During the communist era with a centrally planned economy, dominated by industrial and agricultural sectors, there was neither demand nor government's desire to support the expansion of tertiary education. Thus, the share of academic graduates reminded low. It all changed in the 1990s, when the educational boom started, and the gross enrollment ratio in tertiary education in Poland increased from 20% in 1990 to 67% in 2016.

Investment in higher education was encouraged by a large increase in the education premium. Indeed, Keane and Prasad (2006) document a significant rise in the premium at the beginning of the 1990s, resulting from both more competitive wage settings and increasing demand for skilled labor. Adamczyk and Jarecki (2008) show that the returns to education also rose between 2000 and 2004. As for the period 2005–2010, the empirical literature is not that conclusive. While Majchrowska and Roszkowska (2014) point to an increase in the education premium, Gajderowicz et al. (2012) claim that it declined between 2007 and 2009.

Educational attainment is an important factor that shapes individual earnings, and it does so not only through the education premium. The individual income process of Polish households with an academic degree is found to be more persistent, compared to that for the rest of Polish households (Kolasa, 2017). It means that past earnings affect current income more strongly for the group of more educated individuals.[12]

3. MODEL DESCRIPTION

To analyze the fertility and educational changes, I use a small open economy general equilibrium model. The model economy is populated by heterogeneous households, who differ in age, educational attainment, wealth, and productivity. Households face mortality risk and are hit by uninsured individual income shocks. They smooth consumption by accumulating assets. These are claims on physical capital and foreign assets. Households obligatorily participate in a pay-as-you-go pension system, that is balanced via adjustments in the contribution rate. Consistently with the small open economy assumption, the real interest rate is a sum of exogenous and endogenous components, where the latter responds to the economy's foreign debt.

3.1. Households

The model economy is inhabited by a continuum of households. Households either have or do not have an academic degree ($h = 1$ or $h = 0$), and their level of education is exogenous and predetermined. The less educated individuals enter the model at the age of 20 ($j = 20$), the more educated at the age of 25 ($j = 25$), both with no financial assets. The share of individuals with an academic degree might vary between cohorts and for a cohort aged 25 it is denoted by p_t^h. All households work until the age of 65 ($j = 65$), and then they retire and live up to the maximum age of 85. The conditional survival probability (s_j) depends on age so that an individual lives at least up to the age of m with probability $S_m = \Pi_{i=20}^{m} s_i$. Each period a new cohort aged 20 is born, and the size of this cohort is $1 + n_t$ times the mass of people born in the previous period.

Labor income. Each period working-age households receive an endowment of efficiency labor units described by the following formula:

$$z(h, j, e_h) = \bar{z}(h, j)e_h. \qquad (1)$$

and hence it consists of two components. The first one $\bar{z}(h, j)$ is deterministic and depends on age j and the level of education h, so that more educated households are on average more productive within their age group. The second component e_h is stochastic and follows a Markov process. Thus, it is independent and identically distributed across individuals with the same educational attainment level h. The conditional probability matrix takes a form $\pi^h_{kl} = P(e_h = e^l_h | e_h = e^k_h)$, where $\pi^h_{kl} \geq 0$ and $\sum_{l=1}^{N} \pi^h_{kl} = 1$ for each $k, l = 1, 2 \ldots, N$ and $e^k_h, e^l_h \in E_h = \{e^1_h, e^2_h, \ldots, e^N_h\}$. Although there is no direct unemployment in the model, it is incorporated in the structure of productivity shocks.

Pension payments received by households are a fraction of wages earned just before retirement, that is at the age of 65.

As a result, total net labor income at time t is summarized by the formula:

$$y_t(h, j, e_h) = \begin{cases} (1 - \tau_t)w_t z(h, j, e_h) & \text{for } j \leq 65, \\ (1 - \tau_t)w_t \theta \bar{z}(h, j = 65)e_h(j = 65) & \text{for } j > 65, \end{cases} \qquad (2)$$

where w_t is the wage rate per efficiency unit of labor, τ_t stands for the contribution rate, and θ is the replacement rate.

The time t share of households with age j, educational attainment level h and hit by productivity shock e^l_h is denoted by $\mu_t(h, j, e_h = e^l_h)$. Aggregates by certain characteristics are defined by dropping them from the bracket.

Preferences. The period utility depends on consumption c in the following way:

$$u(c) = \log(c). \qquad (3)$$

Thus, the risk aversion parameter $\sigma = -cu''(c)/u'(c)$ equals one, that is the value recommended by Chetty (2006).

Budget constraint. During their lifetime, households accumulate assets denoted by $a_t(h, j, e_h)$. The corresponding rate of return equals r_t. Each period and for each household, the following budget constraint is satisfied:

$$(1 + r_t)a_t(h, j - 1, e_h) + y_t(h, j, e_h) + \mathrm{tr}_t = a'_t(h, j, e_h) + c_t(h, j, e_h), \qquad (4)$$

where tr_t stands for accidental bequests, assumed to be equally distributed over surviving households.

Household decision problem. The household's maximization problem at time i is the following:

$$\max E_t \sum_{i=t}^{\infty} \beta^{i-t} S_{j+i-t} u(c_i(h, j + i - t, e_h)), \qquad (5)$$

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

subject to (4) and to $c_i(h, j+i-t, e_h) > 0$ for all $i \geq t$, where the future realizations of $w_i, r_i, \tau_i, \text{tr}_i$ for $i = t, t+1, \ldots$ are known. S_{j+i-t} is a probability of survival up to the age of $j+i-t$.

3.2. Government

The role of the government is reduced to collect contributions, which finance pension expenditures, and to redistribute accidental bequests. The contribution rate balancing the pension system satisfies

$$(1-\tau_t) * \sum_{j>j0} \sum_{h \in H} \sum_{l=1}^{N} pen(h, e_h(j=65) = e_h^l) * \mu_t(h, j, e_h(j=65) = e_h^l) =$$

$$= \tau_t \sum_{j \leq j0} \sum_{h \in H} \sum_{l=1}^{N} z(h, j, e_h = e_h^l) * \mu_t(h, j, e_h = e_h^l), \quad \textbf{(6)}$$

$$pen(h, e_h(j=65)) = \theta * \bar{z}(h, j=65) * e_h(j=65). \quad \textbf{(7)}$$

3.3. Firms

Firms are competitive in the product and factor markets. They are identical and produce final homogeneous good (Y_t) according to the Cobb–Douglas technology and with constant returns to scale:

$$Y_t = K_t^{\alpha} (G_t L_t)^{1-\alpha}, \quad \textbf{(8)}$$

where G_t stands for aggregate productivity, which increases at a constant annual pace g, K_t is the aggregate capital, L_t denotes the aggregate labor input, and α is a capital share in production. Capital depreciates at a constant rate δ and factor prices equal their marginal products:

$$\partial Y_t / \partial L_t = w_t \quad \text{and} \quad \partial Y_t / \partial K_t = r_t + \delta. \quad \textbf{(9)}$$

3.4. Interest Rate Rule

Following Schmitt-Grohé and Uribe (2003), the domestic real interest rate equals world interest rate (r^*) plus a risk premium. The latter reacts to changes in the foreign debt in a way described in the formula:

$$r_t = r^* + \phi \left(\exp\left(\frac{K_t - A_t}{Y_t} \right) - 1 \right), \quad \textbf{(10)}$$

where A_t stands for aggregate households assets.

3.5. Equilibrium

In the next subsections, I will present two definitions of equilibrium: a general competitive equilibrium and its special case—a stationary equilibrium. Since in

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

the model both population and technology are nonstationary, it is convenient to define equilibrium using appropriately detrended variables. I will use the following transformations: $\tilde{c}_t(h,j,e_h,a) = c_t(h,j,e_h,a) * (1+g)^{20-j}$, $\tilde{a}'_t(h,j,e_h,a) = a'_t(h,j,e_h,a) * (1+g)^{20-j}$, $\tilde{\mathrm{tr}}_t = \mathrm{tr}_t * (1+g)^{20-j}$, $\tilde{Y}_t = Y_t/\,(G_tN_t)$, $\tilde{L}_t = L_t/N_t$, $\tilde{K}_t = K_t/\,(G_tN_t)$, $\tilde{w}_t = w_t/G_t$, where N_t is the size of population.

3.5.1. Competitive equilibrium. Let $\mu_t(h,j,e_h,a)$ be a probability measure in time t defined on $X = H \times J \times E_h \times [0,\infty)$. Then, given the initial condition μ_0 and $\tilde{a}'_0(h,j,e_h,a)$, a competitive equilibrium for the model economy is the sets of household's policy functions $\{\tilde{c}_t(h,j,e_h,a), \tilde{a}'_t(h,j,e_h,a)\}_{t=1}^{\infty}$, probability measures $\{\mu_t\}_{t=1}^{\infty}$, a vector of factor prices $\{\tilde{w}_t, r_t\}_{t=1}^{\infty}$, a vector of contribution rates and accidental bequests $\{\tau_t, \tilde{\mathrm{tr}}_t\}_{t=1}^{\infty}$, a vector of macroeconomic aggregates $\{\tilde{K}_t, \tilde{L}_t\}_{t=1}^{\infty}$, and a function Q_t governing the changes in household distribution over X, such that the following conditions hold:

1. The values of aggregate variables result from households' individual choices:

$$\tilde{L}_t = \int z(h,j,e_h)d\mu_t, \tag{11}$$

$$\tilde{A}_t = (N_{t-1}/N_t)\,(1+g)^{-1}\int \tilde{a}'_{t-1}(h,j,e_h,a)d\mu_{t-1}, \tag{12}$$

$$\tilde{C}_t = \int \tilde{c}_t(h,j,e_h,a)d\mu_t, \tag{13}$$

$$\tilde{\mathrm{tr}}_t = (N_{t-1}/N_t)\,(1+g)^{-1}\int (1+r_t)(1-s_j)\tilde{a}'_{t-1}(h,j,e_h,a)d\mu_{t-1}. \tag{14}$$

2. The contribution rate τ_t satisfies (6).
3. Factor prices are equal to their marginal products:

$$\partial\tilde{Y}_t/\partial\tilde{L}_t = \tilde{w}_t \quad \text{and} \quad \partial\tilde{Y}_t/\partial\tilde{K}_t = r_t + \delta. \tag{15}$$

4. Domestic real interest rate r_t satisfies $r_t = r^* + \phi * \left(\exp\left(\frac{\tilde{K}_t - \tilde{A}_t}{\tilde{Y}_t}\right) - 1\right)$.
5. Given the vectors $\{\tilde{w}_t, r_t\}_{t=1}^{\infty}$ and $\{\tau_t, \tilde{\mathrm{tr}}_t\}_{t=1}^{\infty}$, households optimally choose $\tilde{c}_t$ and $\tilde{a}'_t$.
6. Aggregate resource constraint holds ($\tilde{I}_t$ stands for investments at time t)

$$\tilde{Y}_t + (1+r_t)(\tilde{A}_t - \tilde{K}_t) - (1+g)\,(N_{t+1}/N_t)\,(\tilde{A}_{t+1} - \tilde{K}_{t+1}) = \tilde{C}_t + \tilde{I}_t, \tag{16}$$

$$\tilde{I}_t = (N_{t+1}/N_t)\,(1+g)\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t. \tag{17}$$

7. Population distribution changes according to the rule:

$$\mu_{t+1} = \int_X Q_t d\mu_t. \tag{18}$$

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

The function Q, defined on the four-dimensional set X, is determined by distributions over educational level, age, productivity shocks, and assets. It also depends on an exogenous vector $\{p_t^h, n_t\}_{t=1}^{\infty}$ and survival probabilities s_j, $j \in J$.

