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Multi-objective comprehensive evaluation approach to a river health system based on fuzzy entropy†

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The process of river health evaluation is subject to uncertainty and complexity. To address this, we establish a river health evaluation system based on entropy and multi-objective space theory. We apply an analytic hierarchy process (AHP) to calculate the weight of the first grade indexes, and then apply the entropy to calculate the weight of the second indexes. In this way, the health measure can be considered in terms of objective data and a subjective classification. The results of the computation show that the whole system provides a good measure of the health. The results express the measurement of the health of the river on different levels and for different aspects, which indicates that the evaluation method is feasible.

1. Introduction

Rivers provide various ecological services such as water supply and nature conservation. However, with populations increasing, city expansion and industrial and agricultural development, river ecological systems are degenerating (Geng *et al.* 2006). People have come to see that the river is not only a provider of resources, but also a carrier of life, so they should not just pay attention to the river's resource function, but also show concern for its ecological function. In this way, the concept of river health arose (Geng *et al.* 2006). In order to give this concept meaning, we need to be able to carry out a health evaluation of river systems that is reliable, practical, transferable and informative to decision makers. Since river systems are complex, with multiple factors and targets, too narrow a focus or a simple subjective analysis is not enough, so we need a reasonable scientific theory and method for analysing the river system synthetically, and obtaining a final evaluation result for qualitative and quantitative analysis. In this paper, we start from systematic issues and the complexity of river health and establish an evaluation model according to matter-element analysis and information entropy theory, but avoiding the influence of subjective weights so that we can evaluate the river health system objectively, rationally and accurately.

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2. Index system for river health evaluation

To determine the function describing the river's health, we obtained a primary index by brainstorming, and then clustered and filtered the indexes using cluster analysis and the Delphi Method, which are based on filtration principles (independence, measurability and typicality), and with reference to the relevant literature. From the point of view of application and operability, we established a river health system evaluation index system and defined evaluation grades on the basis of expert consultation and grading.

River health can be classified on a five point scale as:

— very good health;
— health;
— semi-health;
— unhealthy; and
— morbidity.

There are five subsystem functions:

— environmental;
— ecological;
— utilisable;
— service;
— and flood control.

We also use 25 property indexes in this paper. The river health system evaluation index system and evaluation grade are shown in Table 1 later in the paper (Geng *et al.* 2006).

3. Defining the weight

When evaluating natural environmental impacts using an integrative evaluation method, the process of defining the weights is extremely important, and often influences the objectivity of the results. By considering both the subjective factors given by the knowledge, experience and value judgements of experts and the objective information given by measurement data, we define the index weights by combining a subjective weight method (AHP) with an objective weight method to reflect the evaluation index's importance and the relevant factual data more objectively and completely. The AHP is used to define the subsystem weights and the entropy method to define the weights of the indexes.

3.1. *Defining the evaluation index weights*

There are many methods for determining index weights, such as the expert investigation method. For this paper, we chose the analytic hierarchy process (AHP) with the weight set

$$w = (w_1, w_2, w_3, w_4, w_5). \tag{1}$$

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Generally speaking, all the indexes are normalised to the unit interval 0–1. In order to take account of the critical effect of target state and index, the fuzzy mathematics membership degree was introduced for standardisation:

— **Positive indicators** (bigger always better): use the upper semi-trapezoid distribution function for standardisation:

$$N(v_i) = \begin{cases} 0 & v_i \leqslant C(v_i) \\ \dfrac{v_i - C(v_i)}{T(v_i) - C(v_i)} & C(v_i) < v_i < T(v_i) \\ 1 & v_i \geqslant T(v_i). \end{cases} \tag{2}$$

— **Negative indicators** (smaller always better): use the lower semi-trapezoid distribution function for standardisation:

$$N(v_i) = \begin{cases} 1 & v_i \leqslant T(v_i) \\ \dfrac{C(v_i) - v_i}{C(v_i) - T(v_i)} & T(v_i) < v_i < C(v_i) \\ 0 & v_i \geqslant C(v_i). \end{cases} \tag{3}$$

Where v_i, $C(v_i)$, $T(v_i)$, $N(v_i)$, respectively, are the original, critical, target and standardised values of index i. In the current paper, we take the upper bound of 'excellent' as the target value and the lower bound of 'bad' as the critical value.