3.5.2. Stationary equilibrium. If all aggregate variables in the model are constant over time, we can speak of a stationary equilibrium. The formal definition is presented below. Assume that $p_t^h = p^h$ and $n_t = n$ for all $t = 0, 1, \ldots$. Let $\mu(h, j, e_h, a)$ be a probability measure defined on $X = H \times J \times E_h \times [0, \infty)$. A stationary competitive equilibrium for the model economy is the sets of household's policy functions $\tilde{c}(h, j, e_h, a)$ and $\tilde{a}'(h, j, e_h, a)$, probability measure μ, factor prices $(\tilde{w},\ r)$, contribution rate and the value of accidental bequests $(\tau,\ \tilde{\text{tr}})$, macroeconomic aggregates $(\tilde{K},\ \tilde{L})$, and a function Q, such that the following conditions hold:

1. The values of aggregate variables result from households' individual choices:

$$\tilde{L} = \int z(h, j, e_h)d\mu, \tag{19}$$

$$\tilde{A} = (1+n)^{-1}(1+g)^{-1} \int \tilde{a}'(h, j, e_h, a)d\mu, \tag{20}$$

$$\tilde{C} = \int \tilde{c}(h, j, e_h, a)d\mu, \tag{21}$$

$$\tilde{\text{tr}} = (1+n)^{-1}(1+g)^{-1} \int (1+r)(1-s_j)\tilde{a}'(h, j, e_h, a)d\mu. \tag{22}$$

2. The contribution rate τ satisfies (6).
3. Factor prices are equal to their marginal products:

$$\partial\tilde{Y}/\partial\tilde{L} = \tilde{w} \text{ and } \partial\tilde{Y}/\partial\tilde{K} = r + \delta. \tag{23}$$

4. Domestic real interest rate r satisfies $r = r^* + \phi * \left(\exp\left(\frac{\tilde{K}-\tilde{A}}{\tilde{Y}}\right) - 1\right)$.
5. Given $\tilde{w}$, r, τ, and $\tilde{\text{tr}}$, households optimally choose $\tilde{c}$ and $\tilde{a}$.
6. Aggregate resource constraint holds

$$\tilde{Y} + (r - (1+n)(1+g) + 1) * (\tilde{A} - \tilde{K}) = \tilde{C} + \tilde{I}, \tag{24}$$

$$\tilde{I} = \tilde{K} * (\delta + (1+n)(1+g) - 1). \tag{25}$$

7. The population distribution is constant over time:

$$\mu = \int_X Q d\mu. \tag{26}$$

Since the model does not have a closed form solution, an equilibrium has to be found numerically. The algorithms used for finding transitional dynamics and stationary equilibria are described in Appendix A.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

4. MODEL DISCUSSION AND CALIBRATION

The model is design to study the macroeconomic and redistributive impact of changes in educational attainment and fertility. More specifically, I use it to study transition from the equilibrium with low educational attainment (low p^h) and high population growth (high n) to the equilibrium with high educational attainment (high p^h) and low population growth (low n). The model is calibrated to the Polish economy and simulations start in 1990.

As it was discussed in Section 2, in the 1990s, Poland went through transition to a market economy, and its income per capita started converging toward the levels observed in Western Europe. These processes had a significant impact on economic developments in the country. Studying them, however, is beyond the scope of this paper, so the model abstracts from them. It will not replicate, in particular, the inflow of foreign capital or dramatic swings in macroeconomic variables, including GDP and unemployment, in the early 1990s. The goal instead is to capture the key features of the economy after most of the transition shock is absorbed. This is because, even though the simulations start in 1990, their considerable horizon covers the future. It is then a deliberate decision not to calibrate the model to the early 1990s and, thus, not to introduce estimates that reflect only temporary effects of transformation.

Both the educational boom and fertility decline are exogenous in the model. This approach is typical in papers that rely on comparable modeling techniques and are designed to study macroeconomic consequences of selected processes, and not their underlying causes (Gourinchas and Parker (2002), Börsch-Supan et al. (2006), Díaz-Giménez and Díaz-Saavedra (2009)).

As it is also common in this line of literature, the model abstracts from international migration. From the Polish perspective, the two important flows since 2000 are the outflow of Polish workers after the EU accession and the recent inflow of Ukrainian workers. While analyzing separately, their impact on the Polish economy might be substantial, but as they largely offset each other, their joint effect is much less profound, at least over the medium run. Moreover, before incorporating migration into the model, the scale and future trends of Ukrainian immigration need to be assessed, and this is itself a challenging task, warranting a separate study.

There is also no endogenous choice of working hours in the model. Nevertheless, this assumption is quite in line with the characteristics of the Polish labor market. According to the Polish Labor Force Survey, in the second quarter of 2016, only 7% of employees were part-time workers, from which around half would prefer to work full-time if possible. It is also consistent with evidence on the discrepancy between workers' actual and desired number of working hours (see Wyszyński (2016)).

For the questions raised in this paper, replicating the key characteristics of income distribution is crucial. Therefore, in the model, the life cycle income profile and the individual income process are calibrated very carefully based on the empirical evidence from Polish household level data. Importantly, the calibration

TABLE 1. Educational structure of population during the educational change

Periods	Share of households with an academic degree in a group aged 25–29
before 1995	0.180
1995–1999	0.225
2000–2004	0.300
2005–2009	0.375
2010–2014	0.450
after 2014	0.480

explicitly distinguishes between more and less educated individuals. In the rest of this section, I discuss these calibration choices in more detail.

4.1. Calibration

I consider the following four scenarios for stationary equilibria (steady states):

- Scenario 1 *baseline*—low share of households with an academic degree, high population growth rate ($p^h = 18\%,\ n = 0.9\%$),
- Scenario 2 *educational change*—high share of households with an academic degree, high population growth rate ($p^h = 48\%,\ n = 0.9\%$),
- Scenario 3 *fertility change*—low share of households with an academic degree, low population growth rate ($p^h = 18\%,\ n = -0.6\%$),
- Scenario 4 *educational and fertility changes*—high share of households with an academic degree, low population growth rate ($p^h = 48\%,\ n = -0.6\%$).

Between scenario 1 and one of the scenarios from 2 to 4, the model economy is assumed to undergo transitional dynamics, during which vector $\{(p_t^h, n_t)\}_{t=1}^{t_0}$ evolves as described below. In the case of the educational change, the population growth rate remains constant and equals to that in the baseline scenario, but the share of more educated households gradually changes as described in Table 1. During the fertility change, there are no shifts in the educational structure of the population, but n_t changes permanently in 1990 and remains at the new level afterwards. Both the assumed population growth rates and the shares of more educated households match the empirical data for Poland. The former is based on the Eurostat historical data (mean for 1960–1990) and its projections (for 2066–2080), and the latter corresponds to statistics taken from the Polish HBS (Figure 2).[13]

Regarding the model structural parameters, the assumed values are summarized in Table 2. The replacement rate θ and the world real interest rate r^* match the observed averages, while the aggregate productivity growth rate g corresponds to the European Commission's long-term projection of total factor productivity for Poland.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 2. Model calibration

Parameters	Values	Sources
Discount factor β	0.977	
Capital share in production function α	0.307	
Depreciation rate δ	0.08	
Replacement rate θ	43%	OECD (Organisation for Economic Co-operation and Development) Pension at Glance 2015
Aggregate productivity growth rate g	1.4%	European Commission Aging Report 2015 Projected mean for the period 2013–2060
Interest rate risk premium ϕ	0.0232	
Global real interest rate $r*$	2.3%	OECD 1956–2015 Inflation-adjusted long-term interest rate in the US
Income process		
Households with an academic degree		
Autocorrelation coefficient ρ	0.92	Kolasa (2017)
Variance of the permanent component of income σ_μ^2	0.02	Kolasa (2017)
Variance of the transitory component of income σ_ϵ^2	0.167	Calibrated to the Gini coeff.: 0.303 (OECD 2012)
Households without an academic degree		
Autocorrelation coefficient ρ	0.82	Kolasa (2017)
Variance of the permanent component of income σ_μ^2	0.02	Kolasa (2017)
Variance of the transitory component of income σ_ϵ^2	0.167	Calibrated to the Gini coeff.: 0.303 (OECD 2012)
Productivity over the life cycle		Average income profile, Kolasa (2017)
Survival probabilities		Eurostat 2012

The deterministic component of life cycle productivity is approximated by average household income over the life cycle, estimated for Poland by Kolasa (2017). As regards the stochastic component of income, it is assumed that its logarithm (u_{ij}) is given by the following process:

$$u_{ij} = \epsilon_{ij} + v_{ij}, \tag{27}$$

$$v_{ij} = \rho v_{i,j-1} + \mu_{ij}, \tag{28}$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2), \quad \mu_{ij} \sim N(0, \sigma_\mu^2), \quad \epsilon_{ij} \perp \mu_{ij} \; i.i.d., \quad v_{i0} = 0, \quad E_i(u_{ij}) = 0. \tag{29}$$

To solve the model, this process needs to be discretized. For the permanent income shock (v_{ij}), I use the Tauchen and Hussey (1991) method and assume three states. The transitory income shock (ϵ_{ij}) takes two values: $-\sigma_\epsilon$ and σ_ϵ with equal probability. Parameters ρ and σ_μ of the income process are taken from Kolasa (2017), and σ_ϵ is calibrated to match the Gini coefficient for Poland.

The discount factor β, the share of capital in production function α, and the debt elasticity of the domestic interest rate ϕ are calibrated to target the share of consumption in output, risk premium, and international investment position in the baseline scenario. The resulting parameters are close to the values present in the literature. Finally, the rate of depreciation δ is set at the mean of the estimates taken in life cycle models calibrated for Poland (Rubaszek (2012), Makarski et al. (2016)).

The model period corresponds to five years, which translates into fourteen cohorts of households. All parameters in Table 2 are reported on an annual basis, so, when necessary, they are transformed to match the five-year windows.

The aggregate statistics for Poland are quite well matched by the model (see Table 3). It is worth noting that despite its simplicity, the model does a good job at matching the life cycle profiles of consumption (Figure 3).

5. MODEL IMPLICATIONS

In this section, I present the main findings that were obtained using the previously described model. I start with transitional dynamics and then compare the steady states before and after the changes. The last subsection is exclusively devoted to the development of inequalities.

5.1. Educational Change

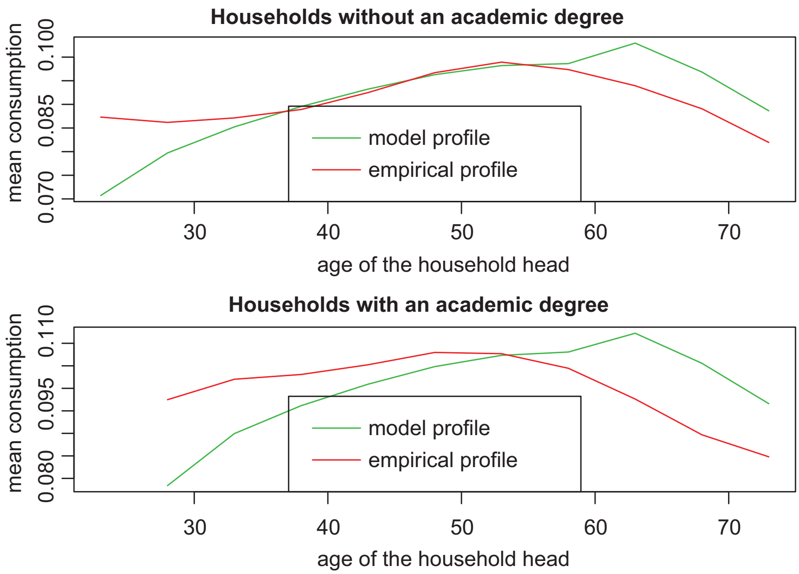
Transitional dynamics (from scenario 1 to scenario 2). In the case of the educational boom, changes in the educational structure of population start from younger households, while within older individuals the old (lower) share of educated workers still persists. From a macroeconomic perspective, we observe an increase in productivity of young workers, which translates into higher effective labor supply, higher output, and slightly lower wage per efficiency unit (Figure 4). Changes in effective labor supply are followed by an increase in capital demand.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 3. Steady-state characteristics

	Model	Data
Consumption to output ratio	0.74	0.74*
Investment to output ratio	0.25	0.25*
Net foreign assets to output ratio	−0.56	−0.56**
Interest rate risk premium	1.8%	1.9%***
Gini coefficient for workers	0.28	0.30****

Sources: *Nonfinancial national accounts 2004–2015, without government expenditures.
**Balance of payments statistics and nonfinancial national accounts 2004–2014.
***Federal Reserve System and OECD data 2001–2014, the difference between the Polish and the US inflation-adjusted 10-year government bonds yields.
****OECD data 2012, based on net income with transfers.