Using (2) and (3), we can calculate the membership consisting of pending samples and each classification standard. Once the membership degrees have been standardised, we can get r_{ji}, and then calculate w_i as follows:

$$k = \frac{1}{\ln(m)}$$

$$e_i = -k \sum_{j=1}^{m} r_{ji} \ln(r_{ji})$$

$$h_i = 1 - e_i$$

$$w_i = \frac{h_i}{\sum_{i=1}^{n} h_i}.$$

4. Multi-objective comprehensive evaluation method for a river health system

4.1. *Multi-dimensional space of a river health system*

River health systems are made up of numerous subsystems and factors, where each subsystem and factor is a one-dimensional property of the river health system. Hence, taking all the subsystems and factors together forms a multi-dimensional space. River health evaluation is thus, in essence, the description and evaluation of the geometric position (state point) of the system state in this multi-dimensional space – see Figure 1 for an example, where A and B are two different states of the natural environmental system, and x, y and z are three dimensions of the river health system (the example is restricted to three dimensions so that it can be represented graphically – there are many more dimensions in the real system).

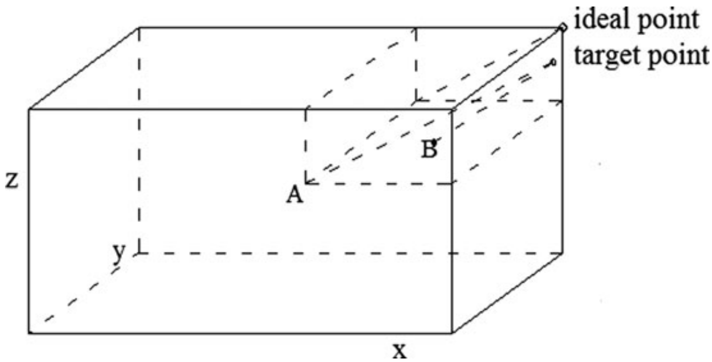

Fig. 1. Multidimensional space of the river health system

4.2. *River health system ideal state and target state*

From a theoretical point of view, there should always be a defined ideal state for a certain region. The distance between the state point and the ideal point could then be used as a measure of the current relative quality or impact level for this region. However, the dynamic nature of systems means it is hard to define the ideal value for each index, so we usually set a target value to evaluate the distance between the current state and the ideal state.

4.3. *Health level*

The distance between the state point and the target point in the multi-dimensional space indicates the degree to which the river health system achieves the target state – the smaller this distance, the better.

The distance between the state point and the critical point indicates how far the river health system's is away from a hazardous critical condition – the larger this distance, the better.

After weighting the membership degree matrix using Equations (2) and (3), we get

$$M = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \tag{4}$$

The system's critical and target points are then the vectors O_1 and O_2 as follows:

$$O_1 = (0\ 0\ \cdots\ 0) \tag{5}$$

$$O_2 = (w_1\ w_2\ \cdots\ w_n) \tag{6}$$

where $w_1,\ w_2, \cdots,\ w_n$ are the weights of the indexes 1–n.