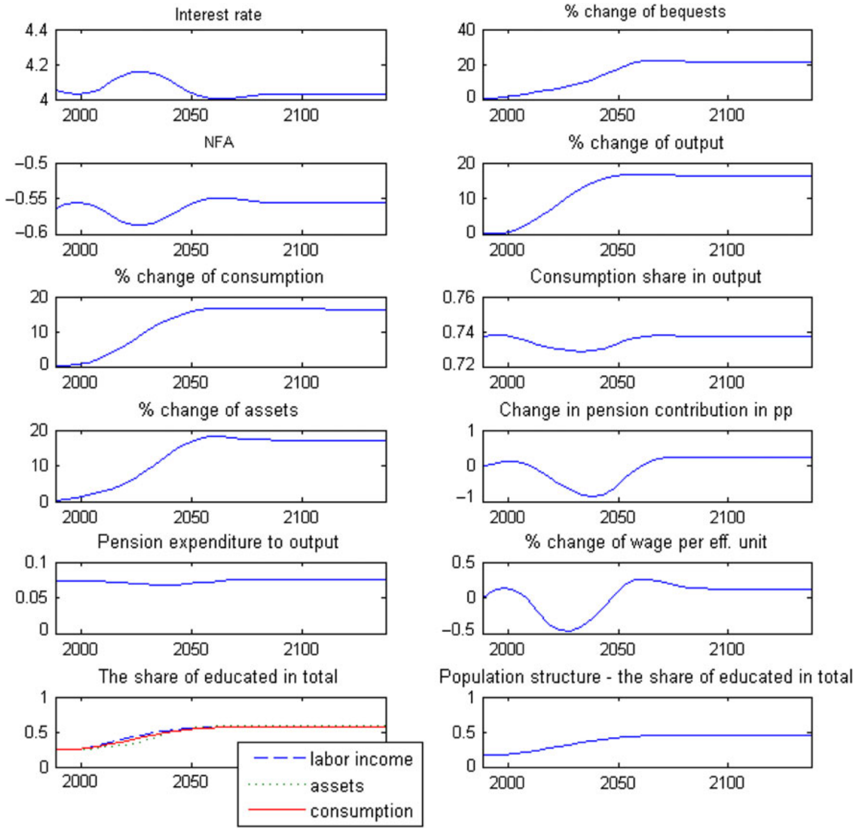
Note: Profiles are scaled with means equal one. The model profiles correspond to the baseline scenario, while the empirical profiles are calculated on the basis of Kolasa (2017) estimates.

FIGURE 3. Life cycle consumption profiles, model, and data.

Domestic assets are slightly higher (more educated individuals save more, even at the early stage of their life cycle); nevertheless, a significant part of capital has to be financed from abroad. In response for higher debt, the domestic interest rate goes up. During the initial periods of the educational change, pension system revenues increase but pension expenditures remain unchanged, which results in a lower contribution rate.

In the later phases of the educational change (after 2030), the higher share of more educated individuals is also observed for older cohorts, and the economy

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



Note: Changes to the baseline scenario. $n_t = 0.9\%$, $p^h = 18\%$ before 1995, $p^h = 22.5\%$ 1995–1999, $p^h = 30.0\%$ 2000–2004, $p^h = 37.5\%$ 2005–2009, $p^h = 45.0\%$ 2010–2014, $p^h = 48.0\%$ 2015 and after.

FIGURE 4. Educational change—transitional dynamics.

gradually converges to the new steady state. The contribution rate and wage per efficiency unit are steadily increasing, the domestic interest rate is declining, and the net foreign investment position is improving, until they come back to the levels close to the baseline scenario.

The propensity to consume of an individual household does not change much. In the short term, it decreases only slightly due to higher interest rate despite an expected rise in bequests. Therefore, the dynamics of macrovariables reflects mainly changes in composition of population between better and less educated workers.

New steady state. When the economy reaches its new steady state, there is a higher share of more educated (and on average more productive) individuals in all age groups compared to the situation before changes. This higher effective labor supply results in an increase in the main macroeconomic aggregates (Table 4).

TABLE 4. New steady states—compared to the baseline scenario (no. 1)

	Scenario 2	Scenario 3	Scenario 4
	Educational change	Fertility change	Educational and fertility changes
Δy per capita (%)	16.4	−10.2	4.6
Δc per capita (%)	16.3	−6.0	9.4
Δa per capita (%)	17.0	−5.7	10.4
Δi per capita (%)	16.7	−24.7	−12.1
Δnfa/y (pp)	0.4	5.1	5.6
Δr (pp)	0.0	−0.2	−0.2
Δc per household			
With an academic degree (%)	0.2	−6.7	−6.5
Without an academic degree (%)	0.4	−6.1	−5.7
Δa per household			
With an academic degree (%)	−1.3	−6.4	−7.8
Without an academic degree (%)	−1.7	−6.8	−8.7
Δl per household (income)			
With an academic degree (%)	−0.2	−11.0	−11.0
Without an academic degree (%)	−0.2	−10.3	−10.4
$\Delta\tau$ (contribution rate, pp)	0.3	5.6	5.8
Δw (wage per productivity unit, %)	0.1	1.0	1.1
ΔGini (income of workers, in scale 0–100)	1.9	0.1	1.9
ΔGini (assets, in scale 0–100)	1.0	−2.1	−1.0

According to the model, after the educational change output per capita is more than 16% bigger than in the baseline scenario. Consumption, investment, and assets grow by similar magnitude. As a result, in the long term, the domestic interest rate stabilizes at the baseline scenario's level. There is no permanent shift in the contribution rate either.

The new (higher) ratio of educated, and thus better paid, households in the population is responsible for a rise in inequalities. Indeed, the income Gini coefficient increases from 0.28 to 0.30 in the model (Table 5).

Households make their consumption-saving decisions not only based on their individual characteristics, but also on the expected realizations of the domestic interest rate, wage, and the contribution rate. The model indicates that these variables should not change significantly in the long term. Therefore, the steady-state life cycle profiles of income, consumption, and assets, analyzed separately for more and less educated households, do not move much (Figures 5, 6, and 7). Let us take a "newborn" household with or without an academic degree. Its economic situation before and after the educational change would be about the same, so

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 5. Macroeconomic aggregates, different scenarios

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
	Baseline, before changes	Educational change	Fertility change	Educational and fertility changes
$C/Y(\%)$	73.7	73.7	77.2	77.1
I/Y (%)	25.2	25.3	21.1	21.2
nfa$/Y(\%)$	−56.3	−55.9	−51.2	−50.7
Gini (income of workers, in scale 0–100)	28.2	30.1	28.3	30.1
Gini (assets, in scale 0–100)	58.5	59.5	56.4	57.5

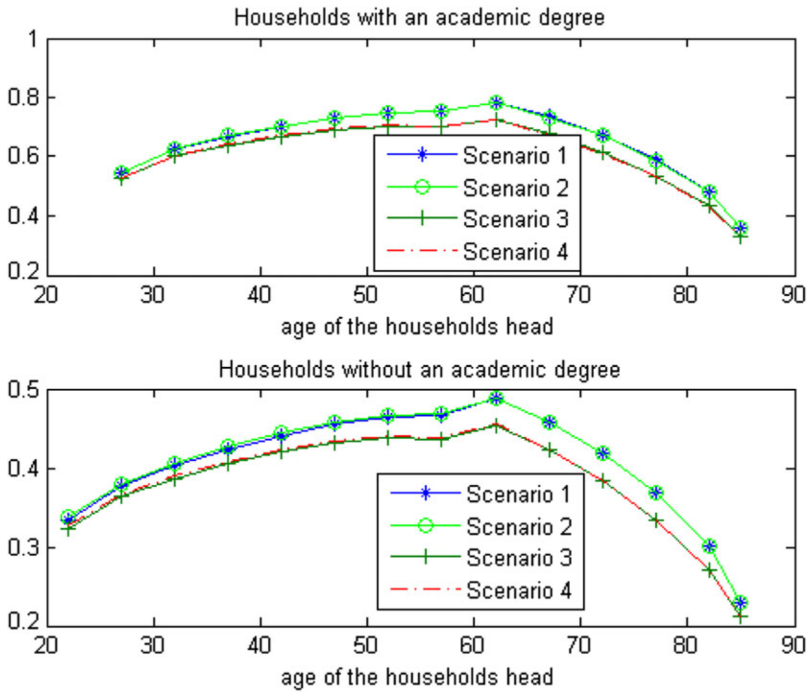
the substantial increase in macroeconomic variables observed after the transition results almost entirely from the changes in educational structure.

An interesting question is how the educational change affects less educated households. What they gain in the new steady state are higher bequests. In the model, it is assumed that bequests are equally distributed. However, in reality, there exists an association between parents' and their children's education (Black et al. (2005), Ermisch and Pronzato (2010)). In the extreme case, households might inherit only from individuals with the same educational level and the economic situation of less educated households might not improve at all. By assuming equally distributed bequests, the model might also underestimate changes in income inequality. As the propensity to consume is lower for more educated households, the long-term increase in assets (consumption) might also be larger (smaller) than that indicated by the model.

5.2. Fertility Change

Transitional dynamics (from scenario 1 to scenario 3). During the fertility change, (effective) labor supply steadily declines. It causes, on the one hand, an upward trend in wage per efficiency unit, but, on the other hand, deterioration in output per capita and demand for capital (Figure 8). Therefore, demand for foreign capital also decreases and so does the domestic interest rate. Finally, as expected, the share of pension expenditure in output has an upward trend, which results in a rising contribution rate.

New steady state. In the new steady state, the old dependency ratio is higher and the effective labor supply is lower compared to the baseline scenario. As a consequence, output per capita is 10% lower (Table 4). Since households start to consume their assets relatively late in the life cycle, a smaller decline is expected for assets (slightly less than 6%). As a result, domestic debt shrinks and the domestic interest rate falls by 0.2 pp. With lower labor supply, less investment is required and its share in output drops from 25 to 21% (Table 5).



Note: Scenario 1: $p^h = 18\%$ $n = 0.9\%$ Scenario 2: $p^h = 48\%$ $n = 0.9\%$ Scenario 3: $p^h = 18\%$ $n = -0.6\%$ Scenario 4: $p^h = 48\%$ $n = -0.6\%$.

FIGURE 5. Life cycle consumption profiles, different scenarios.

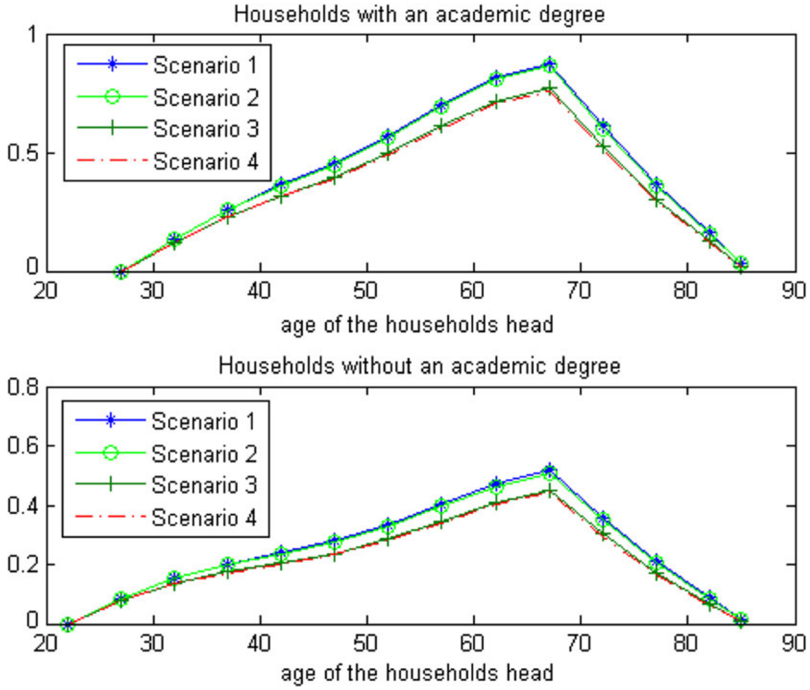
One of the most disturbing questions associated with population aging is how it would affect the pension system. In the model, pension expenditures always equal pension revenues, which is ensured by the adjustments in the contribution rate. The fertility change increases the contribution rate substantially, that is by 5.6 pp. As a result, in the new steady state, households earn, consume, and save proportionally less over the life cycle (Figures 5, 6, and 7).

The fertility change significantly affects assets inequality. The associated Gini coefficient is 2.1 pp smaller compared to the situation before changes.

5.3. Educational and Fertility Changes

Transitional dynamics (from scenario 1 to scenario 4). In the case of both changes, one can notice the divergent trends imposed by the fertility shift and the educational boom (Figure 9). Due to the fertility change, the share of workers in the population decreases, but, thanks to the educational change, those who enter the labor market are more productive. Eventually output per capita increases, but the pace of this increase is smaller than in the case of the educational change alone.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



Note: Scenario 1: $p^h = 18\%$ $n = 0.9\%$ Scenario 2: $p^h = 48\%$ $n = 0.9\%$ Scenario 3: $p^h = 18\%$ $n = -0.6\%$ Scenario 4: $p^h = 48\%$ $n = -0.6\%$.