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The Euclidean distance between any state point i and the critical point O_1 and target point O_2 are:

$$IO_1 = \sqrt{(y_{j1}-0)^2+(y_{j2}-0)^2+\cdots+(y_{jn}-0)^2} = \left[\sum_{i=1}^{n}(y_{ji})^2\right]^{1/2}$$
$$IO_2 = \sqrt{(y_{j1}-w_1)^2+(y_{j2}-w_2)^2+\cdots+(y_{jn}-w_n)^2} = \left[\sum_{i=1}^{n}(y_{ji}-w_i)^2\right]^{1/2}. \quad (7)$$

The system's impact level is a comprehensive measure of the performance of the system in keeping far away from the critical state and close to the target status, and is defined for state j by

$$SD_j = IO_1/(IO_1+IO_2) \quad (8)$$

so

$$0 \leqslant SD_j \leqslant 1.$$

5. Example

5.1. *The research sample*

For ease of comparison of the evaluation results of our multi-objective comprehensive evaluation based on fuzzy entropy, we used the data given for a set pair analysis model of a health assessment of the Lantsang river in Hu *et al.* (2008)) – the data is shown in Table 1.

5.2. *Membership*

Because of the limited space, for this paper we just used the service function as an example to show the calculation process.

By inserting the pending samples and classification standards' lower bound (shown in Table 1) into Equations (2) and (3), we can get the corresponding membership degrees shown in Table 2.

5.3. *Defining the weights*

(1) **River function weight**
The hierarchical structure of the river health system is shown in Table 1, and the weights given by the AHP method are

$$w = (0.2565,\ 0.2410,\ 0.0790,\ 0.1515,\ 0.2720). \quad (9)$$

(2) **Evaluation index weight**
The membership values $u(x_{ji})$ in Table 2 for the factors are first standardised, to give the r_{ij} values shown in Table 3.

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Table 1. *The river health evaluation index system and evaluation grades (VG = very good; H = health; SH = semi-health; U = unhealthy; M = morbidity)*

River function	Evaluation index		Classification boundaries					Sample
			VG	H	SH	U	M	
u_1 Service	u_{11}	Probability of water supply (%)	90–100	80	70	60	50	96
	u_{12}	Probability of irrigation (%)	90–100	80	70	60	50	82
	u_{13}	Probability of navigable depth (%)	90–100	80	70	60	50	75
	u_{14}	Probability of drinking water safety (%)	90–100	80	70	60	50	98
	u_{15}	Rate of runoff coefficient (%)	0–5	10	20	30	40	1.47
u_2 Environmental	u_{21}	Attainment rate of water quality	80–100	65	50	40	30	94
	u_{22}	Rate of water self-purification	92–100	84	76	68	60	61.1
	u_{23}	Rate of soil erosion area	0–10	20	30	40	50	28.7
	u_{24}	Rate of river length	0–4	8	12	16	20	0
	u_{25}	Rate of cutout	0–0.70	0.85	0.93	0.96	0.98	0
u_3 Flood control	u_{31}	Rate of flood control project	95–100	90	80	70	0	85
	u_{32}	Rate of other flood control measure	95–100	90	80	70	60	70
	u_{33}	Rate of silt-carrying	0–5	10	15	25	40	35.5
	u_{34}	Modulability index	0.15–0.20	0.10	0.06	0.02	0.01	0.012
	u_{35}	Rate of flood carrying capacity	0–10	20	30	40	50	0.54
u_4 Utilisable	u_{41}	Utilisation rate of water	5–20	35	45	54	60	2.82
	u_{42}	GDP of pre-stere water (yuan/m3)	40–50	30	20	10	1	8.51
	u_{43}	Proportion of life/produce/ecologic	0.80–1.0	0.6	0.4	0.2	0.05	0.6
	u_{44}	Rate of sewage treatment	80–100	60	40	20	5	30
	u_{45}	Rate of water bill collection	90–100	80	70	60	50	18.2
u_5 Ecological	u_{51}	Rate of fish variety	0–5	10	15	20	25	18.98
	u_{52}	current condition of rare aquatic animal	0.80–1.0	0.6	0.4	0.2	0.05	0.8
	u_{53}	index of water purity in water head area	4–5	3	2	1	0	4
	u_{54}	Rate of natural vegetation	40–50	30	20	10	1	38.5
	u_{55}	Guarantee rate of ecological water requirement	90–100	80	65	50	30	100