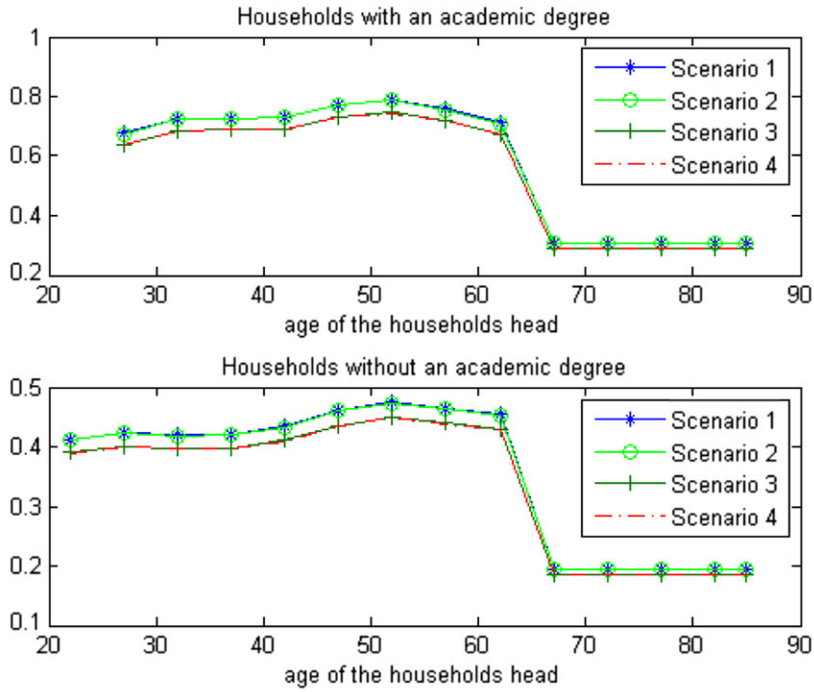
FIGURE 6. Life cycle assets profiles, different scenarios.

Similarly to the fertility change, in the case of both processes occurring simultaneously, the net foreign assets position shows an upward trend, the interest rate is declining, and the contribution rate is rising.

New steady state. In the new steady state, output per capita is higher by 4.6% compared to the baseline scenario. Consumption and assets rise more than twice as much. Therefore, the aggregate variables indicate a general improvement of households' economic situation. However, income and consumption per household by educational attainment are lower in the new steady state regardless of the level of education. This can also be seen in the life cycle profiles of more and less educated households, both of which shift downwards compared to the baseline. What worsens households' situation is a higher contribution rate, which increases by 5.8 pp. in the new steady state.

Lower effective labor supply leads to a decline in demand for capital, which improves the net foreign assets position and slightly (by 0.2 pp) decreases the domestic interest rate in the long term. Finally, when it comes to inequalities, they increase for income but fall in the case of assets.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



Note: Scenario 1: $p^h = 18\%$ $n = 0.9\%$. Scenario 2: $p^h = 48\%$ $n = 0.9\%$. Scenario 3: $p^h = 18\%$ $n = -0.6\%$. Scenario 4: $p^h = 48\%$ $n = -0.6\%$.

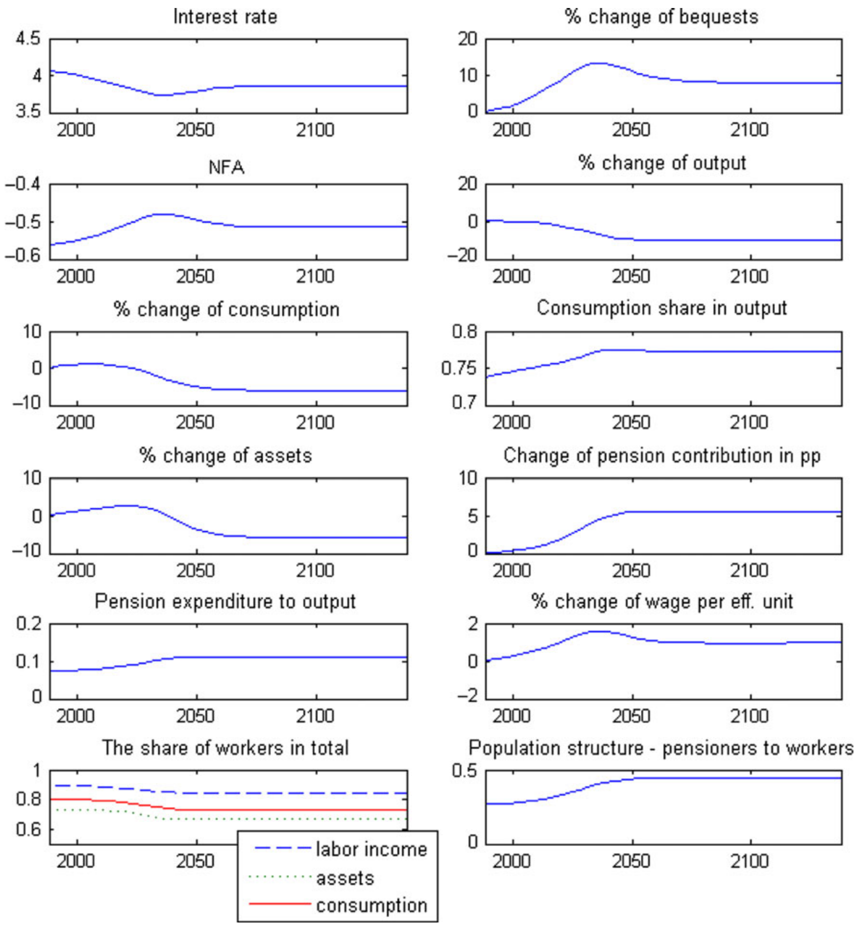
FIGURE 7. Life cycle income profiles, different scenarios.

5.4. Impact on Inequalities

To examine the changes in the distribution of income, consumption, and assets in more detail, I use several inequality measures, the exact formulas of which are presented in Table 6. The outcomes are depicted in Figures 10–13. According to all analyzed indices, the effect of the fertility change on income inequality is negligible. However, the model indicates that during the educational change income inequality continuously rises until 2060.[14] The Theil (1967) decomposition shows that the main source of the variation in income distribution is inequality within educational groups, but what causes the major increase in income inequality are changes in between group inequality. Eventually, in the new steady state (scenario 4), the Gini coefficient is 1.9 pp higher than in the baseline scenario.

Similarly, consumption and assets inequalities are also higher in the long term due to the educational change. However, between 1990 and 2020, the model predicts their decline. During that period, the share of young educated households is increasing at the expense of young but less educated individuals. Since consumption and accumulated assets increase over the life cycle (at least up to a certain age), the higher share of young individuals among all educated workers translates

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



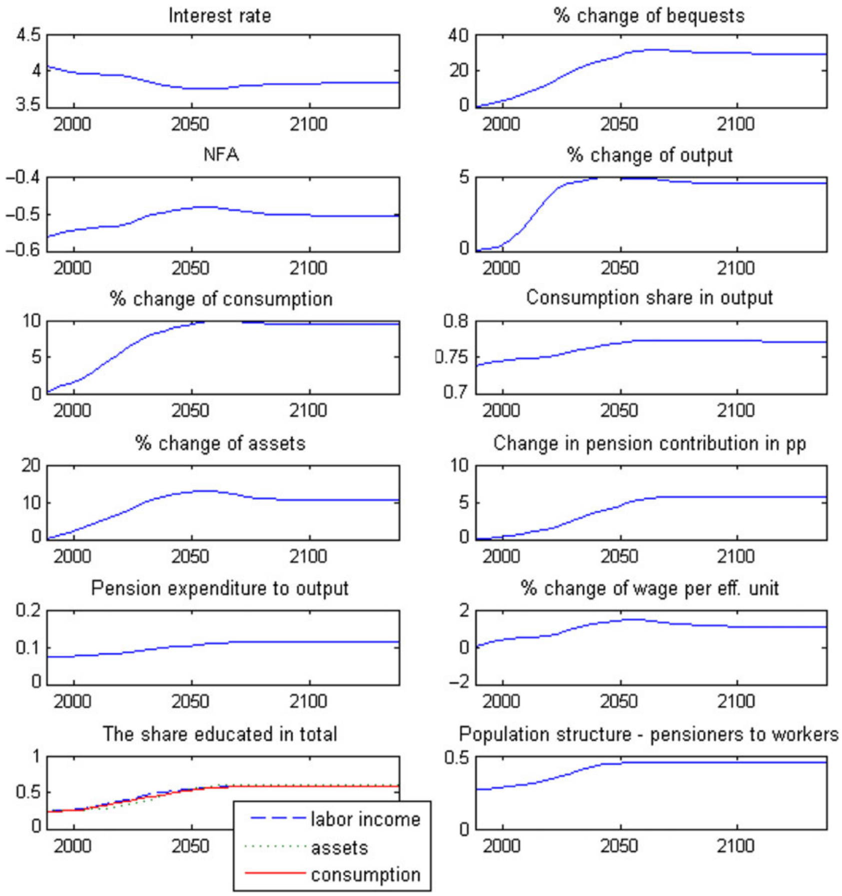
Note: Changes to the baseline scenario. $n_t = 0.9\%$ before 1990, $n_t = -0.6\%$ after 1990, $p^h = 0.18\%$.

FIGURE 8. Fertility change—transitional dynamics.

into lower mean consumption and assets within this group. The opposite is true for less educated households, where the share of young workers decreases. As a result, the between-groups inequality shrinks.

In contrast to income and consumption inequalities, the variation in assets distribution is significantly and negatively affected by the lower fertility. The impact of the fertility change is stronger than that of the educational change, which results in a lower value of the Gini coefficient in the new steady state (scenario 4). In the case of assets distribution, there is a substantial share of households with no assets. It constitutes around one-fifth of the population in the baseline scenario, and it drops by 4 pp after the fertility change. This substantial shift accounts for a large part of the decline in assets inequality.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



Note: Changes to the baseline scenario. $p^h = 18\%$ before 1995, $p^h = 22.5\%$ 1995–1999, $p^h = 30.0\%$ 2000–2004, $p^h = 37.5\%$ 2005–2009, $p^h = 45.0\%$ 2010–2014, $p^h = 48.0\%$ 2015 and after, $n_t = 0.9\%$ before 1990, $n_t = -0.6\%$ after 1990.

FIGURE 9. Educational and fertility changes—transitional dynamics.

6. ALTERNATIVE MODEL ASSUMPTIONS

This section aims to expand the analysis of different model assumptions. First, I define the demographic change by combining the fertility change with the process of rising longevity and study its long-term impact on the Polish economy. Second, I allow the less and more educated workers to be imperfect substitutes. Next, to examine the importance of assumptions associated with the interest rate, two extreme cases are considered: a closed economy, and an open economy with no interest rate risk premium. Finally, the effects of the educational and fertility changes are quantified for an economy in which the replacement rate adjusts to balance the pension system with a fixed contribution rate.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

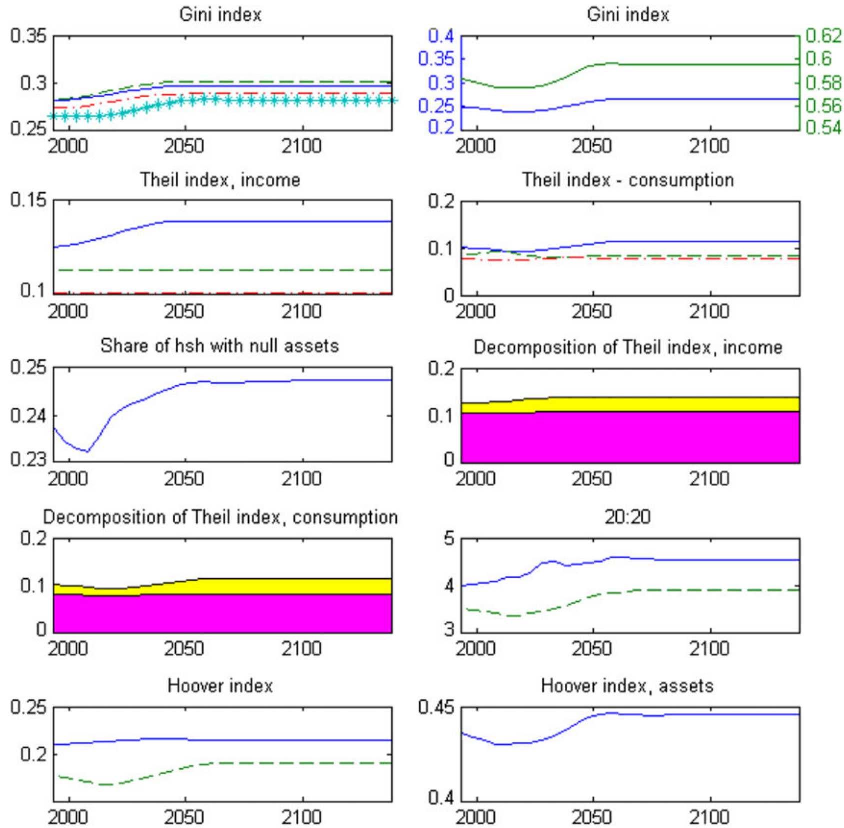
TABLE 6. Definitions of the selected inequality measures

Measures	Formulas
Gini index	$\left(\sum_{i=0}^{N}\sum_{j=0}^{N} f_k(k(i)) * f_k(k(j)) * \lvert k(i)-k(j)\rvert\right)$ ... ... $\Big/ \left(2 * \sum_{j=0}^{N}\sum_{i=0}^{N} k(i) f_k(k(i))\right)$, where f_k- density function on the set $K = \{k(0), k(1), \ldots, k(N)\}$
Theil index	$\sum_{i=0}^{N} f_k(k(i)) * \frac{k(i)}{\mu} \log\left(\frac{k(i)}{\mu}\right)$,
Decomposition:	where $\mu = \sum_{i=0}^{N} f_k(k(i)) * k(i)$, log—the natural logarithm
- Inequality within groups	$\sum_{m=1}^{M} s_m * \left(\sum_{i=0}^{N} f_{k\mid m}(k(i)) * \frac{k(i)}{\mu_m} \log\left(\frac{k(i)}{\mu_m}\right)\right)$, where $s_m = \left(\sum_{i=0}^{N} f_k(k(i)) * k(i) * I_m(k(i))\right) \Big/ \left(\sum_{i=0}^{N} f_k(k(i)) * k(i)\right)$ M—the number of groups $m \subset K$, μ_m—mean for the group m, $f_{k\mid m}$—conditional density function, I_m—characteristic function of the set m
- Inequality between groups	$\sum_{m=1}^{M} s_m \log\left(\frac{\mu_m}{\mu}\right)$
Hoover index (Robin Hood index)	$\frac{1}{2} * \sum_{i=0}^{N} \lvert \frac{f_k(k(i)) * k(i)}{k_{\text{total}}} - f_k(k(i))\rvert$, where $k_{\text{total}} = \sum_{i=0}^{N} f_k(k(i)) * k(i)$
20:20	The ratio between a mean of a quintile with the highest values of k and a mean of a a quintile with the lowest values of k

Higher life expectancy. Until now, I here analyzed only one component of the demographic change occurring in Poland, which is lower fertility. However, another important source of population aging is rising life expectancy. I do not include rising longevity in the main simulations for the following reasons. First, both processes, that is, the fertility decline and the rising share of university graduates, occurred in Poland more or less in the same periods. The time needed by the economy to converge to the new steady state in these two cases is also roughly the same. On the other hand, it is difficult to determine when and/or whether life expectancy would finally stabilize. The available projections point to their continuous rise at least up to the year 2080. Therefore, this process has a clearly different time horizon. Another argument in favor of narrowing the study to fertility and educational attainment is their possible causal relation.

In order to quantify the effect of rising longevity on the Polish economy, I compare the economy with higher survival probabilities to the baseline scenario. The new mortality tables correspond to the Eurostat projections for the year 2080. According to the model, an increase in life expectancy translates into a higher share of pensioners and, thus, a decline in output per capita by 3.9% and an increase in the contribution rate of 3.0 pp (column 2 of Table 7). Therefore, when it comes to changes in aggregate output, the effect of rising longevity is not as

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press



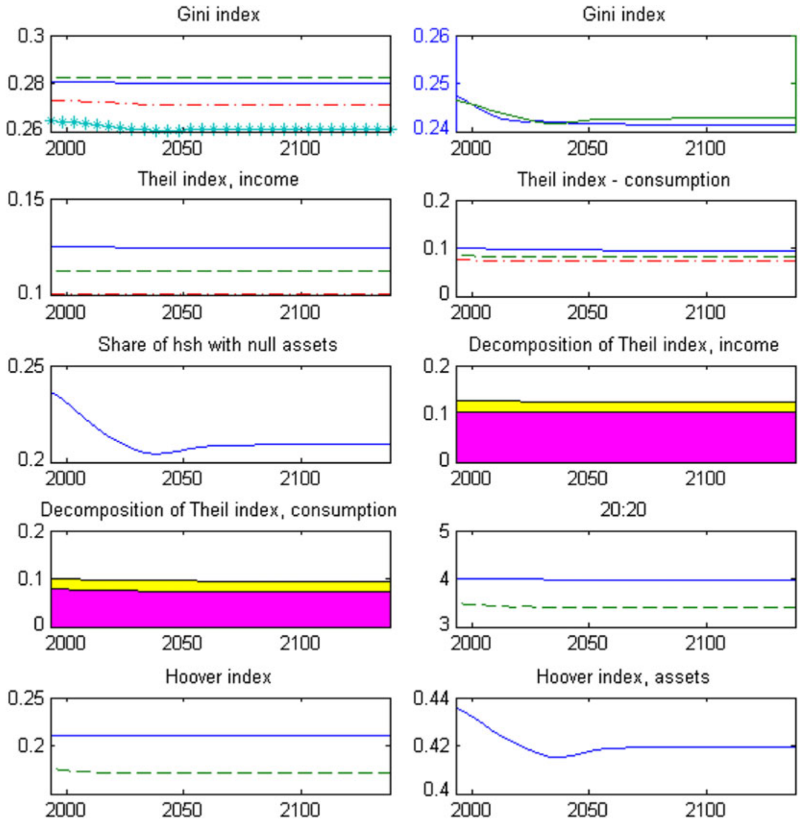
Note: Before 1995: $p^h = 18\%$, 1995–1999: $p^h = 22.5\%$, 2000–2004: $p^h = 30.0\%$, 2005–2009: $p^h = 45.0\%$, 2010–2014: $p^h = 45.0\%$, $p^h = 45.0\%$, 2015 and after: $p^h = 48.0\%$, $n_t = 0.9\%$. Gini index (left panel): dark blue line—labor income (inc. pensioners), green line—labor income (only workers), red line—labor income and bequests, blue line—disposable income. Gini index (right panel): dark blue line—consumption, green line—assets. Theil index: dark blue line—all households, green line—households with an academic degree, red line—households without an academic degree. Decomposition of Theil index: pink—within groups, yellow—between groups. 20:20 and Hoover index: dark blue line—income, green line—consumption. If not specified income consists of labor income and pensions.

FIGURE 10. Changes in inequalities, educational change.

substantial as of fertility decline. Nevertheless, their combined negative impact on output cannot be compensated by higher productivity imposed by the educational change (column 4 of Table 7).

Moreover, an increase in life expectancy significantly and positively affects domestic assets. As a consequence, in the new steady state, net foreign assets position improves and the domestic interest rate declines by 0.6 pp, that is stronger than in the case of fertility shift.[15]

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

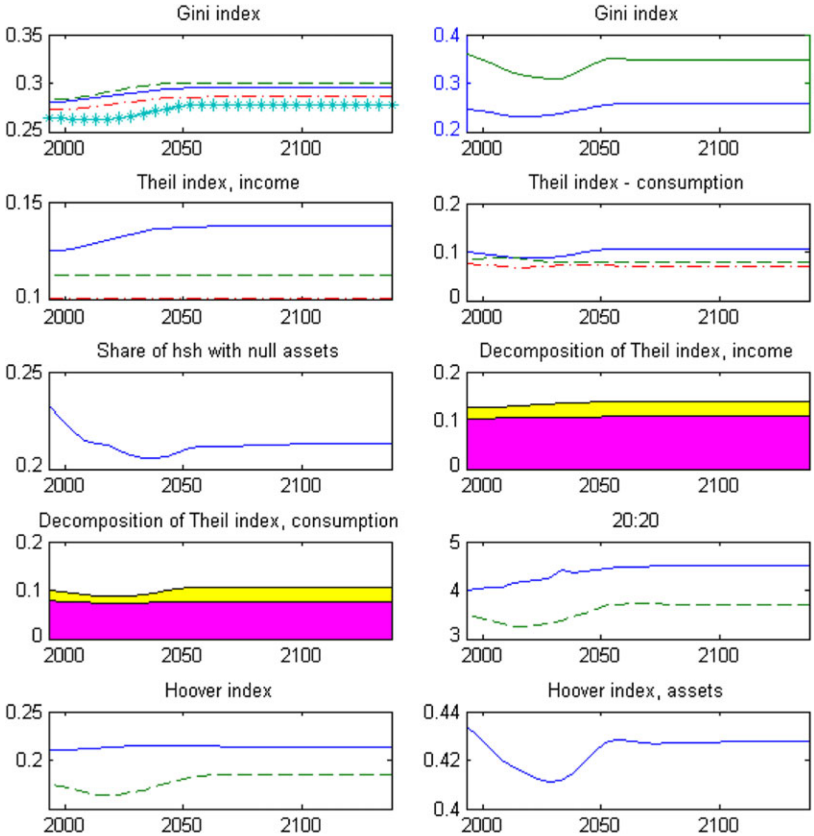


Note: $p^h = 18\%$, before 1990: $n_t = 0.9\%$, after 1990: $n_t = -0.6\%$. Gini index (left panel): dark blue line—labor income (inc. pensioners), green line—labor income (only workers), red line—labor income and bequests, blue line—disposable income. Gini index (right panel): dark blue line—consumption, green line—assets. Theil index: dark blue line—all households, green line—households with an academic degree, red line—households without an academic degree. Decomposition of Theil index: pink—within groups, yellow—between groups. 20:20 and Hoover index: dark blue line—income, green line—consumption. If not specified income consists of labor income and pensions.

FIGURE 11. Changes in inequalities, fertility change.

Imperfect substitution between less and more educated workers. In the model, the aggregate effective labor supply is a simple summation of the productivity of individual workers. However, as pointed in many studies (see i.a. Krusell et al. (2000), Caselli and Coleman (2006), Ottaviano and Peri (2012)), people with different level of educational attainment are not necessarily perfect substitutes in the labor market. In order to quantify to what extent the results presented so far are sensitive to the assumption of perfect substitution between workers, I recalculate the effects of the educational change, but this time the effective labor supply is aggregated using the following constant elasticity of substitution function (Ottaviano and Peri (2012)):

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

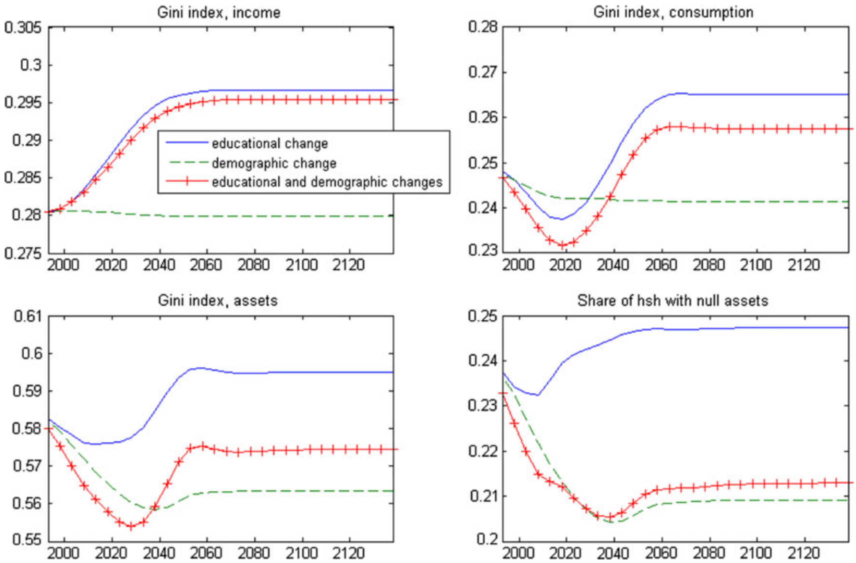


Note: Before 1995: $p^h = 18\%$, 1995–1999: $p^h = 22.5\%$, 2000–2004: $p^h = 30.0\%$, 2005–2009: $p^h = 45.0\%$, 2010–2014: $p^h = 45.0\%$, $p^h = 45.0\%$, 2015 and after: $p^h = 48.0\%$, before 1990: $n_t = 0.9\%$, after 1990: $n_t = -0.6\%$. Gini index (left panel): dark blue line—labor income (inc. pensioners), green line—labor income (only workers), red line—labor income and bequests, blue line—disposable income. Gini index (right panel): dark blue line—consumption, green line—assets. Theil index: dark blue line—all households, green line—households with an academic degree, red line—households without an academic degree. Decomposition of Theil index: pink—within groups, yellow—between groups. 20:20 and Hoover index: dark blue line—income, green line—consumption. If not specified income consists of labor income and pensions.