Table 2. *Degrees of membership of evaluation indexes*

Evaluation Index	Sample	Excellent	Fine	Medium	Qualified
u_{11}	0.9200	0.8000	0.6000	0.4000	0.2000
u_{12}	0.6400	0.8000	0.6000	0.4000	0.2000
u_{13}	0.5000	0.8000	0.6000	0.4000	0.2000
u_{14}	0.9600	0.8000	0.6000	0.4000	0.2000
u_{15}	0.9633	0.8750	0.7250	0.5000	0.2500

Table 3. r_{ji} *of evaluation indexes*

Evaluation Index	Sample	Excellent	Fine	Medium	Qualified
u_{11}	0.3150	0.2470	0.2055	0.1370	0.0685
u_{12}	0.2424	0.3030	0.2273	0.1515	0.0758
u_{13}	0.2000	0.3200	0.2400	0.1600	0.0800
u_{14}	0.3243	0.2703	0.2027	0.1351	0.0676
u_{15}	0.2907	0.2641	0.2188	0.1509	0.0755

Table 4. *River system health levels*

SD	Sample	Excellent	Fine	Medium	Qualified	Multi-objective method	Set pair analysis
U_1	0.7786	0.8098	0.6570	0.4153	0.2054	Fine close to excellent	excellent
U_2	0.6853	0.5404	0.3163	0.1762	0.0903	excellent	fine
U_3	0.0782	0.4316	0.4270	0.2789	0.0549	qualified	qualified
U_4	0.3676	0.7860	0.5658	0.3694	0.1809	Qualified close to medium	medium
U_5	0.6412	0.7910	0.5971	0.3965	0.1961	fine	excellent
U_6	0.6011	0.7063	0.5560	0.3348	0.1595	fine	fine

There are 5 evaluation levels, so $m = 5$ and

$$k = \frac{1}{\ln(m)} = 0.62133. \tag{10}$$

From this value of k and the data in Table 3, we then calculate each evaluation factor weight using Equations (3–5) to give

$$w_1 = (0.2427,\ 0.1753,\ 0.1736,\ 0.2336,\ 0.1748). \tag{11}$$

5.4. *Health level*

After weighting the membership degree matrix using Equations (2) and (3), we get

$$M = \begin{bmatrix} 0.2233 & 0.1942 & 0.1456 & 0.0971 & 0.0485 \\ 0.1122 & 0.1402 & 0.1051 & 0.0701 & 0.0351 \\ 0.0868 & 0.1389 & 0.1042 & 0.0934 & 0.0467 \\ 0.1684 & 0.1530 & 0.1267 & 0.0874 & 0.0437 \end{bmatrix} \tag{12}$$

According to equations (7–9), the result is now

$$SD_1 = (0.7786, 0.8098, 0.6570, 0.4153, 0.2054). \tag{13}$$

We can at the same time obtain the impact level of the environmental, flood control, utilisable and ecological functions shown in Table 4. When the weight of the river function is given, we can obtain the health level of the river system. Table 4 compares our results with the results produced by the set pair analysis model.

Table 4 shows the results of both methods are consistent, though some function levels are different because the guiding concepts of the evaluation methods are different, and the evaluation index in Hu *et al.* (2008) uses an average weight, which does not reflect the utility value of the data.

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6. Conclusions

In this paper:

(1) We have applied the AHP method to calculate the weight of the subsystem and the entropy method to calculate the weight of the indexes so that the level of the environmental impact can be considered objectively through the data and subjectively through the classification system.
(2) The evaluation method effectively unifies the river health system with each of the river functions. It not only characterises the overall health level of the river system and the relative health levels of the individual functions, but also identifies any factors and symptoms of stress in the system.
(3) We have used multi-objective space analysis to establish a multi-objective analysis method founded on fuzzy entropy weight that provides a new method of river health system evaluation, and proved it to be viable through an example.

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