FIGURE 12. Changes in inequalities, educational and fertility changes.

$$L = \left(\mu_y^{1/\sigma_y} \left(L^1 \right)^{(\sigma_y - 1)/\sigma_y} + (1 - \mu_y)^{1/\sigma_y} \left(L^0 \right)^{(\sigma_y - 1)/\sigma_y} \right)^{(\sigma_y/(\sigma_y - 1)}. \qquad \textbf{(30)}$$

In the above formula, L^1 and L^0 stands for aggregated labor supply of more and less educated households, while μ_y and σ_y are parameters, where the latter equals the elasticity of substitution between workers with and without an academic degree. Following Krusell et al. (2000), σ_y is calibrated at 1.67.



Note: Income consists of labor income and pensions.

FIGURE 13. Changes in inequalities, summary.

In the case of $\sigma_y \to \infty$ or $\mu_y = \frac{L^1}{L^1+L^0}$, equation (30) simplifies to $L = L^1 + L^0$, that is the form used in the main simulations of this paper. In the above situation, wage per efficiency unit is the same for more and less educated workers: $1 = \left(w^1\right) / \left(w^0\right) = \left(\partial Y/\partial \left(L^1\right)\right) / \left(\partial Y/\partial \left(L^0\right)\right)$ and the reason why households with higher educational attainment earn more is that they are on average more productive in all kinds of jobs.

The reality, however, is not that simple and there is a certain demand for workers with different skills. The structure of this demand depends among others things on the level of economic development. In the presence of imperfect substitution, we can think of optimal allocation, in which case L^1 and L^0 should satisfy $\mu_y = \frac{L^1}{L^1+L^0}$, and that is what I assume for the model economy before the changes. Thus, the initial steady state is identical to what was previously called the baseline scenario (Table 5).

During the educational change, the labor market structure might respond to changes in workers educational attainment and, as a result, the relative demand for more and less educated workers might adjust. In the new steady state, one can think of two extreme cases. In the first one, labor is optimally allocated between low- and high-skilled jobs (model adjustment 5), so the effect of the educational change is the same as in the main simulations. In the second one, there are no shifts in relative demand (μ_y does not change at all during the transition, model adjustment 1) and the existing imbalance reduces the benefits of educational change. In this exercise, I also consider three additional scenarios, with different μ_y.

TABLE 7. New steady states, including higher survival probabilities, compared to the baseline scenario (no. 1)

	Higher survival probabilities	Fertility change and unchanged survival probabilities	Educational and demographic changes (that including higher survival probabilities)
Δy per capita (%)	−3.9	−10.2	−2.8
Δc per capita (%)	−5.2	−6.0	0.8
Δa per capita (%)	9.8	−5.7	18.8
Δi per capita (%)	2.0	−24.7	−12.6
$\Delta \mathrm{nfa}/y$ (pp)	14.5	5.1	22.8
Δr (pp)	−0.6	−0.2	−0.8
Δc per household			
With an academic degree (%)	−5.0	−6.7	−13.6
Without an academic degree (%)	−5.4	−6.1	−13.6
Δa per household			
With an academic degree (%)	14.5	−6.4	2.8
Without an academic degree (%)	11.6	−6.8	−0.1
Δl per household (income)			
With an academic degree (%)	−4.4	−11.0	−17.6
Without an academic degree (%)	−3.9	−10.3	−16.5
$\Delta \tau$ (contribution rate, pp)	3.0	5.6	10.3
Δw (wage per productivity unit, %)	2.7	1.0	4.1
ΔGini (income of workers, in scale 0–100)	0.0	0.1	1.8
ΔGini (assets, in scale 0–100)	−1.4	−2.1	−3.6

The change in the educational structure of population occurring in Poland is so dramatic that with little or no adjustments in labor demand, there will be a substantial deficit of less educated workers in the future. Therefore, the positive impact of the educational change, i.a., on output per capita will be notably reduced (Tables 8 and 9). Moreover, the relative wage per efficiency unit of more and less educated individuals will change greatly in favor of the latter, so that in the new steady-state households without an academic degree will earn more, even taking into account the differences in productivity. Since in reality the level of educational attainment is not given exogenously but is subject to individual's choice, this situation will not persist in the long term. Nevertheless, it is important to note that the positive impact of the educational change might materialize with some lags.

How has the Polish labor market adjusted to changes in education so far? One way to look at it is through the education premium. The ratio of mean disposable income of more educated households to less educated individuals increased

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 8. New steady states, the educational change and imperfect substitution between more and less educated workers

μ_y	Model adjustment 1 0.241	Model adjustment 2 0.324	Model adjustment 3 0.407	Model adjustment 4 0.489	Model adjustment 5 0.572
C/Y (%)	73.7	73.7	73.7	73.7	73.7
I/Y (%)	25.2	25.2	25.2	25.2	25.2
nfa$/Y$ (%)	−56.3	−56.1	−56.0	−56.0	−55.9
The ratio of mean consumption between households with and without an academic degree (%)	70.0	88.8	109.2	132.6	160.9
The ratio of mean income between households with and without an academic degree (%)	69.3	88.6	109.8	134.1	163.7
The ratio of mean assets between households with and without an academic degree (%)	71.8	92.2	114.7	140.9	172.7

TABLE 9. New steady states, educational change and imperfect substitution between more and less educated workers—compared to the baseline scenario (no. 1)

μ_y	Model adjustment 1 0.241	Model adjustment 2 0.324	Model adjustment 3 0.407	Model adjustment 4 0.489	Model adjustment 5 0.572
Δy per capita (%)	0.8	7.8	12.6	15.4	16.4
Δc per capita (%)	0.8	7.8	12.5	15.4	16.4
Δa per capita (%)	1.0	8.1	13.0	15.9	17.0
Δi per capita (%)	0.9	7.9	12.8	15.7	16.7
$\Delta \text{nfa}/y$ (pp)	0.2	0.3	0.3	0.4	0.5
Δr (pp)	0.0	0.0	0.0	0.0	0.0
Δc per household					
With an academic degree (%)	−44.3	−31.3	−19.5	−9.0	0.2
Without an academic degree (%)	28.3	24.9	18.9	10.7	0.4
Δa per household					
With an academic degree (%)	−46.1	−33.0	−21.2	−10.6	−1.3
Without an academic degree (%)	29.3	25.1	18.3	9.3	−1.7
Δl per household (income)					
With an academic degree (%)	−45.3	−32.1	−20.2	−9.5	−0.2
Without an academic degree (%)	29.2	25.5	19.1	10.5	−0.2
$\Delta \tau$ (contribution rate, pp)	0.1	0.1	0.2	0.2	0.3
Δw^1 (wage per productivity unit, %)	−45.3	−32.0	−20.0	−9.3	0.1
Δw^0 (wage per productivity unit, %)	29.4	25.7	19.3	10.7	0.1

TABLE 10. Changes in the ratio of income between households with and without an academic degree

Year	$\Delta(\text{income}^1/\text{income}^0)$, %
2000	
2005	4.6
2010	−5.4
2014	−5.0

Notes: Based on the data from the Polish Household Budget Survey. The disposable equivalent income, that is, the household's income divided by the squared number of the household's members, is used. The values of income^1 and income^0 equal the means from the means of income for more and less educated households calculated for the following age groups: 25–30 year olds, 30–35 year olds,. . . , 60–65 year olds.

TABLE 11. Changes in the ratio of wages (per productivity unit) between households with and without an academic degree: $\Delta(w^1/w^0)$, imperfect substitution between more and less educated workers

	Model adjustment 1	Model adjustment 2	Model adjustment 3	Model adjustment 4	Model adjustment 5
μ_y	0.241	0.324	0.407	0.489	0.572
Years 2010–2014 (%)	−10.0	−8.3	−6.7	−5.3	−3.9

Notes: In calculations, the impact of the fertility change on labor supply and its structure is included.

between 2000 and 2005 and declined afterwards (Table 10).[16] If we compare the pace of change of the ratio in question with the model's implications, we can conclude that so far the model adjustment 4 gives the most accurate predictions (Table 10 and 11).[17] Therefore, if this trend continues, the positive impact of the educational change on GDP obtained in the main simulations should not be significantly overestimated.

Closed economy. If we change the assumptions about the interest rate and consider a closed economy with no borrowing from abroad, in the initial steady state (before changes), the interest rate would be 0.4 pp higher. This translates into a higher level of domestic assets and slightly lower assets inequality (Table 12). Households consume less per capita and their consumption is not that smooth over the life cycle (Figure 14). With the higher interest rate, demand for capital is lower, and so do output per capita and effective wage. Although the initial steady states differ, the estimated impact of educational and demographic changes remains roughly as in the main simulations (column 3 in Table 13).

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 12. Steady states before changes and different model assumptions, compared to the baseline scenario (no. 1)

	Closed economy	No interest rate risk premium
Δy per capita (%)	−1.9	9.2
Δc per capita (%)	1.0	2.7
Δa per capita (%)	17.7	−47.5
Δi per capita (%)	−6.0	33.4
$\Delta \text{nfa}/y$ (pp)	56.3	−177.3
Δr (pp)	0.4	−1.8
Δw (wage per productivity unit, %)	−1.9	9.2
ΔGini (assets, in scale 0-100)	−1.9	7.7

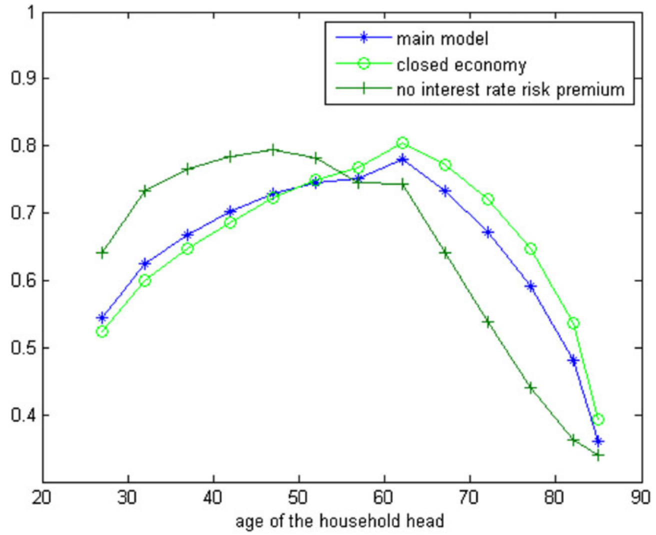


FIGURE 14. Life cycle consumption of households with an academic degree, different model assumptions.

No interest rate risk premium. In an open economy with no risk premium and no borrowing constraints, capital is cheaper ($r = r^*$) and so the demand for it is higher. Therefore, in contrast to the closed economy, in this case, we observe higher output, consumption, and investment per capita. On the other hand, lower interest rate translates into lower, but more unequally distributed, domestic assets, and significantly higher indebtedness (Table 12). With no interest rate risk premium, the household life cycle profiles change, so that young individuals consume relatively more. Since the interest rate does not adjust to changes in net foreign assets, the negative impact of the fertility change on output

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

TABLE 13. The long-term impact of the educational and fertility changes, different model assumptions

	Main model	Closed economy	No interest rate risk premium	Fixed contribution rate
Δy per capita (%)	4.6	4.8	3.4	7.0
Δc per capita (%)	9.4	9.9	6.4	11.3
Δa per capita (%)	10.4	8.0	19.8	26.9
Δi per capita (%)	−12.1	−11.4	−15.1	−5.1
Δnfa/y (pp)	5.6	0.1	17.1	18.9
Δr (pp)	−0.2	−0.3	0.0	−0.7
Δc per household				
With an academic degree (%)	−6.5	−5.9	−9.2	−5.1
Without an academic degree (%)	−5.7	−5.1	−8.5	−3.8
Δa per household				
With an academic degree (%)	−7.8	−8.9	−1.3	6.3
Without an academic degree (%)	−8.7	−10.2	−1.1	3.1
Δl per household (income)				
With an academic degree (%)	−11.0	−10.9	−12.0	−9.0
Without an academic degree (%)	−10.4	−10.2	−11.4	−8.2
$\Delta\tau$ (contribution rate, pp)	5.8	5.8	5.8	0.0
Δw (wage per productivity unit, %)	1.1	1.3	0.0	3.5
ΔGini (income of workers, in scale 0–100)	1.9	1.9	1.9	1.9
ΔGini (assets, in scale 0–100)	−1.0	−1.1	−1.1	−1.3

is stronger compared to the results of the main simulations (Figure 14 and column 4 in Table 13).

Constant contribution rate. The negative impact of the fertility change on output, consumption and assets would be much smaller if the contribution rate (τ) was fixed and, in order to balance the pension system, the replacement rate (θ) was used (column 5 in Table 13). In this case, households would respond to the expected lower pensions by accumulating more assets, which, in turn, would improve net foreign assets position and lower the interest rate (by 0.7 pp compared to 0.2 pp in the main simulations). Thus, in this case, the output deterioration would not be that substantial. Nevertheless, since cutting the pensions expenditures is not the preferred option for the current generations (see Kudrna et al. (2019)), implementing this alternative pension system would be difficult.

7. CONCLUSIONS

This paper investigates the demographic and educational changes in Poland within a general equilibrium framework with heterogeneous agents and idiosyncratic uninsured productivity shocks.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

According to the results, both permanently lower fertility and rising longevity have a significant impact on macroeconomic variables, which include lower equilibrium real interest rate, notably higher contribution rate and deterioration in output and investment per capita. Fortunately, the positive effect of increased productivity due to the educational change should more than offset the negative consequences of falling fertility on output per capita. Nevertheless, if the total impact of population aging (i.e., lower fertility and rising life expectancy) is included, the net effects become negative. Importantly, one should not expect dramatic shifts in inequalities. Due to the educational change, income and consumption inequalities rise modestly, while fertility decline translates into the slightly more equal distribution of assets.

The currently occurring changes in the ratio of more to less educated workers are rather substantial. Therefore, making a labor demand and supply compatible produces a challenge for the economy. The empirical evidence on the education premium suggests that so far Poland has been doing pretty well in meeting this goal, but for the economy to enjoy the benefits of the educational boom, this trend needs to be continued. In particular, it is essential that skills gained by academic graduates address market needs. For meeting this goal, the following actions are important: monitoring the quality of education, promoting fields of study that equip graduates with the most in-demand skills, and providing assistance for students that would help them choose an appropriate career/education path. Another crucial task is to secure a friendly and compatible environment for foreign investment and to support local businesses, which create jobs that fit with the educational structure of local labor force.

Population aging puts a pressure on the Polish pension system. According to my results, in order to keep the replacement rate unchanged, the contribution rate will have to be more than 10 pp higher in the long term. As a result, the households' consumption over the life cycle would be significantly lower compared to the situation before the changes. Therefore, appropriate reforms of the pension system should be considered. On the one hand, both raising the retirement age and/or cutting the replacement rates would increase the effectiveness of the Polish economy, measured by output per capita. On the other hand, the problem is more complicated and, for instance, lower replacement rates cause a significant welfare loss of current generations (Nishiyama, 2015).

NOTES

1. For the foundations and possible causes of the demographic transition, refer to Landry (1987), Coale (1989), Chesnais (1992), Caldwell (1976), Kirk (1996), Galor and Weil (2000), Woods (2000), Demeny and McNicoll (2003).

2. Whether this shift in fertility trends is indeed so distinct from what was observed in the past is disputable. See for example Bailey et al. (2014).

3. The reason for distinguishing this particular threshold is that it implies a reduction of the annual number of births by 50% and a halving of the population size in less than 45 years (Billari (2008), pp. 2–3).

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

4. Similarly, according to the World Bank Database, the ratio of those who at least completed short-cycle tertiary education among population aged 25 and older dobbled between 2002 and 2014, increasing from 12% to 24%.

5. For studies on over-education, see among others Leuven and Oosterbeek (2011), Galasi (2008), Walker and Zhu (2005), Groot and Maasen Van Den Brink (1997), Li et al. (2008), Budria and Moro-Egido (2008).

6. Another important (earlier) paper is Becker et al. (1990), where the authors link returns on human capital with fertility choices.

7. The quantative simulations of the model can be found, i.a., in Lagerlöf (2006) and Cervellati and Sunde (2015). In the latter, the authors also find that the mortality environment might be one of the explanations for why the timing of the take-off of the demographic transition differs so much across countries.

8. Various aspects of the Polish economic transition and different viewpoints are presented, among others, in Sachs (1992), Gomułka (2016), Balcerowicz (1997), Kołodko (1992).

9. For a more detailed discussion on the challenges facing the Polish labor market, see Lewandowski and Magda (2018).

10. The presented estimate comes from the Polish CSO calculations based on the HBS.

11. One exception when a surge in TFR was observed was in the mid-1980s and was related to echo boomers.

12. The same result was obtained for Sweden (Flodén and Lindé (2001)) but is not present in the US data (Cooper and Zhu (2016)).

13. The values presented in Table 1 are based on the cross-section observed in 2013. More educated households are those whose household head has an academic degree. The underlying assumption made in Table 1 is that from the age of 24 the educational structure of a cohort does not change much over time.

14. Due to a finite set of income realizations, the graph of quintile ratio might have steps.

15. Carvalho et al. (2016) also estimate a lower equilibrium interest rate due to population aging.

16. Similar estimates can be obtained, when instead of disposable income the labor and self-employed income is used.

17. Here I assume the smooth adjustment process, that is, the linear trajectory of parameter μ_y.

REFERENCES

Adamczyk, A. and W. Jarecki (2008) Evolution of the internal rate of return for investment in higher education (in Polish). *Gospodarka Narodowa* 11–12, 77–93.

Aiyagari, S. R. (1994) Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics* 109(3), 659–684.

Bailey, M. J., M. Guldi and B. J. Hershbein (2014) Is there a case for a 'second demographic transition'? In: L. P. Boustan, C. Frydman and R. A. Margo (eds.), *Human Capital in History: The American Record,* pp. 273–312. Chicago: University of Chicago Press.

Balcerowicz, L. (1997) *Socialism, Capitalism, Transformation.* Warszawa: Wydawnictwo Naukowe PWN.

Becker, G. S., K. M. Murphy and R. Tamura (1990) Human capital, fertility, and economic growth. *Journal of Political Economy* 98(5, Part 2), 12–37.

Billari, F. C. (2008) Lowest-low fertility in Europe: Exploring the causes and finding some surprises. *The Japanese Journal of Population* 6(1), 2–18.

Black, S. E., P. J. Devereux and K. G. Salvanes (2005) Why the apple doesn't fall far: Understanding intergenerational transmission of human capital. *American Economic Review* 95(1), 437–449.

Börsch-Supan, A., A. Ludwig and J. Winter (2006) Ageing, pension reform and capital flows: A multi-country simulation model. *Economica* 73(292), 625–658.

Brzeziński, M., B. Jancewicz and N. Letki (2013) Growing inequalities and their impacts in Poland. GINI Growing Inequalities Impact Country Report for Poland, Amsterdam.

Brzozowska, Z. (2014) Fertility and education in Poland during state socialism. *Demographic Research* 31, 319–336.

Budria, S. and A. I. Moro-Egido (2008) Education, educational mismatch, and wage inequality: Evidence for Spain. *Economics of Education Review* 27(3), 332–341.

Caldwell, J. C. (1976) Toward a restatement of demographic transition theory. *Population and Development Review* 2(3/4), 321–366.

Carvalho, C., A. Ferrero and F. Nechio (2016) Demographics and real interest rates: Inspecting the mechanism. *European Economic Review* 88(C), 208–226.

Caselli, F. and W. J. Coleman (2006) The world technology frontier. *American Economic Review* 96(3), 499–522.

Catalano, M. and E. Pezzolla (2016) The effects of education and aging in an OLG model: Long-run growth in France, Germany and Italy. *Empirica* 43(4), 757–800.

Cervellati, M. and U. Sunde (2015) The economic and demographic transition, mortality, and comparative development. *American Economic Journal: Macroeconomics* 7(3), 189–225.

Chesnais, J.-C. (1992) *The Demographic Transition: Stages, Patterns, and Economic Implications*. Oxford: Oxford University Press.

Chetty, R. (2006) A new method of estimating risk aversion. *American Economic Review* 96(5), 1821–1834.

Chmielewska, I., G. Dobroczek and A. Panuciak (2018) Citizens of Ukraine working in Poland (in Polish). Department of Statistics, Narodowy Bank Polski.

Coale, A. J. (1989) Demographic transition. In: J. Eatwell, M. Milgate and P. Newman (eds.), *Social Economics*, pp. 16–23. London: Palgrave Macmillan.

Cooper, R. and G. Zhu (2016) Household finance over the life-cycle: What does education contribute? *Review of Economic Dynamics* 20, 63–89.

De Nardi, M., S. İmrohoroglu and T. J. Sargent (1999) Projected U.S. demographics and social security. *Review of Economic Dynamics* 2(3), 575–615.

Demeny, P. and Geoffrey M. (2003) *Encyclopedia of Population*. London: Palgrave Macmillan.

Díaz-Giménez, J. and J. Díaz-Saavedra (2009) Delaying retirement in Spain. *Review of Economic Dynamics* 12(1), 147–167.

Diebolt, C., A.-R. Menard and F. Perrin (2016) Behind the Fertility-Education Nexus: What Triggered the French Development Process? Working Papers 03-16, Association Française de Cliométrie.

Ermisch, J. and C. Pronzato (2010) Causal Effects of Parents' Education on Children's Education. ISER Working Paper Series 2010-16.

Flodén, M. and J. Lindé (2001) Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance? *Review of Economic Dynamics* 4(2), 406–437.

Frejka, T. (2008) Determinants of family formation and childbearing during the societal transition in Central and Eastern Europe. *Demographic Research* 19, 139–170.

Gajderowicz, T., G. Grotkowska and L. Wincenciak (2012) Wage premium from higher education by occupational groups (in Polish). *Ekonomista* (5), 577–603.

Galasi, P. (2008) The Effect of Educational Mismatch on Wages for 25 Countries. Budapest Working Papers on the Labour Market 2008/8.

Galor, O. and D. N. Weil (1999) From Malthusian stagnation to modern growth. *American Economic Review* 89(2), 150–154.

Galor, O. and D. N. Weil (2000) Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review* 90(4), 806–828.

Goldstein, J. R., T. Sobotka and A. Jasilioniene (2009) The end of "lowest-low" fertility? *Population and Development Review* 35(4), 663–699.

Gomułka, S. (2016) Poland's economic and social transformation 1989–2014 and contemporary challenges. *Central Bank Review* 16(1), 19–23.

Gourinchas, P.-O. and J. A. Parker (2002) Consumption over the life cycle. *Econometrica* 70(1), 47–89.

Grejcz, K. and Z. Żółkiewski (2017) Household wealth in Poland: The results of a new survey of household finance. *Bank i Kredyt* 48(3), 295–326.

Groot, W. and H. M. Van Den Brink (1997) Allocation and the returns to over-education in the UK. *Education Economics* 5(2), 169–183.

Hadjar, A. and R. Becker (2009) *Expected and Unexpected Consequences of the Educational Expansion in Europe and USA: Theoretical Approaches and Empirical Findings in Comparative Perspective*. Vienna, Bern: Haupt.

Hagemejer, J., K. Makarski and J. Tyrowicz (2015) Unprivatizing the pension system: The case of Poland. *Applied Economics* 47(8), 833–852.

Hansen, C. W., P. S. Jensen and L. Lønstrup (2018) The fertility decline in the United States: Schooling and income. *Macroeconomic Dynamics* 22(6), 1584–1612.

Hubbard, R. G. and K. L. Judd (1987) Social security and individual welfare: Precautionary saving, borrowing constraints, and the payroll tax. *American Economic Review* 77(4), 630–646.

Johnson, V. E. (2003) *Grade Inflation: A Crisis in College Education*. New York: Springer-Verlag.

Keane, M. P. and E. S. Prasad (2002) Inequality, transfers, and growth: New evidence from the economic transition in Poland. *The Review of Economics and Statistics* 84(2), 324–341.

Keane, M. P. and E. S. Prasad (2006) Changes in the structure of earnings during the Polish transition. *Journal of Development Economics* 80(2), 389–427.

Kirk, D. (1996) Demographic transition theory. *Population Studies* 50(3), 361–387.

Kohler, H.-P., F. C. Billari and J. A. Ortega (2002) The emergence of lowest-low fertility in Europe during the 1990s. *Population and Development Review* 28(4), 641–680.

Kolasa, A. (2017) Life cycle income and consumption patterns in Poland. *Central European Journal of Economic Modelling and Econometrics* 9(2), 137–172.

Kołodko, G. W. (1992) *Transformation of the Polish Economy: Success or Failure? (in Polish)*. Warszawa: Polska Oficyna Wydawnicza BGW.

Kotowska, I., J. Jóźwiak, A. Matysiak and A. Baranowska (2008) Poland: Fertility decline as a response to profound societal and labour market changes? *Demographic Research* 19, 795–854.

Krueger, D. and A. Ludwig (2007) On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics* 54(1), 49–87.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull and G. L. Violante (2000) Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica* 68(5), 1029–1054.

Kudrna, G., C. Tran and A. Woodland (2019) Facing demographic challenges: Pension cuts or tax hikes? *Macroeconomic Dynamics* 23(2), 625–673.

Lagerlöf, N.-P. (2006) The Galor–Weil model revisited: A quantitative exercise. *Review of Economic Dynamics* 9(1), 116–142.

Landry, A. (1987) Adolphe Landry on the demographic revolution. *Population and Development Review* 13(4), 731–740.

Lesthaeghe, R. J. (2007) Second demographic transition. In: G. Ritzer (ed.), *The Blackwell Encyclopedia of Sociology*, pp. 4123–4127. Malden, MA: Blackwell.

Leuven, E. and H. Oosterbeek (2011) Overeducation and mismatch in the labor market. In: E. A. Hanushek, S. Machin and L. Woessmann (eds.), *Handbook of the Economics of Education*, vol. 4, pp. 283–326. Amsterdam: Elsevier.

Lewandowski, P. and I. Magda (2018) The labor market in Poland, 2000–2016. *IZA World of Labor* 426, 1–9.

Li, F., J. Morgan and X. Ding (2008) The expansion of higher education, employment and over-education in China. *International Journal of Educational Development* 28(6), 687–697.

Ludwig, A., T. Schelkle and E. Vogel (2012) Demographic change, human capital and welfare. *Review of Economic Dynamics* 15(1), 94–107.

Mackenbach, J. P. (2013) Political conditions and life expectancy in Europe, 1900–2008. *Social Science and Medicine* 82, 134–146.

Majchrowska, A. and S. Roszkowska (2014) Returns to Education and Experience by Sex in Poland (in polish). Materialy i Studia 302, Narodowy Bank Polski.

Makarski, K., J. Hagemejer and J. Tyrowicz (2016) Analyzing the efficiency of pension reform: The role of the welfare effects of fiscal closures. *Macroeconomic Dynamics* 21(5), 1–30.
Matysiak, A. (2009) Is Poland really 'immune' to the spread of cohabitation? *Demographic Research* 21, 215–234.
McDonald, P. (2000a) Gender equity in theories of fertility transition. *Population and Development Review* 26(3), 427–439.
McDonald, P. (2000b) Gender equity, social institutions and the future of fertility. *Journal of the Australian Population Association* 17(1), 1–16.
Medina, L. and F. Schneider (2018) Shadow Economies Around the World: What Did We Learn Over the Last 20 Years? IMF Working Papers 18/17, International Monetary Fund.
Mishtal, J. Z. (2009) Understanding low fertility in Poland: Demographic consequences of gendered discrimination in employment and postsocialist neoliberal restructuring. *Demographic Research* 21, 599–626.
Newell, A. and M. W. Socha (2007) The Polish wage inequality explosion. *The Economics of Transition* 15(4), 733–758.
Nishiyama, S. (2015) Fiscal policy effects in a heterogeneous-agent OLG economy with an aging population. *Journal of Economic Dynamics and Control* 61(C), 114–132.
Osiewalska, B. (2017) Couple's education and their number of children in Poland (in polish). *Studia Demograficzne* 172(2), 71–96.
Ottaviano, G. and G. Peri (2012) Rethinking the effect of immigration on wages. *Journal of the European Economic Association* 10(1), 152–197.
Parro, F. (2017) Understanding the supply and demand forces behind the fall and rise in the US skill premium. *Macroeconomic Dynamics* 23(6), 2191–2220.
Roszkowska, S., K. Saczuk, M. Skibińska, P. Strzelecki and R. Wyszyński (2017) Quarterly report on the labor market, first quarter of 2017 (in Polish). Economic Analysis Department, Narodowy Bank Polski.
Rubaszek, M. (2012) Mortgage down-payment and welfare in a life-cycle model. *Bank i Kredyt* 43(4), 5–28.
Sachs, J. (1992) The economic transformation of Eastern Europe: The case of Poland. *Economics of Planning* 25(1), 5–19.
Schmitt-Grohé, S and M. Uribe (2003) Closing small open economy models. *Journal of International Economics* 61(1), 163–185.
Snopkowski, K., M. C. Towner, M. K. Shenk and H. Colleran (2016) Pathways from education to fertility decline: A multi-site comparative study. *Philosophical Transactions of the Royal Society B: Biological Sciences* 371(1692), 20150156.
Storesletten, K. (2000) Sustaining fiscal policy through immigration. *Journal of Political Economy* 108(2), 300–323.
Tauchen, G. and R. Hussey (1991) Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica* 59(2), 371–396.
Theil, H. (1967) *Economics and Information Theory*. Amsterdam: North Holland.
Valletta, R. G. (2016) Recent Flattening in the Higher Education Wage Premium: Polarization, Skill Downgrading, or Both? Working Paper Series 2016-17, Federal Reserve Bank of San Francisco.
Van de Kaa, D. J. (1987) Europe's second demographic transition. *Population Bulletin* 42(1), 1–59.
Walker, I. and Y. Zhu (2005) The College Wage Premium, Overeducation, and the Expansion of Higher Education in the UK. IZA Discussion Papers 1627.
Walker, I. and Y. Zhu (2008) The college wage premium and the expansion of higher education in the UK. *The Scandinavian Journal of Economics* 110(4), 695–709.
Woods, R. (2000) *The Demography of Victorian England and Wales*. Cambridge: Cambridge University Press.
World Bank Group (2017) Lessons from Poland, Insights for Poland. World Bank Other Operational Studies 28960, The World Bank.
Wyszyński, R. (2016) The phenomenon of underemployment - What is it and what does it reveal about the Polish labor market? (in Polish). *Bank i Kredyt* 47(3), 267–284.

A: APPENDIX

A.1. ALGORITHM FOR FINDING STATIONARY EQUILIBRIUM

Let the space for households' assets be limited to a grid $\mathrm{A} = \{0, a^1, a^2, \ldots, a^M\} \subset [0, \infty)$ with a^M large enough not to constitute a constraint for the optimization problem. In order to cover all household choices of $\tilde{a}'$, also those laying between the grid points, the golden section search method is applied. Given p^h and n, the rules of finding stationary equilibrium are the following.

1. Set the starting value for r and $\tilde{\mathrm{tr}}$.
2. Based on equations (6) and (23), calculate the wage rate $\tilde{w}$, contribution rate τ, demand for aggregate capital $\tilde{K}$, and aggregate labor units. For the last one use the following formula:

$$\tilde{L} = \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} z(h, j, e_h = e_h^l)\mu(h, j, e_h = e_h^l). \tag{A1}$$

3. Given $r,\ \tau,\ \tilde{w}$ and $\tilde{\mathrm{tr}}$, and using backward induction, solve the household's optimization problem (5) and find policy functions $\tilde{c}(h, j, e_h, a)$, $\tilde{a}'(h, j, e_h, a)$ for $h \in H,\ j \in J,\ e_h \in E_h,\ a \in \mathrm{A}$.
4. Calculate aggregate output $\tilde{Y}$, next period assets $\tilde{A}$, and bequest $\tilde{\mathrm{tr}}$ based on the following equations:

$$\tilde{Y} = \tilde{K}^{\alpha}\tilde{L}^{1-\alpha}, \tag{A2}$$

$$\tilde{A} = (1+n)^{-1}(1+g)^{-1} \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} \sum_{a \in \mathrm{A}} \mu(h, j, e_h = e_h^l, a) * \tilde{a}'(h, j, e_h, a), \tag{A3}$$

$$\tilde{\mathrm{tr}} = (1+n)^{-1}(1+g)^{-1} \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} \sum_{a \in \mathrm{A}} (1+r)(1-s_j) * \mu(h, j, e_h = e_h^l, a) * \tilde{a}'(h, j, e_h, a). \tag{A4}$$

5. Check whether the value of $\tilde{\mathrm{tr}}$ is the same as assumed and whether r satisfies $r = r^* + \phi * (\exp(\frac{\tilde{K} - \tilde{A}'}{\tilde{Y}}) - 1)$. If yes, the equilibrium is found. Otherwise, go back to step 1 and update r and $\tilde{\mathrm{tr}}$.

A.2. ALGORITHM FOR FINDING TRANSITIONAL DYNAMICS

In the case of a stationary equilibrium, it was assumed that p^h and n are constant. Let us now consider a situation in which initially (at time $t = 0$) the model economy is in a stationary equilibrium with parameters $(p^{h,\mathrm{old}}, n^{\mathrm{old}})$. At time $t = 1$ an unexpected change occurs that can be described by a new set of parameters $\{(p_t^h, n_t)\}_{t=1}^{\infty}$, which for $t > t_0$ stabilize at the new levels $(p^{h,\mathrm{new}}, n^{\mathrm{new}})$. Before the model economy achieves the new stationary equilibrium, it goes through transitional dynamics, during which all general equilibrium conditions described in Subsection 3.5.1 need to be satisfied. It is assumed that at time $t = 1$ households learn $\{(p_t^h, n_t)\}_{t=1}^{t_0}$ and $(p^{h,\mathrm{new}}, n^{\mathrm{new}})$. Below I present the algorithm of how

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press

to find the transitional dynamics from the stationary equilibrium $(p^{h,\text{old}}, n^{\text{old}})$ to the one described by $(p^{h,\text{new}}, n^{\text{new}})$, along the path given by $\{(p^h_t, n_t)\}_{t=1}^{t_0}$.

1. Using the algorithm from Subsection A.1, find a stationary equilibrium for the pair $(p^{h,\text{new}}, n^{\text{new}})$ and corresponding levels of τ^{new}, $\tilde{w}^{\text{new}}$ and $\tilde{\text{tr}}^{\text{new}}$.
2. Set period $t_1 > t_0$ at which the new stationary equilibrium is to be achieved. Set the maximum number of iterations N_0. Set the number of iterations (iter) to zero.
3. Increase the number of iterations (iter = iter + 1). Guess the starting values for $\{r_t\}_{t=1}^{t_1}$ and $\{\tilde{\text{tr}}_t\}_{t=1}^{t_1}$.
4. Calculate $\{\tilde{w}_t\}_{t=1}^{t_1}$, $\{\tau_t\}_{t=1}^{t_1}$ and $\{\tilde{K}_t\}_{t=1}^{t_1}$ based on the previous guesses, equations (6) and (15), and $\tilde{L}_t = \sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N} z(h,j,e_h = e^l_h)\mu_t(h,j,e_h = e^l_h)$.
5. Given $\{r_t, \tilde{w}_t, \tau_t, \tilde{\text{tr}}_t\}_{t=1}^{\infty}$, for all periods $t = 1, 2, \ldots, t_1$ solve the households maximization problem (5) using backward induction and find policy functions $\tilde{c}_t(h,j,e_h,a)$, $\tilde{a}'_t(h,j,e_h,a)$ for $h\in H,\ j\in J,\ e_h\in E_h,\ a\in \text{A}$.[18]
6. Calculate aggregate output, aggregate next period assets and bequests using the following formulas:

$$\tilde{Y}_t = \tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}, \tag{A5}$$

$$\tilde{A}_{t+1} = (N_t/N_{t+1})\,(1+g)^{-1}\sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N}\sum_{a\in \text{A}}\mu_t(h,j,e_h = e^l_h, a) * \tilde{a}'_t(h,j,e_h,a), \tag{A6}$$

$$\begin{aligned}\tilde{\text{tr}}_{t+1} = (N_t/N_{t+1})\,(1+g)^{-1}\sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N}\sum_{a\in \text{A}}(1+r_{t+1})(1-s_j) * \mu_t(h,j,e_h\\ = e^l_h, a) * \tilde{a}'_t(h,j,e_h,a).\end{aligned} \tag{A7}$$

7. Check whether the starting values $\{r_t\}_{t=1}^{t_1}$ satisfy $r_t = r^* + \phi * (\exp(\frac{\tilde{K}_t - \tilde{A}_t}{\tilde{Y}_t}) - 1)$ and whether $\{\tilde{\text{tr}}_t\}_{t=1}^{t_1}$ are consistent with aggregates calculated in the previous step. If yes, the transitional dynamics is found. Otherwise, if iter $\leq N_0$, go back to step 3 and update the vectors $\{r_t\}_{t=1}^{t_1}$ and $\{\tilde{\text{tr}}_t\}_{t=1}^{t_1}$. If iter $> N_0$ and transitional dynamics is not yet found, go back to step 2 and increase t_1.

https://doi.org/10.1017/S1365100519000944 Published online by Cambridge University Press